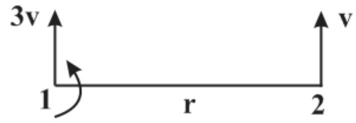
IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 16/04/23**

TOPIC: ROTATIONAL DYNAMICS

SOLUTIONS

1. **(D)**

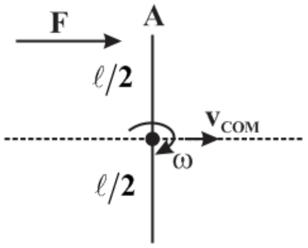


$$\omega_{21} = \frac{\left|\overrightarrow{v_{21_\perp}}\right|}{r}$$

$$= \frac{2v}{r}$$

2. **(C)**

3. **(D)**



$$F\Delta t = mv_{COM} \qquad ...(1)$$

$$F_{\ell/2}\Delta t = \frac{m\ell^2\omega}{12} \qquad ...(2)$$

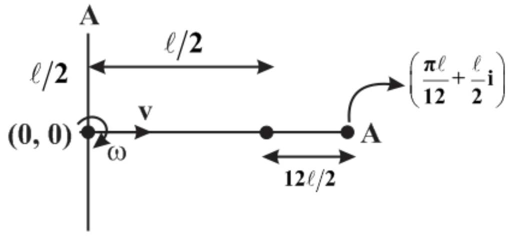
Dividing (1) & (2)

$$\omega = \frac{6v}{\ell}$$

$$S = vt$$

$$S = \frac{v\pi t}{12v}$$

$$= \frac{\pi\ell}{12}$$



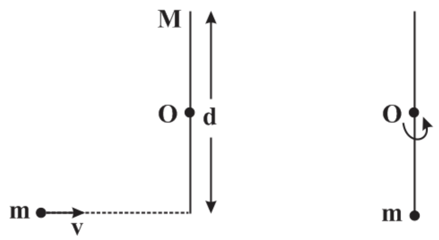
't' is when rod becomes parallel to x-axis.

$\Rightarrow$ any displacement of $\frac{\pi}{2}$

$$\frac{\pi}{2} = \omega t$$

$$t = \frac{\pi}{2\omega}$$

4. **(A)**



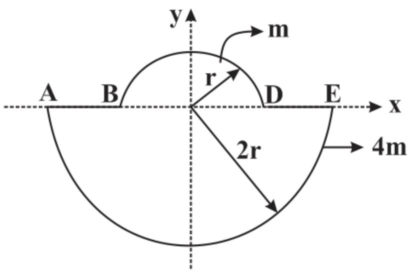
Angular momentum constant 'O'.

$$\frac{mvd}{2} = \left(\frac{6md^2}{12} + \frac{md^2}{4}\right)\omega$$

$$\frac{mvd}{2} = \frac{3}{4}md^2\omega$$

$$\frac{2v}{3d} = \omega$$

5. **(B)**

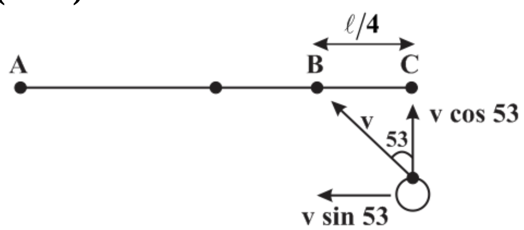


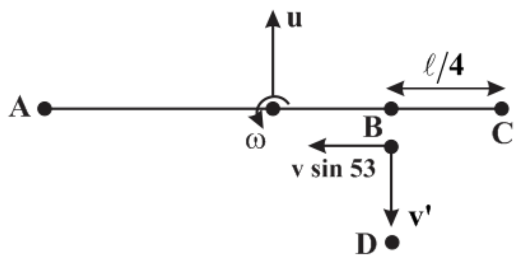
$$I_x = \frac{mr^2}{2} + \frac{4m(2r)^2}{2} = \frac{17mr^2}{2}$$

$$I_y = \frac{mr^2}{2} + \frac{4m(2r)^2}{2} + \left[I_{AB_y}\right]2 > \frac{17mr^2}{2}$$

$$\frac{I_x}{I_y} < 1$$

6. **(ABC)**





Velocity of sep = velocity of app.

$$v\cos 53 = v' + \left(u + \frac{\omega\ell}{4}\right)$$

$\rightarrow$ P constant in a direction perpendicular to rod

$$mv\left(\frac{3}{5}\right) = mu - mv'$$

$\rightarrow$ L constant about D as shown in figure

$$0 = 0 + \left[\frac{mu\ell}{4} - \frac{m\ell^2\omega}{12}\right] \Rightarrow u = \frac{\omega\ell}{3}$$

By solving $u = \frac{24v}{55}; \omega = \frac{72v}{55\ell}$

$\rightarrow$ time taken to rotate by π radians

$$t = \frac{\pi}{\omega}$$

In the same time distance travelled

$$= ut$$

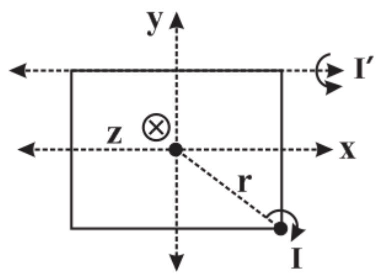
$$= \frac{24v}{55} \times \frac{\pi 55\ell}{72v}$$

$$= \frac{\pi\ell}{3}$$

$\rightarrow \int N dt = mu$

$$= \frac{24}{55} mv$$

7. **(BCD)**



$$I_x = I_y = 2\left[\frac{m\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2\right]$$

$$= \frac{2}{3} m\ell^2$$

$$I_z = I_x + I_y$$

$$= \frac{4m\ell^2}{3}$$

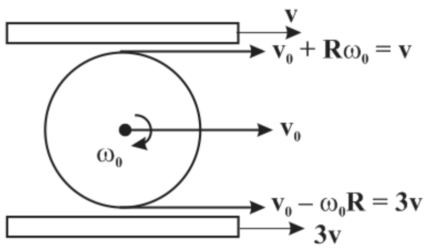
$$I = I_z + 4m\left(\frac{\ell}{\sqrt{2}}\right)^2$$

$$= \frac{4m\ell^2}{3} + \frac{4m\ell^2}{2} = \frac{20}{6}m\ell^2 = \frac{10}{3}m\ell^2$$

$$I' = I_x + 4m\left(\frac{\ell}{2}\right)^2$$

$$= \frac{2m\ell^2}{3} + m\ell^2 = \frac{5m\ell^2}{3}$$

8. **(AD)**

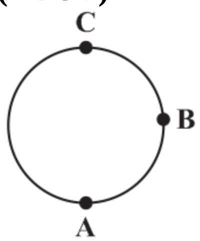


$2v_0 = 4v$

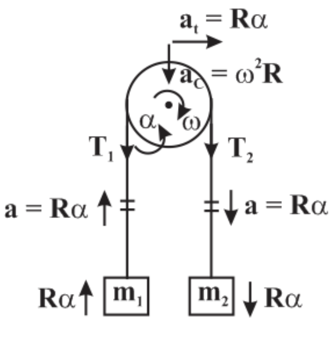
$v_0 = 2v$

$\omega_0 = -\frac{v}{R} \quad \Rightarrow \quad \omega_0 = \frac{v}{R}$ clockwise

9. **(ABCD)**



10. **(BD)**

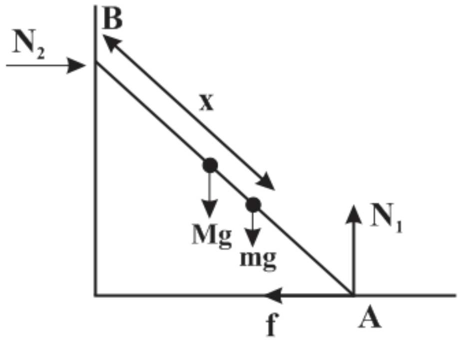


(1) acceleration of the sections are not same.

(2) $T_2 > T_1$ due to friction

(3) may of block's acceleration is same direction is not same

11. **(ACD)**



M = mass of ladder
m = mass of man
$\Rightarrow$ Ladder reaction in equilibrium (Translation rotational)

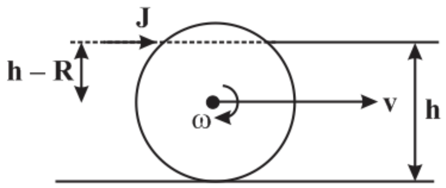
$N_1 = (m+M)g$

$\rightarrow$ Net torque on man + ladder about B = 0
$\rightarrow$ If x decreases torque of mg about B will decreases
Hence f must increase.
Hence N_2 has to increases since torque due to mg about A will increase.

12. **(BD)**

$I = mK^2$ where $\left(K = \frac{R}{2}\right)$ radius of gyration.

$$= m\left(\frac{R}{2}\right)^2 = \frac{mR^2}{4}$$



Ball will roll purely if $v = R\omega$

$$\frac{J}{m} = R\left[\frac{J(h-R)}{\frac{1}{4}mR^2}\right]$$

$$h = \frac{5R}{4}$$

$\Rightarrow$ if the ball hits the centre of mass. There will be no rotation.

13. **(CD)**

14. **(AC)**

$$\frac{K_R}{K_T} = \frac{I}{mr^2}$$

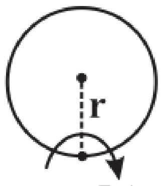
$$\therefore K_T = \left(\frac{mr^2}{I+mr^2}\right)K_{Total}$$

$$K_{Total} = \left(\frac{I+mr^2}{mr^2}\right)\left(\frac{mv^2}{2}\right)$$

$= mgh$

$= mg\left(\frac{3v^2}{4g}\right)$

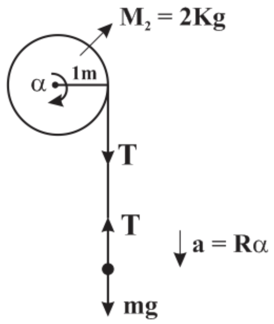
$I = \frac{mr^2}{2}$ Solid cylinder / disc



$I = \frac{mr^2}{2} + mr^2$

$= \frac{3}{2}mr^2$

15. **(ABCD)**



$TR = \frac{MR^2}{2}\alpha \quad \Rightarrow T = \frac{Ma}{2} \Rightarrow T = a$

$mg - T = m(R\alpha) \Rightarrow 10 - T = ma$

$10 = 2a$

$a = 5\,m/s^2$

$T = 5N$

$\theta = \frac{1}{2}\alpha t^2$

$= \frac{5}{2}(4)^2 = 40 \text{ rad}$

$w_{T_{(4s)}} = \int T d\theta$

$= 5(40) = 200 \text{ J}$

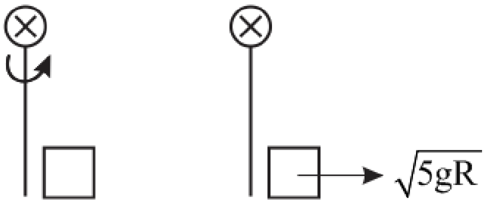
$w = K_f - K_i$

$200J = K_f - K_i = \Delta K$

16. **(5)**

$\frac{MgR}{2} = \frac{MR^2}{3}\frac{\omega^2}{2}$

$\sqrt{\frac{3g}{R}} = \omega$



Angular momentum conservation about hinge

$$I\omega = mR\sqrt{5gR}$$

$$\frac{MR^2}{3}\sqrt{\frac{3g}{R}} = mR\sqrt{5gR}$$

$$\frac{M}{m} = \sqrt{15}$$

17. (2)

$$M_{initial} = \pi R^2 \cdot L \cdot \rho$$

$$M_{final} = \pi\left(\frac{R}{2}\right)^2 L\rho = \frac{M_i}{4}$$

Initial PE of carpet = MgR

Final PE of carpet = $\frac{M}{4}g\frac{R}{2} = \frac{MgR}{8}$

$$\Delta PE = -\frac{7MgR}{8}$$

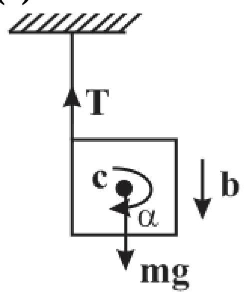
Equal to $\Delta K = KE_{Rot} + KE_{trans}$

$$\frac{1}{2}\frac{M}{4}v^2 + \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2\left(\frac{1}{2}\right)\left(\frac{2v}{R}\right)^2 = \frac{7MgR}{8}$$

$$\frac{3}{16}MV^2 = \frac{7MgR}{8}$$

$$\Rightarrow\ v = \sqrt{\frac{14gR}{3}}$$

18. (5)



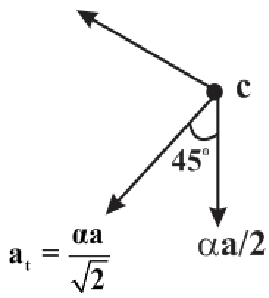
Let b and α be the linear acceleration of COM and average acceleration of plane just after BF is cut.

$$\frac{Ta}{2} = \frac{Ma^2}{6}\alpha \quad (\text{abt COM})$$

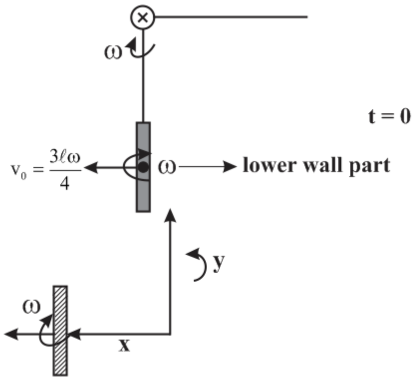
$$Mg - T = mb \quad (\text{for COM})$$

$$b = \frac{\alpha a}{2}$$

$$T = \frac{2mg}{5}$$



19. **(5)**



$$\omega = \sqrt{\frac{3g}{\ell}} = \sqrt{40} \text{ rad/s}$$

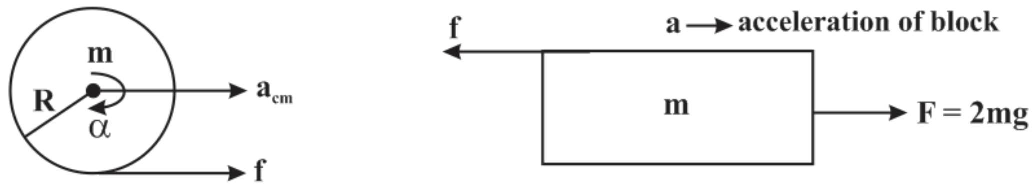
$$t = \frac{\pi}{\omega} = \frac{\pi}{\sqrt{40}} \text{sec}$$

$$x = v_0 t, \ \ y = \frac{gt^2}{2}$$

$$r = \sqrt{x^2 + \left(y + \frac{3\ell}{4}\right)^2} \approx 2.5\text{m}$$

$$2r = 5\text{m}$$

20. **(2)**



Newton's law on Block be spare

F – f = ma

f = ma_{cm}

$$fR = \frac{2}{5} mR^2 \alpha$$

Since the spare does not sleep over the block

$$a = a_{cm} + R\alpha$$

$$\alpha = \frac{10\,g}{9R} \text{rad/s}^2$$

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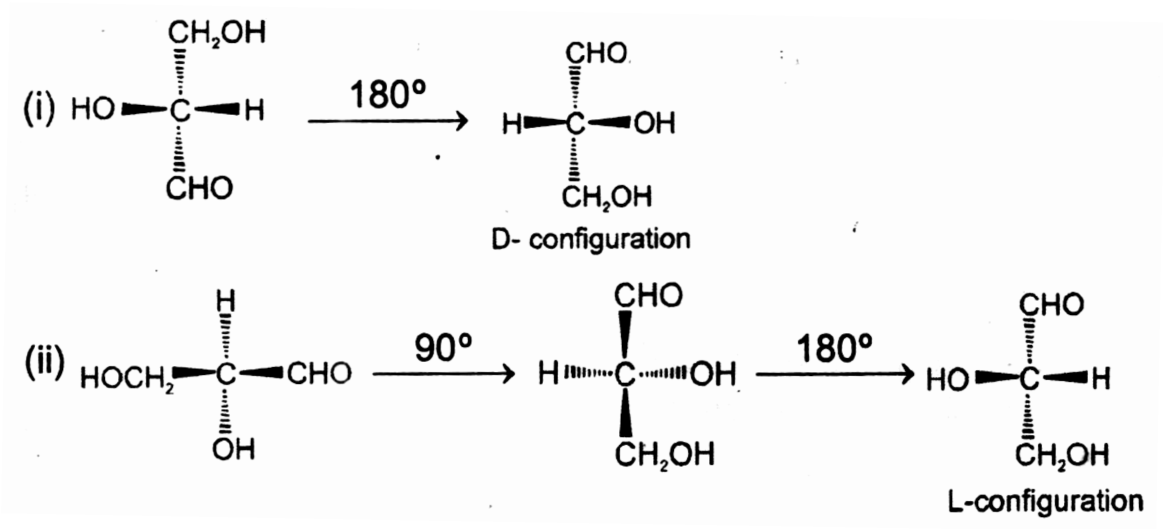
TOPIC: ISOMERISM

SOLUTION

21. **(C)**
Both are enantiomer of each other. So optical rotation will be -12°

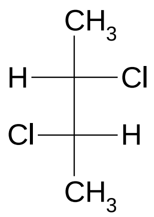
22. **(C)**

23. **(B)**



24. **(A)**
The configuration in a compound is independent of its physical properties(optical activity)

25. **(A)**



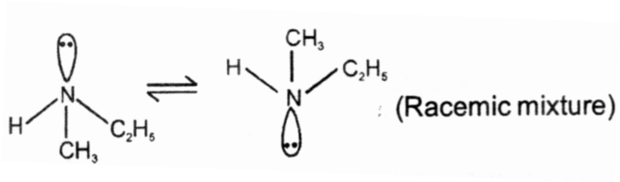
26. **(AB)**

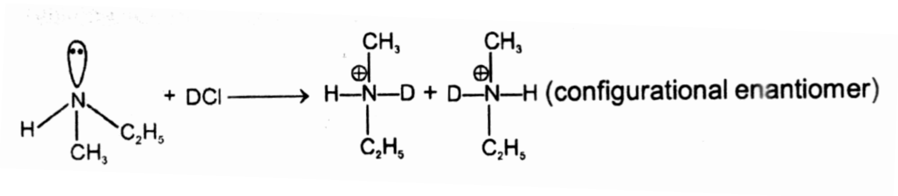
27. **(AD)**
For being functional isomers, functional group should not match.
(Phenol and aliphatic alcohol are considered as different functional groups)

28. **(ABC)**

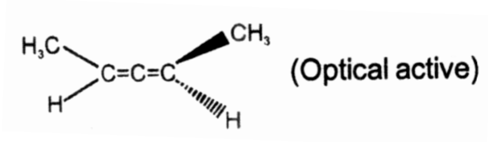
I is trans isomer and II is cis isomers I and III are position isomers similarly II and III are position isomers.

29. **(ABCD)**

(A) 

(B) 

(C) Two gauche form of butane are chiral and have equal energy.

(D) 

30. **(ABD)**

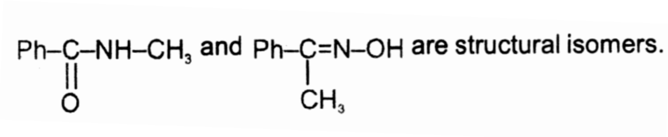
Ortho substituted biphenyls are optically active (c) and exists as enantiomers.

31. **(BCD)**

It is a meso achiral compound and it is no centre of symmetry and no axis of symmetry but plane of symmetry.

32. **(BD)**

33. **(ACD)**

(a) 

(b) These compounds are identicle (c) These are chain isomers

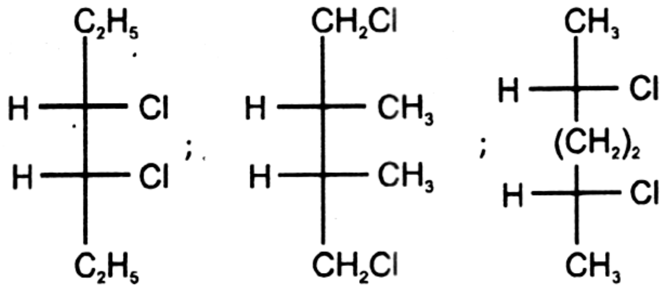
34. **(CD)**

35. **(ABC)**

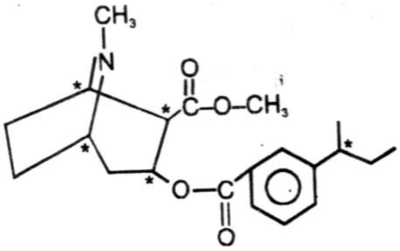
A meso compound has minimum two chiral centres and it has a plane of symmetry and it is inactive.

36. **(5)**

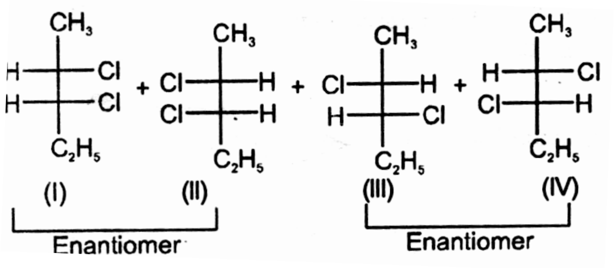
37. **(3)**



38. **(5)**

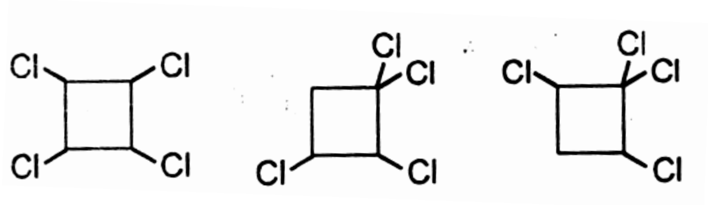


39. **(6)**



$\xrightarrow[\text{Distillation}]{\text{Fractional}} \text{Fraction}_1\left(\text{I}+\text{II}\right)+\text{fraction}_2\left(\text{III}+\text{IV}\right)$

40. **(3)**



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TOPIC: LIMITS

SOLUTIONS

41. (B)

$$\lim_{x\to 0^+}\left[1+[x]\right]^{2/x}$$

$$=[1]^{2/0^+}=1$$

42. (A)

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$$

$$\lim_{x\to 0}\frac{x\left(1+a\left(1-\frac{x^2}{\lfloor 2}\right)\right)-b\left(x-\frac{x^3}{\lfloor 3}\right)}{x^3}=1$$

$$\lim_{x\to 0}\frac{x+ax-\frac{ax^3}{\lfloor 2}-bx+\frac{bx^3}{\lfloor 3}}{x^3}=1$$

For limit to exist and finite

Coefficient of $x=0 \quad \Rightarrow 1+a-b$

And coefficient of $x^2=0$

Coefficient of $x^3=1\Rightarrow \frac{b}{\lfloor 3}-\frac{a}{\lfloor 2}=1$

$b-a=1$ ………. (1)

$b-3a=6$ ……… (2)

$2a=-5$

$a=\frac{-5}{2}$ $\qquad b=\frac{-3}{2}$

43. (B)

$$\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1} \qquad \left(\frac{0}{0}\text{form}\right)$$

$$=\lim_{x\to\frac{\pi}{3}}\frac{\sin\frac{\pi}{3}\cos x-\cos\frac{\pi}{3}\sin x}{2\cos x-1}$$

$$= \lim_{x \to \frac{\pi}{3}} \frac{\sqrt{3}\cos x - \sin x}{2(2\cos x - 1)}$$

$$= \lim_{x \to \frac{\pi}{3}} \frac{\sqrt{3}(-\sin x) - \cos x}{2(-2\sin x)} \qquad \text{(using L'Hopital)}$$

$$= \frac{-\sqrt{3} \times \frac{\sqrt{3}}{2} - \frac{1}{2}}{-4 \times \frac{\sqrt{3}}{2}} = \frac{-\frac{3}{2} - \frac{1}{2}}{-2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

44. (B)

$$\lim_{x \to \infty} \left(\frac{x}{1+x} \right)^x \ (1^\infty)$$

$$\Rightarrow \quad e^{\lim_{x \to \infty} (x)\left(\frac{x}{1+x} - 1\right)}$$

$$\Rightarrow \quad e^{\lim_{x \to \infty} (x)\left(\frac{x-1-x}{1+x}\right)}$$

$$\Rightarrow \quad e^{\lim_{x \to \infty} \frac{-1}{1+\frac{1}{x}}} = e^{-1}$$

45. (B)

$$\lim_{x \to 0} (1 + f(x))^{g(x)} = e^{\lim_{x \to 0} f(x) g(x)}$$

When $f(x) \to 0, g(x) \to 0$

$$L = \lim_{x \to 0} e^{\frac{a \sin bx}{\cos x} \times \frac{1}{x}}$$

$$= e^{\lim_{x \to 0} \frac{a \sin bx}{bx} \times \frac{b}{\cos x}}$$

$$= e^{a \times \frac{b}{\cos 0}} = e^{ab}$$

46. (ABCD)

(A) $\lim_{x \to 1^-} \frac{3x^2 + ax + a + 1}{x^2 + x - 2} = K$

$$\frac{3 + 2a + 1}{0} \left(\frac{0}{0} \text{form} \right)$$

$2a + 4 = 0$

$a = -2$

(B) $\lim_{x \to -2} \frac{3x^2 + ax + a + 1}{x^2 + x - 2} = \frac{12 - 2a + a + 1}{4 - 4} = \text{finite}\ a = 13$

(C) $\lim_{x \to 1} \frac{3x^2 + ax + a + 1}{x^2 + x - 2} = \frac{3x^2 - 2x - 1}{x^2 + x - 2} (\text{L Hopital})$

$$= \frac{6x - 2}{2x + 1} = \frac{4}{3}$$

(D) $\lim\limits_{x\to-2}\dfrac{3x^2+ax+a+1}{x^2+x-2}$

$\lim\limits_{x\to-2}\dfrac{3(-2)^2-13(2)+13+1}{4-2-2}$ $\qquad\left(\dfrac{0}{0}\right)$

$\lim\limits_{x\to-2}\dfrac{3x^2+13x+14}{x^2+x-2}$

$\lim\limits_{x\to-2}\dfrac{6x+13}{2x+1}$

$\dfrac{1}{-3}=\dfrac{-1}{3}$

47. (A, B, C)

(A) $\lim\limits_{t\to0}\dfrac{\sin(\tan t)}{\sin t}$

$\lim\limits_{t\to0}\dfrac{\sin(\tan t)}{\tan t}\cdot\dfrac{\tan t}{t.}\cdot\dfrac{t}{\sin t}=1$

(B) $\lim\limits_{x\to\pi/2}\dfrac{\sin(\cos x)}{\cos x}=1$

(C) $\lim\limits_{x\to0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}\times\dfrac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$\lim\limits_{x\to0}\dfrac{(1+x)-(1-x)}{x(2)}\Rightarrow\lim\limits_{x\to0}\dfrac{2x}{2x}=1$

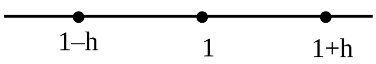
(D) $\lim\limits_{x\to0}\dfrac{\sqrt{x^2}}{x}=\lim\limits_{x\to0}\dfrac{|x|}{x}$

L.H.L. = – 1, R.H.L. = +1

Limit does not exist

48. (D)

f(x) = |x – 1| – [x]



L.H.L = $\lim\limits_{h\to0^-}f(1-h)$

$=\lim\limits_{h\to0^-}\{|1-h-1|-[1-h]\}$

$=\lim\limits_{h\to0^-}$ {|–h| – [Value less than 1]}

= 0 – 0

= 0

R.H.L = $\lim\limits_{h\to0^+}f(1+h)$

$=\lim\limits_{h\to0^+}\{|1+h-1|-[1+h]\}$

$=\lim\limits_{h\to0^+}$ {|h| – [Value greater than 1]}

$=\lim\limits_{h\to0^+}\{h-1\}=-1$

Since limit does not exist.

$\therefore$ Option (A) & (D) are correct Answers.

49. (ABC)

50. (AC)

$f(x)=x+\sqrt{x^2+2x}$ $\qquad g(x)=\sqrt{x^2+2x}-x$

(A) $\lim_{x\to\infty}\sqrt{x^2+2x}-x$

$$\lim_{x\to\infty}\frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x} \qquad =\lim_{x\to\infty}\frac{2x}{\sqrt{x^2+2x+x}}$$

$$=\lim_{x\to\infty}\frac{2}{\sqrt{1+\frac{2}{x}}+1}$$

$=1$

(B) $\lim_{x\to\infty}\sqrt{x^2+2x}+x=\text{does not exist}$

(C) $\lim_{x\to-\infty}\sqrt{x^2+2x}+x \qquad (\infty-\infty)$

$$\lim_{x\to-\infty}\frac{x^2+2x-x^2}{\sqrt{x^2+2x}-x}=\frac{2x}{\sqrt{x^2+2x}-x}$$

$$=\frac{2x}{-(x)\left(\sqrt{1+\frac{2}{x}}+1\right)}$$

$$=\lim_{x\to\infty}\frac{-2}{\sqrt{1+\frac{2}{x}}+1}=-1$$

51. (AC)

$$\lim_{x\to 0}\frac{\left(\sin x-\frac{(\sin x)^3}{\lfloor 3}+\frac{(\sin x)^5}{\lfloor 5}......\right)-\sin x}{ax^3+bx^5+c}=-\frac{1}{12}$$

$$\lim_{x\to 0}\frac{(\sin x)^3}{x^3}\left(\frac{-\frac{1}{\lfloor 3}+\frac{(\sin x)^2}{\lfloor 5}.......}{a+bx^2+\frac{c}{x^3}}\right)=-\frac{1}{12}$$

For limit to exist c = 0

And $-\frac{1}{\lfloor 3\times a}=-\frac{1}{12}$

$a=2$

52. **(ACD)**

$$\lim_{x\to\infty} 2\left(\sqrt{25x^2+x}-5x\right)\times\frac{\left(\sqrt{25x^2+x}+5x\right)}{\sqrt{25x^2+x}+5x}$$

$$\lim_{x\to\infty} 2\frac{\left(25x^2+x-25x^2\right)}{\sqrt{25x^2+x}+5x}$$

$$\lim_{x\to\infty}\frac{2x}{\sqrt{25x^2+x}+5x}=\frac{2}{\sqrt{25+0}+5}$$

$$=\frac{2}{10}=\frac{1}{5}$$

(A) $\lim_{x\to 0}\frac{2x-\ell n\left(1+x^2\right)}{10x}$

$$=\frac{2-\frac{1}{1+x^2}(2x)}{10x}$$

$$=\frac{2-0}{10}=\frac{1}{5}$$

(C) $\lim_{x\to 0}\frac{2\left(2\sin^2\frac{x^2}{2}\right)}{5x^4}$

$$\lim_{x\to 0}\frac{\not{4}\left(\sin\left(\frac{x^2}{2}\right)\right)^2}{5\frac{x^4}{4}\times\not{4}}$$

$$=\frac{1}{5}$$

53. **(AD)**

(A) $\lim_{x\to\infty}\operatorname{cosec}^{-1}\underbrace{\left(\frac{1}{1+7/x}\right)}_{\text{less than one}}$

= limit does not exist

(B) $\lim_{x\to 1^-}\sec^{-1}\left(\sin^{-1}x\right)$

$$\lim_{x\to 1^-}\sec^{-1}\left(\frac{\pi^-}{2}\right)=\text{finite.}$$

(C) $\lim_{x\to 0^+}\left(0^+\right)^{\infty}\qquad\left(0^{\infty}\right)$

$$y=\lim_{x\to 0^+}e^{\frac{1}{x}\ell nx}=0=\text{finite exist}$$

(D) $\lim_{x\to 0^+}\left(\tan\left(\frac{5\pi}{8}+x\right)\right)^{\cot x}$

$\left(\tan\frac{5\pi}{8}\right)^{\infty}$ does not exist.

54. **(ABC)**

$L = \lim_{x\to a}\frac{|2\sin x - 1|}{2\sin x - 1}$, then

(A) If $a = \frac{\pi}{6}$ $L = \lim_{x\to \pi/6}\frac{(|2\sin x - 1|)}{2\sin x - 1}$

L.H.L $= -1$ R. H. L = 1

$\Rightarrow$Limit does not exist

(B) When $a = \pi$

$L = \lim_{x\to\pi}\frac{|2\sin x - 1|}{2\sin x - 1} = -1$

(C) When $a = \pi/2$

$L = \lim_{x\to\pi/2} L = 1$

55. **(ABC)**

$f(x) = \frac{x^2 - 9x + 20}{x - [x]}$

$= \frac{x^2 - 5x - 4x + 20}{\{x\}}$

$= \frac{(x-5)(x-4)}{\{x\}}$

(A) $\lim_{x\to 5^-} f(x) = \frac{0}{1} = 0$

(B) $\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+}\frac{(x-5)(x-4)}{\{x\}}$

$= \lim_{x\to 5^+}\frac{\cancel{(x-5)}(x-4)}{\cancel{(x-5)}} = 1$

(C) From A and B option.

56. (7)

57. (0)

58. (7)

$L = \frac{1^2 + 2^2}{2} = \frac{5}{2}$

59. **(0)**

$h(0^+) = 0^+$ $h(0^-) = 0^+$

$g(0^+) = 1^+$ $\quad$ $g(0^+) = 1^+$

$f(1^+) = 0$ $\quad$ $f(1^+) = 0$

$$\lim_{x\to 0^+} f\left(g\left(x(x)\right)\right) = 0$$

$$\lim_{x\to 0^-} f\left(g\left(x(x)\right)\right) = 0$$

60. **(0)**

$$\lim_{x\to\infty} \frac{\ell n(\ell nx)}{e^{\sqrt{x}}} = \lim_{x\to\infty} \frac{\frac{1}{\ell nx} \times \frac{1}{x}}{e^{\sqrt{x}} \times \frac{1}{2\sqrt{x}}}$$

$$= \lim_{x\to\infty} \frac{2}{\ell nx\, e^{\sqrt{x}}.\sqrt{x}}$$

$= 0$