IIT – JEE: 2023 **TW TEST (ADV)** **DATE: 12/03/23**

TOPIC: HEAT & THERMODYNAMICS

SOLUTIONS

1. (C)

Specific heat of ice is $2.1 J/g^{\circ}C$. Total heat released by water is $10\times4.2\times50=2100J$. Total heat absorbed by ice from $-20^{\circ}C$ to $0^{\circ}C=10\times2.1\times20=420J$

$\Delta\theta=2100-420=mL$ melted ice, $m=\left(2100-420\right)/336\approx5gm$. Hence, at equilibrium , total water is 15 gm, total ice is 5gm.

2. (B)

At temperature t the heat energy required to raise temperature of unit mass by dt is

$dq=at^3\times1\times dt$

So heat required to raise temperature from 1K to 2K is

$$\int_0^Q dq=\int_1^2 at^3dt\Rightarrow Q=a\left.\frac{t^4}{4}\right|_1^2=a\left(16-1\right)$$

$$\Rightarrow Q=15a/4$$

3. (C)

Since specific heat of lead is given in Joules, hence use W = Q instead of W = JQ.

So, $\frac{1}{2}\times\left(\frac{1}{2}mv^2\right)=m.c.\Delta\theta\ \Rightarrow\ \Delta\theta=\frac{v^2}{4c}=\frac{(300)^2}{4\times150}=150^{o}\,C$

4. (D)

$\gamma_{ac}=\gamma_l-\gamma_c$

$\therefore C=\gamma_l-\gamma_c$ ……(1)

$\gamma_{as}=\gamma_l-\gamma_s$

$\therefore S=\gamma_l-\gamma_s$ ………(2)

From (1) and (2)

$S+\gamma_s=C+\gamma_c$

$\gamma_s=C-S+\gamma_c$

$3\alpha_s=C-S+\gamma_c$

$$\Rightarrow\alpha_s=\frac{C-S+\gamma_c}{3}$$

5. (D)

$\because$ $dl=\alpha l_0 dT$

$$\therefore \quad \Delta l = \int dl = \int_{T_1}^{T_2} \left(aT - bT^2\right) l_0 dT$$

$$= l_0 \left[\frac{a}{2}\left(T_2^2 - T_1^2\right) - \frac{b}{3}\left(T_2^3 - T_1^3\right) \right]$$

$$= l_0 \left[\frac{3}{2} a T_1^2 - \frac{7b}{3} T_1^3 \right]$$

6. (B)

We just need to insulate the system, and balance the heat. So this experiment is not dependent on time taken to reach equilibrium. If system is insulated then, heat lost by copper = heat gain by beaker and water.

7. (A,C,D)

$$\frac{\Delta A}{A} \times 100 = 2\left(\frac{\Delta l}{l}\right) \times 100$$

% increase in area = $2 \times 0.2 = 0.4\%$

$$\frac{\Delta V}{V} \times 100 = 3 \times 0.2 = 0.6\%$$

Since

$$\Delta l = l\alpha\Delta T$$

$$\frac{\Delta l}{l} \times 100 = \alpha\Delta T \times 100 = 0.2$$

$$\alpha = 0.25 \times 10^{-4} / {}^{o}C$$

8. (B, C)

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l_0 + \alpha l_0 \Delta\theta_0}{g}}$$

$$= T_0\left(1 + \frac{1}{2}\alpha\Delta\theta\right)$$

At 30^{o}C, fraction loss of time = $\dfrac{T_{30^o} - T_{20^o}}{T_{20^o}}$

$$= 5\alpha = 5 \times 19 \times 10^{-6}$$

Time lost in 24 h = $86400 \times 95 \times 10^{-6} = 8.2\,s$

On a cold day at 10^{o}C, fraction gain of time

$$= \frac{T_{10^o} - T_{20^o}}{T_{20^o}} = -5\alpha$$

Time gains in 24 h = 8. 2 s

9. (B, D)

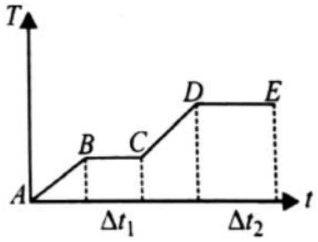
$$R = \frac{t}{\left(\alpha_B - \alpha_C\right)\Delta T}$$

10. (Bonus)

$$dQ = mCdT$$

or $\quad Hdt = mCdT$

$\therefore \qquad C = \left(\frac{H}{m}\right)\frac{1}{(dT/dt)}$



As $\frac{dT}{dt}$ of CD is smaller, so $C_{liquid} > C_{solid}$

$Q_1 = H(\Delta t_1)$ and $Q_2 = H(\Delta t_2)$

AS $\Delta t_2 > \Delta t_1$, $\therefore Q_2 > Q_1$

11. (B)
All parallel faces will expand by same amount. Therefore, there will not be any distortion in shape.
$\beta_{BCGH} = \alpha_y + \alpha_z = 5\times10^{-5} / {}^{o}C$ [Refer to example 9]
Similarly, $\gamma = \alpha_x + \alpha_y + \alpha_z = 6\times10^{-5} / {}^{o}C$

12. (A, B)
Let M and m be masses of water and ice, initially at temperature of 40^{o}C and –40^{o}C, respectively.
To attain a temperature of 0^{o}C, the heat lost by the water would be 4 units.
(Mg) (1 cal/g-^{o}C) (40 – 0^{o}C) = 4 units

or $\quad M \text{ cal} = \frac{1}{10}$ units ….. (i)

Similarly, to attain a temperature of 0^{o}C, the heat gained by ice would be 1 unit.

$(mg)\left(\frac{1}{2}\text{cal/g}^{o}\text{C}\right)\left(0+40^{o}\text{ C}\right) = 1 \text{ unit}$

or $\quad m \text{ cal} = \frac{1}{20}$ unit ….. (ii)

From (i) and (ii), $\frac{M}{m} = 2$

Heat required for the complete ice to melt at 0^{o}C will be (mg) (80 cal/g) = 80 m cal = 4 unit.
By the time the temperature of the entire water had dropped to 0^{o}C, the amount of heat ejected would be 4 unit, out of which 1 unit would be consumed by the ice to get heated up from –40^{o}C to 0^{o}C and the remaining heat of 3 units, would be consumed to melt.
Since of total mass of ice require a heat of 4 unit to melt completely, a heat of 3 units will be able to melt only (3/4)th of the ice.

13. (A, D)
$\Delta V_L = \Delta V_V$

$\gamma_L V_L = \gamma_V V_V$ or $\frac{\gamma_L}{\gamma_V} = \frac{V_V}{V_L}$

$V_V > V_L \Rightarrow \gamma_L > \gamma_V$

14. (A, B)
When the steam at 100^{o}C transforms into water at 100^{o}C, it releases heat given by
$Q_1 = 100\times540 = 54000 \text{ cal}$
200 g ice, for melting at 0^{o}C needs an amount of heat given by

$Q_2 = 200 \times 80 = 16000\,\text{cal}$.

Water formed at 0°C, if heated to 100°, will need a heat given by

$Q_3 = 200 \times 1 \times 100 = 20000\,\text{cal}$

200g water at 55°C, if heated to 100°C, will need a heat given by

$Q_4 = 200 \times 1 \times 45 = 9000\,\text{cal}$

$(Q_2 + Q_3 + Q_4) < Q_1$

This implies that the entire steam will not condense, and the mixture will attain a temperature of 100°C.

Let mass of steam condensed by m

$mL_v = Q_2 + Q_3 + Q_4$

$m \times 540 = 45000 \Rightarrow m = 83.3g$

$\therefore$ Mass of water in the final mixture = 200+ 200+83.3

= 483.3 g

15. (C, D)

Thermal expansion is like photographic enlargement.

16. (8)

$W_0 = mg = 46\ g\ wt,\ \theta_1 = 27^\circ\ C$

$W_1 = 30\ g = W_0 - B_1$

$\Rightarrow\ B_1 = (46 - 30)\, g$

$\Rightarrow\ B_1 = 16\,\text{g-wt} = V_1 \rho_1 g$

$\theta_2 = 42^\circ\ C$

$W_2 = 30.5\ g = W_0 - B_2$

$\Rightarrow\ B_2 = 15.5\ g = V_2 \rho_2 g$

$\therefore\ \frac{B_2}{B_1} = \frac{V_2 \rho_2}{V_1 \rho_1}$

$\frac{15.5}{16} = (1 + 3\alpha_S \times 15) \times \frac{1.2}{1.24}$

$\alpha_S = \left[\left(\frac{15.5}{16} \times \frac{1.24}{1.2}\right) - 1\right] \times \frac{1}{45}$

$\alpha_S = 2.31 \times 10^{-5}\ /\ ^\circ C = \frac{1}{43200}\ /\ ^\circ C$

17. (12)

Heat released by steam = heat absorbed by water

$m_1 L + m_1 \times S(100 - 90) = m_2 \times S(90 - 24)$

$540 m_1 + 10 m_1 = 66\, m_2$

$\Rightarrow\ m_1 = \frac{66 \times 100}{550} = 12g$

18. (15)

$V_C - V_{Hg} = V'_C = V'_{Hg} =$ Volume of air

$\Rightarrow\ V'_C = V_C (1 + 3\alpha_S \Delta\theta)$

$V'_{Hg} = V_{Hg} (1 + \gamma_L \Delta\theta)$

So, $V_C \times 3\alpha_S = V_{Hg} \times \gamma_L$

$$V_{Hg} = \frac{1\times 3\times 9\times 10^{-6}}{1.8\times 10^{-4}} = 0.15\,L$$

19. (0)

(Final mixture is 125g ice and 275 g water at 0^{o}C.)

Say final mixture is 400 g water at 0^{o}C.

$Q_2 = (200)(1)(50) = 10,000$ cal

$Q_1 = (200)(0.5)(40) + (200)(80)$

$= 20,000$ cal

$Q = Q_2 - Q_1 = 10,000 - 20,000$

$= -10,000$ cal

Since, Q < 0

$\therefore \ |Q| = mL_F$

$10,000 = m\times 80$

$m = 125$ g

So final mixture is 125 g ice and 275 g water at 0^{o}C.

20. (90)

Rate of cooling during solid phase: $d\theta, \text{with}\, d\theta/dt$ where s is the specific heat, m is the mass of the substance and $d\theta$ is the temp difference. Rate of cooling during phase change: $\frac{mL}{30}$ where L is the latent heat of fusion.

Given the rate of cooling is same for both

$\therefore \quad d\theta, \text{with}\, d\theta/dt$

$\therefore \frac{s}{L} = \frac{1}{3\times 30} = \frac{1}{90}K^{-1}$

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TOPIC: IONIC EQUILIBIRUM

SOLUTIONS

21. (D)

$$pOH_1 = pK_b + \log\left(\frac{0.1}{0.1}\right); \left(K_b = 2\times10^{-5}\right)$$

$$pOH_2 = pK_b + \log\left(\frac{9+1}{9-1}\right)$$

$$\Delta pH = \log\left(\frac{10}{8}\right)$$

$$\%\text{ change } = \frac{\Delta pH}{14-[pK_b]}\times100$$

$$= \frac{\log10 - 3\log2}{9.3} = \frac{10}{9.3}\%\downarrow$$

22. (A)

23. (A)

24. (D)

25. (D)

26. (ABC)

27. (AD)

28. (ABC)

29. (AD)

$[H^+]_{H_2O}$ is considered for very dilute solutions.

30. (ABCD)

$$[A_2H_4]_0 = [A_2H_4] + [A_2H_5^+] + [A_2H_6^{2+}]$$

$$K_{b_1} = \frac{[A_2H_5^+][OH^-]}{[A_2H_4]};$$

$[A_2H_5^+] = [OH^-] = \sqrt{4\times10^{-5}\times0.1} = 2\times10^{-3}$ M

$\Rightarrow\ [H^+] = 5\times10^{-12}$ M

$$K_{b_1}\times K_{b_2} = \frac{[A_2H_6^{2+}][OH^-]^2}{[A_2H_4]}$$

$$\Rightarrow [A_2H_6^{2+}] = \frac{4\times10^{-5}\times10^{-8}\times0.1}{\left(2\times10^{-3}\right)^2} = 10^{-8}\text{ M}$$

31. (AC)

32. (AB)

33. (BC)

34. (ACD)

$$\underset{1-\alpha}{HA} + OH^- \rightarrow \underset{\alpha}{A^+} + H_2O$$

$$[H^+] = k_1\times\frac{[HA]}{[A^-]} = 1.75\times10^{-5}\times\left[\frac{1-\alpha}{\alpha}\right]$$

$$[H^+] = k_2\times\frac{[H\,Pr]}{[Pr^-]} = 1.3\times10^{-5}\times\frac{(1-\beta)}{\beta}$$

$$\Rightarrow \left[\frac{\alpha}{1-\alpha}\right] = \frac{1.75}{1.3}\times\left[\frac{\beta}{1-\beta}\right]$$

35. (ABC)

(A) $[A^{2-}]$ depends on $[H^+]$.

(B) {pH = 1; $[H^+] = 10^{-1}$}; {pH = 3; $[H^+] = 10^{-3}$}

(C) On dilution, there is no effect on pH of neutral buffer.

36. (12.56)

37. (25)

$$pH = pK_a + \log\frac{\text{salt}}{\text{acid}}$$

$$\Rightarrow 7.4 = 6.7 + \log\frac{\text{salt}}{\text{acid}} \quad \Rightarrow \frac{\text{salt}}{\text{acid}} = 5$$

$\Rightarrow$ $5\times2.5\times10\times10^{-3} = 5\times V$

$\Rightarrow$ required volume = 25 mL

38. (6)

(a), (b), (e), (f), (g), (i)

39. (5)

40. (4)

$$[H^+]_{total} = [H^+]_{HCl} = 0.5\,M$$

$$K_{a_1} + K_{a_2} = \frac{[H^+]^2 \times [S^{2-}]}{[H_2S]}$$

$$10^{-21} = \frac{5^2 \times 10^{-2}[S^{2-}]}{10^{-1}}$$

$$[S^{2-}] = 4 \times 10^{-22}$$

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TOPIC: PARABOLA

SOLUTIONS

41. (B)

Let (x_1, y_1) be mid-point of any chord of the parabola

$y^2 = 4ax$

$\therefore$ its equation is $yy_1 - 2(x + x_1) = y_1^2 - 4x_1$ $\quad (T = S_1)$

It passes through vertex (0, 0).

42. (C)

Solving $x^2 = 4y$ and $y^2 = 4x$, we get $x = 0, y = 0$ and $x = 4, y = 4$.

$\therefore$ P is (4, 4). The = n of the tangents to the two parabolas at (4, 4) are

$2x - y - 4 = 0 \quad ...(1)$

and $x - 2y + 4 = 0 \quad ...(2)$

m_1 = slope of (1) = 2

m_2 = Slope of (2) = $\frac{1}{2}$

Since $m_1 m_2 = 1$ i.e. $\tan\theta_1 \tan\theta_2 = 1$

$\therefore$ θ_1 and θ_2 are such that $\theta_1 + \theta_2 = \frac{\pi}{2}$.

i.e. $\theta_2 = \frac{\pi}{2} - \theta_1$.

43. (C)

The equation of any normal to $y^2 = 4ax$ is

$y = mx - 2am - am^3 \quad ...(1)$

The combined equation of the lines joining the origin (vertex) to the points of intersection of (1) and $y^2 = 4ax$ is

$$y^2 = 4ax\left(y - \frac{mx}{-2am - am^3}\right)$$

or $y^2\left(2am + am^3\right) + 4axy - 4amx^2 = 0$

This represents a pair of $\perp$ lines

$\therefore$ co-eff. of x^2 + co-eff. of $y^2 = 0$

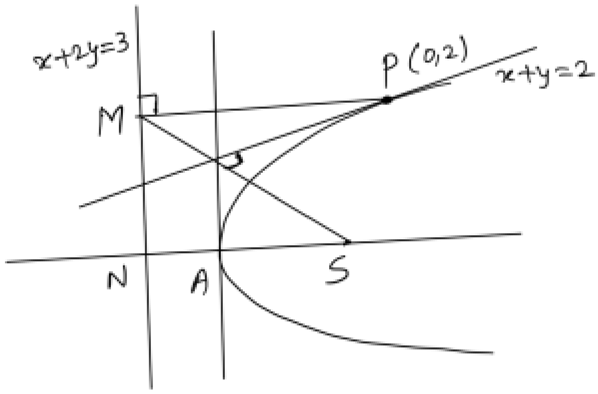
$\Rightarrow$ $2am + am^3 - 4am = 0$

$\Rightarrow\ m^2 = 2\ \Rightarrow\ m = \sqrt{2}$

44. (B)

45. (C)

46. (A, B)



The coordinate of foot of perpendicular from P to directrix are

$$\frac{x_1 - 0}{1} = \frac{y_1 - 2}{2} = -\left(\frac{1}{5}\right)$$

$$\Rightarrow M\left(\frac{-1}{5}, \frac{8}{5}\right)$$

By property focus is image of M with respect to tangent P.

$$\therefore \frac{h + \frac{1}{5}}{1} = \frac{k - \frac{8}{5}}{1} = -\left(\frac{-3}{5}\right)$$

$$h = \frac{2}{5}\ \&\ k = \frac{11}{5} \qquad \therefore\ S\left(\frac{2}{5}, \frac{11}{5}\right)$$

Now axis is $2x - y + \lambda = 0$

Put focus $\frac{4}{5} - \frac{11}{5} + \lambda = 0$

$$\lambda = \frac{7}{5}$$

$$2x - y + \frac{7}{5} = 0$$

Mid pint of M & S : $\left(\frac{1}{10}, \frac{19}{10}\right)$ lies on tangent at vertex

$\therefore$ it equation : $x + 2y = k$

Passes through $\left(\frac{1}{10}, \frac{19}{10}\right) \Rightarrow k = \frac{39}{10}$

$\therefore$ tangent at vertex : $10x + 20y = 39$

47. (B)

48. (AB)

Let (h, k) be the circumcircle of ΔABC

Then, $h = \frac{4+3p}{2p}$; $k = \frac{4-3p}{2q}$

$\Rightarrow\ p = \frac{4}{2h-3}$; $q = \frac{4h-12}{k(2h-3)}$

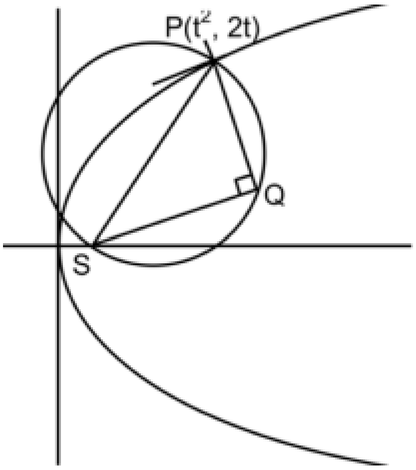
This (p, q) lies on $x^2 = 4ay$

$\Rightarrow$ Locus of (h, k) is $\frac{1}{2a}\left(y + \frac{9}{8}a\right) = \left(x - \frac{9}{4}\right)^2$

49. (AC)

50. (C)
51. (B)

Solution for Que. No. 52 & 53
52. (B)
53. (C)



Let P be $(t^2, 2t)$, then equation of normal is $y + tx = 2t + t^3$

Therefore $SQ = \left|\frac{t(t^2+1)}{\sqrt{1+t^2}}\right| = \left|t\sqrt{1+t^2}\right|$

Now $SP = (1+t^2)$

So, $PQ = \sqrt{SP^2 - SQ^2}$

$= \sqrt{(1+t^2)^2 - t^2(1+t^2)}$

$= \sqrt{(1+t^2)(1+t^2-t^2)}$

$= \sqrt{1+t^2}$

Now $PQ = 2$

Area of ΔPSQ is $\frac{1}{2} \times PQ \times QS = 2\sqrt{3}$

54. (A, B)

$y = 2x - 1$ meets $x^2 = ay$ when $x^2 = a(2x+1) = 2ax + a$

$\Rightarrow x^2 - 2ax - a = 0$

Let x_1, x_2 be the roots

$\therefore x_1 + x_2 = 2a, x_1x_2 = -a$

$\therefore (x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2 = 4a^2 + 4a = 4a(a+1)$

Also, $(y_1 - y_2)^2 = [(2x_1 + 1) - (2x_2 + 2)]^2$

$= (2(x_1 - x_2))^2 = 4(x_1 - x_2)^2 = 16a(a+1)$

Required intercept $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{20a(a+1)} = \sqrt{40}$ (given)

$\therefore 20a(a+1) = 40$

$\Rightarrow a(a+1) = 2 \Rightarrow a^2 + a - 2 = 0$

$\Rightarrow (a+2)(a-1) = 0$

$\Rightarrow a = -2$

55. (A, B, D)

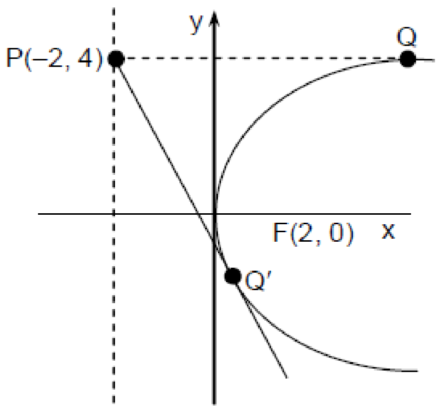
E = $y^2 - 8x = 0$

a = 2

Let $Q(2t_1^2, 4t_1), Q'(2t_2^2, 4t_2)$

$t_1t_2 = -1, t_1 + t_2 = 2$

$t_1 = 1 + \sqrt{2}, t_2 = 1 - \sqrt{2}$



(A) (slope of PF) (slope of FQ) = –1

$\Rightarrow \angle PFQ = \frac{\pi}{2}$

(B) (slope of PQ′)(slope of PQ) = –1

$\Rightarrow \angle QPQ' = \frac{\pi}{2}$

(C) $PF = 4\sqrt{2}$

(D) slope of Q′F = slope of FQ

$\Rightarrow$ Q, F, Q′ are collinear

56. (5)

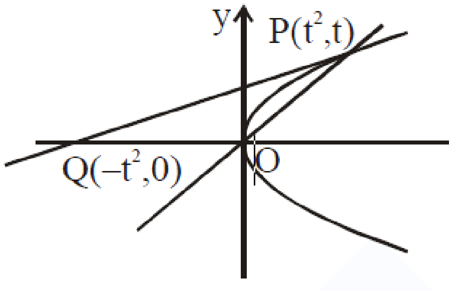
The power is 2/3

57. (2)

58. (0.50)

$\Delta OPQ = 4$

$$\frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{matrix}\right\| = 4$$



$t = 2 \quad (\because\ t > 0)$

$\therefore\ m = \frac{1}{2}$

Ans. 0.50

59. (0)

For focal chord (w.r..t $y^2 = 4ax$)

$$x_1x_2 = a^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$$

$$y_1y_2 = -4a^2 = -4.\left(\frac{5}{4}\right)^2 = -\frac{25}{4}$$

$$4x_1x_2 + y_1y_2 = \frac{25}{4} - \frac{25}{4} = 0$$

60. (24)

Distance of focus from directrix = 2a

$$\Rightarrow \frac{4+8}{\sqrt{1^2}} = 2a \Rightarrow a = 6$$

$\therefore$ length of latus rectum $= 4a = 24$ units.