IIT – JEE: 2025 **TW TEST (3 YRS.)** **DATE: 21/05/23**

TOPIC: KINEMATICS-I

Answer Key & Solution

1. (C)

$$T_1 = T_2 \Rightarrow \sqrt{\frac{2h}{g}} = \frac{2\times 10\times \frac{\sqrt{3}}{2}}{10} \Rightarrow h = 15\,m.$$

2. (A)

$$T_1 = \frac{2u\sin\theta}{g};\; T_2 = \frac{2u\cos\theta}{g}$$

3. (D)

$$t_1 = \sqrt{\frac{2h}{g}};\, t_2 = \sqrt{\frac{4h}{g}} - \sqrt{\frac{2h}{g}};\, t_3 = \sqrt{\frac{6h}{g}} - \sqrt{\frac{4h}{g}}$$

4. (A)

Theory

5. (C)

Displacement $= \sqrt{3}\,a$.

Time taken $= \frac{2a}{v}$

So, average velocity $= \frac{\sqrt{3}av}{2a} = \frac{\sqrt{3}v}{2}$

6. (D)

$$t_1 + t_2 = \frac{2u}{g} \Rightarrow u = \frac{1}{g}\left(t_1 + t_2\right)$$

Now , $h = ut_1 - \frac{1}{2}gt_1^2 = \frac{1}{2}gt_1t_2$

7. (A)

$$\frac{1}{2}\times g\times 3^2 = 0 + \frac{1}{2}\times g\times\left(2n-1\right) \Rightarrow n = 5$$

$$h = \frac{1}{2}g\,5^2 = 125\,m$$

8. (A)

$$\frac{1}{2}gt^2-\frac{1}{2}g\left(t-1\right)^2=10 \Rightarrow 2t-1=2=t=\frac{3}{2}$$

9. (D)

Theory

10. (B)

$$\frac{u_1\cos\theta_1}{u_2\cos\theta_2}=\frac{2}{1} \qquad ...(1)$$

$$\frac{u_1\sin\theta_1}{u_2\sin\theta_2}=\frac{2}{1} \qquad ...(2)$$

11. (B)

$$\frac{u^2\cos^2\theta}{u^2\cos^2\theta+\frac{u^2\sin^2\theta}{2}}=\frac{2}{5}\Rightarrow \tan^2\theta=3$$

12. (C)

In one second, vertical velocity change = 10 m/s

Angle = 45°. So, $v_x=10\,m/s$

So, initially, $v_y=20\,m/s$ (2 sec)

13. (A)

$$y=x\tan\theta-\frac{gx^2}{2u_x^2}=x\times 2-\frac{10x^2}{2\times 1^2}$$

14. (C)

$$\vec{v}_{avg}=\frac{v\cos\hat{i}+v\cos\hat{i}+v\sin\theta\hat{j}}{2} \qquad \text{[constant acceleration]}$$

15. (D)

$$T=\frac{2u_y}{g}.$$

New range, $R'=u_xT+\frac{1}{2}\times\frac{g}{4}\times T^2$

$$=R+\frac{g}{8}\frac{4u_y^2}{g^2}=R+H$$

16. (A)

$$y=u\times 2+5\times 2^2 \qquad ...(1)$$

$$2y=u\times 3+5\times 3^2 \qquad ...(2)$$

$$\Rightarrow u=5=gt\Rightarrow t=0.5\sec$$

17. (D)

$$u_x=10\cos 60^o=5\,m/s$$

$$v=u_x\sec 30^o=\frac{10}{\sqrt{3}}\,m/s.$$

18. (D)
Theory

19. (A)
$$\frac{\frac{1}{2}a\left(2n-1\right)}{\frac{1}{2}an^{2}}=\frac{2}{n}-\frac{1}{n^{2}}$$

20. (D)
Theory

ANSWER KEY

21. (C)
22. (D)
23. (B)
24. (A)
25. (B)
26. (B)
27. (C)
28. (C)
29. (D)
30. (C)
31. (C)
32. (A)
33. (B)
34. (C)
35. (D)
36. (B)
37. (C)
38. (A)
39. (B)
40. (B)

TOPIC: TRIGONOMETRY - I

SOLUTIONS

41. (B)

$\sec\theta + \tan\theta = 3$

$\sec\theta - \tan\theta = \frac{1}{3}$

$2\sec\theta = 3 + \frac{1}{3} = \frac{10}{3}$

$\sec\theta = \frac{5}{3}$

$\therefore \quad \cos\theta = \frac{3}{5}$

42. (B)

$\sin\theta = \frac{4}{5}$ but θ lies in 2^{nd} or 4^{th}

Quadrant Hence, $\sin\theta = \frac{4}{5}$ or $-\frac{4}{5}$

43. (C)

$\Rightarrow \quad \sec^4\theta\left(\cos^2\theta\right)\left(1+\sin^2\theta\right) - 2\tan^2\theta$

$\Rightarrow \quad \sec^2\theta\left(1+\sin^2\theta\right) - 2\tan^2\theta \Rightarrow \sec^2\theta + \tan^2\theta - 2\tan^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

44. (D)

$d = 10\theta$ where $\theta = \frac{360^o}{60^o} \times 20 = \frac{2\pi}{3}$

$d = 10 \times \frac{2\pi}{3} \Rightarrow \frac{3d}{10\pi} = 2$

45. (D)

$\sin 4 < 0$.

46. (C)

$\cos\left(A+B\right) = 1 \Rightarrow A = -B$

$\therefore \ 2 + \tan\left(-B\right)\cot B = 2 - 1 = 1$

47. (C)

$\sin^2 A - \sin^2 B = \sin(A+B)\cdot\sin(A-B)$

$\therefore$ using this $K = \frac{1}{\sqrt{2}}$

$\therefore$ $\sqrt{2}K = 1$

48. (D)

If $A+B=45^{\circ}$ then $(1+\tan A)(1+\tan B)=2$

49. (B)

$\text{cosec}^2 x + 25\sec^2 x$

$1+\cot^2 x + 25(1+\tan^2 x)$

$1+\cot^2 x + 25 + 25\tan^2 x$

$26 + \underbrace{25\tan^2 x + \cot^2 x}$

$\geq 2\sqrt{25}$

≥ 10

$\therefore$ Least value of $\text{cosec}^2 x + 25\sec^2 x$ is 36.

50. (C)

$5\cos\theta + 3\left[\cos\theta\cdot\cos\left(\frac{\pi}{3}\right) - \sin\theta\cdot\sin\left(\frac{\pi}{3}\right)\right] + 3$

$5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$\frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

Now, $\lambda = \sqrt{\frac{169}{4}+\frac{27}{4}} + 3$

$= \sqrt{\frac{196}{4}} + 3$

$= 7+3 = 10$

$\mu = -7+3 = -4$

$\therefore$ $\lambda - \mu = 14$

51. (A)

$\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right)$

$\Rightarrow$ $\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{\pi}{8}\right)$

$\Rightarrow$ $2\left[\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right)\right]$

$$\Rightarrow \quad 2\left[\cos^4\left(\frac{\pi}{8}\right)+\sin^4\left(\frac{\pi}{8}\right)\right]$$

$$\Rightarrow \quad 2\left[1-2\sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)\right]$$

$$\Rightarrow \quad 2\left[1-\frac{1}{2}\left(\sin^2\frac{\pi}{4}\right)\right]$$

$$\Rightarrow \quad 2\left[1-\frac{1}{2}\times\frac{1}{2}\right]=2\left[\frac{3}{4}\right]=\frac{3}{2}$$

52. (A)

$$(\alpha+\beta)-(\alpha-\beta)=2\beta$$

$$\tan\left[(\alpha+\beta)-(\alpha-\beta)\right]=\tan 2\beta$$

$$\frac{\tan(\alpha+\beta)-\tan(\alpha-\beta)}{1+\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}=\tan 2\beta$$

$$\therefore \quad \tan(\alpha+\beta)=\frac{3}{4} \text{ and } \tan(\alpha-\beta)=\frac{5}{12}$$

$$\Rightarrow \quad \tan 2\beta=\frac{\frac{3}{4}-\frac{5}{12}}{1+\frac{3}{4}\times\frac{5}{12}}=\frac{36-20}{48+15}$$

$$\tan 2\beta=\frac{16}{63}$$

53. (C)

$$\sin\theta\sin\left(60^\circ-\theta\right)\sin\left(60^\circ+\theta\right)=\frac{\sin 3\theta}{4}$$

$$\therefore \quad \sin 30^\circ\times\frac{\sin 30^\circ}{4}=\frac{1}{2}\times\frac{1}{8}=\frac{1}{16}.$$

54. (D)

$$\frac{1+\cos 20^\circ}{2}-\cos 50^\circ\left(\cos 10^\circ-\cos 50^\circ\right)$$

$$\frac{1+\cos 20^\circ}{2}-2\cos 50^\circ\sin 30^\circ\sin\left(20^\circ\right)$$

$$\frac{1+\cos 20^\circ}{2}-\cos 50^\circ\sin 20^\circ$$

$$\frac{1+\cos 20^\circ}{2}-\frac{1}{2}\left[\sin\left(70^\circ\right)-\sin\left(30^\circ\right)\right]$$

$$\frac{1+\cos 20^\circ-\sin 70^\circ+\sin 30^\circ}{2}$$

$$\frac{\frac{3}{2}+\cos 20^\circ-\cos 20^\circ}{2}=\frac{3}{4}$$

55. (C)

$$\frac{\sin\left(2^9\times\frac{\pi}{2^{10}}\right)}{2^9\times\sin\left(\frac{\pi}{2^{10}}\right)}\times\sin\left(\frac{\pi}{2^{10}}\right)=\frac{1}{512}$$

56. (A)

$$\sin\alpha+\sin\beta=\frac{21}{65}\ \Rightarrow\ 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{21}{65}$$

$$\cos\alpha+\cos\beta=\frac{-27}{65}\ \Rightarrow\ 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{-27}{65}$$

$$\therefore\quad \tan\left(\frac{\alpha+\beta}{2}\right)=\frac{-21}{27}$$

57. (D)

$$\tan A\cdot\tan B+\tan B\cdot\tan C+\tan C\cdot\tan A=1$$

$$\frac{1}{2}+\frac{\tan C}{2}+\tan C=1$$

$$\frac{3\tan C}{2}=\frac{1}{2}\ \Rightarrow\ \tan C=\frac{1}{3}$$

58. (A)

$$\frac{\cos(A-B)}{\cos(A+B)}=\frac{\cos A\cos B+\sin A\sin B}{\cos A\cos B-\sin A\sin B}$$

$$=\frac{1+\tan A\tan B}{1-\tan A\tan B}$$

$$=\frac{1+\frac{1}{3}}{1-\frac{1}{3}}=2$$

59. (D)

$$\tan\left[2(A+B)\right]=\frac{2\tan(A+B)}{1-\tan^2(A+B)}\qquad\left[\tan(A+B)=\frac{2+1}{1-2}=-3\right]$$

$$\therefore\quad \tan 2(A+B)=\frac{2\times(-3)}{1-(-3)^2}=\frac{-6}{-8}=\frac{3}{4}$$

60. (D)

$$\frac{(\sin\theta+\cos\theta)^2}{(\cos\theta-\sin\theta)^2}=\frac{1+\sin 2\theta}{1-\sin 2\theta}=\frac{1+P}{1-P}$$