IIT – JEE: 2025 **TW TEST (3 YRS.)** **DATE: 09/06/23**

TOPIC: PERIODIC PROPERTIES

SOLUTION

1. (D)
$Na^{+} > Ca^{2+} > Mg^{2+} > Al^{3+}$

2. (C)
E.N.: F > Cl > Br > I
N > P > As
O > S > Se

3. (A)

4. (A)
$Li^{+}(aq.) > Na^{+}(aq.) > K^{+}(aq.) > Rb^{+}(aq.) > Cs^{+}(aq.)$
$Be^{2+}(aq.) > Mg^{2+}(aq.) > Ca^{2+}(aq.)$
$Si^{4+}(aq.) > Al^{3+}(aq.) > Mg^{2+}(aq.)$

5. (A)
After losing one electron it attains noble gas configuration.

6. (C)
P : $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^3$
N : $1s^2\ 2s^2\ 2p^3$
F : $1s^2\ 2s^2\ 2p^5$
Cl : $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^5$
Electron affinity of Cl is maximum.

7. (B)

8. (D)

9. (C)
Ionisation energy increases from left to right & decreases top to bottom.

10. (A)
As positive charge on element increases size decreases because of greater attraction between valence shell electron and nucleus.

11. (B)
Be > B > N
$N^{3-} > O^{2-} > F^{-}$

$Li < Na < K$
$Fe^{4+} < Fe^{3+} < Fe^{2+}$

12. (B)
$Li > Na > K$
$Be > Mg > Ca$
$B < C < N$
$C > Si > Ge$

13. (B)
$O < S$ and $F < Cl$
$O < F$ and $S < Cl$

14. (B)

15. (C)

16. (A)
Cu, Ag & Au are coinage metals.

17. (C)

18. (D)
Second ionisation potential is always greater than 1^{st} ionisation potential.

19. (B)
P & N are non-metal.
Al is metal.
As is metalloid.

20. (B)
$Be(OH)_2$ and $Al(OH)_3$ are amphoteric.
$Si(OH)_4$ is acidic.
$Mg(OH)_2$ is basic.

TOPIC: TRIGONOMETRIC EQUATION

SOLUTION

21. **(A)**

$\cos x + \sec x = 2$ only possible when $\cos x = \sec x = 1$

$\Rightarrow \quad x = 2n\pi; n \in I$

22. **(D)**

23. **(D)**

24. **(D)**

$3\tan^2 x \geq 4\sin^2 x$

$\cos^2 x \leq \frac{3}{4}$

$\frac{-\sqrt{3}}{2} \leq \cos x \leq \frac{\sqrt{3}}{2}$

$\Rightarrow \quad x \in x \in \left[2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right] \cup \left[2n\pi + \frac{7\pi}{6}, 2n\pi + \frac{11\pi}{6}\right] \cup \{0, 2\pi\} \qquad n \in I$

25. **(D)**

$\tan\theta + \tan 4\theta = \tan 7\theta(\tan\theta \tan 4\theta - 1)$

$\frac{\tan\theta + \tan 4\theta}{1 - \tan\theta \tan 4\theta} = -\tan 7\theta$

$\tan 5\theta = \tan(-7\theta)$

$5\theta = n\pi - 7\theta$

$12\theta = n\pi$

$\theta = \frac{n\pi}{12}, \theta \neq \frac{\pi}{2}, \ldots.. \qquad n \in I$

26. **(D)**

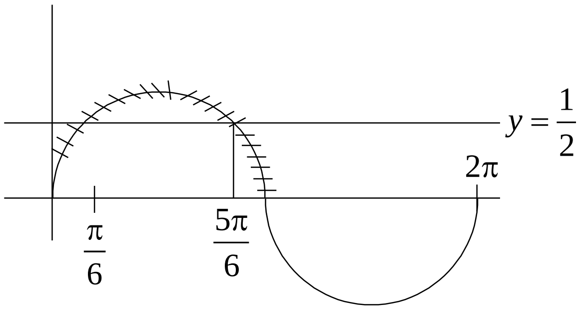
$4\sin^2 x - 6\sin x - 2\sin x + 3 \leq 0$

$2\sin x(2\sin x - 3) - 1(2\sin x - 3) \leq 0$

$(2\sin x - 3)(2\sin x - 1) \leq 0$

$\frac{1}{2} \leq \sin x \leq \frac{3}{2}$

But $\sin x \le 1$ so $\sin x \in \left[\frac{1}{2}, 1\right]$



$x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

27. **(B)**

$\sin 3\alpha = 4\sin\alpha\left(\sin^2 x - \sin^2 x\right)$

$3\sin\alpha - 4\sin^3\alpha = 4\sin\alpha\sin^2 x - 4\sin^3\alpha$

$4\sin^2 x = 3$ if $\alpha \ne 0$

$\sin^2 x = \frac{3}{4}$

$\sin^2 x = \frac{3}{4}$

$x = n\pi \pm \frac{\pi}{3}$

28. **(B)**

A.M. $\ge$ G.M.

$\frac{2^{\sin\theta} + 2^{-\cos\theta}}{2} \ge \left(2^{\sin\theta - \cos\theta}\right)^{\frac{1}{2}}$

$2^{\sin\theta} + 2^{-\cos\theta} \ge 2(2)^{\frac{\sin\theta - \cos\theta}{2}}$

$2^{\sin\theta} + 2^{-\cos\theta} \ge 2 \times 2^{\frac{-\sqrt{2}}{2}}$

$2^{\sin\theta} + 2^{-\cos\theta} \ge 2 \times 2^{\frac{-1}{\sqrt{2}}}$

$2^{\sin\theta} + 2^{-\cos\theta} \ge 2^{1-\frac{1}{\sqrt{2}}}$

Minimum value of $\sin\theta - \cos\theta = -\sqrt{2}$

$\frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta = -1$

$\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta = 1$

$\sin\left(\theta - \frac{\pi}{4}\right) = \sin\left(-\frac{\pi}{2}\right)$

$\theta - \frac{\pi}{4} = 2n\pi - \frac{\pi}{2}$

$\theta = 2n\pi - \frac{\pi}{4}$

$$\theta = 2n\pi + 2\pi - \frac{\pi}{4}$$

$$\theta = 2n\pi + \frac{7\pi}{4} \quad n \in I$$

29. (C)

$\cos^7 x = 1 - \sin^2 x$

$\cos^7 x = \cos^2 x$

$\cos^2 x\left(\cos^5 x - 1\right) = 0$

Either $\cos^2 x = 0$ or $\cos^5 x = 1$

$\cos x = 0$ or $\cos(x) = 1$

$x = \frac{\pi}{2}, -\frac{\pi}{2}$ or $x = 0$

30. (A)

$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$$

$$= \frac{2\tan\phi}{1 + 3\tan^2\phi}$$

$$= \frac{2}{3\tan\phi + \frac{1}{\tan\phi}}$$

$$\tan^2(\theta - \phi) = \frac{4}{9\tan^2\phi + \frac{1}{\tan^2\phi} + 6}$$

A.M. $\geq$ G.M.

$$\frac{9\tan^2\phi + \left(\tan^2\phi\right)^{-1}}{2} \geq (9)^{\frac{1}{2}}$$

$$9\tan^2\phi + \frac{1}{\tan^2\phi} \geq 6$$

Maximum at $\tan^2(\theta - \phi) = \frac{4}{6+6} = \frac{4}{12} = \frac{1}{3}$

31. (A)

$\sin x\cos x\left(\sin^2 x + \cos^2 x\right) + \sin^2 x\cos^2 x = 1$

$\sin x\cos x + \sin^2 x\cos^2 x = 1$

Let $\sin x\cos x = t$

$t^2 + t - 1 = 0$

$t = \frac{-1 \pm \sqrt{5}}{2}$

$2\sin x\cos x = -1 \pm \sqrt{5}$

$\sin 2x = \sqrt{5} - 1$ or $\sin 2x = \sqrt{5} + 1$

Not possible as $\sin 2x \in [-1, 1]$

'0' solutions.

32. **(C)**

$$\sec^2(a+2)x-1+a^2=0$$

$$\tan^2(a+2)x+a^2=0$$

$\Rightarrow$ Only possible when

$\tan(a+2)x=0$ and $a=0$

$a=0 \Rightarrow \tan 2x=0$

$\Rightarrow \quad x=0, \frac{\pi}{2}, -\frac{\pi}{2}$

$(a, x)\equiv(0, 0), \left(0, \frac{\pi}{2}\right), \left(0, -\frac{\pi}{2}\right)$

3 solutions

33. **(B)**

Assume $e^{\sin x}=t$

$t-\frac{1}{t}-4=0$

$t^2-4t-1=0$

$t=\frac{4\pm\sqrt{16+4}}{2}=2\pm\sqrt{5}$

$e^{\sin x}\in\left[e^{-1}, e^{1}\right]$

34. **(C)**

Adding the equations :

$3\cos^3 y+x\sin^3 y+3x(\sin y\cos y)(\sin y+\cos y)=27$

$\cos^3 y+\sin^3 y+3(\cos y\sin y)(\cos y+\sin y)=\frac{27}{x}$

Subtracting equations :

$\cos^3 y-\sin^3 y-3(\cos y\sin y)(\cos y-\sin y)=\frac{1}{x}$

$(\cos y+\sin y)^3=\frac{27}{x}; \ (\cos y-\sin y)^3=\frac{1}{x}$

$(\cos y+\sin y)^2+(\cos y-\sin y)^2=2$

$\left(\frac{27}{x}\right)^{\frac{2}{3}}+\left(\frac{1}{x}\right)^{\frac{2}{3}}=2$

$(27)^{\frac{2}{3}}+1=2x^{\frac{2}{3}}$

$10=2x^{\frac{2}{3}}$

$x^{\frac{2}{5}}=5$

$x^2=12.5$

$x=\pm5^{\frac{3}{2}}=\pm5\sqrt{5}$

35. **(D)**

$\tan\theta=1$

$\theta = n\pi + \frac{\pi}{4}$, But this does not satisfy original equation { $\tan 2\theta$ becomes undefined} so no solution.

36. **(C)**

Minimum value of L.H.S. is 6 and only possible when

$\sin\theta = -1$

$\theta = \frac{3\pi}{2}$ and $\frac{7\pi}{2}$ in $[0, 4\pi]$

Sum = $\frac{3\pi}{2} + \frac{7\pi}{2} = 5\pi$

$K = 5$

37. **(B)**

$x^2 + \frac{1}{x^2} \geq 2$

$\cos^2\left(\frac{x}{2}\right)\sin^2 x \geq 1$

$\cos^2\left(\frac{x}{2}\right) \geq \operatorname{cosec}^2 x$

Which is never possible so no solution.

38. **(D)**

$$\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{2x}{2}}{1+\tan^2\frac{x}{2}} = 1$$

$1 - \tan\frac{2x}{2} + 2\tan\frac{x}{2} = 1 + \tan^2\frac{x}{2}$

$2\tan^2\frac{x}{2} - 2\tan\frac{x}{2} = 0$

$\tan\frac{x}{2} = 0 \qquad \tan\frac{x}{2} = 1$

$x = 0, \pi, 2\pi \qquad \frac{x}{2} = \frac{\pi}{4} \qquad x = \frac{\pi}{2}$

But $x = \pi$ is not satisfying original equation

$x = 0, 2\pi, \frac{\pi}{2}$

So two solution.

39. **(C)**

$\left(1+\sin^2 x\right)\left(1+\cos^2 y\right) \leq 4$

$4\operatorname{cosec}^2 z \geq 4$

They are equal only when

$1+\sin^2 x = 2 \qquad 1+\cos^2 y = 2$

$\sin^2 x = 1 \qquad \cos^2 y = 1$

$\cos^2 x = 0 \qquad \sin^2 y = 0$

$\sin^2 x + \cos^2 y = 2$

40. (C)

$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

$1 + \sin x = 2\cos^2 x$

$(1 + \sin x) = 2(1 - \sin^2 x)$

$(1 + \sin x) = 0$ or $1 - \sin x = \frac{1}{2}$

$\sin x = -1$ $\quad\quad$ $\sin x = \frac{1}{2}$

$x = \frac{3\pi}{2}$ or $x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{3\pi}{2}$ is not possible as $\tan x$ becomes undefined $x = \frac{\pi}{6}, \frac{5\pi}{6}$ (two solutions)