IIT – JEE: 2025 | **TW TEST** | **DATE: 01/07/23**

TOPIC: VECTORS & CALCULUS

Solutions

1. (C)

$R_{max} = |A + B|$ $\quad\quad$ $R_{min} = |A - B|$

$= 6 + 4 = 10$ $\quad\quad$ $R_{min} = |6 - 4| = 2$

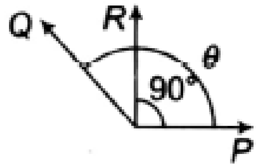
$R_{min} \le R \le R_{max}$

2. (A)

$P + Q = 16, R = 8$, let $P < Q$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$(8)^2 = P^2 + (16 - P)^2 + 2PQ\cos\theta$ $\quad\quad$ ...(i)



$\tan 90° = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

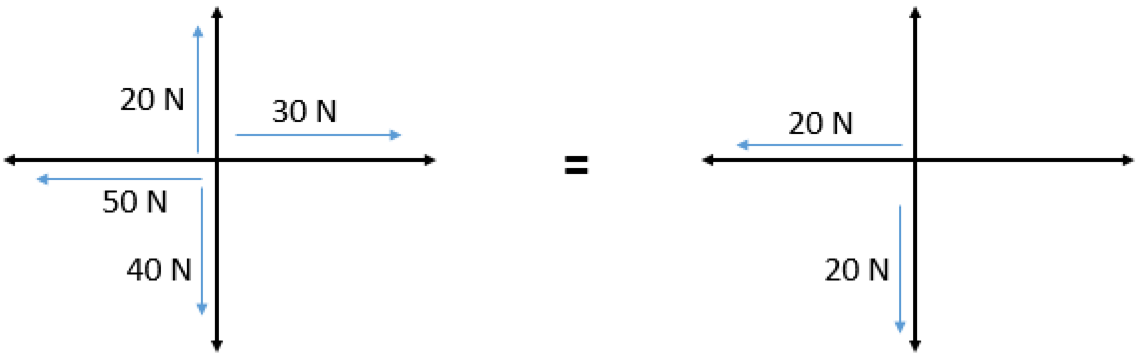
$\dfrac{1}{0} = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$P + Q\cos\theta = 0 \Rightarrow Q\cos\theta = -P$ $\quad\quad$ in (i)

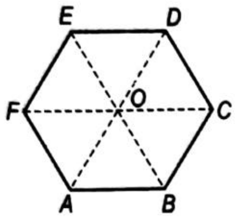
$64 = P^2 + 256 + P^2 - 32P - 2P^2$

$32P = 192 \Rightarrow P = 6N, Q = 10\,N$

3. (B)



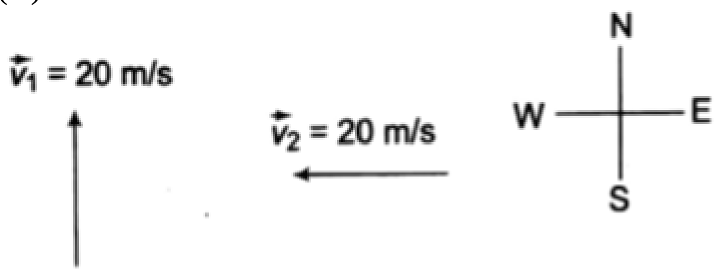
4. (D)



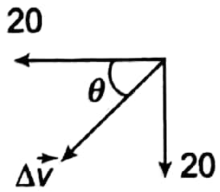
$$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=\overrightarrow{AB}+\left(\overrightarrow{AB}+\overrightarrow{BC}\right)+\left(\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}\right)+\left(\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}\right)$$
$$+\left(\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EF}\right)$$

$$\overrightarrow{AB}=-\overrightarrow{DE},\ \overrightarrow{BC}=-\overrightarrow{EF}$$
$$=3\left(\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}\right)=3\left(\overrightarrow{AD}\right)=6\overrightarrow{AO}$$

5. (D)



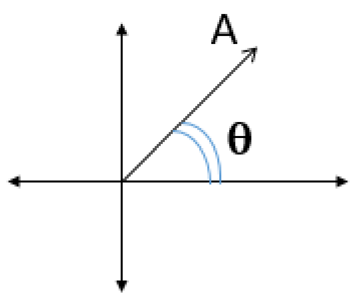
$\Delta\vec{v}=\vec{v}_2-\vec{v}_1=\vec{v}_2+(-\vec{v}_1)$



$\left|\Delta\vec{v}\right|=\Delta v=20\sqrt{2}$ m/s, $\theta=45°$

Change in the velocity: $20\sqrt{2}$ m/s, SW

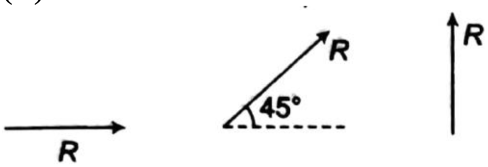
6. (D)



$A_x=A\cos\theta$ $\qquad A_x\le A$

$A_y=A\sin\theta$ $\qquad A_y\le A$

7. (B)



$$X = R + R\cos 45° = R\left(1+\frac{1}{\sqrt{2}}\right)$$

$$Y = R + R\sin 45° = R\left(1+\frac{1}{\sqrt{2}}\right)$$

$$S = \sqrt{X^2+Y^2}$$

$$= \sqrt{2}R\left(1+\frac{1}{\sqrt{2}}\right) = (\sqrt{2}+1)R$$

8. (B)

$$\sqrt{(0.5)^2+(0.8)^2+c^2} = 1$$

9. (C)

$$\vec{r}_1 = i+2j+3k, \vec{r}_2 = 4i+6j+9k$$

$$\vec{d} = \vec{r}_2 - \vec{r}_1 = 3i+4j+6k$$

10. (B)

If $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$, $r=\sqrt{x^2+y^2+z^2}$

α, β, γ : angles made by $\vec{r}$ with coordinate axes

$$\cos\alpha = \frac{x}{r}, \cos\beta = \frac{y}{r}, \cos\gamma = \frac{z}{r}$$

Direction cosines, $\cos\alpha, \cos\beta, \cos\gamma$

$$\vec{A} = 3i+6j-2k, \ |\vec{A}| = A = \sqrt{(3)^2+(6)^2+(-2)^2} = 7$$

$$\cos\alpha = \frac{3}{7}, \cos\beta = \frac{6}{7}, \cos\gamma = \frac{-2}{7}$$

11. (B)

$$\vec{A} = \hat{i}+2\hat{j}+3\hat{k}, \ \vec{B} = 3\hat{i}-2\hat{j}+\hat{k}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = 8\hat{i}-8\hat{j}-8\hat{k}$$

Area of parallelogram $= |\vec{A}\times\vec{B}| = 8\sqrt{3}$

12. (C)

$$\frac{d}{dx}\left[x^3+\frac{1}{x^3}+8\right] = 3x^2-3x^{-4}$$

$$= 3x^2 - \frac{3}{x^4}$$

13. (B)

$x = at^2$ $\qquad\qquad y = 2at$

$\frac{dx}{dt} = 2at$ $\qquad\qquad \frac{dy}{dt} = 2a$

$\frac{\frac{dy}{at}}{\frac{dx}{at}} = \frac{2a}{2at} = \frac{1}{t}$ $\qquad\qquad \frac{dy}{dx} = \frac{1}{t}$

14. (D)

Apply Chain Rule

$y = \sin x^3$

$x^3 = t$

$\frac{dt}{dx} = 3x^2$

$y = \sin t$

$\frac{dy}{dx} = \cos t$

$\frac{dt}{dx} = \cos t 3x^2$

$= \cos\left(x^3\right)3x^2$

$= 3x^2 \cos x^3$

15. (A)

$\int_0^{\pi/2}\left(\sin x + \cos x\right)dx$

$= \int_0^{\pi/2}\sin x\, dx + \int_0^{\pi/2}\cos x\, dx$

$= \left(-\cos x\right)_0^{\pi/2} + \left(\sin x\right)_0^{\pi/2}$

$= 1 + 1 = 2$

16. (B)

$\int_a^b \frac{dx}{x^2} = \left(-\frac{1}{x}\right)_a^b = \frac{1}{a} - \frac{1}{b} = \frac{b-a}{ab}$

17. (D)

$y = 2x^2 - 4x$

$\frac{dy}{dx} = 4x - 4$ $\qquad\qquad \frac{d^2y}{dx^2} = 4$

For Maxima & minima

$\frac{dy}{dx} = 0 = 4x - 4$

At $\left(x = 1\right)$ minima

$y_{\min} = 2 - 4 = -2$

18. (C)

19. (A)

$$\int_0^1 \left(1+e^{-x}\right) dx$$

$$= \left(x + \left(\frac{e^{-x}}{-1} \right) \right)_0^1$$

$$= \left(x - e^{-x} \right)_0^1$$

$$= \left(1 - e^{-1}\right) - \left(0 - e^{0}\right)$$

$$= 1 - e^{-1} + 1 = 2 - e^{-1}$$

$$= 2 - \frac{1}{e}$$

20. (D)

$y = 4x^2$

Slope $= \frac{dy}{dx}$

$$\frac{dy}{dx} = 8x$$

Slope at $x = 1$

$$\frac{dy}{dx} = 8 \times 1$$

$$\frac{dy}{dx} = 8$$

IIT – JEE: 2025 **TW TEST** **DATE: 01/07/23**

TOPIC: MOLE CONCEPT

SOLUTIONS

21. (C)

Mass of $Al_2O_3 = 2\times27+3\times16 = 102$ g

$0.051\,g$ of $Al_2O_3 = \frac{0.51}{102} = 0.0005$ mol (No. of moles $= \frac{\text{Mass}}{\text{Molecular mass}}$)

1 mole of Al_2O_3 contains $2\times6.023\times10^{23}$ Al^{3+} ions

0.0005 mole of Al_2O_3 contains

$2\times0.0005\times6.023\times10^{23}$ Al^{3+} ions

$= 6.023\times10^{20}$ Al^{3+} ions.

22. (C)

Number of molecules of gas at STP

$= \frac{6.023\times10^{23}\times2.8}{22.4} = 7.5\times10^{22}$ molecules

Number of atoms in diatomic molecule $= 2\times7.5\times10^{22}$

$= 15\times10^{22}$ atoms

23. (B)

Total number of atoms represented by the compound

$CuSO_4.5H_2O$ is Cu +S+ 40 + 10H + 50=21

24. (C)

Vapour density = 70

Molecular mass $= 2\times70 = 140$

Formula is $[CO]_x$

Therefore, molecular mass = $(12 + 16)_x = 140$

$= x \times 28 = 140$

$\therefore \quad x = 5$

25. (A)

$$2Ag_2CO_3 \xrightarrow{\Delta} 4Ag + 2CO_2\uparrow + O_2\uparrow$$

$2\times276g \qquad 4\times108g$

$\because \quad 2\times276$ g of Ag_2CO_3 gives $= 4\times108$ g Ag

$\therefore \quad 1$ g of Ag_2CO_3 gives $= \frac{4\times108}{2\times276}$ g Ag

$\therefore \quad 2.76$ of Ag_2CO_3 gives $= \frac{4\times108\times2.76}{2\times276}$

$$= 2.16\text{ g Ag}$$

26. (C)

The unbalanced equation is

$$P_4(s) + O_2(g) \longrightarrow P_4O_{10}(s)$$

It can be balanced as follows :

$$P_4(s) + 5O_2(g) \longrightarrow P_4O_{10}(s)$$

27. (B)

According to Haber's process,

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

Now, according to above equation

2 moles of ammonia (NH_3) require = 3 moles of H_2

$\therefore$ 1 mole of NH_3 require $= \frac{3}{2}$ moles of H_2

or, 20 moles of NH_3 require $= \frac{3}{2} \times 20$ moles of H_2

= 30 moles of H_2

28. (C)

$\because$ Mass of 22400 cm^3. CH_4 = 16 g

$\therefore$ Mass of 112 cm^3 CH_4 = $\frac{16 \times 112}{22400} = 0.08\text{ g}$

29. (D)

No. of Moles of NO = $\frac{4.2}{30} = 0.14$

No. of moles of O_2 = $\frac{3.2}{32} = 0.10$

$2NO_{(g)} + O_2(g) \rightarrow 2NO_2(g)$

Using limiting reagent concept

Mass of NO_2 = 0.14 mol × 46 g mol^{-1}

= 6.44g

30. (A)

Number of moles $= \frac{\text{weight}}{\text{molecular wt.}} = \frac{0.0018}{18} = 1 \times 10^{-4}$ [$\because$ 0.0018 mL = 0.0018 g]

$\therefore$ Number of water molecules $= 1 \times 10^{-4} \times 6.02 \times 10^{23}$

$= 6.023 \times 10^{19}$

31. (B)

Number of C atoms $= \frac{1 \times 10^{-3}}{12} \times N_A = 0.50 \times 10^{20}$

32. (C)

Mass of 1 atom $^{12}C = \frac{\text{Atomic mass of C}}{\text{Avogadro's number}}$

$$= \frac{12g}{6.022 \times 10^{23}} = 1.9927 \times 10^{-23} \text{ g}$$

33. (D)

$3F^- = 1$ Formula unit (AlF_3)

$3.0 \times 10^{24} F^- = 1 \times 10^{24}$ Formula units (AlF_3)

34. (D)

35. (B)

36. (A)

Number of atoms in $N_2 = \frac{11.2 \times 10^{-3} \times 6.023 \times 10^{23} \times 2}{22.4}$

$= 6.023 \times 10^{20}$

Number of atoms in NO $= \frac{0.015 \times 2 \times 6.023 \times 10^{23}}{30}$

$= 6.023 \times 10^{20}$

37. (D)

In a chemical reaction, coefficient represents mole of that substance.

$X + 2Y \rightarrow Z$

This indicates 1 mole of *X* reacts with 2 moles of *y* to form 1 mole of *Z*.
So, 5 moles of *X* will require 10 moles of *Y*.
But we have taken only 9 moles of *Y*.
Hence, *Y* is in limiting quantity.
Hence, we determine product from *Y*.
Thus, 5 moles of *X* reacts with 9 moles of *Y* to form 4 moles of *Z*.

38. (D)

39. (A)

100g $CaCO_3$ = 73g HCl

Since solution is having 50% HCl, therefore amount HCl solution required is 73 × 2 g.

40. (B)

$$\frac{\text{Weight of a compound(w)}}{\text{Molar mass(M)}} = \text{Moles(n)} = \frac{\text{Number of molecules(N)}}{\text{Avogadro number}(N_a)}$$

$\Rightarrow \frac{W(O_2)}{32} = \frac{N(O_2)}{N_A}$ (1)

$\Rightarrow \frac{W(N_2)}{28} = \frac{N(N_2)}{N_A}$ (2)

(1) ÷ (2)

$$\frac{N(O_2)}{N(N_2)} = \frac{1}{4} \times \frac{28}{32} = \frac{7}{32}$$

IIT – JEE: 2025 **TW TEST** **DATE: 01/07/23**

TOPIC: TRIGONOMETRY - I

SOLUTIONS

41. (A)

$\sin x+\operatorname{cosec} x=2 \Rightarrow \sin^2 x-2\sin x+1=0$

$\therefore \sin x=1 \Rightarrow \operatorname{cosec} x=1$

$\therefore \sin^n x+\operatorname{cosec}^n x=2$ always.

42. (B)

$\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}=\sin 10^\circ\sin 50^\circ\sin 70^\circ$

$=\sin 10^\circ\sin(60-10)\sin(60+10)$

$=\frac{1}{4}\sin 3\times 10^\circ=\frac{1}{4}\sin 30^\circ$

$=\frac{1}{8}$

43. $\tan\theta\tan\left(\frac{\pi}{3}-\theta\right)\cos\left(\frac{\pi}{3}+\theta\right)=-1$

$\therefore \tan 3\theta=-1$

$\because \theta\in\left[0,\frac{\pi}{2}\right]$ $\quad 3\theta=135^\circ$ $\quad \theta=45^\circ$

$\therefore 3\sin\theta-4\cos^3\theta=\frac{3}{\sqrt{2}}-\frac{4}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$

44. (C)

$\sin\theta=\frac{1}{\sqrt{5}}$, $\cos\theta=\frac{2}{\sqrt{5}}$ (using triangle)

$\sin\phi=\frac{1}{\sqrt{10}}$

$\sin(\theta+\phi)=\frac{1}{\sqrt{5}}+\frac{3}{\sqrt{10}}+\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{10}}=\frac{1}{\sqrt{2}}$

$\therefore \theta+\phi=45^\circ=\frac{\pi}{4}$

45. (A)

Using graph

$\cos 10^\circ-\sin 10^\circ>0$

46. (B)

Min. value of $a\tan\theta + b\cot\theta = 2\sqrt{ab}$

$\min\left(3\tan\theta + 12\cot\theta\right) = 2\sqrt{3\times 12} = 12$

47. (C)

$4\sin A + \sec A = 0 \Rightarrow 4\sin A = \dfrac{-1}{\cos A}$

$2\times 2\sin A\cos A = -1 \Rightarrow \sin 2A = -\dfrac{1}{2}$

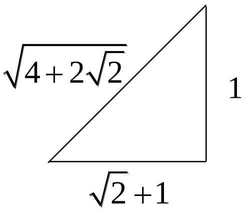
$\because A \in \left[\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right]$ $\qquad 2A = 210°$

$\therefore \cot 2A = \sqrt{3}$

48. (D)

$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$\left(\sqrt{2}+1\right)\sin\theta = \cos\theta$



$\tan\theta = \dfrac{1}{\sqrt{2}+1}$

$\therefore \cos\theta \dfrac{\sqrt{2}+1}{\sqrt{4+2\sqrt{2}}}$

49. (D)

$\cos^2\dfrac{\pi}{12} + \cos^2\dfrac{\pi}{4} + \cos^2\dfrac{5\pi}{12}$

$\cos^2\dfrac{\pi}{12} + \left(\dfrac{1}{\sqrt{2}}\right)^2 + \sin^2\dfrac{\pi}{12}$ $\quad [\therefore$ using $\cos\left(\dfrac{\pi}{2}-\theta\right) = \sin\theta$]

$1 + \dfrac{1}{2} = \dfrac{3}{2}$

50. (D)

$96\cos\dfrac{\pi}{33}\cos\dfrac{2\pi}{33}\cos\dfrac{4\pi}{33}\cos\dfrac{8\pi}{33}\cos\dfrac{16\pi}{33}$

$$\frac{96\sin\dfrac{\pi}{33}\cos\dfrac{\pi}{33}\cos\dfrac{2\pi}{33}\cos\dfrac{4\pi}{33}\cos\dfrac{8\pi}{33}\cos\dfrac{16\pi}{33}}{\sin\dfrac{\pi}{33}}$$

$$\Rightarrow \frac{3\sin\dfrac{32\pi}{33}}{\sin\dfrac{\pi}{33}} \Rightarrow \frac{3\sin\dfrac{\pi}{33}}{\sin\dfrac{\pi}{33}} \qquad \left(\because \sin\left(\pi-\theta\right) = \sin\theta\right)$$

$= 3$

51. (C)

$\tan 15° + \cot 75° + \cot 105° + \tan 105° = 2a$

$\tan 15° + \tan 15° - \tan 15° + \tan 15° = 2a$

$a = \tan 15°$

$a = 2 - \sqrt{3}$

52. (C)
To be remembered.

53. (C)
$\sec x + \tan x = p \qquad ...(1)$

$\sec^2 x - \tan^2 x = 1$

$\therefore \ \sec x - \tan x = \frac{1}{p} \qquad ...(2)$

Using (1) & (2)

$\tan x = \frac{p^2 - 1}{2p}$

54. (A)
$\tan(45°) = \tan(22° + 23°)$

$$1 = \frac{\tan 22° + \tan 23°}{1 - \tan 22 \tan 23}$$
$$= \tan 22° + \tan 23° + \tan 22° \tan 23° = 1$$

55. (A)
$\cot\theta \times \cot(90 - \theta) = 1$

56. (C)
$\sin\alpha = \frac{7}{25} \qquad \therefore \ \tan\alpha = \frac{7}{24}$

$\cos\beta = \frac{9}{41} \qquad \therefore \ \tan\beta = \frac{40}{9}$

$$\therefore \ \tan(\alpha + \beta) = \frac{\frac{7}{24} + \frac{40}{9}}{1 - \frac{7}{24} \times \frac{40}{9}}$$

$\tan(\alpha + \beta) = -\frac{1023}{64}$

57. (D)
$$\cos 10° \cos 50° \cos 70° = \frac{1}{4}\cos(3 \times 10°)$$
$$= \frac{\sqrt{3}}{8}$$

58. (A)
$\max(a \sin x + b \cos x) = \sqrt{a^2 + b^2}$

$\therefore \ \max.(\sin x + 3\cos x) = \sqrt{1 + 9} = \sqrt{10}$

59. (B)
$2\cos^2\theta - 2\sin^2\theta = 1$

$2\cos 2\theta = 1$

$\cos 2\theta = \frac{1}{2}$

$\therefore\ 2\theta = 60^\circ$

$\theta = 30^\circ$

60. (B)

Minimum value of $\left[a\sin^2\theta + b\cos^2\theta\right] = \min(a, b)$

$\therefore\ \min.(4, 5) = 4$