IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 18/06/23**

TOPIC: ELECTROSTAICS

SOLUTIONS

1. (B)

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\sigma r \frac{\pi}{2} .dr}{4\pi\varepsilon_0 r} \qquad \therefore V = \frac{\sigma}{8\varepsilon_0}\int_{R/2}^{R} dr = \frac{\sigma R}{16\varepsilon_0}$$

2. (A)
Due to electrostatic shielding, the ratio will be one

3. (D)

4. (A)

5. (D)

6. (ABCD)
Conceptual

7. (AC)

$$\frac{kq}{r} - \frac{k2Q}{2r} + \frac{k(4Q-q)}{3r} = \frac{k2Q}{3r}$$

$$q - Q + \frac{4Q}{3} - \frac{q}{3} = \frac{2Q}{3} \Rightarrow \frac{2q}{3} = \frac{Q}{3} \Rightarrow q = \frac{Q}{2}$$

$$\frac{kQ}{2R} + \frac{k3Q}{3R} + \frac{kq'}{2R} = 0$$

$$q' = -(Q+2Q) = -3Q$$

$$\therefore \Delta q = Q$$

8. (ABCD)
Electric field in the region $r > r_0$ is given by

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{Q}{4\pi \epsilon_0 r}\right)$$

For $r \le r_0, E = -\frac{d}{dr}\left(\frac{Q}{4\pi \epsilon_0 r_0}\right) = 0$

Hence, the electric field is discontinuous at r = r_0. Therefore, statement (a) is true.
For r < r_0.E = 0. Hence the charge resides only on the spherical surface of radius r = r_0. No charge exists in the region for which r < r_0.. Therefore, statement (c) is also true. Electric energy density is given by

$u = \frac{1}{2} \epsilon_0 \ E^2$

Since for $r < r_0, E = 0, u = 0$ for r < r_0. Hence statement(d) is true.

Let Q1 be the net charge enclosed inside the spherical surface of radius r = $2r_0$. Then from Gausss theorem, we have $\int E.ds = \frac{Q^1}{\epsilon_0}$

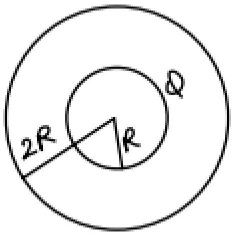
Or Q_1 = Q, which is independent of r as long as r is greater than r0. Hence statement (b) is also true

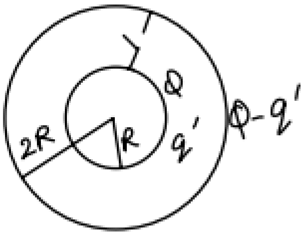
9. (B)

Electric field is discontinuous at the locations of charges Hence (B).

10. (A)

11. (CD)





$$\frac{kq'}{R} + \frac{k(Q-q')}{2R} = \frac{kq'}{2R} + \frac{k(Q-q')}{2R} =$$

$\Rightarrow q' = 0$

At any instant charge on outer sphere = q

$\Rightarrow$ Charge on inner sphere = $Q - q$

$$\Rightarrow \Delta v = \left(\frac{kq}{2R} + \frac{k(Q-q)}{R}\right) - \left(\frac{kq}{2R} + \frac{k(Q-q)}{R}\right)$$

$$= \frac{2k(Q-q)}{2R} - \frac{k(Q-q)}{2R} = \frac{k(Q-q)}{2R}$$

$$\Rightarrow d\omega = (dq)dv = \frac{k(Q-q)}{2R} \cdot dq$$

$$\omega = d\omega = \int_0^Q \frac{k(Q-q)}{2R} dq = \frac{k}{2R} Q[Q] - \frac{k}{2R}\frac{Q^2}{2}$$

$$= \frac{kQ^2}{4R}$$

12. (ABC)

1) $\text{o} + \frac{1}{2}\text{mv}_0^2 = \frac{\text{kq}^2}{\text{R}} + \frac{1}{2}\text{mv}^2$

2) $\text{mv}_0 \frac{\text{R}}{2} = \text{mvR}$

13. (AB)

14. (AB)

15. (BCD)
Potential at each point on y-z plane is zero. The electric field will be zero on y-z plane at a distance $\sqrt{2}a$ from origin.

16. (2)

$$\tan\theta = \frac{F}{mg} \qquad \tan\theta^1 = \frac{F}{kmg\left(1-\frac{\sigma}{\rho}\right)}$$

$$\frac{F}{mg} = \frac{F}{kmg\left(1-\frac{\sigma}{\rho}\right)}$$

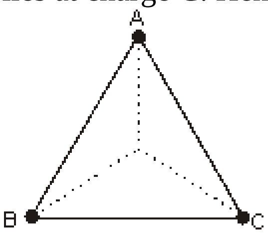
As $\theta' = \theta$ we have $\frac{F}{mg} = \frac{F}{long\left(1-\frac{\sigma}{\rho}\right)}$

$\Rightarrow k = 2$

17. (6)

$$\frac{K(4Q)h}{\left(h^2+\frac{a^2}{2}\right)^{3n}} = mg$$

18. (2)
Since $\vec{F}_{ext} = 0$ and $\vec{P}_{initial}$, the centre of mass of system remains at rest. The electrostatic potential energy is minimum when all three charges are collinear and at that instant centre of mass of system lies at charge C. Hence in final state charge C must be at centroid O of triangle (in initial state)



Hence $OC = \frac{2}{3}L\sin 60^0 = \frac{L}{\sqrt{3}}$

19. (4)

E at $AB = \frac{a}{\ell}(\ell+\ell) = 2a \quad \therefore$ on $AB = 2a\lambda\ell$

E at $CD = \frac{a}{\ell}(2\ell+\ell) = 3a \quad \therefore F$ on $CD = 3a\lambda\ell$

on BC & AD electric field is nonuniform x is not constant. But on BC & AD electric field will have the same type of variation.

$$\therefore F_{AD} = F_{BC} = \int_{x=\ell}^{2\ell} (\lambda dx).\,\frac{a}{\ell}(x+\ell)$$

$$= \frac{a\lambda}{\ell}\left[\frac{x^2}{2}+\ell x\right]_{\ell}^{2\ell} = \frac{a\lambda}{\ell}\left[\frac{3\ell^2}{2}+\ell^2\right] = \frac{5}{2}a\lambda\ell$$

$\therefore$ total force on the loop $= 2a\lambda t + 3a\lambda\ell + 2\left(\frac{5}{2}a\lambda t\right)$

$F = 10a\lambda\ell$

Using values $F = 4\times10^6 N$

20. (2)

$$\frac{1}{2} m v_r^2 = \left(\frac{3}{2}\frac{KQq}{R} - \frac{KQq}{R}\right) + \int_0^R m\,\omega^2 x\,dx = \frac{KQq}{2R} + \frac{m\omega^2 R^2}{2}$$

Substituting values,

$v_r^2 = 2 + 1 = 3$

$v_t\ \omega = r = 1$

$v_{net} = \sqrt{V_r^2 + V_t^2} = \sqrt{3+1} = 2$ m/sec.

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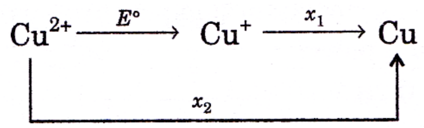
TOPIC: ELECTROCHEMISTRY

SOLUTIONS

21. (C)
$E^{\circ}_{cell} = +0.14 + 0.77 = 0.91V$

22. (A)

23. (D)



$-2x_2 = -E^{\circ} - x_1$
$\Rightarrow E^{\circ} = 2x_2 - x_1$

24. (D)
$$\% \text{ efficiency } = \frac{|\Delta G|}{|\Delta H|} \times 100$$
$$84 = \frac{|\Delta G| \times 100}{285 \times 10^3} \Rightarrow E = \frac{84 \times 285 \times 10}{96500 \times 2} = 1.24V$$

25. (C)
$$E_{cell} = -\frac{0.059}{2} \log \frac{\left[Zn^{2+}\right]_A}{\left[Zn^{2+}\right]_C} = -\frac{0.059}{2} \log\left(\frac{4}{6}\right)$$
$= +0.0052V$

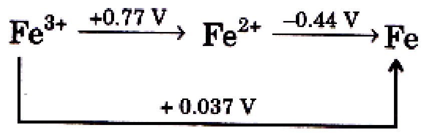
26. (B,C)

27. (B,C)

28. (A,C)

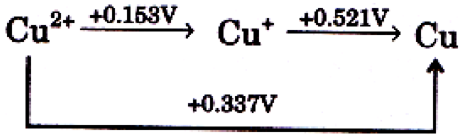
29. (C,D)

30. (B,C)



$Fe^{3+}\downarrow$ and $Fe^{2+}\uparrow$

31. (A,C)



$Cu^{+} \longrightarrow Cu + Cu^{2+}$

$E^{\circ} = 0.521 - 0.153 > 0$

Hence, disproportionation is spontaneous.

32. (A,C,D)

$Hg_2^{2+} \quad + \quad NO_3^{-} \longrightarrow 2Hg^{2+} + HNO_2 + H_2O$

(Oxidised) (Reduced)

(Anode) (Cathode)

$Q = \frac{1}{\left[H^{+}\right]^{3}}$ (Assuming unit active mass for all other species)

$E_{cell} = 0.02 + \frac{0.06}{2}\log\left[H^{+}\right]^{3}$

$= 0.02 - 0.06 \times \frac{3}{2} \times \frac{2}{9} = 0V$

No. of equivalents of $Hg^{2+} = 0.6 \times 1 = 0.6F$ charge

33. (A,C)

34. (B,C)

$E^{\circ}_{cell} = \frac{2.303RT}{nF}\log K;$

For Daniel cell; $n = 2$ as $Cu^{2+} + Zn \longrightarrow Zn^{2+} + Cu$

35. (A,B,D)

Only Au is not oxidised.

36. (1138)

$HIO_3 \rightleftharpoons H^{+} + IO_3^{-}$

$1 - y \qquad y \qquad y$

$\frac{y^2}{1-y} = \frac{1}{6} \Rightarrow y = \frac{1}{3}$

$\left[Ag^{+}\right] = \frac{3 \times 10^{-8}}{1/3} = 9 \times 10^{-8}$

$E_{cell} = -1.56 - \frac{0.06}{2}\log\left\{\frac{\left(9 \times 10^{-8}\right)^{2}}{1}\right\}$

$= -1.138V$

$\left|E_{cell}\right| = 1138 mV$

37. (0.2)

$MnO_4^- + 8H^+ + 2Cl^- + 5e^- \rightarrow Mn^{2+} + 2Cl^- + 4H_2O$

$$Q = \frac{0.2 \times (0.4)^2}{0.1 \times 1^8 \times 1^2} = 0.32$$

$$E_1 = 1.51 - \frac{0.059}{2} \log(0.32) = 1.516 \text{ V}$$

$Cr_2O_7^{2-} + 14H^+ + 6Cl^- + 6e^- \rightarrow 2Cr^{3+} + 6Cl^- + 7H_2O$

$$Q = \frac{(0.2)^2 \times (0.6)^6}{0.1 \times (0.7)^{14} \times (0.7)^6} = 23.39$$

$$E_2 = 1.33 - \frac{0.059}{6} \log\ (23.39) = 1.316 \text{ V}$$

$$\Rightarrow E_1 - E_2 = 1.516 - 1.316 = 0.2 \text{ V}$$

38. (1.5)

$2H^+ + NO_3^- + e^- \longrightarrow NO_2 + H_2O$

$$E_1 = 0.790 - \frac{0.059}{1} \log \frac{1}{\left[H^+\right]^2}$$

$NO_3^- + 6e^- + 8H^+ \longrightarrow NH_3OH^+ + 2H_2O$

$$E_2 = 0.731 - \frac{0.059}{6} \log \frac{1}{\left[H^+\right]^8}$$

Put $E_1 = E_2$, we get, $pH = 1.5$

39. (2)

R.P. of $H_2O_2 >$ R.P. of $\left(I_2 \mid I^-\right), \left(Fe^{3+} \mid Fe^{2+}\right)$

40. (5)

$E^\circ_{cell} = 1.51 - 0.78 = 0.73V$

$$X = 0.73 + \frac{0.06}{5} \log\left[H^+\right]^8 = 0.73 - 0.48 = 0.25$$

$\Rightarrow 20X = 5$

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TOPIC: METHOD OF DIFFERENTIAION

SOLUTIONS

41. (B)

We have $y=\cos^{-1}\cos x$ at $x=5\pi/4$ $y=\begin{cases} x & ,x\in(0,\pi) \\ \pi-\cos^{-1}\cos x & ,x\in\left(\pi,\dfrac{3\pi}{2}\right)\end{cases}$

$\Rightarrow y=\pi-\cos^{-1}\cos x$

$\Rightarrow y=\pi-x$

$\therefore \dfrac{dy}{dx}=-1$

42. (B)

We have $\sin(xy)+\cos(xy)=0$

$\Rightarrow \sin(xy)=-\cos(xy)$

$\Rightarrow \tan(xy)=-1$

$\Rightarrow xy=\tan^{-1}(-1)$

On differentiating w.r.t. x we get

$x\dfrac{dy}{dx}+y=0$

$\therefore \dfrac{dy}{dx}=\dfrac{-y}{x}$

43. (C)

We have $y=\sin^{-1}\left(\dfrac{x^2-1}{x^2+1}\right)+\sec^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)$

$y=\operatorname{cosec}^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)+\sec^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)$

$y=\dfrac{\pi}{2}$ $[\because \operatorname{cosec}^{-1}x\sec^{-1}x=\pi/2]$

$\therefore \dfrac{dy}{dx}=0$

44. (B)

We have,

$y=x-x^2$

$\Rightarrow \dfrac{dy}{dx}=1-2x$

Let $u = y^2$ *and* $v = x^2$

$\frac{du}{dx} = 2y\frac{dy}{dx}$

And $\frac{dy}{dx} = 2x$

$\therefore \frac{du}{dv} = \frac{2ydy/dx}{2x} = \frac{(x-x^2)(1-2x)}{x}$ [From Equs. (i) and (iii)]

$= (1-x)(1-2x) = 2x^2 - 3x + 1$

45. (A)

We have $ax^2 + 2hxy + by^2 = 0$

On differentiating w.r.t. x we get

$2ax + 2h\left(x\frac{dy}{dx} + y\right) + 2by\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-(ax+hy)}{(hx+by)}$

$\left[\begin{array}{l} \because ax^2 + 2hxy + by^2 = 0 \\ ax^2 + hxy = -(by^2 + hxy) \\ \Rightarrow -\left(\frac{ax+hy}{by+hx}\right) = \frac{y}{x} \end{array}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

46. (A)

47. (AC)

48. (A, B, C)

We have $y + \log_e(1+x) = 0$

$\Rightarrow y = -\log_e(1+x) \Rightarrow y = \log_e(1+x)^{-1}$

$\Rightarrow e^y = \frac{1}{1+x}$

$\Rightarrow xe^y + e^y = 1$

On differentiating w.r.t. x we get

$xe^y y' + e^y + e^y y' = 0$

$xy' + y' + 1 = 0$ …(ii)

$\Rightarrow y' = -\frac{1}{x+1} \Rightarrow y' = -e^y$ $\left[\because e^y = \frac{1}{1+x}\right]$

$\Rightarrow y' + e^y = 0$

From Eq. (ii)

$xy' + y' + 1 = 0 \Rightarrow xy' + 1 = -y'$

$\Rightarrow xy' + 1 = e^y$

49. (B, C)

We have $y^2 + b^2 = 2xy$

$\Rightarrow 2xy - y^2 = b^2$

$$2x\frac{dy}{dx}+2y-2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}(x-y)=-y$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{y-x}$$

On multiping by (y – x) both numerator and denominator, we get

$$\Rightarrow \frac{dy}{dx}=\frac{y(y-x)}{(y-x)^2}=\frac{y^2-xy}{(y-x)^2}=\frac{2xy-b^2-xy}{(y-x)^2}=\frac{xy-b^2}{(y-x)^2}$$

50. (A, B)

Let $u=\sin^{-1}\frac{t}{\sqrt{1+t^2}}$

And $v=\cos{-1}\frac{1}{\sqrt{1+t^2}}$

$$\Rightarrow u=\begin{cases}\tan^{-1}t, & t\ge 0\\ -\tan^{-1}t & t<0\end{cases}$$

And $v=\begin{cases}\tan^{-1}t, & t\ge 0\\ -\tan^{-1}t & t<0\end{cases}$

$$\Rightarrow \frac{du}{dt}=\begin{cases}\frac{1}{1+t^2}, & t\ge 0\\ -\frac{1}{1+t}, & t<0\end{cases}$$

And $\frac{dv}{dt}=\begin{cases}\frac{1}{1+t^2}, & t\ge 0\\ \frac{1}{1+t^2}, & t>0\end{cases}$

$$\therefore \frac{du}{dv}=\begin{cases}1 & t\ge 0\\ -1 & t<0\end{cases}$$

51. (A, D)

We have $x=\cos t, y=\log_e t$

$$\frac{dx}{dt}=-\sin t, \frac{dy}{dt}=\frac{1}{t}$$

$$\frac{dy}{dx}=\frac{-1}{t\sin t}$$

$$\therefore \left(\frac{dy}{dx}\right)_{t=}=\frac{-2}{\pi}$$

And $\left(\frac{dy}{dx}\right)=\frac{-12}{\pi}$

52. (A, B, C, D)

We have $2^x+2^y=2^{x+y}$

On differentiating w.r.t. x we get

$$2^x\log 2+2^y\log 2\frac{dy}{dx}=2^{x+r}\left(1+\frac{dy}{dx}\right)\log 2$$

$\Rightarrow 2^x + 2^y \dfrac{dy}{dx} = 2^{x+y}\left(1+\dfrac{dy}{dx}\right)$

$\Rightarrow \dfrac{dy}{dx}(2^x - 2^{x+y}) = 2^{x+y} - 2^x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2^{x+y} - 2^x}{2^r - 2^{x+y}}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2x(2^x - 1)}{2^y(1-2^x)} = \dfrac{2^x(1-2^y)}{2^y(2^x - 1)}$

Or $\dfrac{dy}{dx} = \dfrac{2^x + 2^y - 2^x}{2^y - 2^x - 2^y}$ $[\because 2^{x+y} = 2^x + 2^y]$

$\dfrac{dy}{dx} = \dfrac{-2^y}{2^x} \Rightarrow \dfrac{dy}{dx} = \dfrac{2^{x+y} - 2^x}{2^y(1-2^x)}$

$\dfrac{dy}{dx} = -\dfrac{2^y}{2^x} \Rightarrow \dfrac{dy}{dx} = \dfrac{2^{x+y} - 2^x}{2^y(1-2^x)}$ $[\because 2^x = 2^{x+y} - 2^y]$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2^x(2^y - 1)}{2^y - 2^{x+y}}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2^x(2^y - 1)}{-2^x}$ $\therefore \dfrac{dy}{dx} = 1 - 2^y$

53. (A, B, D)

We have $x^p . x^q = \{x+y\}^{p+q}$

On taking log both sides, we get

$p \log x + q \log y = (p+q)\log(x+y)$

On differentiating w.r.t.x,we get

$\dfrac{p}{x} + \dfrac{q}{y}\dfrac{dy}{dx} = \dfrac{p+q}{x+y}\left(1+\dfrac{dy}{dx}\right)$

$\Rightarrow \dfrac{dy}{dx}\left(\dfrac{q}{y} - \dfrac{p+q}{x+y}\right) = \dfrac{p+q}{x+y} - \dfrac{p}{x}$

$\therefore \dfrac{dy}{dx} = \dfrac{y}{x}$

54. (A, C)

$y = e^{\sqrt{x}} + e^{-\sqrt{x}}$

$\therefore \dfrac{dy}{dx} = e^{\sqrt{x}} \times \dfrac{1}{2\sqrt{x}} = \dfrac{e^{-\sqrt{x}}}{2\sqrt{x}}$

$= \dfrac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}} = \dfrac{\left[\left(e^{\sqrt{x}} - e^{-\sqrt{x}}\right)^2\right]^{1/2}}{2\sqrt{x}}$

$= \dfrac{\sqrt{\left(e^{\sqrt{x}}\right)^2 + \left(e^{-\sqrt{x}}\right)^2 - 2}}{2\sqrt{x}}$

$= \dfrac{\sqrt{\left(e^{\sqrt{x}}\right)^2 + \left(e^{-\sqrt{x}}\right)^2 + 2 - 4}}{2\sqrt{x}}$

$$= \frac{\sqrt{\left(e^{\sqrt{x}} + e^{-\sqrt{x}}\right)^2 - 4}}{2\sqrt{x}} = \frac{\sqrt{y^2 - 4}}{2\sqrt{x}}$$

55. (A, B, C)

We have, $f(0) = \frac{2}{g(0)} \cdot f'(0) = 2g'(0) = 4g(0)$

$g''(0) = 5f''(0) = 6f(0) = 3$

Now, on solving these equations, we get

$$f(0) = \frac{1}{2} \cdot g(0) = 4,\ f'(0) = 16,\ g'(0) = 8$$

$$f''(0) = \frac{3}{5} \cdot g''(0) = 3$$

(A) We have $h(x) = \frac{f(x)}{g(x)}$

$$h'(x) = \frac{g(x)\, f'(x) - g'(x)\, f'(x)}{g(x)^2}$$

$$\therefore \quad h'(0) = \frac{g(0)\, f'(0) - g'(0)\, f(0)}{g(0)^2}$$

$$= \frac{4 \times 16 - 8 \times \frac{1}{2}}{(4)^2} = \frac{15}{4}$$

(B) $k(x) = f(x) \cdot g(x) \cdot \sin x$

$$k'(x) = f(x) g(x) \cos x + f(x) \sin x g'(x) + g(x) \sin x f'(x)$$

$$k'(0) = f(0) g(0) \cos \theta + f(0) \sin \theta g'(0) + g(0) \sin \theta\, f'(0)$$

$$k'(0) = \frac{1}{2} \times 4 \times 1 + 0 + 0 = 2$$

(C) $\lim_{x \to 0} \frac{g'(x)}{f'(x)}$

$$\Rightarrow \quad \frac{g'(0)}{f'(0)} = \frac{1}{2}$$

56. (5)

We have, $f(x) = 2\log_e(x-2) - x^2 + 4x + 1 \qquad [x \neq 2,\ x > 2]$

$$\Rightarrow \quad f'(x) = \frac{2}{x-2} - 2x + 4$$

$$= \frac{2}{x-2} - 2(x-2)$$

$$= 2\left[\frac{1}{x-2} - x - 2\right]$$

$$= 2\left[\frac{1 - (x^2 - 4x + 4)}{x-2}\right]$$

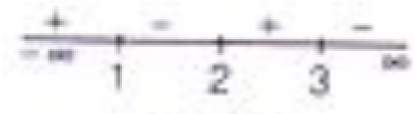
$$f'(x) = 2\frac{\left(-x^2+4x-3\right)}{x-2}$$

$$f'(x) = \frac{-2(x-3)(x-1)}{x-2}$$

$\because \; f'(x) \geq 0$

$\therefore \; \frac{-2(x-3)(x-1)}{x-2} \geq 0$

$\frac{2(x-3)(x-1)}{x-2} \leq 0$



$x \in (2, 3]$ $\quad\quad\quad [\because \; x > 2]$

$\Rightarrow \quad a = 2, b = 3$

$\therefore \; a + b = 2 + 3 = 50$

57. (1)

we have

$$f(x) = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + ...\infty}}}},$$

$\Rightarrow f(x) = x + \frac{1}{x + f(x)}$

$\Rightarrow f(x) - x = \frac{1}{x + f(x)}$

$\Rightarrow \left[f(x)\right]^2 - x^2 = 1$

On differentiating w.r.t x we get

$2f(x).f'(x) - 2x = 0$

$f(x)f'(x) = x \Rightarrow \frac{f(x).f'(x)}{x} = 1$

$\therefore \frac{f(100).f'(100)}{100} = 1$

58. (1)

we have $y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}, x = \sec^{-1}\frac{1}{2u^2-1}$

$y = \sin^{-1} u$

And $x = 2\cos^{-1} u$

$\Rightarrow \frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}$

And $\frac{dx}{du} = \frac{-2}{\sqrt{1-u^2}}$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

$\therefore \left|2\frac{dy}{dx}\right| = \left|2\times\frac{-1}{2}\right| = 1$

59. (1)

We have $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right) + \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$ .....(i)

On putting x = $\cos\theta$ in Eq. (i) we get

$$y = \tan^{-1}\frac{\cos\theta}{1+\sin\theta} + \sin\left[2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right]$$

$$y = \tan^{-1}\left(\frac{\cos\theta/2 - \sin\theta/2}{1+\sin\theta+\sin\theta/2}\right) + \sin\left(2\tan^{-1}\tan\frac{\theta}{2}\right)$$

$$y = \tan^{-1}\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right) + \sin\theta$$

$$y = \frac{\pi}{4} - \frac{\theta}{2} + \sin\theta$$

$$y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x + \sin(\cos^{-1}x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}} - \frac{\cos(\cos^{-1}x)}{\sqrt{1-x^2}}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\Rightarrow \left|2\frac{dy}{dx}\right| = \left|2\times\frac{-1}{2}\right| = 1$$

60. (1)

We have $xe^{xy} = y + \sin^2 x$

When x =0, then y =0

Now, on differentiating w.r.t. x we get

$$xe^{xy}\left(x\frac{dy}{dx} + y\right) + e^{xy} = \frac{dy}{dx} + 2\sin x\cos x$$

$$\Rightarrow \frac{dy}{dx}(x^2e^{xy} - 1) = 2\sin x\cos x - e^{xy} - xye^{xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin 2x - e^{xy}(1+xy)}{x^2e^{xy} - 1}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = \frac{x - e^0}{0-1} = e^0 = 1 \qquad [\because x = 0 \text{ and } y = 0]$$