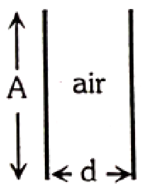
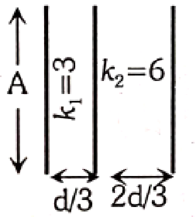
IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 29/07/23**

TOPIC: CAPACITORS

SOLUTIONS

1. (B)





$$C_{eq} = \frac{\varepsilon_0 A}{d} = 9,\ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{k_1\varepsilon_0 A} + \frac{d_2}{k_2\varepsilon_0 A}$$

$$\Rightarrow\ C_{\text{med}} = \frac{k_1 k_2 \varepsilon_0 A}{k_1 d_2 + k_2 d_1}$$

$$= \frac{3\times 6\times \varepsilon_0 A}{3\times 2d/3 + 6\times d/3} = \frac{18}{4}\times 9 = 40.5\ pF$$

2. (B)

The charge flowing through C_4 is

$$q_4 = C_4 \times V = 4CV$$

The series combination of C_1, C_2 and C_3 given

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C}$$

$$\frac{1}{C'} = \frac{6+3+2}{6C} = \frac{11}{6C}\ \Rightarrow\ C' = \frac{6C}{11}$$

Since, C_1, C_2, C_3 are in series combination hence, charge flowing through these will be same.

Hence, $q_2 = q_1 = q_3 = q' = \frac{6CV}{11}$

Thus, $\frac{q_2}{q_4} = \frac{6CV/11}{4CV} = \frac{3}{22}$

3. (A)

Common potential $= \frac{C_1V_0 + C_2\times 0}{C_1 + C_2} = \frac{C_1V_0}{C_1+C_2}$

$$U_{\text{before}} = \frac{1}{2}C_1V_0^2$$

$$U_{\text{after}} = \frac{1}{2}C_1\left(\frac{C_1V_0}{C_1+C_2}\right)^2 + \frac{1}{2}\left(\frac{C_1V_0}{C_1+C_2}\right)^2$$

$$= \frac{1}{2}\left(\frac{C_1V_0}{C_1+C_2}\right)^2 (C_1+C_2) \Rightarrow \frac{U_{Before}}{U_{After}} = \frac{C_1+C_2}{C_1}$$

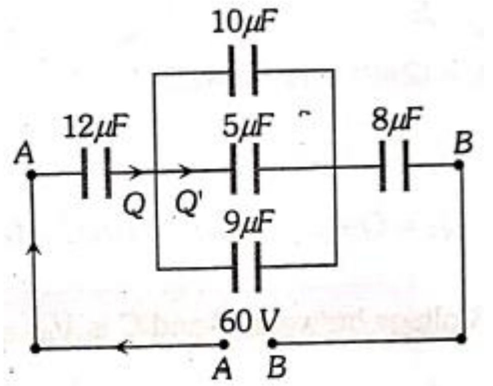
4. (A)

$C_{AB} = 3+\frac{3}{3} = 4\,\mu F,\ C_{AC} = \frac{3}{2}+\frac{3}{2} = 3\,\mu F$

$\therefore\ C_{AB} : C_{AC} = 4:3$

5. (D)

The given circuit can be redrawn as follows



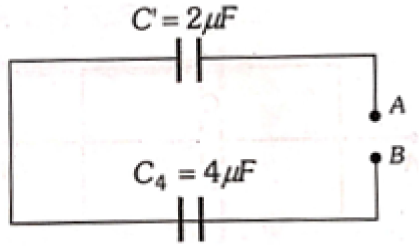
Equivalent capacitance of the circuit $C_{AB} = 4\mu F$

Charge given by the battery $Q = C_{eq}V = 4\times 60 = 240\ \mu C$

Charge in $5\mu F$ capacitor $Q' = \frac{5}{(10+5+9)}\times 240 = 50\,\mu C$

6. (A)

C_2 and C_3 are connected in series, parallel with C_1, then their equivalent capacitance $C' = \frac{2\times 2}{2+2}+1 = 2\mu F$.



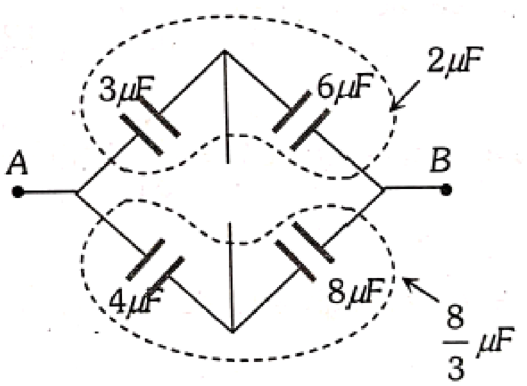
C_4 and C' are in series.

$\therefore$ Effective capacity $C_{eff} = \frac{4}{3}\mu F$.

7. (D)

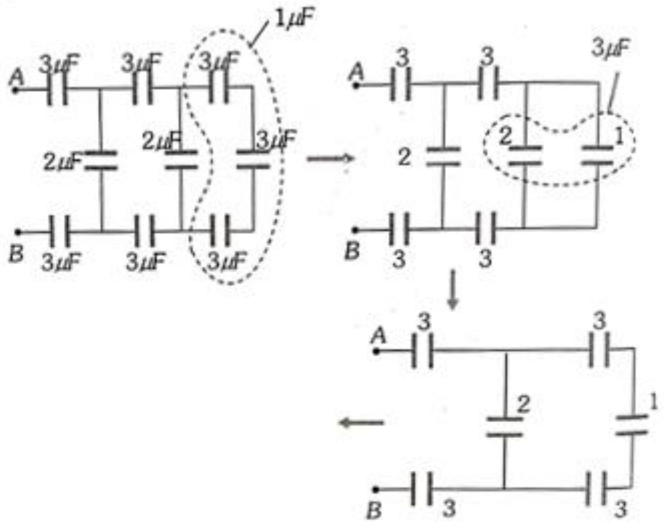
Given circuit is balanced Wheatstone bridge.

So capacitor of $2\mu F$ can be dropped from the circuit



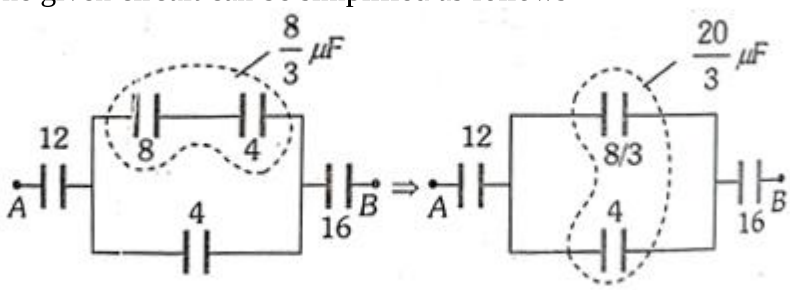
$\Rightarrow \quad C_{AB} = 2 + \frac{8}{3} = \frac{14}{3}\mu F$

8. (A)
The given circuit can be redrawn as follows



On further solving the network in similar manner equivalent capacitance obtained between A and B will be $1\mu F$.

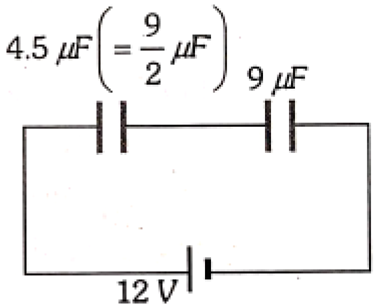
9. (D)
The given circuit can be simplified as follows



Hence equivalent capacitance between A and B

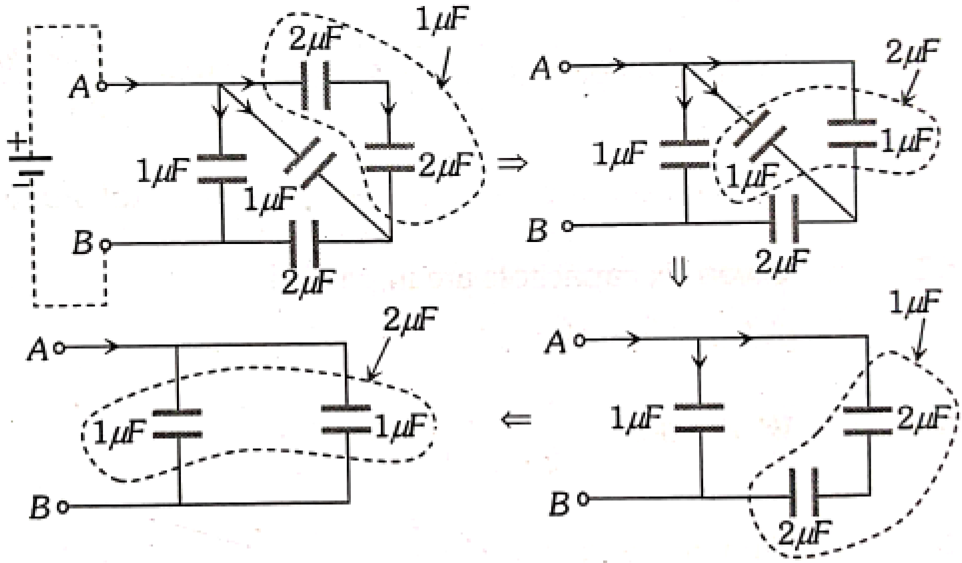
$\frac{1}{C_{AB}} = \frac{1}{12} + \frac{1}{20/3} + \frac{1}{16} \Rightarrow C_{AB} = \frac{240}{71}F$

10. (D)
The given circuit can be redrawn as follows potential difference across $4.5\mu F$ capacitor



$$V = \frac{9}{\left(\frac{9}{2}+9\right)} \times 12$$
$$= 8\text{ V}$$

11. (B)
The given circuit can be simplified as follows

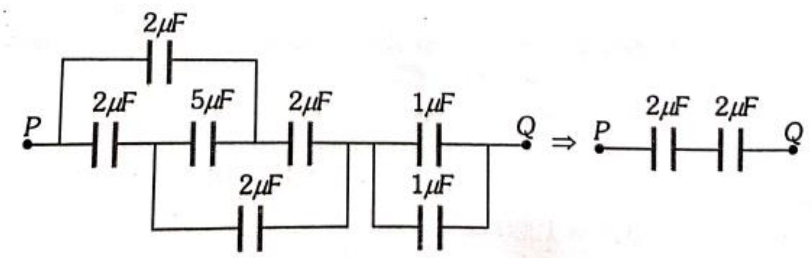


Hence equivalent capacitance between A and B is $2\mu F$

12. (C)
Work done $= \frac{1}{2}\left(\frac{3C}{2}\right)V^2 = \frac{3CV^2}{4}$

13. (C)
Common potential $V = \frac{6\times 20 + 3\times 0}{(6+3)} = \frac{120}{9}$ Volt
So, charge on $3\,\mu F$ capacitor
$Q_2 = 3\times 10^{-6} \times \frac{120}{9} = 40\,\mu C$

14. (B)
The given circuit can be redrawn as follows



$\Rightarrow C_{PQ} = 1\,\mu F$

15. (B)

Initially potential difference across each capacitor.

$V_1 = \dfrac{20}{(10+20)} \times 200 = \dfrac{400}{3}\text{V}$ and $V_2 = \dfrac{10}{(10+20)} \times 200 = \dfrac{200}{3}\text{V}$

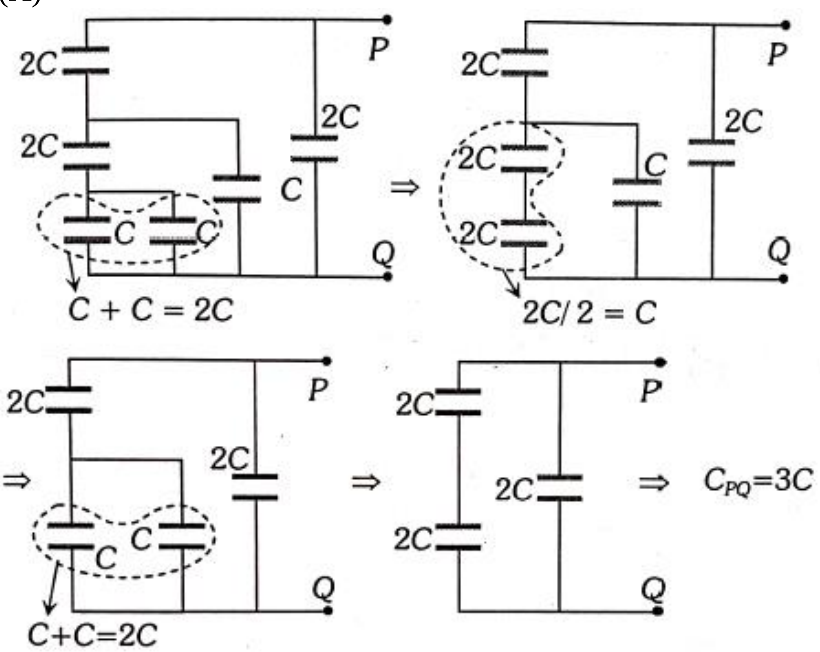
Finally common potential $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$$V = \frac{10 \times \frac{400}{30} + 20 \times \frac{200}{3}}{(10+20)} = \frac{800}{9}\text{V}$$

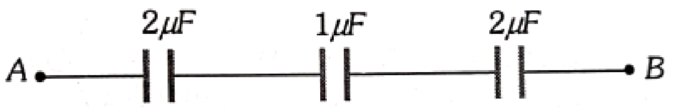
16. (D)

The given figure is equivalent to a balanced Wheatstone's bridge, hence $C_{eq} = 6\,\mu F$

17. (A)

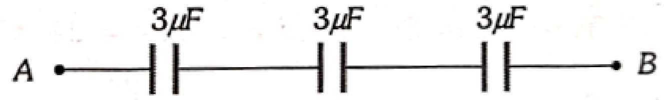


18. (D)



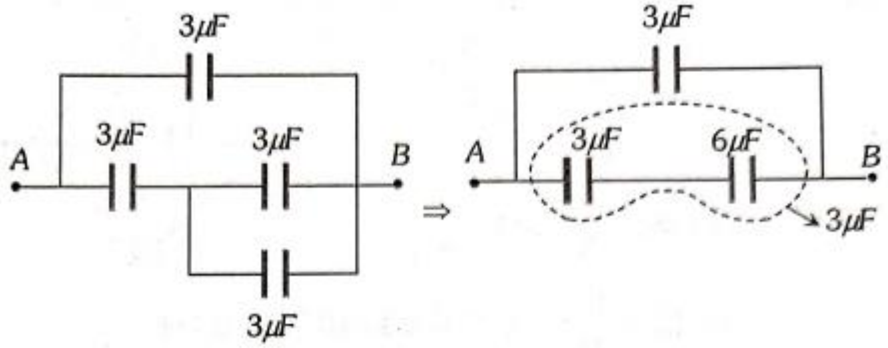
$\dfrac{1}{C} = \dfrac{1}{2} + \dfrac{1}{1} + \dfrac{1}{2} = \dfrac{1+2+1}{2} = \dfrac{4}{2} = 2 \Rightarrow C_{AB} = 00.5\,\mu F$

19. (A)



$$\frac{1}{C_{AB}}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1 \Rightarrow C_{AB}=1\,\mu F$$

20. (D)



$\Rightarrow C_{AB}=5\,\mu F$

21. (6)

In series combination of capacitors, voltage distributes on them, in the reverse ratio of their capacitance i.e.

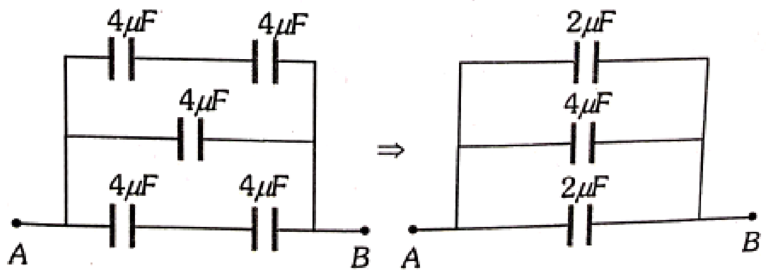
$\frac{V_A}{V_B}=\frac{3}{2}$ ….. (i)

Also $V_A+V_B=10$ ….. (ii)

On solving (i) and (ii)

$V_A=6V, V_B=4V$

22. (8)

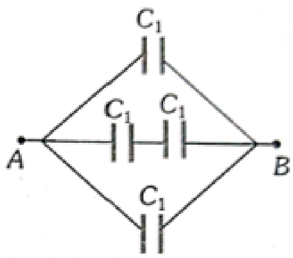


$\Rightarrow C_{AB}=8\,\mu F$

23. (1)

The capacitance across A and B

$=\frac{C_1}{C_2}+C_1+C_1=\frac{5}{2}C_1$



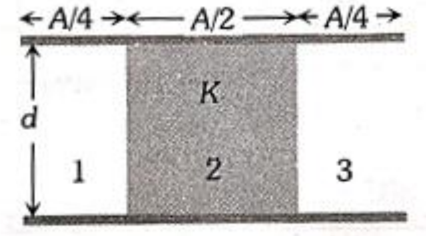
As $Q=CV$,

$1.5\,\mu C=\frac{5}{2}C_1\times 6$

$\Rightarrow C_1=\frac{1.5}{15}\times 10^{-6}=0.1\times 10^{-6}F=0.1\,\mu F$

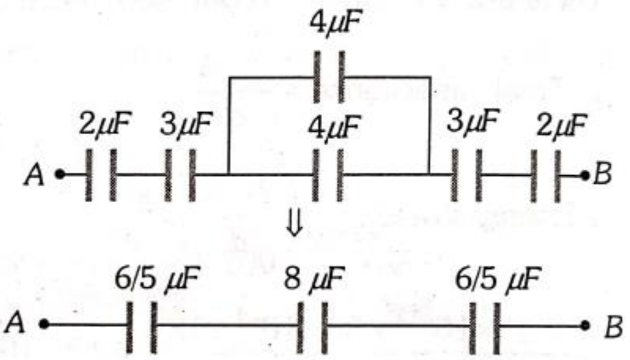
24. (25)

$$C_1 = \frac{\varepsilon_0\left(\frac{A}{4}\right)}{d}, C_2 = \frac{K\varepsilon_0\left(\frac{A}{2}\right)}{d}, C_3 = \frac{\varepsilon_0\left(\frac{A}{4}\right)}{d}$$



$$C_{eq} = C_1 + C_2 + C_3 = \left(\frac{K+1}{2}\right)\frac{\varepsilon_0 A}{d} = \left(\frac{4+1}{2}\right)\times 10 = 25\ \mu F$$

25. (24)



$$\frac{1}{C_{eq}} = \frac{5}{6} + \frac{1}{8} + \frac{5}{6} = \frac{20+3+20}{24} \Rightarrow C_{eq} = \frac{24}{43}\mu F$$

26. (4)

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} \Rightarrow C_{eq} = 1\ \mu F$$

Total charge $Q = C_{eq}.V = 1\times 24 = 24\ \mu C$

So, p.d. across $6\ \mu F$ capacitor $= \frac{24}{6} = 4$ volt

27. (4)

$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2} = \frac{6\times 12 - 3\times 12}{3+6} = 4 \text{ volt}$$

28. (3)

C_2 and C_3 are in series so equivalent capacitance

$$= \frac{2\times 2}{2+2} = 1$$

$\Rightarrow$ Now $1\ \mu F$, $1\ \mu F$ and $1\ \mu F$ are in parallel

$\therefore\ C_{eq} = 1+1+1 = 3\ \mu F$.

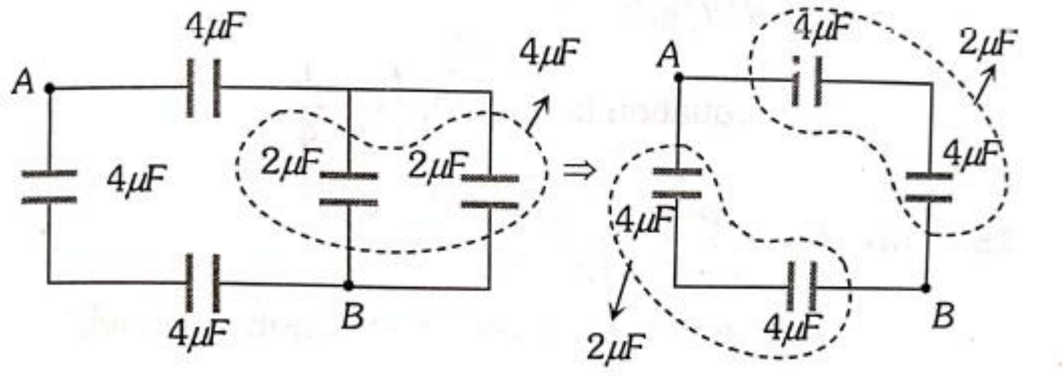
29. (6)

In series $C' = \frac{C}{n}$ i.e. $C = nC' = 2\times 3 = 6\ \mu F$

In parallel $C' = nC$ *i.e.* $C = \frac{C'}{n} = \frac{12}{2} = 6\ \mu F$

30. (4)

The given circuit can be simplified as follows



Equivalent capacitance between *A* and *B* is $C_{AB} = 4\ \mu F$

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TOPIC: HYDROCARBONS

Answer Key & Solution

31. (A)
Stability is checked by hyperconjugation here. More number of alpha H implies more stable double bond.

32. (B)
Oxidative Cleavage of >C=C<. Acetic acid forms.

33. (B)
Water addition by anti-Markovnikov's rule.

34. (A)
Dehydrohalogenation by E_2 mechanism.

35. (A)
4,5-Dimethyloct-4-ene

36. (D)
Aromatization of n-Heptane gives Toluene (Homologue of benzene).

37. (D)
Water addition by anti-Markovnikov's rule.

38. (C)
Carbocation mechanism.

39. (D)
Reductive ozonolysis of alkene.

40. (B)
Oxidative Cleavage of Alkyne.

41. (B)
Oxidative Cleavage of >C=C<. Acetic acid forms.

42. (B)
Anti-Markovnikov rule.

43. (D)
Epoxidation followed by hydrolysis. Anti-addition of two -OH groups.

44. (A)
Oxidative Cleavage of >C=C<.

45. (B)
$F_2 > Cl_2 > Br_2 > I_2$.

46. (C)
Dehydration via carbocation mechanism. Ring expansion.

47. (A)
Carbene addition to alkene.

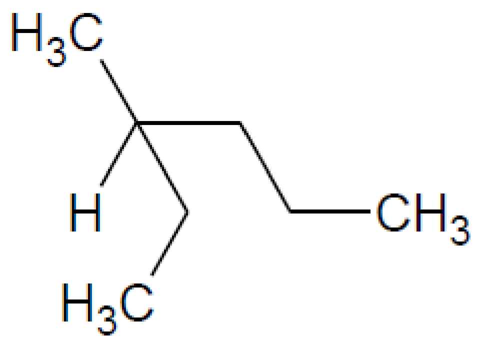
48. (B)
Anti-addition.

49. (C)
Water addition to alkyne by Markovnikov rule.

50. (B)

51. (0)
$CH_3CH_2CH_2CH_3$. No optical activity.

52. (7)



53. (5)
Two rings, 1 double bond and 1 triple bond.

54. (2)
(b) & (d)
Hydrogenation reaction s syn addition.

55. (3)
Three acidic H present.

56. (5)
Three double bonds and one triple bond.

57. (2)
Two acidic H present in acetic acid.

58. (2)
Beryllium and aluminum carbide.

59. (4)
Three double bonds and one ring.

60. (1)
Both OH and triple bond CH will release H_2 (Half mole each)

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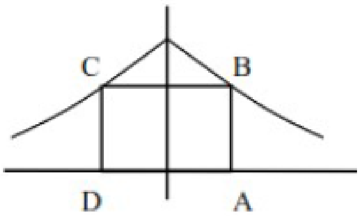
TOPIC: APPLICATIONS OF DERIVATIVES

Answer Key & Solution

61. (B)



$A=(t,0), B=(t,e^{-t})$

$C=(-t,e^{-t}), D=(-t,0)$

$\Delta = 2t\,e^{-t}$

$\Delta^1 = 0, \Delta^{11} < 0$ at $t=1$

$\therefore$ max $\Delta = \frac{2}{e}$s.u

62. (B)

Draw the graph

63. (B)

Let $a+b+c+d=x \Rightarrow e+f+g+h=8-x$

Let $y = x^2 + (8-x)^2 \Rightarrow y^1 = 0 \Rightarrow x = 4$

64. (D)

Let $a=\left(x+\frac{1}{x}\right)^3, b = x^3+\frac{1}{x^3}, b^2 = x^6+\frac{1}{x^6}+2$

$\therefore\ f(x) = \left|\frac{a^2-b^2}{a+b}\right| = |a-b| \Rightarrow \left|3\left(x+\frac{1}{x}\right)\right| \geq 6$

65. (D)

Maximum $f(x)=2$

$\therefore \sin\frac{x}{3} = 1$ and $\sin\frac{x}{11} = 1$

$\frac{x}{3} = 2n\pi + \frac{\pi}{2}$,

$\frac{x}{3} = 90 + 360n$ similarly $\frac{x}{11} = 90 + 360m, (m, n \in z)$

Then $m = 2, n = 8$ for smallest +ve solution

$\therefore x_0 = 8910^o \quad \therefore \alpha = 8910$

66. (C)

Let α, β be roots $\Rightarrow \alpha + \beta = b$

Take $\alpha = 2$ ($\therefore \alpha, \beta$ are prime b is odd)

$\Rightarrow f(2) = 0$

$\Rightarrow 2b - c = 4,\ b + c = 35$

$\therefore b = 13, c = 22$

67. (D)

Locus of P is the chord of contact of A w.r.t. circle i.e., $3x + 4y - 1 = 0$

68. (A)

Draw the graph f is strictly increasing and concave down

69. (C)

$f'(x) = 2x - 1 + \cos x$

$f''(x) = 2 - \sin x > 0$

$f'(x) = 0$

$x = 0$

$f(0) = 1$ (minimum value)

70. (B)

$f^1(x) = \frac{x(400 - x^3)}{(x^3 + 200)^2} = 0 \Rightarrow x = 0,\ x = 400^{\frac{1}{3}}$

$f^1(x) > 0$ for $0 < x < 400^{\frac{1}{3}}$

$f^1(x) < 0$ for $x > 400^{\frac{1}{3}}$

$f(7) = \frac{49}{543},\ f(8) = \frac{64}{712}$

$f(7) > f(8)$

71. (A)

$\theta = \tan^{-1}\sqrt{2}$

$\frac{da}{dx} = \left(\frac{-4}{\sin^2 x} + \frac{1}{(1 - \sin x)^2}\right)\cos x = 0$

$\sin x = \frac{2}{3} (\because \cos x \neq 0)$

$\min a = \frac{4}{2/3} + \frac{1}{1-\frac{2}{3}} = 9$

72. (C)

$f\left(\frac{3}{2}\right) = 0$

$\therefore \lim_{x \to \frac{3}{2}} \left|x^2 - 3x\right| + a \leq 0$

$\Rightarrow a \leq \frac{-9}{4}$

$\Rightarrow |4k| = 9$

73. (C)

$\ell = 2x + 2r + \pi r$

$A = 2rx + \frac{1}{2}\pi r^2$

$\frac{dA}{dr} = 0 \Rightarrow r = \frac{1}{4+\pi}$

74. (C)

Let $y = x - x^P$

$\frac{dy}{dx} = 1 - Px^{P-1} = 0 \Rightarrow x = P^{\frac{1}{1-P}}$

75. (B)

$P \propto v^3 \Rightarrow P = kv^3$

$t = \frac{S}{V-6}$

Cost of fuel $= \frac{S}{V-6} \times kv^3 = f(v)$

$f^1(v) = 0 \Rightarrow v = 9$

76. (A)

$f^1(x) > 0$ for $1 \leq x \leq 2$

$f^1(x) = 4x - k,\ f^{11}(x) = 4 > 0 \qquad \forall x \in [1,2]$

$\therefore$ $f^1(1)$ is the least value of $f^1(x)$ in $[1,2]$

$\Rightarrow f^1(1) > 0 \Rightarrow k < 4$

77. (B)

$f^1(x) > 0, f(0) = 0, f\left(\frac{\pi}{4}\right) > 2$

$\therefore$ By IVT, $f(c) = 2$ for some c in $\left(0, \frac{\pi}{4}\right)$

78. (A)

Draw the graph $\Rightarrow a \geq 1$

79. (B)

$$\cos^{-1}\left(2x^2 - 1\right) = \begin{cases} 2\pi - 2\cos^{-1} x, & -1 \leq x \leq 0 \\ 2\cos^{-1} x, & 0 \leq x \leq 1 \end{cases}$$

$f^1(0-) > 0, f^1(0+) < 0 \Rightarrow$ 'B' is correct

80. (D)

$$f^1(x) = \left(\frac{\sqrt{a+4}}{1-a} - 1\right)5x^4 - 3 < 0 \ \forall x \in R$$

$$f^1(x) = \left(\frac{\sqrt{a+4}}{1-a} - 1\right) \leq 0 \Rightarrow \frac{\sqrt{a+4}}{1-a} \leq 1 \text{ solve}$$

81. (2)

82. (2)

$$5\left(\frac{1}{2^2 \times 3^3}\right)^{1/5}$$

Consider

$$\frac{f^3(x)}{2} + \frac{f^3(x)}{2} + \frac{1}{3f^2(x)} + \frac{1}{3f^2(x)} + \frac{1}{3f^2(x)} \geq 5\left(\frac{1}{2^2 \times 3^3}\right)^{1/5} \quad (\text{A.M.} \geq \text{G.M})$$

83. (9)

$$\frac{4}{x} + \frac{9}{y} + \frac{16}{z} = (x+y+z)\left(\frac{4}{x} + \frac{9}{y} + \frac{16}{z}\right)$$

$$= 29 + \left(\frac{9x}{y} + \frac{4y}{x}\right) + \left(\frac{16x}{z} + \frac{4z}{x}\right) + \left(\frac{16y}{z} + \frac{9x}{y}\right) \geq 81$$

84. (5)

Let $\bar{\alpha} = ai + bj + ck$

$\bar{\beta} = 3i + 4j + 5k$

& $|\bar{\alpha} \times \beta| \leq |\alpha||\beta|$

85. (2)

Hint:

$$\frac{x^2+2-\sqrt{x^4+4}}{x}=\frac{4x}{x^2+2+\sqrt{x^4+4}}=\frac{4}{x+\frac{2}{x}+\sqrt{x^2+\frac{4}{x^2}}}$$ apply AM

86. (6)

87. (5)

88. (9)

89. (3)

Hint : Solving them simultaneously and take $D=0$ we get $b=\frac{1}{e}$ & $a=\frac{1}{a\sqrt{1-e^2}}$, then minimize πab.

90. (9)

Hint : $f(-1)=0\Rightarrow a=-1$, $\underset{x\to\pm\infty}{\text{Lt}} f(x)=1\Rightarrow b=0, c=1$

$f(x)=\frac{x+1}{x+2}$ and $f^{-1}(x)=\frac{1-2x}{x-1}$