IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 05/03/23**

TOPIC: CENTRE OF MASS & COLLISION

Solutions

1. (B)

Let Δx_1 be the shift in position of C.M. of ball and Δx_2 be the shift in position of C.M. of shell. As there is no external force acting on the system in horizontal direction, so there is no shift in position of C.M. of the system (ball + shell). Thus we have,

$$\Delta x_{cm} = 0 = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2}$$

From the figure $\Delta x_1 = R - \Delta x_2$

$$\therefore \quad 0 = \frac{M(R - \Delta x_2) + M(-\Delta x_2)}{M + m}$$

Or $\Delta x_2 = \frac{R}{2}$.

This is the displacement of the shell when ball moves down to its lowest position. When ball moves to the right of the centre of the shell, the shell further displaces by R/2.

Therefore the total displacement of the shell

$$= \frac{R}{2} + \frac{R}{2} = R$$ **Ans.**

2. (D)

Total distance moved by the bodies,

$$x_1 + x_2 = 12R - 3R = 9R \quad \text{........ (i)}$$

Also $Mx_1 = 5Mx_2$ .... (ii)

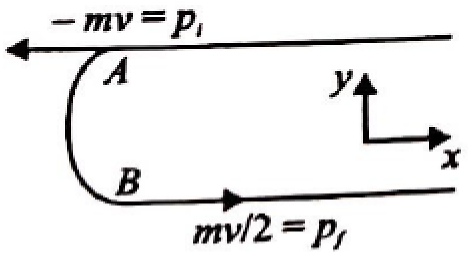
After solving above equations, we get

$x_1 = 7.5R$

$x_2 = 1.5R$.

3. (B)

The momentum of the bead at *A* is $p_i = -mv\hat{i}$



The momentum of the bead at B is $\vec{p}_f = \left(\frac{mv}{2}\right)\hat{i}$

Therefore, the magnitude of the change in momentum between A and B is

$$\Delta p = \left|\vec{p}_f - \vec{p}_i\right| = \left(\frac{3}{2}\right)mv$$

Average force exerted by the bead on the wire is

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{3mv}{2T}$$

4. (C)

$$\Delta P = 2mv\sin 60^{o}$$

$$F = \frac{\Delta P}{\Delta t} = \frac{2mv \times \sqrt{3}/2}{2\times 10^{-3}}$$

$$= \frac{2\times 0.1\times 5\times \sqrt{3}/2}{2\times 10^{-3}}$$

$$= 250\sqrt{3}\,N.$$

5. (C)

Range, $R = \frac{2u_x u_y}{g} = \frac{2\left(u\cos\theta\right)\left(eu\sin\theta\right)}{2}$

$$= \frac{eu^2\sin 2\theta}{2g}$$

6. (BD)

The total momentum and total energy of (earth + ball) system remain conserved.

7. (AD)

The initial momentum of the bomb is zero, and so after explosion, it must be zero. Some internal energy will convert into external K.E.

8. (ABD)

System has non–zero initial momentum, so it must be after collision. Therefore both the bodies can not be at rest after collision.

9. (ABD)

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = \frac{2}{4-0}$$

Now $3\times 4 - 2\times 0 = 3v_1 + 2v_2$

And, $0.5 = -\left[\frac{v_2 - v_1}{0-4}\right]$

After simplifying above equations, we get

$$v_1 = 1.5\,m/s.$$

Thus, $J = m_1\left(v_1 - u_1\right) = 3\left(4 - 1.6\right)$

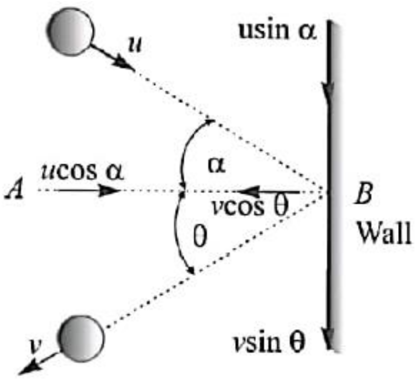
$$= 7.2\,N - s.$$

10. (ABD)

Since the line of collision is AB, therefore the velocity of the ball along a line parallel (normal to AB) to wall does not change.

$\therefore \quad u\sin\alpha = v\sin\theta$ ……… (i)

And $\quad e = -\dfrac{(-v\cos\theta - 0)}{(u\cos\alpha - 0)}$ ………. (ii)



Where u and v are the velocities of the ball before and after collision practically be zero.

Solving above equations, we get

$e = \tan\alpha / \tan\theta$

11. (CD)

COLM : $mu + 0 = (m+M)v \Rightarrow v = \left(\dfrac{m}{M+m}\right)u$

KE after collision

$= \dfrac{1}{2}(m+M)\times\left(\dfrac{m}{m+M}\right)^2 u^2 = \dfrac{m^2v^2}{2(m+M)}$

12. (C)

Let x = displacement of ring to the left.

$\Rightarrow \Delta x_{cm} = \dfrac{2mx + m(x + L - L\cos\theta)}{3m} = 0$

$\Rightarrow x = -\dfrac{L}{3}(1-\cos\theta)$

13. (A)

$v_1 = \dfrac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \left(\dfrac{2m_2}{m_1+m_2}\right)u_2$

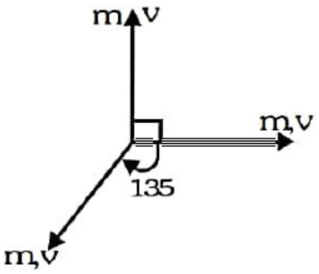
$v_2 = \dfrac{2m_1}{(m_1+m_2)}u_1 + \left(\dfrac{m_2 - m_1}{m_1+m_2}\right)u_2$

For $C : v_C = \dfrac{2mu}{3m} = \dfrac{2}{3}u$

14. (A)

$$\text{COLM} \Rightarrow mv\hat{i} + mv\hat{j} + m\left(\frac{-v}{\sqrt{2}}\hat{i} - \frac{v}{\sqrt{2}}\hat{j}\right) + m\vec{v}_4 = 0$$

$$\vec{v}_4 = -v\left(1 - \frac{1}{\sqrt{2}}\right)\hat{i} - v\left(1 - \frac{1}{\sqrt{2}}\right)\hat{j}$$



Total energy released

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}m\left[v^2\left(1 - \frac{1}{\sqrt{2}}\right)^2 \times 2\right]$$

$$= mv^2\left(3 - \sqrt{2}\right)$$

15. (D)

Mass of removed disc, $m' = \frac{m}{\pi r^2} \times \pi\left(\frac{r}{2}\right)^2 = \frac{m}{4}$

Thus, $x_{cm} = \dfrac{m \times 0 - \frac{m}{4}\left(\frac{r}{2}\right)}{\left(m - \frac{m}{4}\right)} = \frac{r}{6}$

16. (10)

COME:– $MV + m\left(v\cos 60 - V\right) = 0 \Rightarrow v = 10\,m/s$

17. (0.2)

$$\Delta\bar{x} = \frac{m\Delta x_1 + M\Delta x_2}{m + M} = 0$$

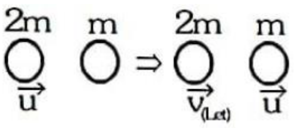
$$\Rightarrow \frac{1\left(\ell\sin 30^\circ + \ell\sin 30^\circ - x\right) - 4x}{1 + 4} = 0$$

$\Rightarrow$ Displacement of bar $= x = 0.2$

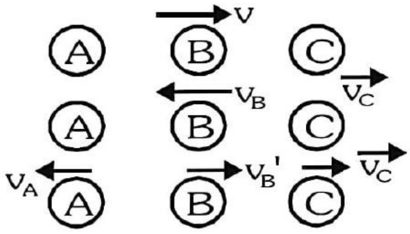
18. (0.5)

$\text{COLM} \Rightarrow 2mu + 0 = 2mv + mu \qquad \Rightarrow v = \frac{u}{2}$

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\left(\frac{u - u/2}{0 - u}\right) = \frac{1}{2}$$



19. (2)



For collision between B and C:

$$v_B = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \frac{m_2(1+e)}{(m_1+m_2)}u_2$$

$$= \left(\frac{m - 4m}{5m}\right)v + 0 \qquad = -3/5\,v$$

$$v_C = \frac{m_1(1+e)}{(m_1+m_2)}u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)v_2 = \frac{m(1+1)}{4m}v + 0 = \frac{v}{2}$$

For collision between A and B:

$$v_A = 0 + \frac{m(1+1)}{5m} \times \left(\frac{3v}{5}\right) = \frac{6}{25}v$$

$$v_B' = 0 + \left(\frac{m - 4m}{5m}\right)\left(-\frac{3v}{5}\right) = -\frac{9v}{25}$$

$\because v_B' < v_C$

$\therefore$ B will not collide with C.

Therefore there will be only two collisions.

20. (30)

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) \Rightarrow 1 = -\frac{5 - v_1}{5 - (-10)} = \frac{v_1 - 5}{15}$$

$\Rightarrow v_1 = 20\,m/s$

$\therefore$ Impulse on ball $= m(\vec{v} - \vec{u}) = 1\left[20 - (10)\right] = 30\,N - s$

IIT – JEE: 2024 | **TW TEST (ADV)** | **DATE: 05/03/23**

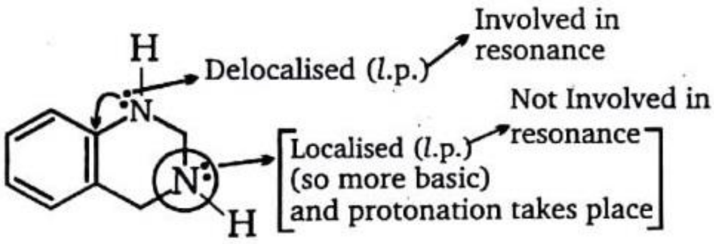
TOPIC: GOC

Solutions

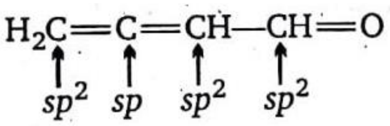
21. (B)

Acidic strength $\propto -I; -H; -M \propto \dfrac{1}{+I; +H; +R}$

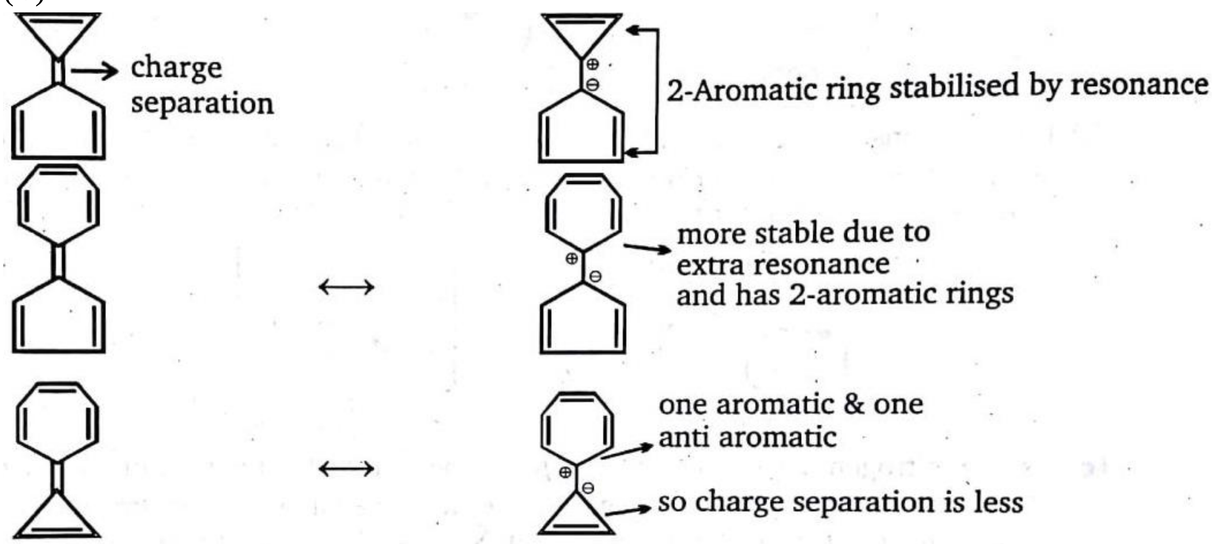
22. (B)



23. (C)



24. (C)



So bond rotation energy C > A > B

So order of rotation is B > A > C

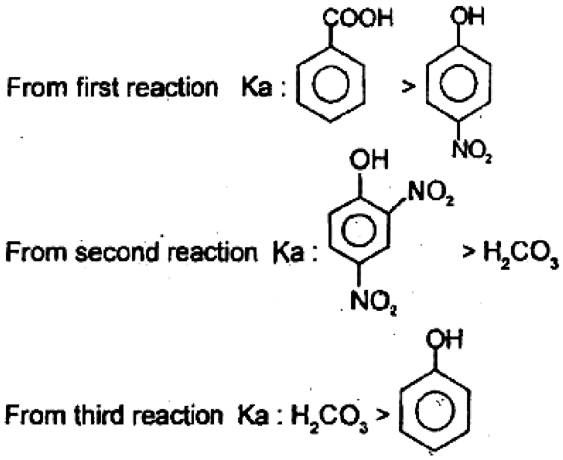
25. (B)

$\propto$ 'H' atoms w.r.t. C=C bond take part in hyper conjugation.

26. (ABD)

Most Stable resonating structure contribute maximum & least stable resonating structure Contribute minimum in resonance hybrid.

27. (B,C,D)

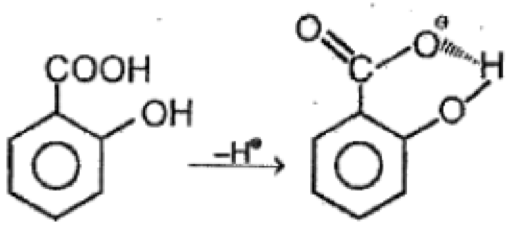


Since a strong acid displaces a weak acid from its salt and forms its own salt.

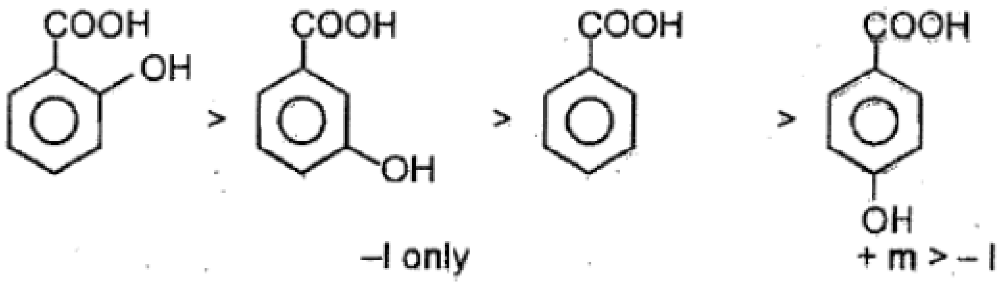
28. (ABD)

Due to steric hinderance of three groups $-NO_2$ group to out of the plane with benzene ring and so conjugation of $-NO_2$ group with benzene is slightly diminished. So bond length of $C_1-N\&C_5-N$ increases

29. (ABD)

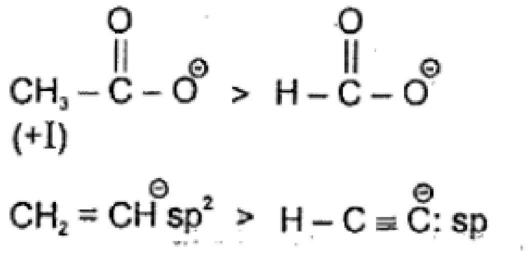


Anion is ore stable due to H-bonding ∴ shows ortho effect.



(D) Resonating structures are hypothetical

30. (ACD)



31. (ABD)

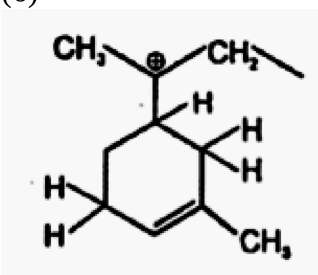
32. (ABC)

33. (BC)

34. (AB)

35. (BD)

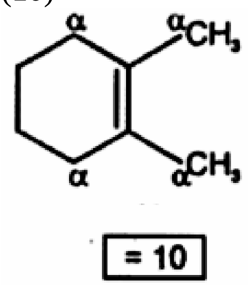
36. (6)



Total number of hydrogen involved in hyperconjugation with carbocation $= 6$

37. (3)
Compound has three acidic hydrogen.

38. (10)



39. (4)

40. (6)
1^{st}, 3^{rd}, 5^{th}, 6^{th}, 8^{th}, 9^{th}

IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 05/03/23**

TOPIC: FUNCTION

Solutions

41. **(D)**
We know that for a relation to be function every element of first set should be associated with one and only one element of second set but elements of first set can have same f-image in second set which is given in (D).

42. **(D)**
$$f(y) = \frac{y}{y-1} = \frac{(x-1)/x}{\frac{x-1}{x}-1} = \frac{x-1}{x-1-x} = 1 - x.$$

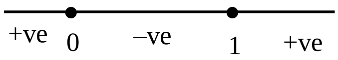
43. **(A)**
Domain = $\{x;\ x \in R;\ x^3 - x \neq 0\}$
$= R - \{-1, 0, 1\}$

44. **(C)**
$[x]$ is an integer, $\cos(-x) = \cos x$ and
$\cos\left(\frac{\pi}{2}\right) = 0$, $\cos 2\left(\frac{\pi}{2}\right) = -1$.
$\cos 0\left(\frac{\pi}{2}\right) = 1$, $\cos 3\left(\frac{\pi}{2}\right) = 0$,.....
Hence range = $\{-1, 0, 1\}$

45. **(B)**
$fog = f\left[g\left(-\frac{1}{4}\right)\right] f(-1) = 1$
and $gof\left(-\frac{1}{4}\right) = g\left[f\left(-\frac{1}{4}\right)\right] = g\left(\frac{1}{4}\right) = [1/4] = 0$
Required value = 1 + 0 = 1.

46. **(A, B, D)**
$f(x) = \sqrt{x^2 - |x|}$, $g(x) = \frac{1}{\sqrt{9 - x^2}}$
f(x) to be defined if
$x^2 - |x| \geq 0 \Rightarrow x^2 \geq |x|$
$\Rightarrow x^2 - x \geq 0$

$\Rightarrow\ x(x-1) \ge 0$

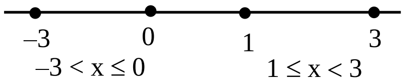


i.e. $x \le 0$ or $x \ge 1$

g(x) to be defined if $9 - x^2 > 0$ and $9 - x^2 \ne 0$

$\Rightarrow x^2 < 9$ and $x^2 \ne 9$

$\Rightarrow -3 < x < 3$ and $x \ne \pm 3$



Required Domain $\in (-3, 0] \cup [1, 3)$

$\therefore$ Option (A), (B) and (D) are correct answers.

47. **(A, B, C)**

Given $A = R - \{2\}$

$B = R - \{1\}$

$f : A \to B$; $f(x) = \dfrac{x-3}{x-2}$

For one-one : $f(x) = f(y)$

$\Rightarrow \dfrac{x-3}{x-2} = \dfrac{y-3}{y-2}$

$\Rightarrow (x-3)(y-2) = (y-3)(x-2)$

$\Rightarrow xy - 3y - 2x + 6 = xy - 3x - 2y + 6$

$\Rightarrow\ x = y$

i.e. f(x) is one-one mapping

For onto : $y = \dfrac{x-3}{x-2} \Rightarrow x - 3 = yx - 2y$

$\Rightarrow 2y - 3 = x(y-1)$

$\Rightarrow x = \dfrac{2y-3}{y-1}$

$$f(x) = \frac{x-3}{x-2} = \frac{\frac{2y-3}{y-1} - 3}{\frac{2y-3}{y-1} - 2} = \frac{2y-3-3y+3}{2y-3-2y+2}$$

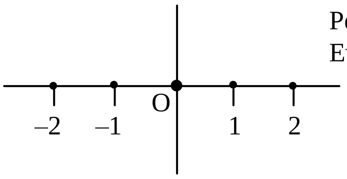
$f(x) = y$

i.e, f(x) is onto mapping

$\therefore$ Option (A), (B) and (C) are correct Answer.

48. **(A, B, C, D)**

$$f(x) = \frac{0}{\{x\}} = \begin{cases} \text{not defined} & x \in I \\ 0 & x \notin I \end{cases}$$



Period =1
Even function

$$g(x) = \underbrace{\operatorname{sgn}\left(\underbrace{\text{signum}\frac{\{x\}}{\sqrt{\{x\}}}}_{1}\right)}_{1} - 1$$

= 0 → same domain & range

49. **(A, B)**

f : [–1, 1] ⟶ [0, 2]

f (x) = ax+b ⟶ onto ∴ range = codomain

Checking options

50. **(ABD)**

$f(x) = |\cos x| + |\sec x|$.

51. **(B, C, D)**

f : [–1, 1] ⟶ [–1, 1]

Checking equations

52. **(ACD)**

53. **(ABCD)**

f(x) = x + 1

$g(x) = x + \frac{1}{x}$ for x > 0

one-one function

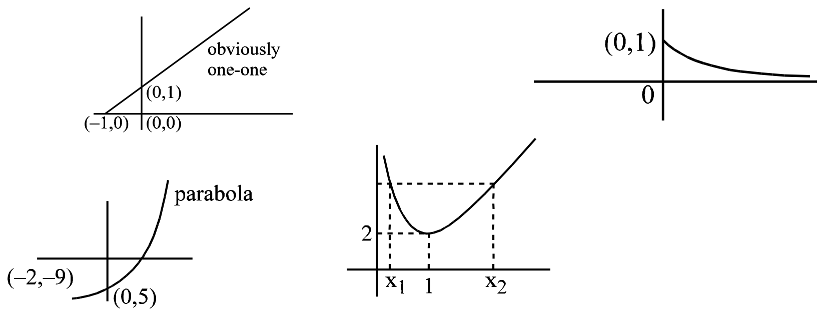
$h(x) = x^2 + 4x - 5$ for x > 0

$= (x + 2)^2 - 9$

Hence One - one

$f(x) = e^{-x}$ for $x \geq 0$

Obviously one-one.



54. **(BC)**

55. **(ACD)**

(A) $\operatorname{sgn}(|x| + 1) = 1 \ \forall \ x \in R$.

(B) $\sin^2(\ln x) + \cos^2(\ln x) = 1 \ \forall \ x \in R^+$

(C) $\frac{2}{\pi}\left(\sin^{-1}\{x\} + \cos^{-1}\{x\}\right) = 1 \ \forall \ x \in R$

(D) $\sec^2[\{x\}] - \tan^2\{[x]\} = \sec^2 0^\circ - \tan^2 0^\circ = 1 \ \forall \ x \in R$.

56. **(1)**

Draw graph of $y = 0$ and $y = \ln x$

57. **(6)**

$gof(x) = [x^2 + 2] = 3$

$fog(x) = [x]^2 + 2 = 3$

58. **(1)**

$\left|\sin\frac{x}{2}\right| + \cos\frac{x}{2} = \sqrt{1+|\sin x|}$

Hence, range is 1.

59. **(2)**

$f(x) - f(-x) = \frac{2}{x}$

Put $x = 2$

$\therefore\ f(2) - f(-2) = 1$

$\Rightarrow\ f(-2) = 2$

60. **(4)**

$f(x) = 0\ \forall\ x \in R$

Now draw graph of $g(x)$ to find the solutions.