IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 22/04/23**

TOPIC: SHM

SOLUTIONS

1. **(A)**

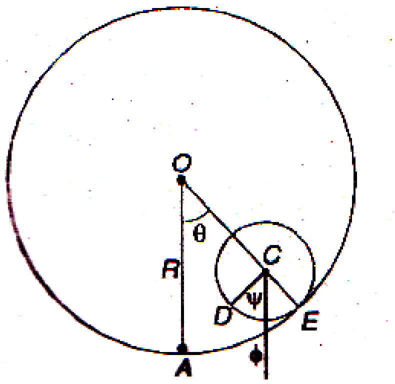
Let at equilibrium position point A of the spherical shell and point D of sphere are in contract.

Then, $r\psi = R\theta$

Where ψ is the angle by which sphere is distributed.

Angle of rotation of sphere from vertical direction is ϕ.

So, $\theta + \phi = \psi$ $\qquad \phi = \dfrac{R-r}{r}\theta$



Equation of motion of centre of mass of small sphere in the tangential direction

$$m(R-r)\frac{d^2\theta}{dt^2} = -mg\sin\theta - f \qquad \text{(i)}$$

For the rotation of sphere

$$\frac{2}{5}mr^2\frac{d^2\phi}{dt^2} = fr \qquad \text{(ii)}$$

From Eqs. (i) and (ii)

$$m(R-r)\frac{d^2\theta}{dt^2} = -mg\sin\theta - \frac{2}{5}mr\frac{d^2\phi}{dt^2}$$

$$= -mg\sin\theta - \frac{2}{5}m(R-r)\frac{d^2\theta}{dt^2} \qquad \text{(iii)}$$

$$\frac{7}{5}(R-r)\frac{d^2\theta}{dt^2} = -\sin\theta$$

Since, θ is small so $\sin\theta = \theta$

$$\frac{7}{5}(R-r)\frac{d^2\theta}{dt^2} = -g\theta \qquad \frac{d^2\theta}{dt^2} = -\frac{5g}{7(R-r)}\theta$$

$$\omega^2 = \frac{2\pi}{\omega} \qquad T = \frac{2\pi}{\omega}$$

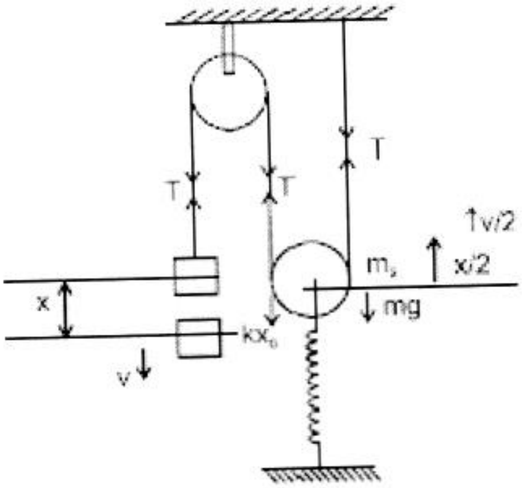
$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

2. **(B)**

The small block oscillates along the inclined plane with an amplitude *A*. As a result the centre of mass of the system undergoes SHM along the horizontal direction:

$$x_{cm} = \frac{mA\sin\omega t}{m+M}\cos 60^{o}$$

3. **(B)**



(i) Equilibrium position determination

$$M_1g = T \rightarrow (\text{from FBD of } M_1)$$

$$2T = M_2g + kx_0 \ (\text{from FBD of } M_2)$$

$$\therefore 2M_1g = M_2g + kx_0$$

$$\therefore kx_0 = 2M_1g - M_2g$$

(ii) Displacement block M_1 by small disp. x by

At new displacement position, energy of system is given by

$$-Mgx + \frac{1}{2}M_1V^2 + \frac{1}{2}M_2\left(\frac{v}{2}\right)^2 + M_2g\left(\frac{x}{2}\right) + \frac{1}{2}K\left(x_0 + \frac{x}{2}\right)^2 = C$$

Differentiating equation $-M_1g\frac{dx}{dt} + \frac{1}{2}M_1 2v\frac{dv}{dt} + \frac{M_2}{8}2v\frac{dv}{dt} + \frac{M_2g}{2}\frac{dt}{dx} + \frac{K}{2}2\left(x_0 + \frac{x}{2}\right)\left(\frac{1}{2}\frac{dx}{dt}\right) = 0$

$$\Rightarrow \ -M_1g + M_1a + \frac{M_2a}{4} + \frac{M_2g}{2} + \frac{K}{2}\left(x_0 + \frac{x}{2}\right) = 0 \qquad \left(\text{Where } a = \frac{dv}{dt}\right)$$

$$\Rightarrow \ -M_1g + \frac{M_2g}{2} + M.a + \frac{M_2a}{4} + \frac{Kx_0}{2} + \frac{Kx}{4} = 0 \quad \left(\text{From equilibrium } -M_1g + \frac{M_2g}{2} + \frac{Kx_0}{2} = 0\right)$$

Hence, $\frac{4M_1 + M_2}{4}a = \frac{-Kx}{4}$

$$\therefore \ a = -\left(\frac{K}{4M_1 + M_2}\right)x \qquad \omega^2 = \frac{K}{(4M_1 + M_2)}$$

$$\omega = \sqrt{\frac{K}{(4M_1 + M_2)}}; \qquad T = \frac{2\pi}{\omega}$$

$$\therefore \ T_2 = 2\pi\sqrt{\frac{4M_1 + M_2}{K}}$$

4. **(A)**

For pure rolling to take place,

$$v = R\omega$$

$\omega' =$ angular velocity of COM of sphere C about O

$$= \frac{v}{4R} = \frac{R\omega}{4R} = \frac{\omega}{4}$$

$$\therefore \frac{d\omega'}{dt} = \frac{1}{4}\frac{d\omega}{dt} \Rightarrow \alpha' = \frac{\alpha}{4}$$

Or $\alpha' = \frac{a}{R}$ for pure rolling

Where, $a = \frac{g\sin\theta}{1+\frac{1}{mR^2}} = \frac{5g\sin\theta}{7}$

As, $I = \frac{2}{5}mR^2$

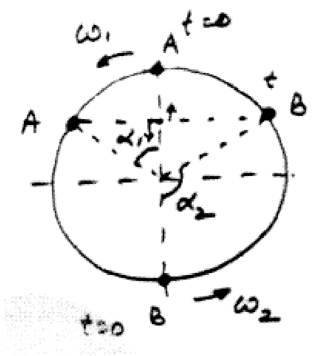
For small $\theta, \sin\theta \approx \theta$, being restoring in nature.

$$\alpha' = -\frac{5g}{28R}\theta$$

$$\therefore \quad T = 2\pi\sqrt{\left|\frac{\theta}{\alpha'}\right|} = 2\pi\sqrt{\frac{28R}{5g}}$$

5. **(C)**

If we map both SHM on a circle



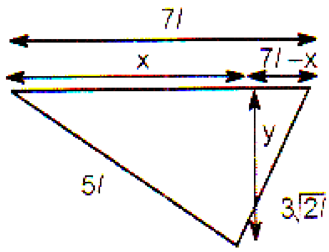
$$\omega_1 = \sqrt{\frac{g}{l}} \ \& \ \omega_2 = \sqrt{\frac{g}{\left(l/4\right)}} = 2\sqrt{\frac{g}{l}}$$

If after time t both strings become parallel

$$t = \frac{\alpha_1}{\omega_1} = \frac{\alpha_2}{\omega_2} = \frac{\alpha_1+\alpha_2}{\omega_1+\omega_2} = \frac{\pi}{3\sqrt{\frac{g}{l}}}$$

$$t = \frac{\pi}{3}\sqrt{\frac{l}{g}}$$

6. **(BC)**



$$\Rightarrow y^2 = (5l)^2 - x^2$$

$$\Rightarrow \left(3\sqrt{2}l\right)^2 - (7l - x)^2$$

$$\Rightarrow x = 4l \Rightarrow y = 3l$$

$$\Rightarrow T = 2\pi\sqrt{\frac{3l}{g}}$$

7. **(AB)**

Restoring torque for small angle θ

$$\tau = \frac{5}{4}k\ell^2\theta + \frac{mg\ell}{2}\theta$$

8. **(BC)**

Amplitude is obtained for v = 0

$\therefore$ Amplitude $=\sqrt{\dfrac{E}{A}}$

Maximum velocity is obtained for x = 0

$$v_{max}=\sqrt{\frac{E}{B}}$$

$$v_{max}=\text{amplitude}\times\omega$$

$$\Rightarrow\ \omega\sqrt{\frac{A}{B}}\ \Rightarrow\ T=2\pi\sqrt{\frac{B}{A}}$$

9. **(AD)**

$$T=\frac{T_0}{2}+2t=\pi\sqrt{\frac{\ell}{g}}+2t \qquad ...(1)$$

$t\rightarrow$ time to travel from O to β angle; $\beta=\alpha\sin\omega t$

$$t=\frac{1}{\omega}\sin^{-1}\left(\frac{\beta}{\alpha}\right)\Rightarrow t=\frac{T_0}{2\pi}\sin^{-1}\left(\frac{\beta}{\alpha}\right)$$

Putting value of t in equation (1)

$$T=2\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2}+\sin^{-1}\left(\frac{\beta}{\alpha}\right)\right]$$

$$=2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(-\frac{\beta}{\alpha}\right)\right]$$

10. **(ABC)**

The given equation is

$$x=A\sin^2\omega t+B\cos^2\omega t+C\sin\omega t\cos\omega t$$

Rearranging the equation in a meaningful form (for interpretation of SHM)

$$x=\frac{A}{2}\left(2\sin^2\omega t\right)+\frac{B}{2}\left(2\cos^2\omega t\right)+\frac{C}{2}\left(2\sin\omega t\cos\omega t\right)$$

$$=\frac{A}{2}[1-\cos2\omega t]+\frac{B}{2}[1+\cos2\omega t]+\frac{C}{2}[\sin\omega t]$$

(A) For A = 0 and B = 0, $x=\dfrac{C}{2}\sin(2\omega t)$

The above equation is that of SHM with amplitude $\dfrac{C}{2}$ and angular frequency 2ω. Thus option (A) is correct.

(B) If $A=B$ and C = 2B then $x=B+B\sin2\omega t$

This is equation of SHM. The mean position of the particle executing SHM is not at the origin. Option (B) is correct. $[x=B=x'=b\sin2\omega t]$

(C) $A=-B,\ C=2B;$ Therefore

$x=B\cos2\omega t+B\sin2\omega t$

Let $B=X\cos\phi=X\sin\phi$ then

$x=X\sin2\omega t\cos\phi+X\cos2\omega t\sin\phi$

This represents equation of SHM.

(D) A = B, C = 0 and x = A. This equation does not represents SHM.

11. **(A)** **12.** **(A)**

$$\frac{1}{2}mv_\ell^2 = \frac{1}{2}k\ell^2 - mg\frac{\ell}{2}$$

Substituting values

$$v_\ell^2 = \frac{k}{m}\ell^2 - g\ell = \frac{25}{0.2}(0.4)^2 - 4$$

$v_\ell = 4$ m/s.

For min v_ℓ

$$\frac{1}{2}mv_\ell^2 = (2 - 2\times 0.4)m\times g\sin 30^0$$

$$v_{\ell 2}^2 = 2\times 1.2\times\frac{g}{2}$$

$$\Rightarrow v_{\ell\,min} = \sqrt{12} = 2\sqrt{3}\ \text{m/s}$$

if $v_\ell = 4$ m/s,

$$L_{max} = \frac{4^2}{2(g\sin 30^0)}$$

$$= 0.8 + \frac{16}{2\times 5} = 1.6\text{m}$$

Maximum speed will be at equilibrium, because beyond that gravitational pull will dominate spring force.

$$\text{So, } x = 0.4 - \frac{0.2\times g\sin 30^0}{25}$$

$$= 0.4 - \frac{0.2\times 5}{25} = 0.36$$

13. **(B)**

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{800}{2}} = 20\ \text{rad/s}$$

14. **(C)**

$$Kx_0 - Kx_0' = ma \Rightarrow A = x_0 - x_0' = \frac{ma}{K} = 2.5\ \text{cm}$$

15. **(D)**

16. **(8)**

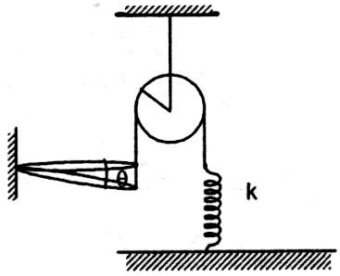
When displacement of the ring is θ, then extension in spring $= (2a\theta + x_0)$ Energy of system,

$$E = \frac{1}{2}k(2a\theta + x_0)^2 - mga\theta + \frac{1}{2}I\omega^2 \qquad \text{Where, } \omega = \frac{d\theta}{dt}$$

$$E = \frac{1}{2}k(2a\theta + x_0)^2 - mga\theta + \frac{1}{2}\left(\frac{1}{2}ma^2 + ma^2\right)\omega^2$$

$$\Rightarrow \frac{dE}{dt} = k(2a\theta + x_0).2a\frac{d\theta}{dt} - mga\frac{d\theta}{dt} + \frac{3}{2}ma^2\omega\frac{d^2\theta}{dt^2}$$

$$\text{As } \frac{dE}{dt} = 0, k(2a\theta + x_0)2a - mga = -\frac{3}{2}ma^2\frac{d^2\theta}{dt^2}$$



$$4ak\theta + 2akx_0 - mga = -\frac{3}{2}ma^2\frac{d^2\theta}{dt^2}$$

$\therefore \quad \frac{8}{3}\frac{k}{m}\theta = -\frac{d^2\theta}{dt^2}$

$\therefore \quad \omega = \sqrt{\frac{8k}{3m}}$

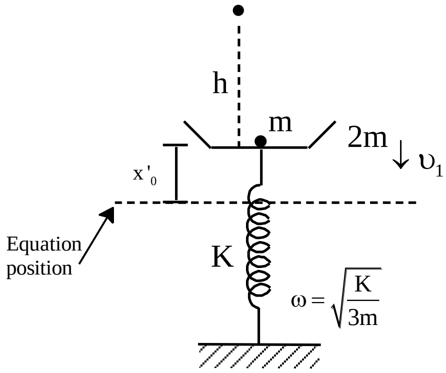
17. (7)

Initial compression in spring $x_0 = \frac{2mg}{k}$

After, m falls, further compression to equilibrium position in spring is $x'_0 = \frac{mg}{k}$

Collision speed of mass is $\upsilon = \sqrt{2gh}$ & we use $m\upsilon = 3m\upsilon_1$

$\upsilon_1 = \frac{1}{3}\sqrt{2gh}$ = common velocity just after collision

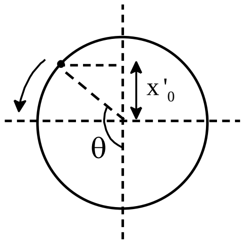


If A be the osc amp of $(m + 2m)$

$\Rightarrow \frac{1}{3}\sqrt{2gh} = \sqrt{\frac{k}{3m}}\left(A^2 - x'^2_0\right)$

$3\frac{2gh}{g} = \frac{k}{3m}\left[A^2 - \left(\frac{mg}{k}\right)^2\right]$

$A = \sqrt{\frac{7}{3}}\frac{mg}{k}$



$\theta = \frac{h}{2} + \sin^{-1}\left(\frac{x'_0}{A}\right)$

$\theta = \frac{P}{2} + \sin^{-1}\sqrt{\frac{3}{7}}$

$t = \sqrt{\frac{2h}{g}} + \frac{\theta}{\omega} = 2\sqrt{\frac{m}{k}} + \sqrt{\frac{3m}{k}}\left(\frac{\pi}{2} + \sin^{-1}\sqrt{\frac{3}{7}}\right)$

18. (2)

For adiabatic pressure

$PV^\gamma = C$

$\therefore\ dp = \frac{-\gamma P}{V}dV$

$\therefore\ dF = dp \cdot A$

Tally $V = V_0,\ P = P_0$ and $dV = Adx$

We get $K_{eff} = \left|\frac{dF}{dx}\right| = \frac{\gamma P_0 A^2}{V_0}$

$\therefore\ \omega = \sqrt{\frac{K_{eff}}{m}} = \sqrt{\frac{\gamma P_0 A^2}{mV_0}}$

19. **(7)**

$T = t_{AB} + t_{BC}$

$t_{AB} = \pi\sqrt{\frac{L}{g}}$ ...(1)

For, t_{BC} $\beta = \theta\sin(\omega t_{BC})$

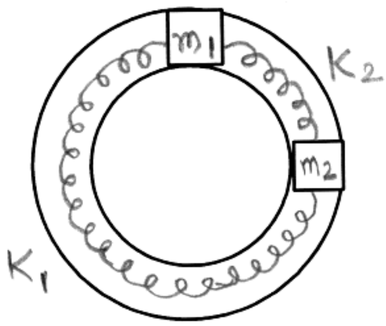
$\therefore\ t_{BC} = \frac{1}{\omega}\sin^{-1}\left(\frac{\beta}{\theta}\right)$ ...(2)

Putting values,

$T = \frac{7\pi}{60}\sec$

20. **(8)**

$K_1 = 4K,\ K_2 = \frac{4}{3}K$



$\therefore\ K_{eq} = \frac{16}{3}K$

The reduced mass $\ell_1 = \frac{m_1m_2}{m_1+m_2} = 2m$

$\omega = \sqrt{\frac{K_{eq}}{\mu}}$

$= \sqrt{\frac{8K}{3m}}$

IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 22/04/23**

TOPIC: VOLUMETRIC ANALYSIS

SOLUTIONS

21. (D)

$Mn^{+7} \longrightarrow Mn^{+2} \quad \Delta OS = 5$

$Fe_{(0.93)}O$

$\hookrightarrow \; Fe^{+2}, \; Fe^{+3}$

Let Fe^{+3} content be x

$\Rightarrow \quad Fe^{+2}$ would be $(0.93 - x)$

Since oxygen is in 2–

$\Rightarrow \quad (x \times (+3)) + ((0.93 - x) \times (+2)) = (+2)$

$\Rightarrow \quad x = 0.14 = Fe^{+3}$

$\Rightarrow \quad (0.93 - x) = 0.79 = Fe^{+2}$

$\Rightarrow \quad 5Fe^{+2}$ requires $1MnO_4^-$

$\Rightarrow \quad 0.79\ Fe^{+2}$ requires $\frac{0.79}{5} = (0.158)$

22. (B)

$I_2 + 2S_2O_3^{2-} \longrightarrow 2I^- + S_4O_6^{2-}$

$n_{S_2O_3^{2-}}$, consumed = 6m moles

η_{I_2}, consumed = 3m moles

$3I_2 + 6NaOH \longrightarrow 5NaI + NaIO_3 + 3H_2O$

η_{I_2}, consumed = $n_{NaOH,\,added} - n_{NaOH,\,reacted\ with\ H_2SO_4}$

$= \frac{(100)(0.3) - (10)(0.3)(2)}{2}$

= 12m moles

Total I_2 consumed = 15×10^{-3} moles

$[I_2] = \frac{15 \times 10^{-3}\,moles}{150 \times 10^{-3}\,L} = 0.1M$

23. (D)

$Al(MnO_4)_3 \downarrow Mn^{2+}$, $\Delta O.S. = (5)3$

$FeC_2O_4 \downarrow Fe^{+3} + CO_2$, $\Delta O.S. = (+1) + (+1)2 = 3$

$(V)(0.2)(15) = (30)(0.5)(3)$

V = 15 mL

24. **(A)**

$KHC_2O_4 \cdot H_2C_2O_4 \cdot 2H_2O$ (3 Replaceable H)

$\Rightarrow \quad n_{KHC_2O_4 \cdot H_2C_2O_4 \cdot 2H_2O} = \frac{4.62}{3} = 1.54$

Reaction with MnO_4^-

$MnO_4^- \longrightarrow Mn^{+2} \quad \Delta O.S = 5$

$C_2O_4^{2-} \longrightarrow CO_2 \quad \Delta O.S = 2$

$n_{MnO_4^-} = n_{C_2O_4^{2-}} \times \frac{2}{5}$

$= \left[2 \times n_{KHC_2O_4 \cdot H_2C_2O_4 \cdot 2H_2O}\right] \times \frac{2}{5}$

$= 2 \times 1.54 \times \frac{2}{5} = 1.232 \text{ mole } MnO_4^-$

25. **(D)**

No. of container	1	2	3	………	n
Volume	V	2V	3V	………	nV
Moles	$1^2 n$	$2^2 n$	$3^2 n$	………	$n^2 n$

$(\text{Pressure})_{\text{initial}} = P$

$\Rightarrow \quad P = \frac{nRT}{V}$

$\Rightarrow$ When stopcocks opened : $V_T = V_1 + V_2 + ... + V_n$

$n_T = n_1 + n_2 + ... + n_n$

$V_T = (1 + 2 + 3 + ... + n)V$

$n_T = (1^2 + 2^2 + 3^2 + ... + n^2)n$

$P_T = (1^2 + 2^2 + ... + n^2)nRT$

$$= \frac{(n)(n+1)(2n+1)\left(\frac{1}{6}\right)nRT}{(n)(n+1)\left(\frac{1}{2}\right)V}$$

$$= \frac{2n+1}{3}P$$

26. **(BC)**

$Cl_2 \rightarrow Cl^- \quad ClO_3^-$

$\qquad\quad \downarrow \qquad\quad \downarrow$

$\qquad\;\; -1 \qquad +5$

Balancing : $3Cl_2 + 6OH^- \rightarrow 5Cl^- + ClO_3^-$

(B) $\quad E_{ox} = \frac{M}{10} = 7.1$

(C) Stoichiometric Ratios.

27. **(ABD)**

$Al \longrightarrow Al^{+3} \quad \Delta O.S. = +3$ oxidation

$Mn^{+7} \longrightarrow Mn^{+2} \quad \Delta O.S. = -5$ Reduction

$\Rightarrow \quad 10Al + 6KMnO_4 + 27H_2SO_4 \rightarrow 6KHSO_4 + 5Al_2(SO_4)_3 + 24H_2O$

Steps (i) Balancing ΔO.S. of reduction & oxidation reactions
(ii) Balancing atoms of oxidized and reduced elements
Step (iii) balancing potassium
Step (iv) Balancing $\left(SO_4^{2-}\right)$
Step (v) Balancing and verifying H & O atoms on the both sides
step (vi) final check for charge balance

$\Rightarrow$ $x=27, \quad y=6, \quad z=24, w=10$

(A) $(2x-z)=30$

(B) $(z+y-w)=20$

(C) $3w \neq 18$

(D) $\frac{x}{y}-\frac{z}{w}=\frac{9}{2}-2.4=2.1$

28. (AD)

$SO_2+O_3 \longrightarrow SO_3+O_2$

$SO_3+H_2O \longrightarrow H_2SO_4$

$\frac{n_{NaOH}}{2}=n_{H_2SO_4}=n_{SO_3}=n_{SO_2}$

$\Rightarrow \quad n_{SO_2}=\frac{40\times10^{-3}\times0.01}{2}$

$=2\times10^{-4}$

$V_{SO_2 9 STP}=44.8\times10^{-4}\,L$

(A) % by volume of $SO_2=0.448$

(C) milimoles of $SO_2=0.2, \quad \neq 2$

(D) V_{SO_2} at 273ºC and 2 atm = 4.48 mL

29. (AD)

$6n\ CO_2 \longrightarrow (C_6H_{10}O_5)_n$

$(M.W)_{C_6H_{10}O_5}=12(6)+1(10)+16(5)$

$=72+10+80=162g$

100% increase in mass $=2\times10^{-3}g$

Moles of starch monomer absorbed $=\left[\frac{2\times10^{-3}}{162}\right]$

$\Rightarrow$ 6 moles CO_2 gives 1 monomer starch

$\Rightarrow$ moles of CO_2 absorbed $=\left[10^{-3}/81\right]\times6$

7×10^{-4} moles CO_2 absorbed in 60 min.

$\frac{2}{27}\times10^{-3}$ moles CO_2 absorbed in $\left[\left(2/27\right)\times10^{-3}\right]\times 60/7\times10^{-4}$

$\Rightarrow \quad \frac{2}{9}\times\frac{200}{7}=6.34\,\text{mins}$

$\Rightarrow$ a is correct ,d is correct……

30. (C)

$H_2O_2+2KI \longrightarrow I_2+2KOH$ Reaction (i)

$I_2 + 2S_2O_3^{2-} \longrightarrow S_4O_6^{2-} + 2I^-$ Reaction (ii)

$H_2O_2 + H_2S \longrightarrow S + 2H_2O$ Reaction (iii)

$\frac{n_{S_2O_3^{2-}}}{2} = n_{I_2}$ Reaction (ii)

$n_{I_2} = n_{H_2O_2}$ Reaction (i)

$\Rightarrow$ $n_{H_2O_2} = \frac{n_{s_2o_3^{2-}}}{2} = 50\text{m moles}$

$\Rightarrow$ $n_{H_2S} = n_{h_2o_2}$ Reaction (iii)

$n_{H_2O_2} = \frac{3.4}{34} = 0.1\text{moles}$

$\Rightarrow$ 50mL H_2O_2 solution contains 50m moles H_2O_2

$\Rightarrow$ 100m moles H_2O_2 will be present in 100mL H_2O_2 solution

31. **(AC)**

$2Na_2CO_3 + H_2SO_4 \longrightarrow 2NaHCO_3 + Na_2SO_4$

$n_{Na_2CO_3} = 2 \times n_{H_2SO_4}$

$= 2 \times 2.5 \times 0.1\text{m moles}$

$(NaHCO_3)_{\text{original present}} = (NaHCO_3)_{\text{produced from } Na_2CO_3}$

Since $(n_{H_2SO_4})_{\text{required}} = 2.5 \times 0.2\text{m moles}$

$2NaHCO_3 + H_2SO_4 \longrightarrow 2H_2CO_3 + Na_2SO_4$

$\Rightarrow$ originally present $NaHCO_3$

$= 2 \times 2.5 \times 0.1\text{m moles}$

$= 2 \times 2.5 \times 0.1 \times 10^{-3} \times 84\text{g}$

$= 42 \times 10^{-3}\text{g}$ in 10 mL solution

$\Rightarrow$ 4.2g in 1 L solution

32. (B, C)

No. of meq of $KMnO_4 = 10 \times 0.1 \times 5 = 5$ meq

No. of meq of $Ca\left(HC_2O_4\right) = 10 \times 0.1 \times 4 = 4$ meq.

$\underset{(4\ meq)}{Ca\left(HC_2O_4\right)_2} \longrightarrow \underset{(4\ meq)}{CO_2}$

$4\ meq\ CO_2 \Rightarrow 4\ mmoles$

1 meq $KMnO_4$ is left = 1/5 th millimoles

33. **(BD)**

Theoretical

34. **(BC)**

$K_xH_y(C_2O_4)_z \cdot nH_2O + NaOH \longrightarrow \text{Pdts.}$

$\frac{9.15 \times 30 \times y}{(M \cdot M) \times 1000} = \frac{27 \times 0.12}{1000}$ equation (1)

$K_xH_y(C_2O_4)_z \cdot nH_2O + KMnO_4 \longrightarrow CO_2 + \text{other Pdts.}$

$\frac{9.15 \times 30 + 2z}{(1000) \times (M \cdot M)} = 36 \times 0.12 \times 10^{-3}$ equation (2)

$$\frac{\text{equation }2}{\text{equation }1} = \frac{2z}{y} = \frac{4}{3} \Rightarrow \frac{z}{y} = \frac{2}{3}$$

Balancing charge : $x + y - 2z = 0$

$\Rightarrow \quad x : y : z = 1 : 3 : 2$

$\Rightarrow \quad \frac{x}{y} = \frac{1}{3}$

using equation (1) $M \cdot M = 254$, n = 2 (formula: $KH_3(C_2O_4)_2 2H_2O$)

35. **(ABCD)**

Theoretical

36. (8)

3MV = $20 \times \frac{1}{20} \times 2 = 2$

Molarity of $KHC_2O_4 . H_2C_2O_4 . 2H_2O$ = 2/3V

With $KMnO_4$

Meq of $KHC_2O_4 . H_2C_2O_4 . 2H_2O$ =Meq of $KMnO_4$

$\frac{2}{3V}\ (4V) = x \times \frac{1}{15} \times 5$

$x = 8$ ml

37. **(6)**

$KMnO_4 \longrightarrow Mn^{+2} \qquad \Delta O.S = -5$

$FeC_2O_4 \longrightarrow Fe^{+3} + 2CO_2 \qquad \Delta O.S = (+1) + (2 \times (+1)) = +3$

$n_{FeC_2O_4} \times 3 = n_{KMnO_4} \times 5$

$\Rightarrow n_{KMnO_4} = 10 \times 3/5 = 6$

38. **(2)**

$MnO_4^- + M^{+x} \rightarrow MnO_2 + MO_3^- + \frac{1}{2} O_2$

Given: $\frac{1}{3}$ mole of $Al(MnO_4)_3$

$\Rightarrow$ 1 mole of MnO_4^-

$\Rightarrow \Delta O.S$ for $MnO_4^- = +7 - 4 = 3$

$\Rightarrow n_{MnO_4^-} \times |\Delta O.S| = n_{M^{+x}} \times |\Delta O.S|$

$\Rightarrow 1 \times 3 = 1 \times (5 - x)$

$x = 2$

39. **(2)**

$2N_2H_4 \longrightarrow X + 16e^-$

$1\ N^{2-} \longrightarrow 1\ N^{y} + 4e^-$

$\Rightarrow -2 = y - 4$

$\Rightarrow y = +2$

40. **(1)**

$KMnO_4 + KHC_2O_4.HCOONa.2H_2O$

$Mno_4^- \longrightarrow Mn^{+2} \qquad \Delta O.S = -5$

$C_2O_4^{2-} + COO^{2-} \rightarrow CO_2 \qquad \Delta O.S = 4$

$n_{KMnO4} \times |\Delta O.S| = n_{KHC_2O_4.HCOON.2H_2O} \times |\Delta O.S|$

$n_{KMnO_4} = (1 \times 4) = 4/5 = A/B$

$\Rightarrow ANS = |B - A| = 1$

IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 23/04/23**

TOPIC: CONTINUITY

SOLUTIONS

41. **(D)**

$$f(x)=\left| x[x]+\frac{[x]}{2} \right|$$

$[x]$ is discontinuous at integers

$$f(x)=\begin{cases} -2\left(x+\frac{1}{2}\right); -2\le x\le -1 \\ x+\frac{1}{2} \quad ; -1\le x< -\frac{1}{2} \\ 0 \quad ; \quad x=\frac{-1}{2} \\ x+\frac{1}{2}; \quad \frac{-1}{2}<x<0 \\ 0 \quad ; \quad x=0 \\ 0 \quad ; \quad 0<x<1 \\ x+\frac{1}{2} \quad ; \quad 1<x<2 \\ 2\left(x+\frac{1}{2}\right) \quad ; \quad x=2 \end{cases}$$

$\therefore$ discontinuous at $x = 2, 1, -1, 0$

42. **(B)**

$$f(x)=\frac{\tan x\log(x-2)}{(x^2-4x+3)}$$

$\tan x$ not defined at $x \in n\pi + \pi/2$; $n \in$ Integer

$\log(x-2)$ not defined at $x \in (-\infty, 2]$ and $f(x)$ not defined at $x \in \{1, 3\}$

Hence, set of discontinuous points would be

$x \in (-\infty, 2] \cup \{3, n\pi + \pi/2 ; n \in I\}$

Hence, option (B) is correct Answer.

43. **(B)**

$$f(x)=\begin{cases} 1 \quad \text{if} \quad x \quad \text{is} \quad \text{rational} \\ -1 \quad \text{if} \quad x \quad \text{is} \quad \text{irrational} \end{cases}$$

$\lim_{x\to Q} f(x) = 1$; $x \in Q$ (Rational Number)

$\lim_{x\to Q^C} f(x) = -1$; $x \in Q^C$ (Irrational number)

Since, there is no unique limit. Hence set of continuous points would be empty.

$\therefore$ Option (B) is correct Answer.

44. **(D)**

$f(x) = \sec 2x + \text{cosec } 2x$

$f(x) = \dfrac{\sin 2x + \cos 2x}{\sin 2x . \cos 2x} \times \dfrac{2}{2}$

$f(x) = \dfrac{\sin 2x + \cos 2x}{\sin 4x} \times 2$

$f(x)$ not to be defined at $\sin 4x = 0 = \sin n\pi$; $n \in I$

$4x = n\pi$

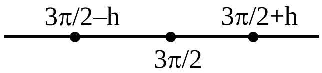
$\Rightarrow \; x = \dfrac{n\pi}{4}; n \in I$

$\therefore$ Option (D) is correct Answer.

45. **(D)**

$f(x) = [\sin x] + |x|$

At $x = \dfrac{3\pi}{2}$



L.H.L. = $\lim\limits_{x \to 3\pi/2^-} f(x) = \lim\limits_{h \to 0} f\left(\dfrac{3\pi}{2} - h\right)$

$= \lim\limits_{h \to 0} \left\{ \left[\sin\left(\dfrac{3\pi}{2} - h\right) \right] + \left| \dfrac{3\pi}{2} - h \right| \right\}$

$= \lim\limits_{h \to 0} \left\{ \left[-\sin\left(\dfrac{\pi}{2} - h\right) \right] + h - \dfrac{3\pi}{2} \right\}$

$= \lim\limits_{h \to 0} \left\{ [-(\text{value less than } 1)] + h - 3\pi/2 \right\}$

$= -1 - 3\pi/2$

R.H.L = $\lim\limits_{x \to \frac{3\pi}{2}^+} f(x) = \lim\limits_{h \to 0} f\left(\dfrac{3\pi}{2} + h\right)$

$= \lim\limits_{h \to 0} \left\{ \left[\sin\left(\dfrac{3\pi}{2} + h\right) \right] + \left| \dfrac{3\pi}{2} + h \right| \right\}$

$= \lim\limits_{h \to 0} \left\{ \left[-\sin\left(\dfrac{\pi}{2} + h\right) \right] + \dfrac{3\pi}{2} + h \right\}$

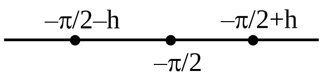
$= \lim\limits_{h \to 0} \left\{ [-\text{Value less than } 1] + \dfrac{3\pi}{2} + h \right\}$

$= -1 + 3\pi/2$

Since, L.H.L $\neq$ R.H.L

Hence, $f(x)$ is Discontinuous at $x = 3\pi/2$

At $x = -\pi/2$



L.H.L = $\lim_{x \to \frac{-\pi}{2}^-} f(x) = \lim_{h \to 0} f\left(-\frac{\pi}{2} - h\right)$

$= \lim_{h \to 0} \left\{\left[\sin\left(-\frac{\pi}{2} - h\right) + \left|-\frac{\pi}{2} - h\right|\right]\right\}$

$= \lim_{h \to 0} \left\{\left[-\sin\left(\frac{\pi}{2} + h\right) + \frac{\pi}{2} + h\right]\right\}$

$= \lim_{h \to 0} \left\{\left[-\text{value less than } 1\right] + \frac{\pi}{2} + h\right\}$

$= -1 + \pi/2$

R.H.L = $\lim_{h \to \frac{-\pi}{2}^+} f(x) = \lim_{h \to 0} f(-\pi/2 + h)$

$= \lim_{h \to 0} \left\{[\sin(-\pi/2 + h) + \left|\frac{-\pi}{2} + h\right|]\right\}$

$= \lim_{h \to 0} \left\{\left[-\sin\left(\frac{\pi}{2} - h\right)\right] + h - \pi/2\right\}$

$= \lim_{h \to 0} \left\{\left[-\text{Value less than } 1\right] + h - \pi/2\right\}$

$= -1 - \pi/2$

Sine, L.H.L $\neq$ R.H.L

$\therefore$ $f(x)$ is Discontinuous at $x = -\pi/2$. In general, we can say that greatest Integer function is discontinuous at all points, $x \in n\pi + (-1)^n \frac{\pi}{2}$; $n \in$ Integer

46. **(A, B)**

$f(x) = \frac{1}{1-x}$; $x \neq 1 \Rightarrow f(f(x)) = 1/1 - f(x)$

$= 1/1 - \frac{1}{1-x} = 1/\frac{1-x-1}{1-x}$

$= \frac{1-x}{-x} = \frac{x-1}{x}$; $x \neq 0$

$f(f(f(x)) = 1/1 - f(f(x)) = 1/1 - \frac{x-1}{x} = 1/\frac{x-x+1}{x}$

$= x$; $x \in R$ except $x \neq 0, 1$

Hence, $y = f(f(f(x)))$ will be discontinuous at $x = 0,1$

47. **(B, D)**

$f(x) = \left[x\left[\frac{1}{x}\right]\right]$ for $x > 0$

At $x = 1/2$ $f\left(\frac{1}{2}\right) = \lim_{x \to \frac{1}{2}} f(x) = \lim_{x \to \frac{1}{2}} \left[\frac{1}{2}[2]\right]$

$= \lim_{x \to \frac{1}{2}} \left[\frac{1}{2} \times 2\right] = 1$

L.H.L. = $\lim_{x \to \frac{1}{2}} f(x) = \lim_{h \to 0} f\left(\frac{1}{2} - h\right)$

$$= \lim_{h\to 0}\left[\left(\frac{1}{2}-h\right)\times\left[\frac{1}{\frac{1}{2}-h}\right]\right] = \lim_{h\to 0}\left[\left(\frac{1}{2}-h\right)\times 1\right]$$

$$\left(\text{Because}\,\frac{1}{\frac{1}{2}-h}>1\right)$$

$$= \lim_{h\to 0}\,[\text{value less than } 1] = 0$$

R.H.L. $= \lim_{x\to \frac{1^+}{2}} f(x) = \lim_{h\to 0} f\left(\frac{1}{2}+h\right)$

$$= \lim_{h\to 0}\left[\left(\frac{1}{2}+h\right)\left[\frac{1}{\frac{1}{2}+h}\right]\right]$$

$$= \lim_{h\to 0}\left[\left(\frac{1}{2}+h\right)\times 1\right]$$

$$\text{Because}\left[\frac{1}{\frac{1}{2}+h}\right] = 1 = 0$$

Since, L.H.L. = R.H.L. $\neq f\left(\frac{1}{2}\right)$

Hence, f(x) is not continuous at x = $\frac{1}{2}$

At x = 3/4

$f\left(\frac{3}{4}\right) = \lim_{x\to 3/4} f(x) = \lim_{h\to 0} f\left(\frac{3}{4}-h\right)$

$$= \lim_{h\to 0}\left[\left(\frac{3}{4}-h\right)\left[\frac{1}{\frac{3}{4}-h}\right]\right]$$

$$= \lim_{h\to 0}\left[\left(\frac{3}{4}-h\right).1\right]$$

Because $\left[\frac{1}{\frac{3}{4}-h}\right] = 1$

$= \lim_{h\to 0}$ [value less than 1] = 0

R.H.L. $= \lim_{x\to \frac{3^+}{4}} f(x) = \lim_{h\to 0} f(3/4 + h)$

$$= \lim_{h\to 0}\left[\left(\frac{3}{4}+h\right)\left[\frac{1}{\frac{3}{4}+h}\right]\right]$$

$$= \lim_{h\to 0}\left[\left(\frac{3}{4}+h\right).1\right]$$

Because , $\left[\dfrac{1}{\frac{3}{4}+h}\right] = 1$

$= \lim_{h\to 0}$ [Value less then 1] = 0

Since, L.H.L. = R.H.L. = f(3/4) = 0

Hence, f(x) is continuous at x = 3/4.

Therefore, we can make conclusions that there will be infinite number of Points where f(x) will be discontinuous.

Hence, Options (B) and (D) are correct answers.

48. **(B, D)**

$$f(x) = \begin{cases} 3-\left[\cot^{-1}\left(\dfrac{2x^3-3}{x^2}\right)\right] & \text{for } x > 0 \\ \{x^2\}\cos(e^{1/x}) \text{for } x < 0 \end{cases}$$

$$\underset{0-h \quad\quad 0 \quad\quad 0+h}{\longleftarrow\!\!\!-\!\!\!-\!\!\!\longrightarrow}$$

L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} ((0-h)^2-[(0-h)^2]) \times \cos\left(\dfrac{1}{e^{(0-h)}}\right)$

$= \lim_{h\to 0} [h^2 - [h^2]] \times \cos (e^{-1/h})$

$= \lim_{h\to 0} [h^2 - [h^2]] \times \cos e^{-\infty}$

$= \lim_{h\to 0} [h^2 - h^2] \times \cos 0 = 0 \times 1 = 0$

R.H.L. $= \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0 + h)$

$= \lim_{h\to 0}\left\{3-\left[\cot^{-1}\left(\dfrac{2(0+h)^3-3}{(0+h)^2}\right)\right]\right\}$

$= \lim_{h\to 0}\left\{3-\left[\cot^{-1}\left(\dfrac{2h^3-3}{h^2}\right)\right]\right\}$

$= \lim_{h\to 0} \{3-[\cot^{-1}(-\infty)]\}$

$= \lim_{h\to 0} \{3-[3.14]\} = 0$

$f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0}\left\{3-\left[\cot^{-1}\left(\dfrac{2x^3-3}{x^2}\right)\right]\right\}$

$= \lim_{x\to 0} \{3-[\cot^{-1}(-\infty)]\}$

$= \lim_{x\to 0} \{3-[\pi]\} = 0$

Hence, L.H.L. = R.H.L. = f(0) = 0

$\therefore$ f(x) is continuous at x = 0

49. **(A, B, C)**

For A : $f(x) = 1/1+2^{1/x}$

$$\underset{0-h \quad\quad 0 \quad\quad 0+h}{\longleftarrow\!\!\!-\!\!\!-\!\!\!\longrightarrow}$$

L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$$= \lim_{h\to 0} \frac{1}{1+2^{-\frac{1}{h}}} = 1$$

R.H.L.$= \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$$= \lim_{h\to 0} \frac{1}{1+2^{\frac{1}{h}}}$$

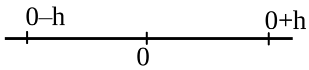
$$= \lim_{h\to 0} \frac{1}{1+2^{\infty}} = 0$$

$f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1}{1+2^{1/x}} = \lim_{x\to 0} \frac{1}{1+2^{\infty}}$

$= 0$

Since, L.H.L. $\neq$ R.H.L.

Hence. It is Discontinuous as well as have non removable discontinuity .



For B : $f(x) = \tan^{-1} \frac{1}{x}$

L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$$= \lim_{h\to 0} \tan^{-1}\left(\frac{1}{0-h}\right)$$

$$= \lim_{h\to 0} \tan^{-1}\left(\frac{-1}{h}\right)$$

$$= \tan^{-1}(\infty) = -\pi/2$$

R.H.L.$= \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

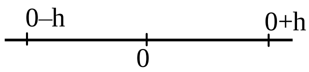
$$= \lim_{h\to 0} \tan^{-1}\left(\frac{1}{0+h}\right)$$

$$= \tan^{-1}(\infty) = \pi/2$$

Since, R.H.L. $\neq$ L.H.L.

Hence, f(x) is discontinuous and have non-removable discontinuity.

For C : $f(x) = \frac{e^{1/x}-1}{e^{1/x}+1}$



L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$$= \lim_{h\to 0} \frac{e^{-1/h}-1}{e^{-1/h}+1}$$

$$= \frac{e^{-\infty}-1}{e^{-\infty}+1} = \frac{0-1}{0+1} = -1.$$

R.H.L.$= \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$$= \lim_{h\to 0} \frac{e^{1/h}-1}{e^{1/h}+1}$$

$$= \lim_{h\to 0} \frac{1-e^{-1/h}}{1+e^{-1/h}}$$

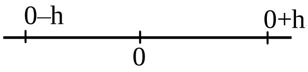
$$= \frac{1-e^{-\infty}}{1+e^{-\infty}} = \frac{1-0}{1+0} = 1.$$

Since, R.H.L. $\neq$ L.H.L.

Hence, f(x) is discontinuous and have non-removable discontinuity

For D : $f(x) = 1/\ell n|x|$

$= \begin{cases} 1/\ell n x; x \geq 0 \\ 1/\ell n(-x); x < 0 \end{cases}$



L.H.L. = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$

$= \lim_{h \to 0} 1/\ell n\,(-(0-h))$

$= \lim_{h \to 0} 1/\ell n\, h = 1/-\infty = 0$

R.H.L. = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h)$

$= \lim_{h \to 0} 1/\ell n h = 1/(-\infty) = 0$

$f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} 1/\ell n|0| = 1/-\infty = 0$

Hence, f(x) is continuous at x = 0 as
L.H.L. = R.H.L. = f(0) = 0.
$\therefore$ options (A), (B) and (C) are correct answers.

50. **(B, C, D)**

For A: $f(x) = \frac{1}{1+2^{\cos x}}$

$f(0) = \lim_{h \to 0} f(x) = \lim_{x \to 0} \frac{1}{1+2^{\cos 0}} = \frac{1}{3}$

L.H.L. = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$

$= \lim_{h \to 0} \frac{1}{1+2^{\cos(0-h)}}$

$= \lim_{h \to 0} \frac{1}{1+2^{\cos h}} = \frac{1}{1+2} = \frac{1}{3}$

R.H.L. = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h)$

$= \lim_{h \to 0} \frac{1}{1+2^{\cos(0+h)}}$

$= \lim_{h \to 0} \frac{1}{1+2^{\cos h}}$

$= \frac{1}{1+2} = \frac{1}{3}$

Since, L.H.L. = R.H.L. = f(0) = 1/3

Therefore, $f(x) = \frac{1}{1+2\cos x}$ is continuous at x =0

For B : $f(x) = \cos\left(\frac{|\sin x|}{x}\right)$

It is discontinuous function at x = 0
have removable discontinuity as follows :

L.H.L = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$

$= \lim_{h \to 0} \cos\left(\frac{-\sin(0-h)}{0-h}\right)$

$= \lim_{h \to 0} \cos\left(\frac{\sin h}{-h}\right)$

As $\lim_{h \to 0} \frac{\sin h}{-h} = -1$

$= \lim_{h \to 0} \cos(-1) = \cos 1.$

R.H.L. $= \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h)$

$= \lim_{h \to 0} \cos\left(\frac{\sin(0+h)}{h}\right)$

$= \lim_{h \to 0} \cos\left(\frac{\sin h}{h}\right)$

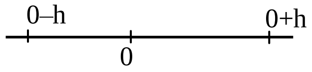
As $\lim_{h \to 0} \frac{\sin h}{h} = 1$

$= \cos 1$

Since, R.H.L. = L.H.L. = cos1.

Hence, it has removable discontinuity

For C : $f(x) = x \sin\pi/x$. It is not defined at x = 0.



L.H.L. $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$

$= \lim_{h \to 0} (0-h) \sin\left(\frac{\pi}{0-h}\right)$

$= \lim_{h \to 0} (-h) \times \sin\left(\frac{\pi}{-h}\right)$

$= \lim_{h \to 0} h \times \sin\pi/h$

$= \lim_{h \to 0} \frac{\sin \pi/h}{\pi/h} \times \pi$

$= 1 \times \pi \quad = \pi$

R.H.L. $= \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h)$

$= \lim_{h \to 0} (0 + h)\sin\left(\frac{\pi}{0+h}\right)$

$= \lim_{h \to 0} h \sin \frac{\pi}{h}$

$= \lim_{h \to 0} \frac{\sin \pi/h}{\pi/h} \times \pi$

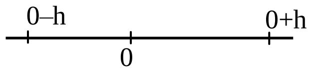
$= 1 \times \pi = \pi$

Since, L.H.L. = R.H.L. = π

Hence, it has removable discontinuity.

For D : $f(x) = 1/\ell n|x| = \begin{cases} 1/\ell n x; x > 0 \\ 1/\ell n(-x); x < 0 \end{cases}$

It is not defined at x = 0.



L.H.L. $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$

$= \lim_{h \to 0} 1/\ell n\,(-(0-h))$

$= \lim_{h \to 0} 1/\ell n\, h$

$= 1/(-\infty)$

$= 0$

R.H.L. $= \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h)$

$= \lim_{h \to 0} 1/\ell n\,(0 + h)$

$= \lim_{h\to 0} 1/\ell nh$

$= 1/(-\infty) = 0$

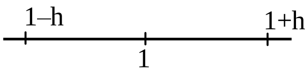
Since, R.H.L. = L.H.L. = 0

Hence, it has removable discontinuity.

options (B), (C), and (D) are correct answers.

51. **(B, D)**

For A : $f(x) = 1/\ell n\,|x|$



L.H.L $= \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$

$= \lim_{h\to 0} 1/\ell n\,(1-h) = -$ ve quantity (as $\ell n\,(1-h) = -$ve)

R.H.L. $= \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$

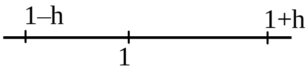
$= \lim_{h\to 0} 1/\ell n\,(1+h) = +$ ve quantity

Since, R.H.L. $\neq$ L.H.L.

Hence, f(x) is discotinuous and have non-removable discontinuity

$\therefore$ Option (A) is not correct answer.

For B : $f(x) = \dfrac{x^2-1}{x^3-1}$. It is not defined at x = 1.



L.H.L. $= \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$

$= \lim_{h\to 0} \dfrac{(1-h)^2-1}{(1-h)^3-1} \left(\dfrac{0}{0}\text{form}\right)$

Apply L–H Rule, we get.

$= \lim_{h\to 0} \dfrac{2(1-h)(-1)-0}{3(1-h)^2(-1)-0} = 2/3$

R.H.L.$= \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$

$= \lim_{h\to 0} \dfrac{(1+h)^2-1}{(1+h)^3-1} \left(\dfrac{0}{0}\text{form}\right)$

Apply L – H Rule, we get

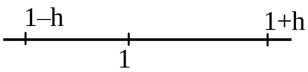
$= \lim_{h\to 0} \dfrac{2(1+h)-0}{3(1+h)^2-0} = 2/3$

Since, L.H.L. = R.H.L. = 2/3

Hence, it has removable discontinuity.

$\therefore$ Option (B) is correct answer.

For C : $f(x) = 2^{-2^{\frac{1}{1-x}}}$



L.H.L. $= \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$

$= \lim_{h\to 0} 2^{-2^{\frac{1}{1-(1-h)}}} = \lim_{h\to 0} 2^{-2^{\frac{1}{h}}}$

$= 2^{-2^{\infty}} = 2^{-\infty} = 0$

R.H.L. $= \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$

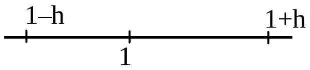
$= \lim_{h\to 0} 2^{-2^{\frac{1}{1-(1+h)}}} = \lim_{h\to 0} 2^{-2^{-1/h}} = 2^{-2^{-\infty}} = 2^{-0} = 1$

Since, R.H.L. $\neq$ L.H.L.

Hence, It has non removable discontinuity.

For D : $f(x) = \dfrac{\sqrt{x+1}-\sqrt{2x}}{x^2 - x}$.

It is not defined at x = 1.



L.H.L. $= \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$

$= \lim_{h\to 0} \dfrac{\sqrt{1-h+1}-\sqrt{2(1-h)}}{(1-h)^2-(1-h)}$

$= \lim_{h\to 0} \dfrac{\sqrt{2-h}-\sqrt{2(1-h)}}{(1-h)^2-(1-h)} \left(\dfrac{0}{0}\text{form}\right)$

Use L–H Rule, weget.

$= \lim_{h\to 0} \dfrac{\dfrac{1}{2\sqrt{2-h}}(-1) - \dfrac{1}{2\sqrt{2(1-h)}}\times 2(-1)}{2(1-h)(-1)+1}$

$= \lim_{h\to 0} \dfrac{\dfrac{-1}{2\sqrt{2-h}} + \dfrac{1}{\sqrt{2(1-h)}}}{2(h-1)+1}$

$= \dfrac{-\dfrac{1}{2\sqrt{2}} + \dfrac{1}{\sqrt{2}}}{-1} = -1/2\sqrt{2}$

R.H.L. $= \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$

$= \lim_{h\to 0} \dfrac{\sqrt{1+h+1}-\sqrt{2(1+h)}}{(1+h)^2-(1+h)}$

$= \lim_{h\to 0} \dfrac{\sqrt{2+h}-\sqrt{2(1+h)}}{(1+h)^2-(1+h)} \left(\dfrac{0}{0}\text{form}\right)$

Use, L–H rule, we get

$= \lim_{h\to 0} \dfrac{\dfrac{1}{2\sqrt{2+h}} - \dfrac{1}{2\sqrt{2(1+h)}}\times 2.1}{2(1+h)-1}$

$= \dfrac{\dfrac{1}{2\sqrt{2}} - \dfrac{1}{\sqrt{2}}}{1} = -1/2\sqrt{2}$

Since, L.H.L. = R.H.L. = $-1/2\sqrt{2}$.

$\therefore$ It has removable discontinuity.

52. **(B, C, D)**

53. **(B, C)**

$f(x) = \sqrt{x}$ and $g(x) = x - 1$

(fog) (x)= f(g(x)) = $\sqrt{(x-1)}$

(fog) (x) is defined when $(x-1) \geq 0$

$\Rightarrow x \geq 1$

$\therefore$ (fog) (x) is continuous in $[1,\infty)$

(gof) (x) = g (f(x))
$= \left(\sqrt{x}-1\right)$
(gof) (x) is define when x ≥ 0.
i.e. (gof) (x) is continuous in $[0,\infty)$
∴ Options (B) and (C) are correct answers.

54. **(B, C, D)**
$f(x) = \lim_{n\to\infty} (\sin x)^{2n}$
We have to check for every option
At x = π/2, it is $(1)^\infty$ type. Which is indeterminate form.
∴ f(x) is discontinuous at x = π/2
At $x = \frac{3\pi}{2}$, It is $(1)^\infty$ type which is also undefined form.
∴ f(x) is discontinuous x = 3π/2.
Hence, there will be infinite number of points where f(x) will be undefined.
∴ Options, (B), (C) and (D) are correct answers.

55. **(A)**
$f(x) = \sqrt{\sin^{-1}x} + \sqrt{\cos^{-1}x}$ defined in [0, 1]
Replace x by (x – 3), we get
$f(x-3) = \sqrt{\sin^{-1}(x-3)} + \sqrt{\cos^{-1}(x-3)}$
then $0 \le x-3 \le 1$
$\Rightarrow 3 \le x \le 4$
$\Rightarrow x \in [3, 4]$
∴ Option (A) is correct Answer.

56. (2)
$$\lim_{x\to0} f(x) = \frac{1}{3} \Rightarrow \lim_{x\to0}\frac{x\cdot\alpha\cot x+\beta}{x^2} = \frac{1}{3} \Rightarrow \lim_{x\to0}\frac{x\alpha+\beta\tan x}{x^2\cdot\tan x} = \frac{1}{3}$$
$$\Rightarrow \lim_{x\to0}\frac{\alpha x+\beta\left(x+\frac{x^3}{3}+\ldots\ldots\infty\right)}{x^3\left(\frac{\tan x}{x}\right)} = \frac{1}{3} \Rightarrow \lim_{x\to0}\frac{(\alpha+\beta)x+\left(\frac{\beta}{3}\right)x^3+\ldots\ldots\infty}{x^3} = \frac{1}{3}$$
So, $(\alpha+\beta) = 0$
Also, $\beta = 1 \Rightarrow \alpha = -1$
Hence, $(\alpha^2+\beta^2) = 2$]

57. (0)
$f(x) = \operatorname{sgn}\left((x-2)^2+k^2\right)$. Clearly, f (x) is discontinuous at exactly one point i.e., x = 2 for k = 0.
$$\therefore \tan^{-1}k + \cos^{-1}k + \operatorname{cosec}^{-1}(2k-1) = 0 + \frac{\pi}{2} + \left(\frac{-\pi}{2}\right) = 0$$ **Ans.**

58. (1)
$$\lim_{x\to0^+}\frac{e^{(x-2)\ln\sqrt{5^x}+\frac{1}{3x}\log_2 2^{6x}}-e^2}{(x-2)\tan x} \Rightarrow \lim_{x\to0^+}\frac{e^{(x-2)\ln\left(5^{x/2}\right)+2}-e^2}{(x-2)\tan x}$$

$$\lim_{x\to 0^+} \frac{e^2\left(e^{\ln\left(5^{\left(\frac{x(x-2)}{2}\right)}\right)}-1\right)}{(x-2)\tan x} = \lim_{x\to 0^+} e^2 \frac{\left(5^{\left(\frac{x(x-2)}{2}\right)}-1\right)}{\frac{2x(x-2)}{2}\cdot\frac{\tan x}{x}} = \frac{1}{2}e^2\ln 5$$

$$\lim_{x\to 0^-} \lambda \frac{e^2\left(\left(\sqrt{5}\right)^x-1\right)\left(e^{x^2}-1\right)}{x\cdot\frac{2\sin x}{x}\cdot\frac{(1-\cos x)}{x^2}\cdot x^2} = \lambda\, e^2\cdot\ln\sqrt{5} = \lambda\frac{e^2}{2}\ln 5$$

$$\lambda\, e^2\cdot\frac{1}{2}\ln 5 = \frac{1}{2}e^2\ln 5 \;\Rightarrow\; \lambda = 1$$

59. (0)

Define $f\left(x\right)=\begin{cases} -2x; & -\infty < x < \frac{-3}{2} \\ 2-\frac{2x}{3}; & \frac{-3}{2} \le x \le \frac{3}{4} \\ 2x; & \frac{3}{4} < x < \infty \end{cases}$

So, f (x) is continuous for all x $\in$ R.

60. (1)

Using continuity at x = 0

$$\lim_{x\to 0^-}\frac{1+x-e^x+\cos x-1}{x^2} = b \;\Rightarrow\; b=-1$$

Using continuity at x = 1

$$\lim_{x\to 1^+}\frac{x^2-2px+1}{(x-1)(x-4)} = a-1 \text{ for } \left(\frac{0}{0}\right) \text{ from p = 1}$$

$\Rightarrow$ a – 1 = 0 $\Rightarrow$ a = 1

$\Rightarrow$ a + b + p = 1 **Ans.**]