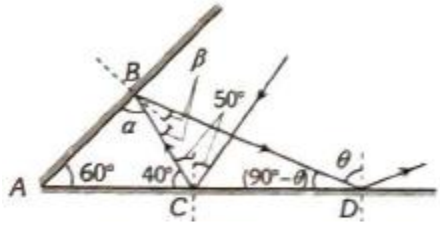
IIT – JEE: 2024 | **TW TEST (ADV)** | **DATE: 05/01/23**

TOPIC: RAY OPTICS

SOLUTIONS

1. (C)
Let required angle be θ



From geometry of figure
In $\Delta ABC; a = 180^0 - (60^0 + 40^0) = 80^0$
$\Rightarrow \beta = 90^0 - 80^0 = 10^0$
In
$\Delta ABD; \angle A = 60^0 \angle B = (a + 2\beta)$
$= (80 + 2 \times 10) = 100^0$ and $\angle D = (90^0 - \theta)$
$\because \angle A + \angle B + \angle D = 180^0 \Rightarrow 60^0 + 100^0 + (90^0 - \theta)$
$= 180^0 \Rightarrow \theta = 70^0$

2. (A)
$m = +\frac{1}{n} = -\frac{\upsilon}{u} \Rightarrow \upsilon = -\frac{u}{n}$
By using mirror formula
$\frac{1}{f} = \frac{1}{-\frac{u}{n}} + \frac{1}{u} \Rightarrow u = -(n-1)f$

3. (C)
Here focal length = f and u = -f
On putting these values in $\frac{1}{f} = \frac{1}{u} + \frac{1}{\upsilon}$
$\Rightarrow \frac{1}{f} = -\frac{1}{f} + \frac{1}{\upsilon} \Rightarrow \upsilon = \frac{f}{2}$

4. (A)
According to the mirror formula $\frac{1}{\upsilon} + \frac{1}{u} = \frac{1}{f}$ or $\upsilon = \frac{fu}{u-f}$
When an object is at a distance of 9m from the concave mirror .
$\therefore u = -9m, f = -1m \Rightarrow \upsilon = \frac{(-1)(-9)}{-9+1} = -\frac{9}{8} m$

As the object moves at a constant speed of $5ms^{-1}$, after 1s the position of image is

$$u' = -9m + 5m = -4m \Rightarrow \upsilon' = \frac{(-1)(-4)}{-4+1} = -\frac{4}{3}m$$

The shift in the position of image in 1 s is

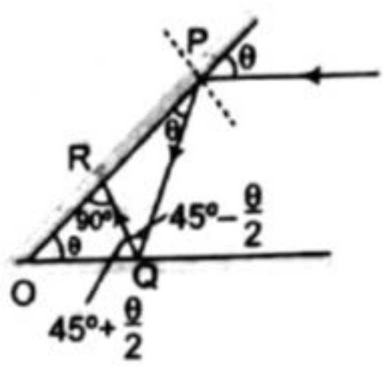
$$= \upsilon - \upsilon' = \frac{-9}{8} + \frac{4}{3} = \frac{-27+32}{24} = \frac{5}{24} = \frac{1}{4.8} \approx \frac{1}{5}$$

Average speed of the image = $\frac{1}{5}ms^{-1}$

5. (D)
As laws of reflection to be true tor all points of the remaining part of the mirror, the image will be that of the whole object. However, as the area of the reflecting surface has been reduced, the intensity of the image will reduce (in this case half)

6. (B)

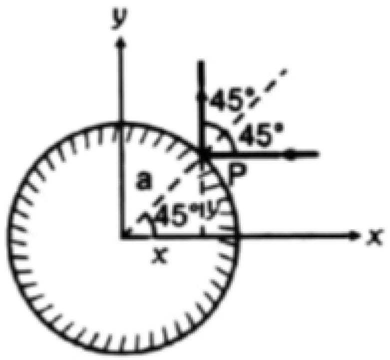
7. (ACD)
In ΔORQ



$$\theta + 90^0 + 45^0 + \frac{\theta}{2} = 180^0$$

$$\therefore \theta + 30^0$$

8. (D)
The ray diagram is as shown



$$x = \frac{a}{\sqrt{2}}$$

and $y = \frac{a}{\sqrt{2}}$

$$\therefore P \equiv \left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$$

9. (AC)

10. (CD)

In the given situation image distance is greater then object distance, size of image increases.

11. (AC)

Given, u = –15 cm, f = –10 cm, O = 1 cm

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}, \frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-10}-\frac{1}{-15}$$

$$\therefore \quad v=-30\,\text{cm}$$

$$\frac{I}{O}=-\frac{v}{u}=-\frac{-30}{-15}=-2$$

$$I=-2\times 1=-2\,\text{cm}$$

Image is inverted and on the same side (real) of size 2 cm.

12. (BD)

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{u}$$

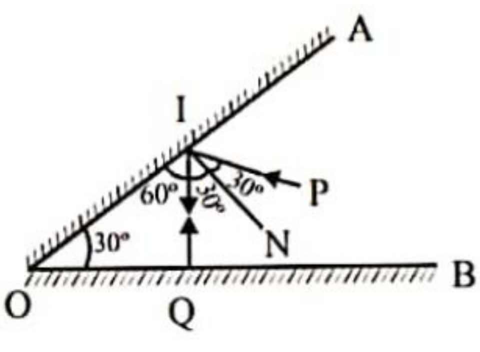
$$\frac{dv}{dt}=-\frac{v^2}{u^2}\frac{du}{dt}=-8\text{ m/s}$$

$$m_T=-\frac{v}{u}=\frac{f}{f-u}$$

$$h_i=\frac{f}{f-u}h_0$$

$$\frac{dh_i}{dt}=\frac{fh_0}{(f-u)^2}\frac{du}{dt}=\frac{-20\times 0.2\times 2}{100}=\frac{-2}{25}\text{ cm/s}$$

13. (A)

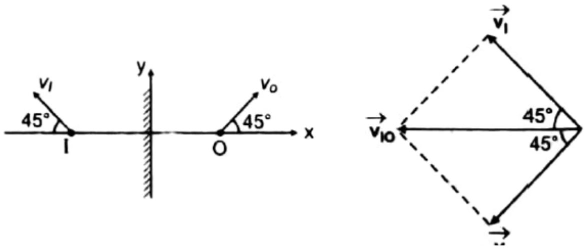


$$\angle i=\angle r=30^\circ$$

$$\therefore \quad \angle OIQ=60^\circ$$

$$\therefore \quad IOQ=90^\circ-60^\circ=30^\circ$$

14. (AC)



$|\vec{v}_0| = |\vec{v}_I|$

$= \sqrt{(2)^2 + (2)^2} = 2\sqrt{2} m/s$

Relative velocity of image with respect to object is in negative x-direction as shown in figure

15. (A)

In the first case. Let x be the distance of object from the mirror, Then

u = -x

v = +2x

f= -f

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Or $\frac{1}{2x} - \frac{1}{x} = -\frac{1}{f}$ or $x = \frac{1}{f}$

In the second case, let y be the distance of object from the mirror. Then

U = -y, v = -2y

And f = -f

So,

$\frac{1}{-2y} - \frac{1}{y} = -\frac{1}{f}$

$\therefore y = \frac{3}{2} f$

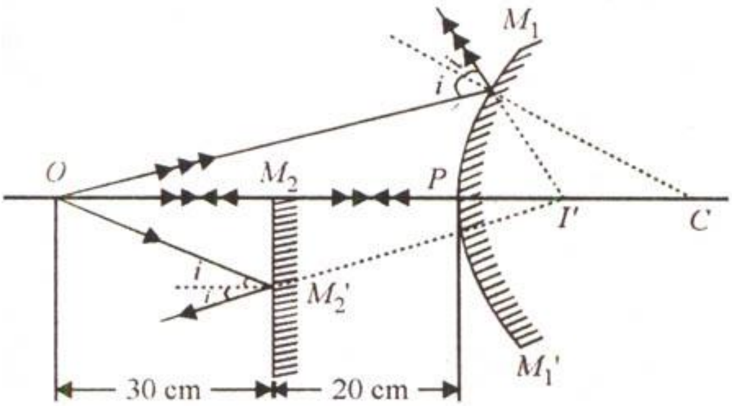
So object will have to be moved by a distance of y –x of f

16. (15)

$m = \frac{f}{f - u} \Rightarrow -3 = \frac{f}{f - (-20)} \Rightarrow f = -15cm$

17. (5)

In the case of a plane mirror distance of object from mirror = distance of image from mirror.



Therefore the distance of mage I' from convex mirror = PI' = 10 cm.

Now in the mirror formula $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$

Substituting u = PO = –50 cm, v = +10 cm

We have

$$\frac{1}{f}=\frac{1}{10}-\frac{1}{50}=\frac{5-1}{50}=\frac{4}{5}$$

$$f=\frac{50}{4}\text{cm}=12.5$$

Radius of curvature of convex mirror,

r = 2f = 2 × 12.5 = 25 cm

18. (9)

$$m=\frac{f}{f-u}$$

$$\Rightarrow -2=\frac{-6}{-6+u}$$

$$\Rightarrow -6+4=3$$

$$\Rightarrow u=9\text{cm}$$

19. (20)

$$v=\frac{uf}{uf}=\frac{(-10)(10)}{-10-10}=\frac{-100}{-20}=5\text{cm}$$

= 0.05 m

20. (4)

M = 2

$\therefore$ Velocity of image (perpendicular to principal axis)

=|m| (object speed)

=4mm/s

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TOPIC: ENERGETIC

SOLUTIONS

21. (B)

$$P_{ext} = 0; \omega = 0; \frac{1\times1}{T} = \frac{1\times2}{T'} \Rightarrow T' = 2T$$

$$T = \frac{1.0\times1}{0.083\times1} = 12.05K$$

$$\Delta U = 1\times\frac{3}{2}\times8.314\times12.05 = 150J$$

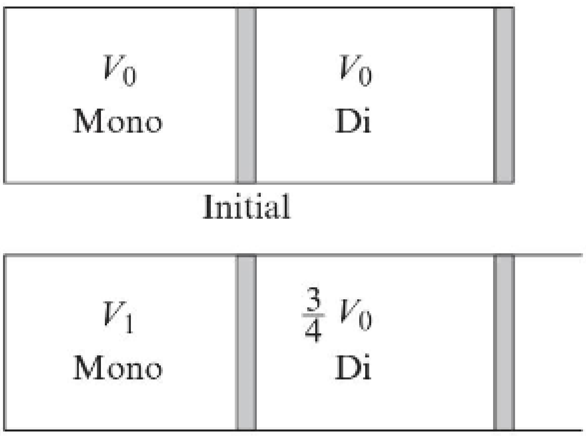
$$\Delta H = 150 + \underbrace{1\times100}_{\Downarrow \; \Delta(PV)} = 250J$$

22. (C)

$$q_{\upsilon} = x = 5\times C_{\upsilon}\times5;$$

$$q_p = \frac{2x}{5} + 2R\times5 = \frac{2x}{5} + 2\times5\times\frac{2x}{75} = \frac{10x}{15} = \frac{2x}{3} J$$

23. (C)



Monoatomic : $P_1V_0^{5/3} = P_2V_1^{5/2}$

Diatomic : $P_1.V_0^{7/5} = P_2.\left(\frac{3}{4}V_0\right)$

$$\therefore \frac{V_1}{V_0} = \left(\frac{3}{4}\right)^{\frac{21}{25}}$$

24. (A)

As $\Delta T = 0, \Delta U = 0$

25. (C)

$$2\times\frac{3}{2}R(T_f-300)+3\times3R(T_f-400)=12000$$

$$T_f=875K$$

$$P_f=\frac{n_{total}RT}{V}=\frac{5\times0.08\times875}{10}=35atm$$

26. (AC)

$$q=-2\Delta H$$

$$\Rightarrow nC\Delta T=-2\times nC_p\Delta T$$

$$\Rightarrow C=-2C_P$$

$$C=C_\upsilon+\frac{R}{1-n}$$

$$\frac{-2\gamma R}{\gamma-1}=\frac{R}{\gamma-1}\frac{R}{1-n}$$

$$\frac{2\gamma+1}{1-\gamma}=\frac{1}{n-1}$$

$$\Rightarrow n=1+\frac{\gamma-1}{2\gamma+1}=\frac{3\gamma}{2\gamma+1}$$

$$PV^n=\text{constant}$$

$$\Rightarrow PV^{\frac{3\gamma}{2\gamma+1}}=\text{constant}$$

$$\Rightarrow TV^{\left(\frac{3\gamma}{2\gamma+1}-1\right)}=\text{constant}$$

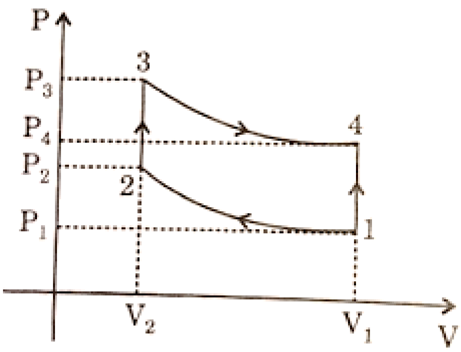
27. (ABD)

28. (AD)

29. (CD)

30. (AC)

31. (ABCD)



$$V_2=V_3$$

$$\omega_{1-2}=\frac{nR(T_2-T_1)}{\gamma-1}$$

$$\omega_{3-4} = \frac{nR(T_4 - T_3)}{\gamma - 1}$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}; P_1V_1^{\gamma-1} = P_2V_2^{\gamma-1}$$

$$\frac{P_3}{T_3} = \frac{P_2}{T_2}; T_3V_2^{\gamma-1} = T_4V_1^{\gamma-1}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{P_4}{P_3}; \frac{P_4}{P_1} = \frac{T_4}{T_1}; \frac{T_3}{T_2} = \frac{T_4}{T_1}$$

$$\Rightarrow \frac{T_3 - T_2}{T_2} = \frac{T_4 - T_1}{T_1}$$

$$\Rightarrow \frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2}$$

$$q \text{ supplied } = nC_\upsilon(T_3 - T_2) = \frac{nR}{\gamma - 1}(T_3 - T_2)$$

Work obtained = $\omega_{1-2} + \omega_{3-4}$

$$= \frac{nR}{\gamma - 1}[T_2 - T_1 + T_4 - T_3]$$

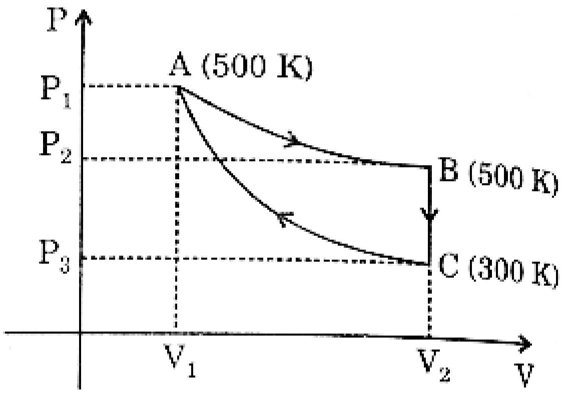
$$\eta = \frac{|\omega|}{q} = 1 - \left(\frac{T_4 - T_1}{T_3 - T_2}\right)$$

$$= 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$q_{4-1} = \frac{nR}{\gamma - 1}(T_1 - T_4)$$

$$\Rightarrow \eta = 1 - \left|\frac{q_{4-1}}{q_{2-3}}\right|$$

32. (ABCD)



$$V_1 = 8L$$

$$5 \times 8 = P_2 \times V_2$$

$$\frac{P_2}{500} = \frac{P_3}{300}$$

$$500 \times 8^{\left(\frac{5}{3} - 1\right)} = 300 \times (V_2)^{2/3}$$

$$8 = (0.6)^{1.5} \times V_2$$

$$\Rightarrow V_2 = \frac{0.6 \times 8}{0.3} = 16L$$

$$\Rightarrow P_2 = 2.5atm, P_3 = 1.5atm$$

$$\Rightarrow W = -1 \times 2 \times 500 \ln 2 + \frac{2 \times 200}{\frac{2}{3}} = 100cal$$

$$\Delta H = 1 \times \frac{5}{2} \times 2 \times \frac{200}{1000} = +1kcal$$

33. (ABC)

$$T \propto V^{-1/2} \text{ or } TV^{1/2} = \text{constant}$$

$$\gamma = \frac{3}{2}$$

Also $P.V. \propto V^{-1/2}$

$$P \propto V^{-3/2}$$

$$V \propto P^{-2/3}$$

Also $T \propto \frac{T^{-1/2}}{P^{-1/2}}$

$$P \propto T^3$$

34. (ACD)

$$\underset{\substack{1atm \\ 300K}}{\text{State A}} \xrightarrow{V=\text{constant}} \underset{\substack{2atm \\ 600K}}{\text{State B}} \xrightarrow{q=0} \underset{\substack{64atm \\ 2400K}}{\text{State C}}$$

$$\Delta H_{AB} = nC_p\Delta T$$

$$= 1 \times \frac{5}{2} \times 2 \times [600 - 300] = 1500cal$$

$$\Delta U_{AC} = nC_\upsilon\Delta T$$

$$= 1 \times \frac{3}{2} \times 2 \times [2400 - 300] = 6300cal$$

$$\Delta U_{BC} = W$$

$$= 1 \times \frac{3}{2} \times 2 \times [2400 - 600] = 5400cal$$

35. (BCD)

36. (131)

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\frac{273}{3_{2/3}} = T_2$$

$$\Rightarrow T_2 = 131K$$

37. (98)

38. (5)

Pressure, molar entropy, density, boiling point, molality are intensive properties.

39. (7700)

$T \propto V^3$ $\quad$ T = 300K

$\Rightarrow PV^{-2} = K$ $\quad$ n = 1 mole

$\gamma = \frac{5}{3}$

$C = \frac{3R}{2} + \frac{R}{3} = \frac{11R}{6}; T_f = 2400K$

$q = 1 \times \frac{11}{6} \times 2 \times 2100 = 7700 cal$

40. (25)

$0.1 \times 1 = P_2 \times 2$

$P_2 = \frac{0.1}{2}$

$T_1 = \frac{0.1 \times 1}{0.083 \times 0.1}$

$\frac{P_2}{T_1} = \frac{P_3}{T_2}$

$\Rightarrow T_2 = \left(\frac{0.1 \times 2}{0.1}\right) \times \frac{1}{0.083}$

$\Delta H = 0.1 \times \frac{5}{2} \times 8.3 \times \frac{1}{0.083} = 25J$

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TOPIC: CIRCLE

SOLUTIONS

41. (B)

$A\left(2+r_1\cos\theta, 2+r_1\sin\theta\right)$ and $B\left(2+r_2\cos\theta, 2+r_2\sin\theta\right)$

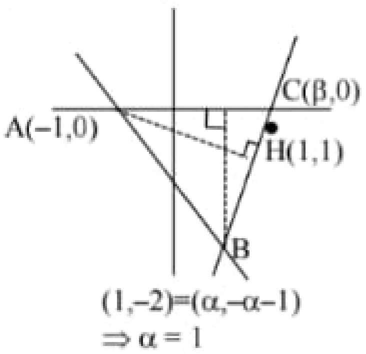
$\Rightarrow r_1, r_2$ are roots of equation $\left(2+r\cos\theta\right)^2+\left(2+r\sin\theta\right)^2=2$

$\Rightarrow r^2+4\left(\cos\theta+\sin\theta\right)+6=0$

$\Rightarrow r_1+r_2=-4\left(\cos\theta+\sin\theta\right)$ and $r_1r_2=6$ also $\dfrac{r_2}{r_1}=3$

$\Rightarrow \theta=\pi/4 \quad \Rightarrow y-2=1\left(x-2\right)$

42. (B)



One of the vertex is intersection of x-axis and $x+y+1=0 \Rightarrow A(-1, 0)$

Let vertex B be $(\alpha, -\alpha-1)$

Line $AC \perp BH \Rightarrow \alpha=1 \Rightarrow B(1, -2)$

Let vertex C be $(\beta, 0)$

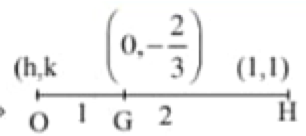
Line $AH \perp BC$

$m_{AH}.m_{BC}=-1$

$\dfrac{1}{2}\cdot\dfrac{2}{\beta-1}=-1 \Rightarrow \beta=0$

Centroid of ΔABC is $\left(0, -\dfrac{2}{3}\right)$

Now G(centroid) divides line joining circum centre (O) ortho centre (H) in the ratio 1:2

$\Rightarrow$ 

$2h+1=0 \qquad 2k+1=-z$

$h=-\frac{1}{2} \qquad k=-\frac{3}{2}$

$\Rightarrow$ circum centre is $\left(-\frac{1}{2},-\frac{3}{2}\right)$

Equation of circum circle is (passing through C(0, 0)) is

$x^2+y^2+x+3y=0$

43. (B)

Let two circles are $S=0$ and $S'=0$ having radius r_1 and r_2 respectively.

$\Rightarrow \frac{r_1}{\sqrt{S_1}}=\frac{r_2}{\sqrt{S_1^{'}}} \Rightarrow S_1^1 r_1^2 = S_1 r_2^2$

$\because r_1 \neq r_2$

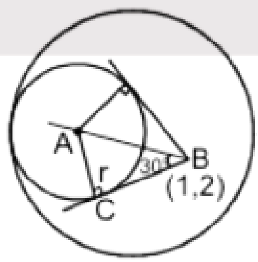
$\therefore$ Locus is a circle

44. (B)

Let centre $A(h,k)$ and r and R be radius of required and given circle

$$\sqrt{(h-1)^2+(k-2)^2}=R-r \qquad \text{.......(i)}$$

Now $\tan 30=\frac{r}{AB}$



$r=(R-r)\frac{1}{2} \quad \therefore r=\frac{R}{3}$ ....(ii)

By (i) & (ii)

$\sqrt{(h-1)^2+(k-2)^2}=R-\frac{R}{3}=\frac{2R}{3}$ & $R=3 \Rightarrow (x-1)^2+(y-2)^2=4$

45. (A)

Let required equation of circle is $x^2+y^2+2gx+2fy+c=0$ it cuts the circle $x^2+y^2-9=0$ orthogonally

$$\therefore 2g(0)+2f(0)=c-9 \Rightarrow c=9$$

It also touches straight line $\ell+my+n=0$

$\therefore \left|\frac{\ell(-g)+m(-f)+n}{\sqrt{\ell^2+m^2}}\right|=\sqrt{g^2+f^2-9}$

Locus of centre $(-g,-f)$ is $(\ell x+my+n)^2=(x^2+y^2-9)(\ell^2+m^2)$

46. (A,B)

Let $y = mx$ be the chord

Points of intersection of chord and circle are given by $\left(1+m^2\right)x^2-\left(3+4m\right)x-4=0$

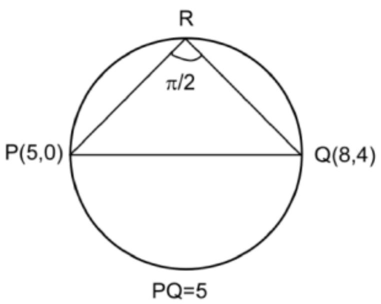
$\Rightarrow x_1+x_2=\frac{3+4m}{1+m^2}$ and $x_1x_2=\frac{-4}{1+m^2}$

As $x_2=-4x_1 \quad \Rightarrow 9+9m^2=9+16m^2+24m$

$\Rightarrow 7m^2+24m=0$

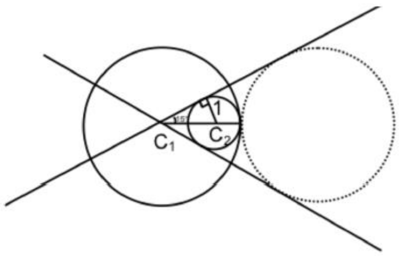
$\Rightarrow m=0,-\frac{24}{7}$

47. (A,B,C)



Maximum area of $\Delta PQR \qquad =\frac{1}{2}\times 5\times\frac{5}{2}=6.25\,\text{sq.units}$

48. (A,B)



$\cos ec15^\circ=\frac{C_1C_2}{1}$

$\Rightarrow r\pm 1=\text{cosec}15^\circ$

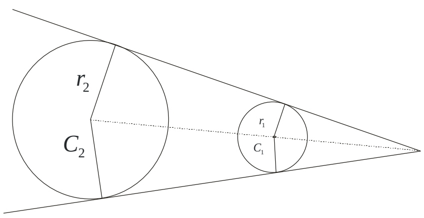
$\Rightarrow r=\sqrt{6}+\sqrt{2}\pm 1$

49. **(BC)**

Centres and radii of the given circles are

$C_1\left(-7,2\right): r_1=5\,and\,C_2(7,-2), r_2=9$

$\because \qquad C_1C_2=\sqrt{\left(14\right)^2+\left(4\right)^2}=\sqrt{212}>r_1+r_2$

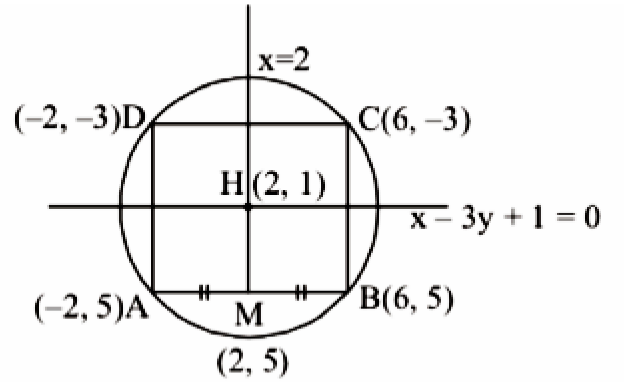


For common tangent length of perpendicular from centre on tangent = radius

Of centre $C_1(-7,2)$ *and* $r_1=5,$ then (B) and (C) are correct.

50. (A,B,C)

Area of rectangle $=64\,\text{sq units}$



Let $D(x,y)$

$\therefore \frac{x+6}{2}=2$ and $\frac{y+5}{2}=1$

$\therefore D(-2,-3)$, Similarly $C(6,-3)$

51. (B,C)

$4\ell^2-5m^2+6\ell+1=0$

$\Rightarrow (3\ell+1)^2=5(\ell^2+m^2)$

$\Rightarrow \left|\frac{3\ell+0.m+1}{\sqrt{\ell^2+m^2}}\right|=\sqrt{5}$

Hence centre (3,0), radius $=\sqrt{5}$

52. (A,C)

We have $(x-5)^2+(y+8)^2=25+64+r^2-89$

and $(x+3)^2+(y-7)^2=(4)^2$

$\Rightarrow (x-5)^2+(y+8)^2=r^2$

$\Rightarrow \sqrt{64+225}=\sqrt{289}=17=$ distance between their centres

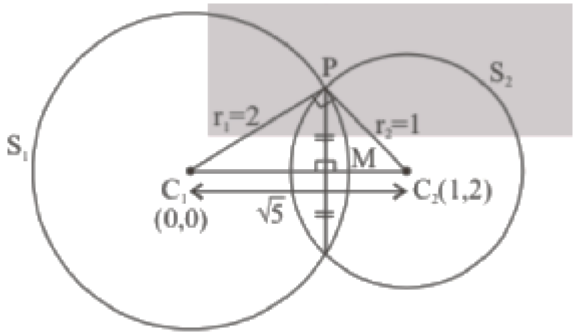
Now, $|r-4|<17<|r|+4$ $\quad |r|+4>17$ $\quad \Rightarrow r>13$ distance between their centres

And $-17<|r|-4<17$ $\quad \Rightarrow -13<|r|<21$

Hence $13<|r|<21$

$\therefore$ Possible values of $|r|$ can be 14,15,,16,17,18,19,20

53. (A,C,D)



Clearly $PC_1^2 + PC_2^2 = (C_1C_2)^2$

$\Rightarrow$ Two circles intersect orthogonally.

Equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow -x + 2y + 4 = 0$

Now $C_1M = \frac{4}{\sqrt{5}}$

$\therefore$ Length of common chord $= 2\sqrt{4 - \frac{16}{5}} = \frac{4}{\sqrt{5}}$

Clearly $S_1(2,3) > 0$ and $S_2(2,3) > 0$

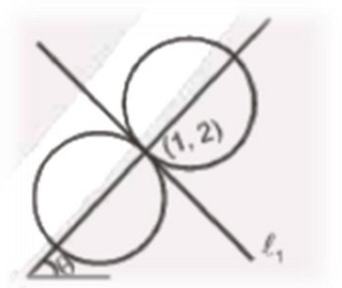
54. (A,B)

$\ell_1 \equiv 4x + 3y = 10$; $\ell_2 \equiv 3x - 4y = -5$

Let θ be the inclination of ℓ_2

$\therefore \tan\theta = \frac{3}{4}$

$\therefore$ equation of ℓ_2 in parametric form $\frac{x-1}{4/5} = \frac{y-2}{3/5} = \pm 5$



Co-ordinates of centres are $(5,5), (-3,-1)$

55. (A,C)

$OP = 5\sqrt{2}\sec\theta, OP_1 = 5\sqrt{2}\operatorname{cosec}\theta$

Area $(\Delta PP_1P_2) = \frac{100}{\sin 2\theta}$, area $(\Delta PP_1P_2)_{min} = 100$

$\Rightarrow \theta = \pi/4 \Rightarrow OP = 10$

$\Rightarrow P = (10,0), (-10,0)$

Hence (A), (C) are correct

56. (4)

$$\frac{1}{2}r_1r_2=\frac{1}{2}\times\left(\frac{\ell}{2}\right)\times\sqrt{r_1^2+r_2^2}$$ {where ℓ is length of common chord)

$$\Rightarrow \ell=\sqrt{2}\sqrt{\frac{2r_1^2r_2^2}{r_1^2+r_2^2}}\Rightarrow k=\sqrt{2}\Rightarrow k^2=2$$

57. (7)

$$x^2+y^2-5x+2y-5=0$$

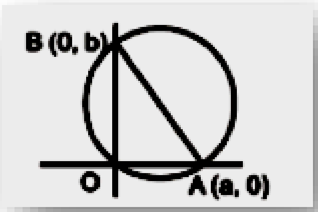
$$\Rightarrow\left(x-\frac{5}{2}\right)^2+\left(y+1\right)^2-5-\frac{25}{4}-1=0$$

$$\Rightarrow\left(x-\frac{5}{2}\right)^2+\left(y+1\right)^2=\frac{49}{4}$$

$\Rightarrow$ So the axes are shifted to $\left(\frac{5}{2},-1\right)$

New equation of circle must be $x^2+y^2=\frac{49}{4}$

58. (2.00)



Equation of circum circle of triangle OAB $\quad x^2+y^2-ax-by=0$

Equation of tangent at origin $ax+by=0$

$d_1=\frac{|a^2|}{\sqrt{a^2+b^2}}$ and $d_2=\frac{|b^2|}{\sqrt{a^2+b^2}}$

$\Rightarrow d_1+d_2=\sqrt{a^2+b^2}$ =diameter

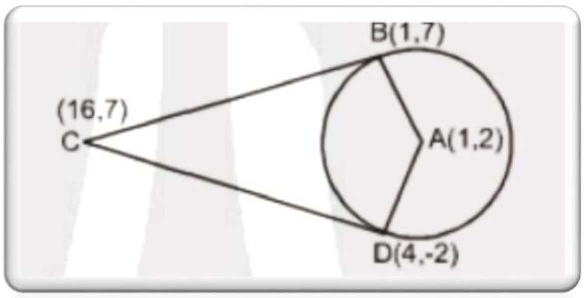
59. (75.00)

Given circle $x^2+y^2-2x-4y-20=0$

Tangents at $(1,7)$ is

$$x+7y-(x+1)-2(y+7)-20=0$$

$$5y-35=0\Rightarrow y=7$$



At $D(4,-2)$

$4x-2y-(x+4)-2(y-2)-20=0$

$3x-4y=20$

Hence $c(16,7)$

Area of quadrilateral $ABCD = AB \times BC = 5 \times 15 = 75$ square units.

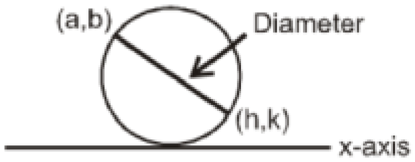
60. (4.00)

Equation of circle whose diameter's end point are (a,b) and (h,k)

$(x-a)(x-h)+(y-b)(y-k)=0$

$x^2+y^2-x(a+h)-y(b+k)+ah+bk=0$

it touches x-axis



Hence $g^2=c \Rightarrow \left(\frac{a+h}{2}\right)^2 = ah+bk \Rightarrow (h-a)^2 = 4bk$

$\therefore$ Locus of (h,k) is $(x-a)^2 = 4by$