IIT – JEE: 2024 **TW TEST (NAPJC – 6, 7)** **DATE: 28/08/22**

TOPIC: CALCULUS IN PHYSICS

Answer Key

1.	(A)	2.	(B)	3.	(B)	4.	(B)	5.	(C)
6.	(A)	7.	(A)	8.	(A)	9.	(C)	10.	(C)
11.	(C)	12.	(B)	13.	(A)	14.	(B)	15.	(A)
16.	(A)	17.	(A)	18.	(A)	19.	(C)	20.	(A)
21.	(D)	22.	(C)	23.	(B)	24.	(C)	25.	(A)

SOLUTIONS

1. (A)

$$\int_0^\infty e^{-5x}dx = \left[\frac{e^{-5x}}{-5}\right]_0^\infty$$

$$= \frac{0-1}{-5} = \frac{1}{5}$$

2. (B)

$$\int \frac{x^3}{4+x^4}dx$$

$$= \frac{\ln\left(4+x^4\right)}{4} + C$$

3. (B)

$$\int_0^1 \frac{dx}{(ax+b)}$$

$$= \left[\frac{\ln(ax+b)}{a}\right]_0^1$$

$$= \left[\ln(a+b) - \ln b\right]\frac{1}{a}$$

$$= \frac{1}{a}\ln\left(\frac{a+b}{b}\right) = \frac{1}{a}\ln\left(1+\frac{a}{b}\right)$$

4. (B)

$$3y = 2x^3 + 1$$

$$\frac{3dy}{dt} = 6x^2 \frac{dx}{dt}$$

$$dy = 2x^2 \cdot dx$$

$$8 = 2x^2 \Rightarrow x = \pm 2$$

$$y = \frac{2(\pm 2)^3 + 1}{3} = \pm \frac{16+1}{3}$$

$$= \frac{17}{3}, -\frac{15}{3}$$

5. (C)

$$y = x \cdot \cos x$$

$$\frac{dy}{dx} = x \cdot (-\sin x) + \cos x (1)$$

$$= -x \sin x + \cos x$$

6. (A)

$$\frac{dy}{dx} = \cos x - \sin x$$

For maxima

$$\frac{dy}{dx} = 0$$

$$\cos x - \sin x = 0 \Rightarrow \cos x = \sin x \Rightarrow \tan x = 1$$

$$\Rightarrow x = \frac{\pi}{4} \ \& \ x = \frac{3\pi}{4} \quad x \in [0, 2\pi]$$

$$\frac{d^2y}{dx^2} = -\sin x - \cos x$$

$$\left(\frac{d^2y}{dx^2}\right)_{\text{at } x = \frac{\pi}{4}} < 0$$

Hence, $x = \frac{\pi}{4}$ is a point of maxima.

$$y_{\text{maximum}} = \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

7. (A)

$$\int_2^5 (3x^2 + 4x + 1) dx$$

$$= \left[x^3 + 2x^2 + x\right]_2^5$$

$$= 180 - 18 = 162$$

8. (A)

$$\int_0^{\pi/4} \sec^2\theta \cdot d\theta = \left[\tan\theta\right]_0^{\pi/4} = 1$$

9. (C)

$$\frac{dy}{dx} = 3x^2 - 2$$

$$\left(\frac{dy}{dx}\right)_{\text{at } x=1} = 3 - 2 = 1$$

10. (C)

$$\frac{dy}{dx} = \cos x$$

$$\left(\frac{dy}{dx}\right)_{\text{at } x=2\pi} = \cos(2\pi) = 1$$

11. (C)

$$y = e^x - \ln x + \frac{1}{x}$$

$$\frac{dy}{dx} = e^x - \frac{1}{x} - \frac{1}{x^2}$$

12. (B)

$$\frac{dy}{dx} = x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$\frac{d^2y}{dx^2} = 2x + 1$$

At $x = -4; \frac{d^2y}{dx^2} = -7 < 0$ (maximum)

At $x = 3; \frac{d^2y}{dx^2} = 7 > 0$ (minimum)

13. (A)

$$\frac{dy}{dx} = 3x^2 - 2$$

$$\frac{d^2y}{dx^2} = 6x$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 6$$

14. (B)

$$\frac{dy}{dx} = \sec x \cdot \tan x + \sec^2 x$$

$= 2\sqrt{3} + (2)^2$

$= 2\left(\sqrt{3} + 2\right)$

15. (A)

16. (A)

$I = \int t\,dt$ $\qquad [\sin x = t, \cos x\,dx = dt]$

$= \frac{t^2}{2}$

$= \frac{\sin^2(x)}{2} + C$

17. (A)

$\frac{dy}{dx} = e^x - e^{-x}$

18. (A)

$x + y = 15$

$z = xy$

$= x(15 - x)$

$z = 15x - x^2$

For maxima

$\frac{dz}{dx} = 15 - 2x = 0$,

$x = \frac{15}{5}$

$\left(\frac{d^2z}{dx^2}\right)_{x=15/2} < 0$

Hence it is a point of maxima.

$x = \frac{15}{2}$ & $y = \frac{15}{2}$

19. (C)

$y = \ln x^2$

$\frac{dy}{dx} = \frac{1}{x^2} \times 2x = \frac{2}{x} = \frac{2}{e}$

20. (A)

$I = \int \left(\frac{1 - \cos 2x}{2}\right) dx$

$= \frac{1}{2}\left[\int dx - \int \cos(2x)\,dx\right]$

$$= \frac{x}{2} - \frac{\sin(2x)}{4} + C$$

21. (D)

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$= 8\pi(2)(10)$$

$$= 160\pi$$

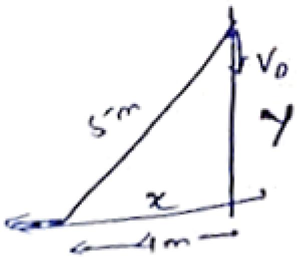
22. (C)

$$x^2 + + y^2 = 5^2$$

$$2x\frac{dy}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

$$= \frac{4}{3}(3) = 4\,\text{m/s}$$



23. (B)

$$I = \int\left(x + 1 + \frac{1}{x^2}\right)dx$$

$$= \int x\,dx + \int dx + \int x^{-2}\,dx$$

$$= \frac{x^2}{2} + x - \frac{1}{x} + C$$

24. (C)

$$\frac{dy}{dx} = 2\sin\left(x^2\right)\cdot\cos\left(x^2\right)\cdot 2x$$

$$= 4x\sin\left(x^2\right)\cos\left(x^2\right)$$

25. (A)

$$P \propto t$$

$$F = \frac{dP}{dt} = \text{constant}$$

i.e. $F \propto t^{o}$

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TOPIC: ATOMIC STRUCTURE

Answer Key

26.	(B)	27.	(A)	28.	(A)	29.	(C)	30.	(C)
31.	(C)	32.	(B)	33.	(C)	34.	(C)	35.	(C)
36.	(B)	37.	(C)	38.	(A)	39.	(A)	40.	(D)
41.	(C)	42.	(A)	43.	(B)	44.	(C)	45.	(B)
46.	(D)	47.	(A)	48.	(C)	49.	(C)	50.	(C)

SOLUTIONS

26. (B)

$$E = \frac{1}{2}(E_1 + E_2) \Rightarrow \frac{1}{\lambda} = \frac{1}{2}$$

$$\therefore\ \lambda = 4800\,\text{Å}$$

27. (A)

$$E = n \cdot \frac{hc}{\lambda}$$

$$\Rightarrow\ \frac{E \cdot \lambda}{hc} = \frac{10^{-7} \times 5000 \times 10^{-10}}{6.626 \times 10^{-34} \times 3 \times 10^{8}} = 2.5 \times 10^{11}$$

28. (A)

Na^{10+} ion has single electron

29. (C)

$$\frac{(2\pi r)_3}{(2\pi r)_2} = \frac{3^2}{2^2} = \frac{9}{4}$$

30. (C)

$$V_{n,Z} = 2.188 \times 10^6 \frac{z}{n}\,\text{m/s}$$

$$19.54 \times 11.2 \times 10^3 = 2.188 \times 10^6 \times \frac{1}{n}\ \Rightarrow\ n \approx 10$$

31. (C)

$$f_{n,Z} = \frac{1}{T_{n,Z}} \alpha \frac{1}{n^3}$$

32. (B)

33. (C)

$$E_{n,Z} = -13.6\frac{z^2}{n^2}\text{eV} \Rightarrow -13.6 = -13.6\times\frac{z^2}{n^2} \Rightarrow z = 3$$

34. (C)

$$\text{P.E.} = -27.2\times\frac{z^2}{n^2} = -27.2 \text{ eV}$$

35. (C)

$\Delta E_{2\to1}$ is even greater that $\Delta E_{\infty\to2}$.

36. (B)

$$\Delta E = 13.6\, z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\text{eV}$$

or $47.2 = 13.6\times z^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow z \approx 5$

37. (C)

$$R = \frac{2\pi^2 me^4}{(4\pi\varepsilon_0)^2 h^3 c}$$

$\Rightarrow R\alpha m$

Here, m becomes $\left(m - \frac{m}{4} = \frac{3m}{4}\right)$, then R becomes $\frac{3}{4}R$.

38. (A)

Now, $\upsilon = \frac{c}{\lambda} = cRz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$= cR\times2^2\left(\frac{1}{3^2} - \frac{1}{5^2}\right)$$

$$= \frac{64cR}{225}$$

39. (A)

$$\frac{\overline{\upsilon_2}}{\overline{\upsilon_1}} = \frac{z_2^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)_1}{z_1^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)_2}$$

$$\Rightarrow \frac{\overline{\upsilon_2}}{2.5\times10^5} = \frac{3^2\left(\frac{1}{3^2}-\frac{1}{5^2}\right)}{4^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}$$

$$\therefore \overline{\upsilon_2} = 7.2\times10^4 \text{ cm}^{-1}$$

40. (D)

$$\Delta x = \frac{h}{4\pi m\cdot\Delta V_{\min}}$$

41. (C)
Number of orbit = Number of waves.

42. (A)
Theoretical

43. (B)
For p-orbital, $l = 1$

$\therefore$ Orbital angular momentum $= \sqrt{l(l+1)}\cdot\frac{h}{2\pi}$

$$= \sqrt{1(1+1)}\cdot\frac{h}{2\pi} = \sqrt{2}\cdot\frac{h}{2\pi}$$

44. (C)
Number of orbitals of a particular type = $2l + 1$ and for g-orbital, $l = 4$

45. (B)
The p-orbital has only one nodal plane.

46. (D)
Informative

47. (A)
Orbitals are $3p$, $4s$.
Number = 3 + 1 = 4.

48. (C)

Pauli's exclusion principle limits the maximum capacity of electrons in an orbital equal to 2.

49. (C)

$$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ \underbrace{3d^5\ 4s^1}_{\text{6 unpaired electrons}}$$

50. (C)

Number of unpaired electron in Fe^{2+} is 4.

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TOPIC: TRIGONOMETRY - I

Answer Key

51.	(B)	52.	(B)	53.	(C)	54.	(D)	55.	(D)
56.	(C)	57.	(C)	58.	(D)	59.	(B)	60.	(C)
61.	(A)	62.	(A)	63.	(C)	64.	(C)	65.	(D)
66.	(C)	67.	(B)	68.	(B)	69.	(A)	70.	(B)
71.	(D)	72.	(A)	73.	(D)	74.	(D)	75.	(D)

SOLUTIONS

51. (B)

$\sec\theta + \tan\theta = 3$

$\sec\theta - \tan\theta = \frac{1}{3}$

$2\sec\theta = 3 + \frac{1}{3} = \frac{10}{3}$

$\sec\theta = \frac{5}{3}$

$\therefore \quad \cos\theta = \frac{3}{5}$

52. (B)

$\sin\theta = \frac{4}{5}$ but θ lies in 2nd or 4th

Quadrant Hence, $\sin\theta = \frac{4}{5}$ or $-\frac{4}{5}$

53. (C)

$\Rightarrow \quad \sec^4\theta\left(\cos^2\theta\right)\left(1+\sin^2\theta\right) - 2\tan^2\theta$

$\Rightarrow \quad \sec^2\theta\left(1+\sin^2\theta\right) - 2\tan^2\theta \Rightarrow \sec^2\theta + \tan^2\theta - 2\tan^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

54. (D)

$d = 10\,\theta$ where $\theta = \frac{360^\circ}{60^\circ} \times 20 = \frac{2\pi}{3}$

$d = 10 \times \frac{2\pi}{3} \Rightarrow \frac{3d}{10\pi} = 2$

55. (D)

$\sin 4 < 0$.

56. (C)

$\cos(A+B) = 1 \Rightarrow A = -B$

$\therefore\ 2 + \tan(-B)\cot B = 2 - 1 = 1$

57. (C)

$\sin^2 A - \sin^2 B = \sin(A+B)\cdot\sin(A-B)$

$\therefore$ using this $K = \frac{1}{\sqrt{2}}$

$\therefore\ \sqrt{2}K = 1$

58. (D)

If $A + B = 45^\circ$ then $(1+\tan A)(1+\tan B) = 2$

59. (B)

$\operatorname{cosec}^2 x + 25\sec^2 x$

$1 + \cot^2 x + 25(1 + \tan^2 x)$

$1 + \cot^2 x + 25 + 25\tan^2 x$

$26 + \underbrace{25\tan^2 x + \cot^2 x}$

$\geq 2\sqrt{25}$

≥ 10

$\therefore$ Least value of $\operatorname{cosec}^2 x + 25\sec^2 x$ is 36.

60. (C)

$5\cos\theta + 3\left[\cos\theta\cdot\cos\left(\frac{\pi}{3}\right) - \sin\theta\cdot\sin\left(\frac{\pi}{3}\right)\right] + 3$

$5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$\frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

Now, $\lambda = \sqrt{\frac{169}{4} + \frac{27}{4}} + 3$

$= \sqrt{\frac{196}{4}} + 3$

$= 7 + 3 = 10$

$\mu = -7 + 3 = -4$

$\therefore \quad \lambda - \mu = 14$

61. (A)

$$\cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{5\pi}{8}\right)+\cos^4\left(\frac{7\pi}{8}\right)$$

$$\Rightarrow \cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{\pi}{8}\right)$$

$$\Rightarrow 2\left[\cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)\right]$$

$$\Rightarrow 2\left[\cos^4\left(\frac{\pi}{8}\right)+\sin^4\left(\frac{\pi}{8}\right)\right]$$

$$\Rightarrow 2\left[1-2\sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)\right]$$

$$\Rightarrow 2\left[1-\frac{1}{2}\left(\sin^2\frac{\pi}{4}\right)\right]$$

$$\Rightarrow 2\left[1-\frac{1}{2}\times\frac{1}{2}\right]=2\left[\frac{3}{4}\right]=\frac{3}{2}$$

62. (A)

$$(\alpha+\beta)-(\alpha-\beta)=2\beta$$

$$\tan\left[(\alpha+\beta)-(\alpha-\beta)\right]=\tan 2\beta$$

$$\frac{\tan(\alpha+\beta)-\tan(\alpha-\beta)}{1+\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}=\tan 2\beta$$

$$\therefore \quad \tan(\alpha+\beta)=\frac{3}{4} \text{ and } \tan(\alpha-\beta)=\frac{5}{12}$$

$$\Rightarrow \tan 2\beta=\frac{\frac{3}{4}-\frac{5}{12}}{1+\frac{3}{4}\times\frac{5}{12}}=\frac{36-20}{48+15}$$

$$\tan 2\beta=\frac{16}{63}$$

63. (C)

$$\alpha=\frac{\pi}{2}-\beta$$

$$\tan\alpha=\cot\beta \Rightarrow \tan\alpha\cdot\tan\beta=1$$

Now, $\alpha-\beta=\gamma$

$$\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}=\tan\gamma$$

$$\tan\alpha-\tan\beta=2\tan\gamma$$

$$\tan\alpha=\tan\beta+2\tan\gamma$$

64. (C)

$$\sin\theta\sin\left(60^\circ-\theta\right)\sin\left(60^\circ+\theta\right)=\frac{\sin 3\theta}{4}$$

$$\therefore \quad \sin 30^\circ\times\frac{\sin 30^\circ}{4}=\frac{1}{2}\times\frac{1}{8}=\frac{1}{16}.$$

65. (D)

$$\frac{1+\cos 20^\circ}{2}-\cos 50^\circ\left(\cos 10^\circ-\cos 50^\circ\right)$$

$$\frac{1+\cos 20^\circ}{2}-2\cos 50^\circ\sin 30^\circ\sin\left(20^\circ\right)$$

$$\frac{1+\cos 20^\circ}{2}-\cos 50^\circ\sin 20^\circ$$

$$\frac{1+\cos 20^\circ}{2}-\frac{1}{2}\left[\sin\left(70^\circ\right)-\sin\left(30^\circ\right)\right]$$

$$\frac{1+\cos 20^\circ-\sin 70^\circ+\sin 30^\circ}{2}$$

$$\frac{\frac{3}{2}+\cos 20^\circ-\cos 20^\circ}{2}=\frac{3}{4}$$

66. (C)

$$\frac{\sin\left(2^9\times\frac{\pi}{2^{10}}\right)}{2^9\times\sin\left(\frac{\pi}{2^{10}}\right)}\times\sin\left(\frac{\pi}{2^{10}}\right)=\frac{1}{512}$$

67. (B)

$\tan^2 x=\frac{2}{3}$ from the given relation.

Hence, (B).

68. (B)

$$\tan 30^\circ+\tan 15^\circ=-p$$

$$\tan 30^\circ\cdot\tan 15^\circ=q$$

$$\therefore \quad \frac{\tan 30^\circ+\tan 15^\circ}{1-\tan 30^\circ\tan 15^\circ}=1$$

$$\Rightarrow \quad -p=1-q$$

$$\Rightarrow \quad q-p=1$$

$$\therefore \quad 2+q-p=3$$

69. (A)

$$\sin\alpha+\sin\beta=\frac{21}{65}\Rightarrow 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{21}{65}$$

$$\cos\alpha+\cos\beta=\frac{-27}{65} \Rightarrow 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{-27}{65}$$

$$\therefore \quad \tan\left(\frac{\alpha+\beta}{2}\right)=\frac{-21}{27}$$

70. (B)

Using componendo and dividendo

$$\frac{1-\cos x}{1+\cos x}=\frac{2-\cos y-2\cos y+1}{2-\cos y+2\cos y-1}$$

$$\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=\frac{3(1-\cos y)}{1+\cos y}=\frac{3\times 2\sin^2\frac{y}{2}}{2\cos^2\frac{y}{2}}$$

$$\therefore \tan^2\frac{x}{2}=3\tan^2\frac{y}{2}$$

$$\tan^2\frac{x}{2}\cdot\cot^2\frac{y}{2}=3$$

$$\tan\frac{x}{2}\cdot\cot\frac{y}{2}=\sqrt{3}$$

71. (D)

$$\tan A\cdot\tan B+\tan B\cdot\tan C+\tan C\cdot\tan A=1$$

$$\frac{1}{2}+\frac{\tan C}{2}+\tan C=1$$

$$\frac{3\tan C}{2}=\frac{1}{2} \Rightarrow \tan C=\frac{1}{3}$$

72. (A)

$$\frac{\cos(A-B)}{\cos(A+B)}=\frac{\cos A\cos B+\sin A\sin B}{\cos A\cos B-\sin A\sin B}$$

$$=\frac{1+\tan A\tan B}{1-\tan A\tan B}$$

$$=\frac{1+\frac{1}{3}}{1-\frac{1}{3}}=2$$

73. (D)

$$\tan\left[2(A+B)\right]=\frac{2\tan(A+B)}{1-\tan^2(A+B)} \qquad \left[\tan(A+B)=\frac{2+1}{1-2}=-3\right]$$

$$\therefore \quad \tan 2(A+B)=\frac{2\times(-3)}{1-(-3)^2}=\frac{-6}{-8}=\frac{3}{4}$$

74. (D)

Let $\tan\frac{\theta}{2} = x$ then

$$\sin\theta + \cos\theta = -\frac{1}{5}$$

$$\frac{2x}{1+x^2} + \frac{1-x^2}{1+x^2} = -\frac{1}{5}$$

$$10x + 5 - 5x^2 = -x^2 - 1$$

$$4x^2 - 10x - 6 = 0$$

$$2x^2 - 5x - 3 = 0$$

75. (D)

$$\frac{(\sin\theta + \cos\theta)^2}{(\cos\theta - \sin\theta)^2} = \frac{1+\sin 2\theta}{1-\sin 2\theta} = \frac{1+P}{1-P}$$