**IIT – JEE: 2024** **TW TEST (MAIN)** **DATE: 28/01/23**

**TOPIC: COM & COLLISION**

## SOLUTIONS

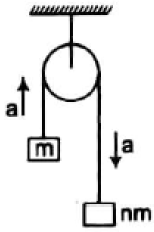
1. (A)

$$\vec{a}_{cm} = \frac{-ma\,\hat{i} + 2ma\,\hat{j} + 3ma\,\hat{i} - 4ma\,\hat{j}}{10m}$$

2. (C)

$$a = \frac{n\,mg - mg}{nm + m}$$

$$= \left(\frac{n-1}{n+1}\right)g$$




Now, $a_{\text{CM}} = \dfrac{(nm)(a) - (ma)}{nm + m}$

$$= \left(\frac{n-1}{n+1}\right)a$$

$$= \left(\frac{n-1}{n+1}\right)^2 g$$

3. (C)

$$P_1 = P_2 = P$$

$$\therefore \quad K_T = K_1 + K_2 = \frac{P^2}{2m_1} + \frac{P^2}{2m_2}$$

$$= \frac{P^2(m_1 + m_2)}{2m_1 m_2}$$

4. (D)

$$2mv_0 + O = 3mv_y \qquad \Rightarrow v_y = {}^{2v_o}\!/_{3}$$

5. (C)

Since, the lower end moves towards positive $x$-axis the force of friction will be along negative $x$-direction. Therefore, centre of mass of the rod will finally fall at $x < 0$. Hence, the lower end will be at $x < \dfrac{l}{2}$.

6. (C)

7. (A)

$V_{cm,y} = O \qquad \Rightarrow 10 \times {}^{3}\!/\!_{5} - 2 \times V \times {}^{5}\!/\!_{13} = O$

8. (B)

Block just reaches the top of the wedge, it implies that velocity of block with respect to wedge at the top of the wedge is zero. Let *v* be the horizontal velocity of both at this instant.

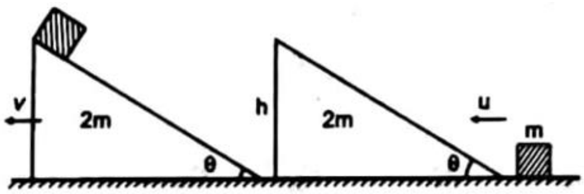
Then, from conservation of linear momentum, we have

$$(2m+m)v = mu$$

or $\quad v = \frac{u}{3}$

Now from conservation of mechanical energy, we get

$$\frac{1}{2}mu^2 = \frac{1}{2}(3m)v^2 + mgh$$




or $\quad u^2 = 3\left(\frac{u^2}{9}\right) + 2gh$

or $\quad \frac{2}{3}u^2 = 2gh$ or $u = \sqrt{3gh}$

9. (C)

Let 50 kg moues x. COM is at rest

$$50x - 70(6-x) = 0$$

10. (C)

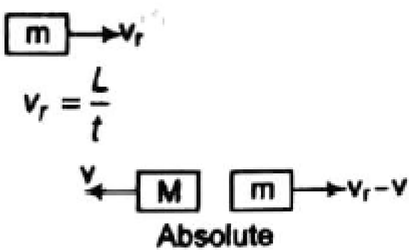
P is centre of mass

$$CP = \frac{m2l + 2m \times l}{3m} = {}^{4l}\!/\!_{3}$$

11. (C)

$$\vec{r}_{cm} = \frac{(\hat{i} + 2\hat{j} + \hat{k}) + (-9\hat{i} - 6\hat{j} + 3\hat{k})}{4} = -2\hat{i} - \hat{j} + \hat{k}$$

12. (B)




$$P_i = P_t$$

$\therefore \quad 0 = m(v_r - v) = -Mv$

$\therefore \quad v = \dfrac{m}{m+m} v_r = \left(\dfrac{m}{M+m}\right)\left(\dfrac{L}{t}\right)$

13. (B)
99m(-x) + m(50 – x) = 0 consternation of momentum

14. (D)

15. (B)
Centre of mass will move in a vertical line if $v_1 \cos\theta_1 = v_2 \cos\theta_2$.
Otherwise for any other values it will follow a parabolic path.

16. (A)
From conservation of mechanical energy

$$\frac{1}{2}kx^2 = \frac{1}{2}\mu v_r^2 \qquad ...(1)$$

Here, $\mu =$ reduced mass of the blocks

$$= \frac{(m)(2m)}{m+2m} = \frac{2}{3}m$$

and $v_r =$ relative velocity of the two.
Substituting in Eq. (1), we get

$$kx^2 = \frac{2}{3}mv_r^2$$

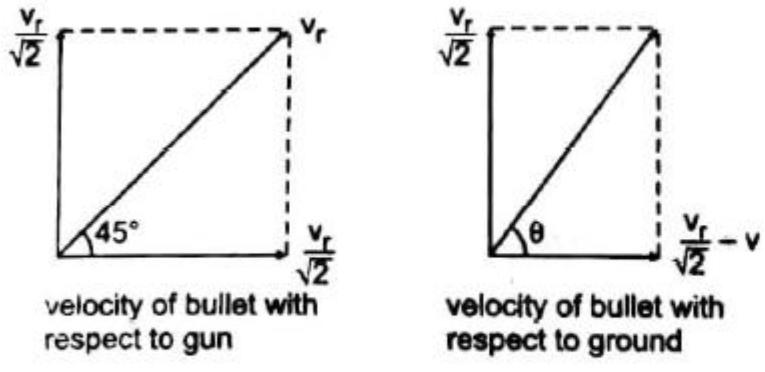
$$\therefore \quad v_r = \left(\sqrt{\frac{3k}{2m}}\right)x$$

17. (B)
Conservation of linear momentum

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = O \qquad \Rightarrow \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$

$\Rightarrow \vec{p}_3 = -\left|\vec{p}_1 + \vec{p}_2\right|$

18. (C)
Let $v_r$ be the velocity of bullet with respect to gun and *v* the velocity of gun. Then,




From the two figures it is clear that $\theta > 45^\circ$.

19. (A)

Let $v'$ be the velocity of block. The from conservation of linear momentum.

$$mu = mv + mnv' \text{ or } v' = \left(\frac{u-v}{n}\right)$$

$\therefore$ Velocity of bullet relative to block will be

$$v_r = v - v' = v - \left(\frac{u-v}{n}\right)$$

$$= \frac{(1+n)v - u}{n}$$

20. (C)

Kinetic energy of the system of particles = KE of centre of mass + KE of difference particles in the frame of reference of centre of mass.

Here, KE of centre of mass is $\frac{1}{2}mv^2$

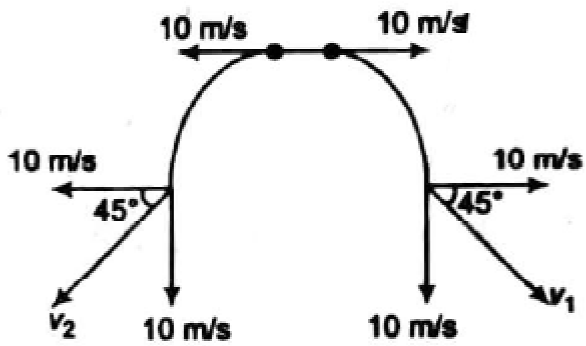
$\therefore$ KE of the system of particles $\geq \frac{1}{2}mv^2$

21. (2)

1(12 – x) + 5 (– x) = O $\Rightarrow$ x = 2 m.

22. (1)

At 1 s




23. (4)

Velocity of block just after collision $= \sqrt{2gh}$

$$= \sqrt{2 \times 10 \times 0.2}$$

$$= 2\,\text{m/s}$$

Now applying conservation of linear momentum just before and just after collision.

$$0.02 \times 600 = 4 \times 2 = 0.02 \times v$$

$\therefore$ $v = 200\,\text{m/s}$

24. (5)

$$KE\ w.r.t.com = \frac{1}{2} * \frac{5 \times 2}{5+2} \times (5+2)^2 = 35J$$

25. (3)

Conservation of linear momentum

$50\times10=(50+M)\times2.5$

$\Rightarrow$ M = 150 kg

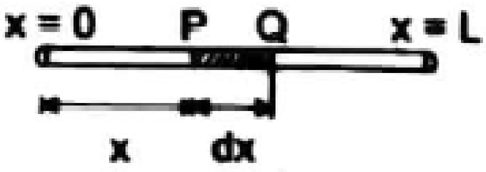
26. (13)

Area under $F-t$ graph = Impulse $\Delta P=m\left(v_f-v_i\right)$

$\therefore \quad v_f=\dfrac{\text{Area under } F-t \text{ graph}}{m}$ as $v_i=0$

$=\dfrac{16-3}{2}=6.5\,\text{m/s}$

27. (3)




Mass of the element $PQ$ is $dm=\dfrac{Kx^2}{L}\cdot dx$

$$\therefore\ x_{\text{com}}=\frac{\int_0^L x\,dm}{\int_0^L dm}=\frac{\int_0^L \frac{Kx^3}{L}dx}{\int_0^L \frac{Kx^2}{L}dx}=\frac{\left(\frac{L^4}{4}\right)}{\left(\frac{L^3}{3}\right)}=\frac{3L}{4}$$

28. (3)

$F=\dfrac{\Delta p}{\Delta t}=n\left(mv\right)$

Here, $n$ = number of bullets fired per second.

$\therefore \quad n=\dfrac{F}{mv}$

$=\dfrac{144}{0.04\times1200}=3$

29. (2)

$$\frac{1}{2}\times50\times x^2=\frac{1}{2}\times\frac{6\times3}{6+3}(10-0)^2$$

30. (6)

Taking $A$ is origin and $AB$ as position $x$-axis

$$x_{\text{CM}}=\frac{A_1x_1-A_2x_2}{A_1-A_2}$$

$$=\frac{\left(\pi R^2\right)(0)-\left(\frac{\pi R^2}{4}\right)\left(\frac{R}{2}\right)}{\pi r^3-\left(\frac{\pi R^2}{4}\right)}=-\frac{R}{6}$$

**IIT – JEE: 2024** **TW TEST (MAIN)** **DATE: 28/01/23**

**TOPIC: GOC**

## ANSWER KEY

31. (A)
32. (A)
33. (A)
34. (A)
35. (B)
36. (B)
37. (B)
38. (D)
39. (C)
40. (B)
41. (D)
42. (C)
43. (C)
44. (A)
45. (B)
46. (B)
47. (B)
48. (D)
49. (D)
50. (D)
51. (6)
52. (5)
53. (3)
54. (5)
55. (5)
56. (9)
57. (3)
58. (6)
59. (5)
60. (6)

**IIT – JEE: 2024** **TW TEST (MAIN)** **DATE: 28/01/23**

**TOPIC: P & C**

## SOLUTIONS

61. (C)

$^{5}C_{2} \cdot {}^{6}C_{3}$

62. (B)

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |

$9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 3 \quad = 3 \times 9^{6}$

63. (B)

Since two families has 3 members each and one family with four members. So that can be seared among themselves, so same family members are not separated in 3!, 3! and 4! Respectively.

Now the groups (means families) can arrange in 3! ways.

So, required number of ways is

$3! \times 3! \times 4! \times 3! = (3!)^{3} \cdot 4!$

64. (C)

Given, $^{n}P_{r} = {}^{n}P_{r+1}$

$\Rightarrow \quad \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow \quad \frac{n!}{(n-r)(n-r-1)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow \quad n-r=1 \qquad \ldots(\text{i})$

and $^{n}C_{r} = {}^{n}C_{r-1}$

$\Rightarrow \quad \frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)!(n-r+1)!}$

$\Rightarrow \quad \frac{1}{r(n-r)!} = \frac{n!}{(n-r+1)!(n-r)!}$

$\Rightarrow \quad n-r+1=r$

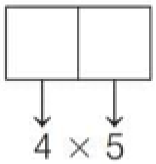
From Eq. (i),

$1+1=r \Rightarrow r=2$

65. (A)

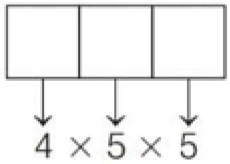
Using the digits 0, 1, 3, 7, 9

Number of one digit natural numbers that can be formed = 4,
Number of two digit natural numbers that can be formed = 20,

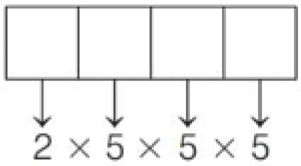



($\because 0$ can not come in Ist box)

Number of three digit natural numbers that can be formed = 100




and number of four digit natural numbers less than 7000, that can be formed = 250




($\because$ only 1 or 3 can come in Ist box)

$\therefore$ Total number of natural numbers formed = 4 + 20 + 100 + 250 = 374

66. (D)

Clearly, number of words start with $A = \frac{4!}{2!} = 12$

Number of words start with L = 4! = 24

Number of words start with M $= \frac{4!}{2!} = 12$

Number of words start with SA $= \frac{3!}{2!} = 3$

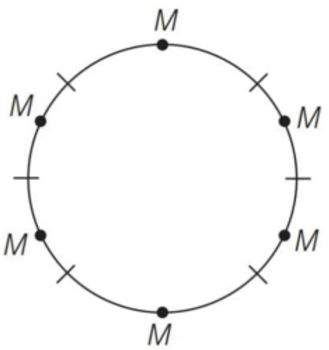
Number of words start with SL $= 3! = 6$

Note that, next word will be “SMALL”.

Hence, the position of word “SMALL” is 58th.

67. (A)

First, we fix the position of men, the number of ways to sit men = 5! And the number of ways to sit women = ${}^6P_5$




$\therefore$ Total number of ways = 5! × ${}^6P_5$

$= 5! \times 6!$

68. (C)

| | Boys | Girls |
|---|---|---|
| Team $A$ | 7 | $n$ |
| Team $B$ | 4 | 6 |

Number of matches between Team $A$ and Team $B$ when a boy play against a boy $\left({}^{7}C_1 \times {}^{4}C_1\right) = 28$

Similarly, number of matches between Team $A$ and Team $B$ when a girl play against a girl $\left({}^{n}C_1 \times {}^{6}C_1\right) = 6n$

According to question,

$$28 + 6n = 52$$
$$6n = 24$$
$$n = 4$$

69. (B)

To make 6-digit numbers from given digits 1, 3, 5, 7 and 9, we must repeat a digit and we can done the same in ${}^{5}C_1$ ways.

Now, the arrangement of these 6-digits in which two are identical is $\frac{6!}{2!}$.

So, required numbers of 6-digit numbers

$$= {}^{5}C_1 \frac{6!}{2!} = \frac{5}{2}(6!)$$

70. (B)

Given, $T_n = {}^{n}C_3$

$$T_{n+1} = {}^{n+1}C_3$$

$\therefore \quad T_{n+1} - T_n = {}^{n+1}C_3 - {}^{n}C_3 = 10 \quad$ [given]

$\Rightarrow \quad {}^{n}C_2 + {}^{n}C_3 - {}^{n}C_3 = 10$

$\Rightarrow \quad {}^{n}C_2 = 10$

$\therefore \quad n = 5$

71. (C)

Required number of ways

$$= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4$$
$$= \frac{12!}{8! \times 4!} \times \frac{8!}{4! \times 4!} \times = \frac{12!}{(4!)^3}$$

72. (B)

The required number of ways $= {}^{8-1}C_{3-1}$

$$= {}^{7}C_2 = \frac{7!}{2!5!} = \frac{7 \cdot 5}{2 \cdot 1} = 21$$

73. (B)

Here, we use ${}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r \qquad .....(\text{i})$

Now, ${}^{n}C_{r+1} + {}^{n}C_{r-1} + 2 \cdot {}^{n}C_r$

$= {}^{n}C_{r+1} + {}^{n}C_{r-1} + {}^{n}C_{r} + {}^{n}C_{r}$
$= {}^{n}C_{r+1} + {}^{n+1}C_{r} + {}^{n}C_{r}$ [using eq. (i)]
$= {}^{n+1}C_{r+1} + {}^{n+1}C_{r}$ [using eq. (i)]
$= {}^{n+2}C_{r+1}$ [using eq. (i)]

74. (C)
Required number of ways = $2^7 - 1 = 127$. {Since the case that no friend be invited i.e., ${}^{7}C_{0}$ is excluded}.

75. (A)
Since as per the given condition $x > -1$,
So *x* is non-negative integer,
$y > -2$ so $y = -1 + b$ and similarly $z > 3$ so $z = -2 + c$ or $(x) + (-1+b) + (-2+c) = 23$ or $x + b + c = 23$ and we need to find the number of non-negative integral solution of the equation $x + b + c = 23$ which is, ${}^{23+3-1}C_{3-1} = {}^{25}C_{2} = {}^{25}C_{23}$

76. (B)
The straight line $\ell_1, \ell_2, \ell_3$ are parallel and lie in the same plane.
Total number of points = $m + n + k$
Total no. of triangles formed with vertices $= {}^{m+n+k}C_3$
By joining three given points on the same line we don't obtain a triangle.
Therefore, the maximum number of triangles
$= {}^{m+n+k}C_3 - {}^{m}C_3 - {}^{n}C_3 - {}^{k}C_3$

77. (A)
Obviously, *A*, *B* and *C* get 4, 5 and 7 objects, respectively.
Then number of distribution ways is equal to number of division of ways, which is given by 16! (4!5!7!)

78. (A)
Number of words in which all the 5 letters are repeated = $10^5$ = 100000 and the number of words in which no letter is repeated are ${}^{10}P_5 = 30240$.
Hence, the required number of ways are 100000 – 30240 = 69760

79. (A)
For *A*, *B*, *C* to speak in order of alphabets, 3 places out of 10 may be chosen first in $1.{}^{3}C_2 = 3$ ways.
The remaining 7 persons can speaks in 7! Ways.
Hence, the number of ways in which all the 10 person can speak is
${}^{10}C_3.7! = \frac{10!}{3!} = \frac{10!}{6}$.

80. (C)
Required number of ways 9! × 2. {By fundamental property of circular permutation}.

81. (2454)

Given word is 'EXAMINATION' having letters (AA), (II), (NN), EXMOT, we have to form 4 letters words, then following cases are possible

(I) 2 same, 2 same and number of words are $^3C_2 \times \frac{4!}{2!2!} = 18$

(II) 2 same, 2 different and number of words are

$$^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 3 \times \frac{7\times 6}{2} \times \frac{4\times 3\times 2}{2} = 21\times 36 = 756$$

(III) All are different and number of words are

$$^8C_4 \times 4! = \frac{8\times 7\times 6\times 5}{4!} 4! = 56\times 30 = 1680$$

So, total number of 4 letter words are

18 + 756 + 1680 = 2454

Hence, answer is 2454.

82. (135)

Number of ways to select four questions from six questions = $^6C_4$

And number of ways to answer these questions correctly = $1\times 1\times 1\times 1 = 1$

And number of ways to answer remains two questions wrongly $= 3\times 3 = 9$

$\therefore$ required number of ways $= {}^6C_4 \times 1\times 9$

$$= \frac{6!}{2!4!}\times 9$$

$$= \frac{6\times 5}{2}\times 9 = 135$$

83. (54)

Let the digits of 3-digit numbers are $x$, $y$, $z$ such that

$x + y + z = 10$ and $x, y, z \in \{0, 1, 2, 3, , ....., 9\}$, but $x \neq 0$

Now, let $x = t + 1, t \in \{0, 1, 2, 3, ......, 8\}$

So, $t + 1 + y + z = 10$

$\Rightarrow t + y + z + = 9$ having non-negative integral solution $= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = 55$

But, it include the case, when $t$ = 9

$\Rightarrow x = 10$, which is not possible, so required number of 3-digit numbers

= 55 – 1 = 54

Hence, answer is 54.00.

84. (300)

Let the number be $xyz, 0 \le x, y, z \le 9$

Case I '3' appears only one time

$\Rightarrow {}^3C_1 \times 9\times 9 = 243$

Case II '3' appears two times

$\Rightarrow {}^3C_2 \times 2\times 9 = 54$

Case III '3' appears three times

$\Rightarrow {}^3C_3 \times 3 = 3$

$\therefore$ Total = 243 + 54 + 3 = 300

85. (777)

Total number of players = 15

Bowlers = 6, Batsman = 7, Wicket keepers = 2

| **Bowlers** | **Batsman** | **Wicket Keepers** | **Total** |
|---|---|---|---|
| 4 + 1 | 5 | 1 | ${}^6C_5 \times {}^7C_5 \times {}^2C_1 = 252$ |
| 4 | 5 + 1 | 1 | ${}^6C_4 \times {}^7C_6 \times {}^2C_1 = 210$ |
| 4 | 5 | 1 + 1 | ${}^6C_4 \times {}^7C_5 \times {}^2C_2 = 315$ |

Total = 252 + 210 + 315 = 777

86. (238)

| 10(5) | 11(6) | 12(8) |
|---|---|---|
| 2 + 1 | 2 | 2 + 3 |
| 2 | 2 + 1 | 2 + 3 |
| 2 | 2 | 2 + 4 |
| | 5 | 5 |

$\Rightarrow \ {}^5C_3 \times {}^6C_2 \times {}^8C_5 = 8400$

$\Rightarrow \ {}^5C_2 \times {}^6C_3 \times {}^8C_5 = 11200$

$\Rightarrow \ {}^5C_2 \times {}^6C_2 \times {}^8C_6 = 4200$

Total = 8400 + 11200 + 4200 = 23800

According to the question, $100k = 23800$

$k = 238$

87. (309)

Given word in MOTHER, now alphabetical order of letters is EHMORT, so number of words start with letter.

E ------ is 5! H ------ is 5!

M E ------ is 4! M H ------ is 4!

M O E ------ is 3! M O H ------ is 3!

M O R ------ is 3! M O T E ------ is 2!

M O T H E R is 1

So, position of the word ‘MOTHER’ is

5! + 5! + 4! + 4! + 3! + 3! + 3! + 2! + 1

= 120 + 120 + 24 + 24 + 6 + 6 + 6 + 2 + 1

= 309

88. (1000)

Let $x$ be four digit number, then $gcd(x, 18) = 3$

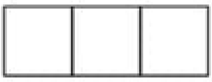
This implies $x$ is divisible by 3 but not divisible by 9.

The 4-digit numbers which is an odd multiple of 3 are 1005, 1011, 1017, …. 9999

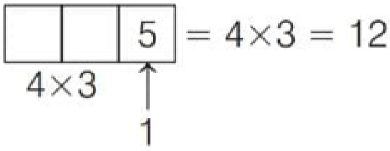
These are 1499 in counting i.e. total number of 4-digit numbers which is odd multiple of 3 are 1499.

Now, the 4-digit numbers which is an odd multiple of 9 are, 1017, 1035, …999
These, are total 499.
Then, required 4-digit numbers
= 1499 – 499 = 1000

89. (32)
Given, digits = {1, 2, 3, 4, 5}
Numbers divisible by 3 (sum of digits divisible by 3).



Case I When sum is 12 $\rightarrow$ 3, 4, 5 $\rightarrow$ 3! = 6
Case II When sum is 9 $\rightarrow$ 2, 3, 4 $\rightarrow$ 3! = 6
Case III When sum is 9 $\rightarrow$ 1, 3, 5 $\rightarrow$ 3! = 6
Case IV When sum is 6 $\rightarrow$ 1, 2, 3 $\rightarrow$ 3! = 6
So, total numbers divisible by
$= 6 \times 4 = 24$
Numbers divisible by 5 (ending with 5)




So, total numbers divisible by 5 = 12
Numbers divisible by 15, are 145, 415, 345, 435
i.e. total 4 numbers are divisible by both 3 and 5.
i.e. divisible by 15.
Hence, the required numbers which are divisible by 3 or 5
= 24 + 12 – 4 = 32

90. (45)
We may write, $7^n = (10-3)^n$ or $7^n = 10K + (-3)^n$ (using expansion)
$\therefore\ 7^n + 3^n = 10K + (-3)^n + 3^n$
$$= \begin{cases} 10k, & n = \text{odd} \\ 10k + 2.3^n, & n = \text{even} \end{cases}$$
Let $n$ = even = $2t,\ t \in N$
Then, $3^n = 3^{2t} = 9^t = (10-1)^t$
$$= 10p + (-1)^t$$
$$= 10p \pm 1$$
If $n$ = even, then $7^n + 3^n$ will never be multiple of 10.
This implies $n$ = odd
$n$ = 11, 13, 15, …. 99 (since, $n$ is two digit)
$\Rightarrow\quad 10 < n < 100$
Total possible '$n$' are 45.