IIT – JEE: 2024 **TW TEST (3 YRS.)** **DATE: 15/10/22**

TOPIC: CIRCULAR MOTION, WPE

SOLUTION

1. (C)

$T = Kx = m\omega^2 r$

where $r = \ell_0 + x$

$\therefore \quad Kx = m\omega^2(\ell_0 + x)$

$\Rightarrow \quad x = \dfrac{m\omega^2 \ell_0}{K - m\omega^2}$

$\therefore \quad r = \ell + x = \dfrac{K\ell_0}{K - m\omega^2}$.

2. (D)

The acceleration vector shall change the component of velocity u_1 $r = \dfrac{v^2}{a}$

Radius of curvature r_{min} means v is minimum and a_n is maximum. This is at point P when component of velocity parallel to acceleration vector becomes zero, that is $u_a = 0$

$\therefore R = \dfrac{u_1^2}{a} = \dfrac{8^2}{4} = 16$ *meters*

3. (C)

For the block to complete the circle, its minimum velocity at the highest point must be $v = \sqrt{g\ R}$

Let h = minimum height of the block.

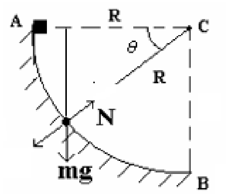
From the initial positon to the topmost position,

Loss in PE = Gain in KE

$\Rightarrow \ mg(h - 2R) = \dfrac{1}{2}mv^2 - 0 = \dfrac{1}{2}m(gR)$

$\Rightarrow 2h - 4R = R \ \ \therefore \ h = 2.5\,R$

4. (D)



$$N - mg\sin\theta = \frac{mv^2}{R} = 2mg\sin\theta$$

$$3mg\sin\theta = N = mg$$

$$\sin\theta = \frac{1}{3}$$

5. (B)

Tension in the string,

$$T = M(g-a) = M\left(g - \frac{g}{2}\right) = \frac{Mg}{2}$$

W = Force × Displacement = $-M\left(\frac{g}{2}\right)h$

Negative sign is because tension is in the upward direction and displacement is in the downward direction.

6. (D)

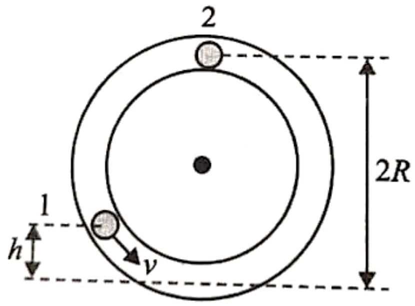
We know that $dU = -dW$

where dU is the change in potential energy

and dW is the work done by conservative forces

Hence, work done by conservative forces on a system is equal to the negative of the change in potential energy.

7. (D)



Loss in KE = Gain in PE

$$\Rightarrow \frac{1}{2}mv^2 - 0 = mg(2R-h) \quad \therefore\ v = \sqrt{2g(2R-h)}$$

8. (A)

Let normal reaction makes an angle θ from vertical, then

$$v^2 = 2gr(1-\cos\theta) \text{ and } \frac{mv^2}{r} = mg\cos\theta$$

$\Rightarrow$ height from ground $h = \frac{2r}{3}$

9. (C)

Let the particle be dropped from a height h and the spring be compressed by y. According to the conservation of mechanical energy, loss in PE of the particle = gain in elastic potential energy of the spring

$$mg\,(h + y) = \frac{1}{2}\,ky^2$$

Now, as the particle and spring are same for second case,

$$\frac{h_1 + y_1}{h_2 + y_2} = \left(\frac{y_1}{y_2}\right)^2 \quad \text{or} \quad \left(\frac{0.24 + 0.01}{h_2 + 0.04}\right) = \left(\frac{0.01}{0.04}\right)^2$$

Solving, we get $h_2 = 3.96$ m

10. (C)

Given $v = k\sqrt{x}$ or $\frac{dx}{dt} = k\sqrt{x}$ or $x^{-\frac{1}{2}}\,dx = k\,dt$

Integrating both sides, we get

$$\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = kt + C\ ; \text{ assuming } x(0) = 0$$

Therefore, $C = 0$

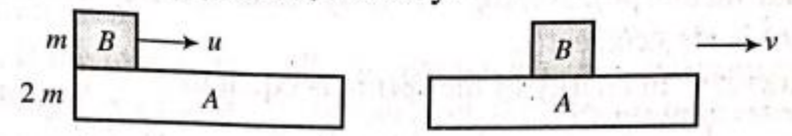
$$2\sqrt{x} = kt \Rightarrow x = \frac{k^2t^2}{4} \text{ or } v = \frac{k^2t}{2}$$

Therefore, work done,
ΔW = Increase in KE

$$= \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}m\left[\frac{k^2t}{2}\right]^2 = \frac{1}{8}mk^4t^2$$

11. (BD)

v is the final common velocity.



$$(m + 2m)v = mu \Rightarrow v = \frac{u}{3}$$

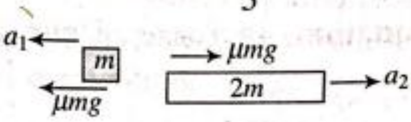


Fig. S8.71

$$a_1 = \frac{\mu mg}{m} = \mu g, a_2 = \frac{\mu mg}{2m} = \frac{\mu g}{2}$$

Acceleration of B relative to A:

$$a_1 + a_2 = \mu g + \frac{\mu g}{2} = \frac{3}{2}\mu g \text{ towards left}$$

Work done against friction:

$$W_1 = K_i - K_f = \frac{1}{2}mu^2 - \frac{1}{2}3mv^2 = \frac{mu^2}{3}$$

Final KE = $\frac{1}{2}3mv^2 = \frac{mu^2}{6}$

Hence, **c.** is not correct.
Option (d) is correct, because momentum is always conserved as there is no external force.

12. (BC)

Consider a system of two point particles which are free to move on a smooth horizontal surface, then the work done by electrical forces acting on the two particles is non-zero. But it is not necessarily true always. For a rigid body, there is no relative displacement of particles, hence, internal forces do not do any work.

13. (AB)

For a body moving in a circular path,
Force towards the centre = force away from the centre

$$\Rightarrow \frac{K}{r^3} = \frac{mv^2}{r}$$

$$\Rightarrow KE = \frac{1}{2}mv^2 = \frac{K}{2r^2}$$

$\therefore$ (A) is correct.

$$PE = U = -\int_{\infty}^{r} F dr = \int_{\infty}^{r} \frac{K}{r^3} dr = -\frac{K}{2}r^2$$

$\therefore$ (B) is correct.

Mechanical enerngy, $E = KE + PE = 0$
Sice, $E \geq 0$, the body is in unbound state.

14. (ABD)

a, b, d. When the plank is shifted, the cord elongates and becomes inclined with vertical. Therefore, a tension is developed in the cord and that tension has two components, a vertical component and a horizontal component: Horizontal component tries to slide the block leftwards, relative to the plank. But friction F_2 prevents that slipping. Hence, the block moves to the right with the plank.

Since the block does not slip over the plank, no energy is lost against friction F_2. Hence, work done by the force F is used to overcome loss against friction F_1 and to elongate the cord. Therefore, option (a) is correct.

The cord elongates due to displacement of the block and the block gets displaced due to friction F_2 acting on it. Hence, the friction F_2 is responsible for the elongation of the cord. Therefore, strain energy stored in the cord is equal to work done by the friction on the block or work done against friction acting on the upper surface of the plank. Hence, the total work done by F is equal to energy lost against friction F_1 plus work done against friction acting on upper surface of the plank. Therefore, option (b) is correct. Hence, it is obvious that option (c) is wrong.

Since, strain energy stored in the cord is equal to work done by friction acting on the block, option (d) becomes same as option (b) Hence, it is also correct.

15. (ABCD)

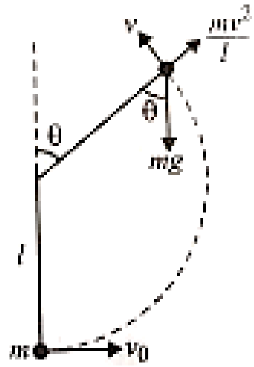
$$\Delta KE = \frac{1}{2} mv^2 = \frac{1}{2} \times 1 \times 2^2 = 2\text{ J}$$

$$W_{cons} = -\Delta U = -5\text{ J} \Rightarrow \Delta U = 5\text{ J}$$

$$W_{ext} = \Delta U + \Delta KE = 5 + 2 = 7\text{ J}$$

16. (8.00)

At the moment the spring becomes slack, tension will become zero. So, at that point, $mg\cos\theta = mv^2/l$



s

$\Rightarrow \cos\theta = v^2/gl$

By energy conservation,

$\Delta U + \Delta K = 0$

$\Rightarrow mgl(1+\cos\theta) + \frac{1}{2}m\left(v^2 - v_0^2\right) = 0$

$\Rightarrow v_0^2 - v^2 = 2gl(1+\cos\theta) = 2gl\left[1 + \frac{v^2}{gl}\right]$

$\Rightarrow 3v^2 = v_0^2 - 2gl = \left(10\sqrt{2}\right)^2 - 2\times10\times5 = 100$

$\Rightarrow v = 10/\sqrt{3}$ m/s. $\quad \therefore n = 3$

17. (7.00)

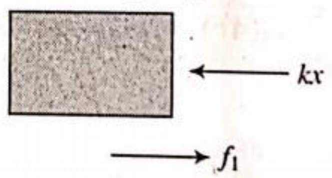
$$5.6 = \frac{1}{2}k(0.3)^2$$

$$W = \frac{1}{2}k\left[(0.3+0.15)^2 - 0.3^2\right]$$

Solve to get: $W = 7$ J

18. (7.00)

Let compression in the spring be x.



19. (4.00)

Speed of block will be maximum when

$\text{mg}\cos\theta = \text{kx} \Rightarrow \text{x} = \frac{\text{mg}\cos\theta}{\text{K}}$

Where x is compression. So n = 4.

20. (4.00)

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TOPIC: GASEOUS STATE

SOLUTION

21. (C)

22. (C)

23. (B)
Most probable speed increase and fraction of molecules possessing most probable speed decreases.

24. (A)
At high temperature and low pressure, real act as an ideal gas.

25. (D)
H_2O molecules have greater attractie forces than O_2 due to their polar nature.

26. (D)

27. (A)
For Vander Waal gas, at any temperature: $Z = \frac{V_m}{V_m - b} - \frac{a}{V_m RT}$

For Vander Waals gas : $\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$

$\therefore\ Z = \frac{PV_m}{RT} = \frac{V_m}{V_m - b} - \frac{a}{V_m RT}$

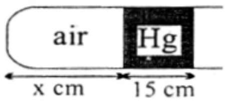
At Boyle temperature,

$$Z = \frac{V_m}{V_m - b} - \frac{a}{V_m \cdot R \cdot \frac{a}{Rb}} = \frac{V_m}{V_m - b} - \frac{b}{V_m} = 1 + \frac{b^2}{V_m(V_m - b)}$$

28. (A)
Critical temperature is lower than Boyle temperatre and below Boyle temperature, the graph is like option (A).

29. (C)

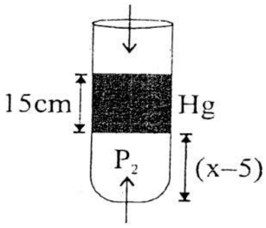
30. (D)



$P_2 = P_{atm} + h$

$= 75 + 15 = 90\,cm$

From Boyle's law



$P_1V_1 = P_2V_2$

$75 \times x \times A = 90 \times (x - 5) \times A$

$75x = 90x - 450$

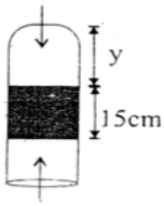
$450 = 15x$

$x = 30$

Now for open end vertical down

$75 = P_3 + 15$

$P_3 = 60\,cm$



Now, $P_1V_1 = P_3V_3$

$75 \times 30 \times A = 60 \times y \times A$

$y = 37.5$

$\therefore$ Hg column will move down by 7.5 cm

31. (ABCD)

At (273^{o}C) = 546 K

1 mole occupy $2 \times 22.4 = 44.8\,L$

$\Rightarrow \frac{1}{2}$ mole H_2 occupy $\rightarrow$ 22.4 L

$\Rightarrow \frac{1}{2}$ mole He occupy $\rightarrow$ 22.4 L

$\Rightarrow \frac{4}{64}$ mole SO_2 occupy $\rightarrow$ 2.8 L

32. (BC)

As M.W. of $D_2 > H_2$, the rate of diffusion of $H_2 > D_2$.

33. (CD)

34. (CD)

35. (ABC)

Average speed of helium will be greater but the momentum will be greater for neon.

Average speed of helium will be greater and hence it will make more collision with the wall. But the momentum and force exerted on wall will be greater for neon.

36. (7.00)

$V = \frac{nRT}{P}$

$$\frac{V_{273}}{V_{273/2}} = \frac{273+273}{273+\frac{273}{2}} = \frac{2}{\frac{3}{2}} = \frac{4}{3}$$

37. (5.00)

$$\frac{P_C V_C}{RT_C} = \frac{3}{8}$$

$$\frac{73.89\times V_C}{0.0821\times 300} = \frac{3}{8}$$

$$V_C = \frac{1}{8} = 3b$$

$$b = \frac{1}{24}$$

$$V_{actual}\times 4 = b$$

$$V_{actual} = \frac{b}{4} = \frac{1}{24\times 4} \text{ for 1 mole}$$

For 24 mole

$$V_{actual} = \frac{24}{24\times 4}\times 1000 = 250 \text{ ml}$$

38. (4.00)

$$H_2(g) + \frac{1}{2} O_2(g) \rightarrow H_2O(g)$$

Start	a	0.5 a	–
80% yeild	0.2a	0.1a	0.8a

For the mix. Before $r\times n$.

$$PV = nRT$$

$$4.5\times V = 1.5a\times R\times 330 \qquad ...(i)$$

For the mix. After $r\times n$

$$PV = nRT$$

$$P\times V = 1.1a\times R\times 400 \qquad ...(ii)$$

Divide (ii) by (i)

$$\frac{P}{4.5} = \frac{1.4\times 400}{1.5\times 330}$$

$$P = 4 \text{ atm}$$

39. (1.00)

40. (4.00)

For gas A, $p_A = \frac{3}{m_A}\times RT$

For gas B, $P_B = \frac{1.5}{m_B}\times RT$

$$\therefore \frac{P_A}{P_B} = \frac{2\times m_B}{m_A} = \frac{2\times 2\times m_A}{m_A} = 4$$