IIT – JEE: 2025 **TW TEST (ADV)** **DATE: 01/07/23**

TOPIC: VECTORS & CALCULUS

SOLUTIONS

1. (B)

$\frac{dx}{dt} = 0 + 3 + 10t$

$\left.\frac{dx}{dt}\right|_{\text{at } t=1} = 3 + 10 = 13\text{ m}$

2. (B)

$y = 4x - 5$

$\left(\frac{dy}{dt}\right) = 4\left(\frac{dx}{dt}\right)$

'y' changes 4 times greater than 'x'.

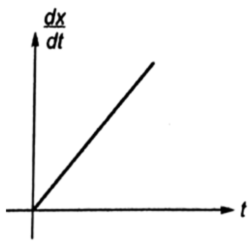
3. (D)

$P = 4a$

$\frac{dP}{dt} = 4\left(\frac{da}{dt}\right) = 4 \times 0.2 = 0.8\text{ cm/s}$

4. (A)

$x = t^2, \frac{dx}{dt} = 2t$



5. (D)

$\frac{dy}{dx} = 4x$, at $x = 1$, $\frac{dy}{dx} = 4$

6. (D)

$\vec{P} + \vec{Q} \neq \hat{P} + \hat{Q}$

7. (C)

$\int_0^3 \left(x^2 + 1\right) dx = \frac{x^3}{3} + x\Big|_0^3 = 9 + 3 - 0 = 12$

8. (C)

$$\int_{\pi/2}^{\pi/2} \cos x\, dx = \sin x \Big|_{-\pi/2}^{\pi/2} = 1-(-1) = 2$$

9. (D)

$$\frac{d}{dx}\left(x^4 - 2\sin x + 3\cos x\right) = 4x^3 - 2\cos x - 3\sin x$$

10. (A)

11. (C)

The resultant of three vectors lying in a plane may be zero if the magnitude of one vector, say A

$$(B-C) \le A \le (B+C)$$

B and C are magnitude of two other vectors.

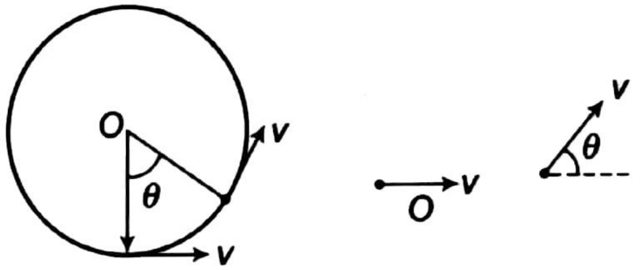
12. (B)

$(\vec{B}\times\vec{A})$ is perpendicular to $\vec{B}$ as well as $\vec{A}$.

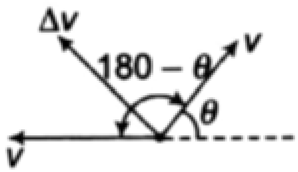
$$\vec{A}\cdot(\vec{B}\times\vec{A}) = 0$$

13. (D)

14. (B)



$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1)$$



$$|\Delta\vec{v}| = \Delta v = \sqrt{v^2 + v^2 + 2v\cdot v\cdot\cos(180-\theta)}$$

$$= \sqrt{2}\, v\sqrt{1-\cos\theta}$$

$$= \sqrt{2}\, v\sqrt{1-\left(1-2\sin^2\frac{\theta}{2}\right)}$$

$$= 2\sin\left(\frac{\theta}{2}\right)$$

The magnitude of the velocity always remains same.

15. (B)

Scalar component of $\vec{A}$ along $\vec{B}$ is

$$|\vec{A}|\cos\theta = |\vec{A}|\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \vec{A}\cdot\frac{\vec{B}}{|\vec{B}|} = \hat{A}\cdot\hat{B}$$

$$\hat{B} = \frac{\vec{B}}{|\vec{B}|} = \frac{i + j}{\sqrt{1+1}\sqrt{2}} = (i + j)$$

$$\vec{A} \cdot \vec{B} = (2i + 3j) \cdot \frac{1}{\sqrt{2}}(i + j) = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

16. (A, C, D)

17. (B, D)

18. (A, B, C)

19. (A, B, C)

20. (A, B)

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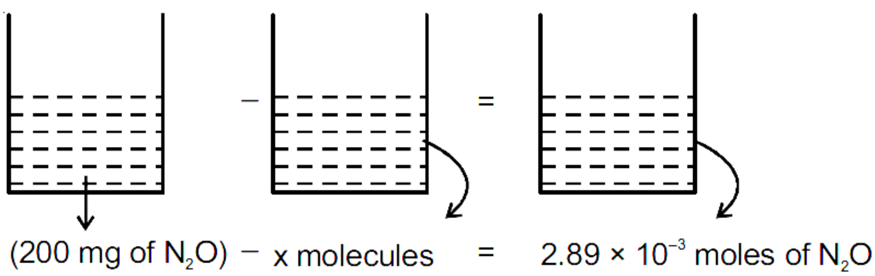
TOPIC: MOLE CONCEPT

SOLUTIONS

21. (D)

From Equation



200 mg of N_2O have molecule $= \frac{200}{44} \times 10^{-3} \times 6.022 \times 10^{23}$

$\approx 2.7 \times 10^{21}$ molecule

$\therefore$ 2.89×10^{-3} moles of N_2O have molecule $= 2.89 \times 10^{-3} \times 6.022 \times 10^{23}$

$= 1.7 \times 10^{21}$ molecules

$\therefore$ 200 mg of N_2O – x molecule = 2.89×10^{-3} moles of N_2O

$[2.7 \times 10^{21} - x = 1.7 \times 10^{21}]$ molecule

$[x = (2.7 - 1.7) \times 10^{21}]$ molecule

$= 10^{21}$ molecule

22. (A)

Weight of Fe in heamoglobin $= \frac{0.334}{100} \times 67200 = 224.48$ u

Mass of one Fe atom = 56u

$\therefore$ Total number of Fe atom $= \frac{224.48}{56} \approx 4$

23. (C)

1 mole $S_8 \equiv 8SO_3 \equiv 8 \times 80$ g = 640 g

24. (B)

Let the atomic mass of B = y g ; A = x g

In B_2A_3

$2y + 3x$ = mol. Mass of $B_2A_3 = \frac{\text{given weight}}{\text{mole}}$

$$2y+3x=\frac{9}{0.05}\text{g}$$

In B_2A

$$\therefore \quad 2y+x=\frac{10}{0.1}\text{g}$$

Solving x and y

$$\begin{Bmatrix} x=40 \\ y=30 \end{Bmatrix}$$

25. (B)

POAC on Ca

$$1\times {}^{n}\text{Ca} = 1\times {}^{n}\text{CaO}$$

$$\frac{1.35}{x}=\frac{1.88}{x+16}$$

$$x=40$$

26. (C)

$KClO_3 \rightarrow KCl + \frac{3}{2}O_2$

$$\frac{10}{122.5}$$

$${}^{n}O_2=\frac{3}{2}\times\frac{10}{122.5}$$

$$\left(W_{O_2}\right)_{\text{Formed}}=\frac{3}{2}\times\frac{10}{122.5}\times 32$$

$$=3.92\text{ kg}$$

27. (A)

Mass of $H_2O = 18\text{g}$.

No. of moles in $H_2O = \frac{\text{given mass}}{\text{molar mass}}$ .....(1)

$$d=\frac{m}{v}$$

Given mass = density × volume

= 0.05 ml × 1

= 0.05 g

Put this in equation (1)

$$\frac{0.05}{18}\times 6.022\times 10^{23}$$

No. of water molecules $=1.67\times 10^{21}$

28. (A)

29. (A)
5*g* atoms of nitrogen = 70*g*

30. (B)
Find out the number of moles of each. Maximum number of moles would contain largest number of molecules.
In 100 g of CO_2, N = w/m = 100/44 = 2.27
In 200 g NH_3, N = w/m = 200/17 = 11.76
150 g of CH_4, N = w/m = 150/16= 9.37
300 g HCl, N = w/m = 300/36.5= 8.03
$\therefore$ Maximum number of molecules is in NH_3

31. (C)
The stoichiometry of decomposition of $CaCO_3$ is
$CaCO_3 \rightarrow CaO + CO_2$
One mol of $CaCO_3$ decomposes to form one mol of CaO and one mol of CO_2.
Molar mass of CaO = (40 + 16) = 56 g mol^{-1}
Mol of CaO formed = $\frac{\text{Mass of CaO}}{\text{Molar Mass of CaO}} = \frac{5.6}{56} = 0.1$
Mol of CO_2 formed = 0.1
Molar mass of CO_2 = (12+ 2x 16) = 44 g mol^{-1}.
Mass of CO_2 formed = Mol of CO_2 x Molar mass of CO_2 = 0.1 x 44 = 4.4 g

32. (D)

33. (C)

34. (C)

35. (B)

36. (C, D)
1g-atom of nitrogen = 1 mol *N* atoms
$= \frac{1}{2}$ mol N_2 gas
$\frac{1}{2}$ mol N_2 gas = 11.2 L N_2 at STP = 14 gm of N

37. (A, B)
1g molecule V_2O_5 = 1 mole V_2O_5 = 2 mole V
atom = 5 mole O atom

38. (A, C, D)

39. (B, C)

Species having same empirical formula (EF) will have same percentage of C.

	EF
CH_3COOH	CH_2O
$C_6H_{12}O_6$	CH_2O
C_2H_5OH	C_2H_6O
$HCOOCH_3$	CH_2O
CH_3OCH_3	C_2H_6O

40. (A, B)

(A) 46g of 70% $\frac{W}{V}$ HCOOH (d_{solution} = 1.4 g/mL)

$70\%\frac{W}{V}$ HCOOH $\rightarrow$ 70 g

HCOOH in 100 mL solution.

Mass of solution = 1.4 × 100 = 140 g

So, in 140 g solution, mass of HCOOH = 70 g in 46 g, mass of HCOOH $=\frac{70}{140}\times 46 = 23\,\text{g}$

(B) 10 M HCOOH $\rightarrow$ 10 mole

HCOOH in 1000 mL solution mass of solution = 1000 g

Mass of HCOOH = 10 × 46 = 460 g

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TOPIC: TRIGONOMETRY - I

SOLUTIONS

41. (B)

Given the diameter of circular wire = 14 cm. Therefore length of wire = 14π cm.

Hence, required angle $=\frac{\text{Arc}}{\text{Radius}}=\frac{14\pi}{12}=\frac{7\pi}{6}$ radian $\Rightarrow \frac{7\pi}{6}\times\frac{180^\circ}{\pi}=210^\circ$

42. (D)

We have, π radians $=180^\circ$

$\therefore \quad 1=\left[\frac{180}{\pi}\right]^\circ; \quad \therefore \left[\frac{2\pi}{15}\right]=\left[\frac{2\pi}{15}\times\frac{180}{\pi}\right]^\circ=24^\circ$.

43. (C)

$\left(\sin^2\theta+\text{cosec}^2\theta\right)=\left(\sin\theta+\text{cosec}\,\theta\right)^2-2\sin\theta\cdot\text{cosec}\,\theta=2^2-2=2$.

44. (C)

$\sqrt{\frac{(1-\sin\theta)^2}{\left(1-\sin^2\theta\right)}}=\frac{1-\sin\theta}{\cos\theta}=\sec\theta-\tan\theta$

45. (D)

$\tan A+\cot A=4 \Rightarrow \tan^2 A+\cot^2 A+2\tan A\cot A=16$

$\Rightarrow \tan^2 A+\cot^2 A=14 \Rightarrow \tan^4 A+\cot^4 A+2=196 \Rightarrow \tan^4 A+\cot^4 A=194$

46. (B)

$\cos x=\sqrt{1-\sin^2 x}=\sqrt{1-\left(\frac{-24}{25}\right)^2}=\frac{7}{25} \Rightarrow \tan x=\frac{\sin x}{\cos x}=\frac{-24}{7}$

47. (A)

$\tan A+\cot A+(-\tan A)+(-\cot A)=0$

48. (D)

We know $|\sin\theta| \le 1$; so, each θ_1, θ_2 and θ_3 must be equal to $\frac{\pi}{2}$

$\therefore\ \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$.

49. (B)

$$\frac{2\left(\sin^2 A - \sin^2 B\right)}{2\sin A\cos A - 2\sin B\cos B} = \frac{2\sin(A+B)\cdot\sin(A-B)}{\sin 2A - \sin 2B} = \frac{2\sin(A+B)\sin(A-B)}{2\sin(A-B)\cos(A+B)} = \tan(A+B)$$

50. (B)

$$\frac{\cos 10° - \sin 10°}{\cos 10° + \sin 10°} = \frac{1-\tan 10°}{1+\tan 10°} = \tan 35° = \tan(90° - 35°) = \cot 55°$$

51. (A)

$$\frac{1-t^2}{1+t^2} = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \qquad \left(\therefore \tan\frac{\theta}{2} = t\right)$$

$$= \cos\left(2, \frac{\theta}{2}\right) = \cos\theta.$$

52. (D)

Minimum value of $3\cos x + 4\sin x = -\sqrt{3^2+4^2} = -5$

Minimum value of $3\cos x + 4\sin x + 5 = -5 + 5 = 0$

53. (B)

$f(x) = 4\sin^2 x + 3\cos^2 x = \sin^2 x + 3$ and $0 \le |\sin x| \le 1$

$\therefore$ Maximum value of $\sin^2 x + 3\cos^2 x$ is 4.

54. (C)

$2\cos(180 - C)\cos(A-B) + 2\cos^2(C) - 1$

$= -2\cos C\left(\cos(A-B) - \cos C\right) - 1$

$= -2\cos C\left(\cos(A-B) + \cos(A+B)\right) - 1$

$= -2\cos C\left(2\cos A\cos B\right) - 1$

$= -4\cos A\cos B\cos C - 1$

55. (B)

$$1 - \sin x + \sin^2 x = \left(\sin - \frac{1}{2}\right)^2 + \frac{3}{4}$$

Minimum $=\frac{3}{4}$ at $\sin x=\frac{1}{2}$

Maximum = 3 at $\sin x=-1$

Ratio of maximum value of minimum value = 4

56. (A, B)

By hypothesis $450°<A<540°$.

This implies $225°<\frac{A}{2}<270°$.

So, $\left|\sin\frac{A}{2}\right|=\sqrt{\frac{1-\cos A}{2}}=\sqrt{\frac{1+\left(\frac{7}{25}\right)}{2}}=\frac{4}{5}$

Now $\frac{A}{2}$ lies in the third quadrant.

This mean $\sin\frac{A}{2}=\frac{-4}{5}$

Again $\left|\cos\frac{A}{2}\right|=\sqrt{\frac{1+\cos A}{2}}=\sqrt{\frac{1-\left(\frac{7}{25}\right)}{2}}=\frac{3}{5}$

Again $\frac{A}{2}$ lies in the third quadrant implies $\cos\frac{A}{2}=\frac{-3}{5}$

57. (A, B, C, D)

$$\frac{1}{\sqrt{2}}=\cos 45°=2\cos^{-2}22\frac{1}{2}^{\circ}-1$$

Therefore, $\cos^2 22\frac{1}{2}^{\circ}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}+1\right)=\frac{1}{2}\left(\frac{\sqrt{2}+2}{2}\right)=\frac{\sqrt{2}+2}{4}$

$$\cos 22\frac{1}{2}^{\circ}=\frac{\sqrt{\sqrt{2}+2}}{2}\left(\because \cos 22\frac{1}{2}^{\circ}>0\right)$$

$$\sin 22\frac{1}{2}^{\circ}=+\sqrt{1-\cos^2 22\frac{1}{2}^{\circ}}$$

$$=+\sqrt{1-\frac{2+\sqrt{2}}{4}}=\frac{\sqrt{2-\sqrt{2}}}{2}$$

58. (A, B, C, D)

(A) $\cos(30°-A)+\cos(30°+A)$

$=2\cos 30°\cos A$

$=2\left(\frac{\sqrt{3}}{2}\right)\cos A=\sqrt{3}\cos A$

Therefore (A) is true.

(B) $\sin(60°+A)-\sin(60°-A)$

$=2\cos 60° \sin A = 2\left(\frac{1}{2}\right)\sin A = \sin A$

Therefore (B) is true.

(C) $\frac{\cos 2\theta - \cos 3\theta}{\sin 2\theta + \sin 3\theta} = \frac{2\sin\left(\frac{5\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}{2\sin\left(\frac{5\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)} = \tan\frac{\theta}{2}$

Therefore (C) is true.

(D) $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$

$\sin^2 5\theta - \sin^2 3\theta = \sin(5\theta+3\theta)\sin(5\theta-3\theta)$

$= \sin 8\theta \sin 2\theta$

Therefore (D) is true.

59. (A, B, C)

(A) If a, b are positive, then

$\frac{a+b}{2} \geq \sqrt{ab}$

and equality holds if and only if $a=b$.

Therefore $27\tan^2\theta + 3\cot^2\theta \geq 2\sqrt{(27\tan^2\theta)(3\cot^2\theta)}$

$=2\times 9 = 18$

Hence the minimum value of $27\tan^2\theta+3\cot^2\theta$ is 18 and it will occur when $27\tan^2\theta = 3\cot^2\theta$
So (A) is true.

(B) It is known that $\cos(180°-\theta) = -\cos\theta$.

Therefore the given expression is

$(\cos 1° - \cos 1°) + (\cos 2° - \cos 2°) + ... + (\cos 89° - \cos 89°) + \cos 90° = 0$

Hence (B) is true.

(C) $\sin^2 85° = \sin^2(90°-5°) = \cos^2 5°, \sin^2 80° = \cos^2 10°$, etc.

The given expression

$8 + \sin 2^2 45° = 8 + \frac{1}{2} = \frac{17}{2}$

Therefore (C) is true.

(D) Range of $\sin\theta$ is $[-1, 1]$

60. (A, C, D)

$(1+\tan A)(1+\tan B) = 2$

If $A+B=45°$. So, A is true

$(1-\cot A)(1-\cot B) = 2$ if $A+B=45°$

So, (C) is true.

$$\frac{\tan^2\theta+\cot^2\theta}{2} \geq \sqrt{\tan^2\theta\cdot\cot^2\theta}$$ (A.M. – G.M. inequality)

$\Rightarrow \tan^2\theta+\cot^2\theta \geq 2$

$\therefore$ (D) is true.