IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 04/01/23**

TOPIC: RAY OPTICS

SOLUTIONS

1. (B)

$$m = \frac{f}{f-u} = \frac{-20}{-20-20} = \frac{1}{2}$$

2. (C)

3. (A)
Real inverted and magnified image

4. (B)

5. (D)

6. (A)
Both images move opposite to O with u

7. (B)
Diminished, erect image is formed by convex mirror.

8. (A)
According to New Cartesian sign convention, O
Object distance u = -15cm
Focal length of a concave lens, f = -10 cm
Height of the object h_0 =2.0cm

According to mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-10}-\frac{1}{-15} \Rightarrow v=-30\text{cm}$$

This image is formed 30 cm from the mirror on the same side of the object. It is a real image

Magnification of the mirror, m= $\frac{-v}{u}=\frac{h_f}{h_0}$

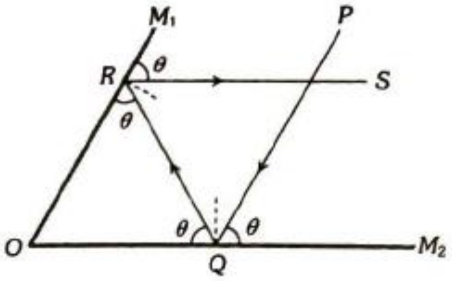
$$\Rightarrow \frac{-(-30)}{-15}=\frac{h_1}{2} \Rightarrow h_f=-4\text{cm}$$

Negative sign shows that image is inverted.
The image is real, inverted, of size 4 cm at a distance 30 cm in front of the mirror.

9. (C)
Number of images = $\left(\frac{360}{\theta}-1\right)=\left(\frac{160}{60}-1\right)=5$

10. (A)
A concave mirror forms real image of virtual object

11. (C)
Let the angle between the two mirror be θ, Ray PQ is parallel to mirror M_1 and RS is parallel to M_2.



So $\angle M_1RS = \angle ORQ = \angle M_1OM_2 = \theta$
Similarly $\angle M_2RS = \angle OQR = \angle M_2OM_2 = \theta$
$\therefore$ In $\Delta ORQ, 3\theta = 180^0, \theta = 60^0$

12. (C)

13. (C)

14. (D)
$\delta = 180^0 - 60^0 = 120^0$

15 (B)
$\frac{1}{O} = \frac{f}{f-u}$; where $u = f + x \therefore \frac{1}{O} = -\frac{f}{x}$

16. (D)
Theory

17. (A)
$\frac{f}{f-u} = 2, u_1 = f/2 - (1), \frac{f}{f-u_2} = -2 \Rightarrow u_1 = 3f/2 - (2)$

18. (A)
Ray should be $\perp$ to mirror

19. (B)
$f = \frac{R}{2}$ and $R = \infty$ for plane mirror

20. (A)
$\frac{300cm \operatorname{cosec} 30^0}{2cm/s} = 300$

21. (11)
$360^0 - 2\theta = 300^0 \Rightarrow \theta = 30^0$
So, 11 images

22. (10)

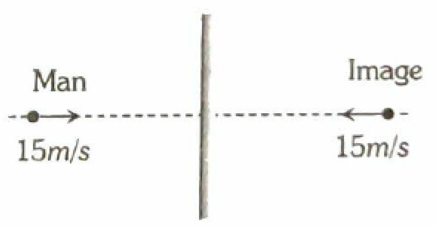
23. (60)

24. (5)

25 (30)

Relative velocity of image w.r.t. man

15 – (-15)= 30 m/s



26. (7)

If $\frac{360}{\theta}$ = fraction, then n is the integer next higher than $\left(\frac{360}{\theta}-1\right)$ $\therefore$ n = 7

27. (7)

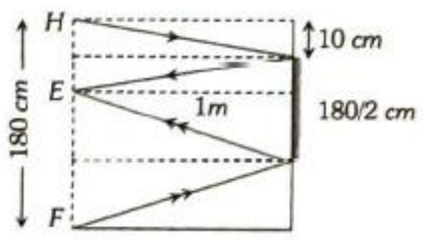
The walls will act as two mirrors inclined to each other at 90^0 and so will form $\left(\frac{360}{90}-1\right)=4-1$, i.e., 3 images of the person. Now these images with person will act as objects for the ceiling mirror and so ceiling mirror will form 4 images further. Therefore total number of images formed = 3 + 3 + 1 =7

Note: He can see, 6 images of himself.

28. (90)

According to the following ray diagram length of mirror

$=\frac{1}{2}(10-170)=90\text{cm}$



29. (9)

Here u = -20cm, f = -15cm

$\frac{1}{\upsilon}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{\upsilon}=\frac{1}{-15}-\frac{1}{-20}=-\frac{1}{15}+\frac{1}{20}=-\frac{1}{60}$

$\Rightarrow \upsilon=-60\text{cm}, u=-20\text{cm}$

$\therefore m=-\frac{\upsilon}{u}=-\frac{(-60)}{(-20)}=-3\text{cm}$ or $|m|=3\text{cm}$

Each side of the square is now 3 cm, area = 9cm^2

30. (25)

For convex mirror, $\upsilon = 0.1\text{m}, u = -0.5\text{m}$

So, $\frac{1}{f} = \frac{1}{0.1} - \frac{1}{0.5} = 8 \Rightarrow f = \frac{1}{8}\text{m}$

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TOPIC: ENERGETIC

SOLUTIONS

31. (A)

32. (A)
As we know that, work done by a system against an external pressure is given as :
$W = -P_{ext.}\,\Delta V$
Given:-
$P_{ext.} = 1\text{atm}$
$V_f = 15\,L$
$V_i = 3\,L$
$\therefore\ W = -1(15-3)$
$\Rightarrow\ W = -12\,L\text{-}atm = -1215.96\,J = 1.215\times10^3\,J$
1 L atm = 101.325 J

33. (D)

34. (C)

35. (D)
$\Delta E = nC_V\Delta T$
For isothermal process, $\Delta T = 0$
Hence, change in internal energy (ΔE) is zero during isothermal expansion of a gas.

36. (B)

37. (A)
In an isochoric process, $\Delta V = 0$, hence, work done P
$\Delta V = W = 0$. So, $\Delta E = q + 0$.
Hence, the increase in internal energy will be equal to heat absorbed by the system.

38. (B)

39. (B)

40. (C)

41. (C)

42. (C)

43. (B)

44. (D)

45. (C)

46. (C)

47. (A)

48. (D)

49. (B)

50. (B)

51. (5)

52. (900)

53. (14)

54. (720)

55. (1200)

56. (350)

57. (150)

58. (100)

59. (900)

60. (100)

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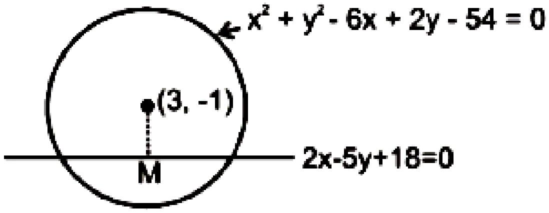
TOPIC: CIRCLE

SOLUTIONS

61. (A)

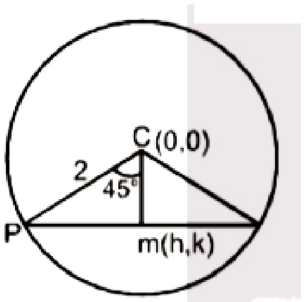
Required point is foot of $\perp$



$$\frac{x-3}{2}=\frac{y+1}{-5}=\left(\frac{6+5+18}{4+25}\right)=-1 \Rightarrow x=1, y=4$$

62. (C)

$$\cos 45^\circ=\frac{cm}{cp}=\frac{\sqrt{h^2+k^2}}{2}$$



Hence locus $x^2+y^2=2$

63. (B)

Let point on line be $(h, 4-2h)$ (chord of contact)

$hx+y(4-2h)=1$

$\Rightarrow h(x-2y)+4y-1=0$

$\Rightarrow$ Point $\left(\frac{1}{2},\frac{1}{4}\right)$

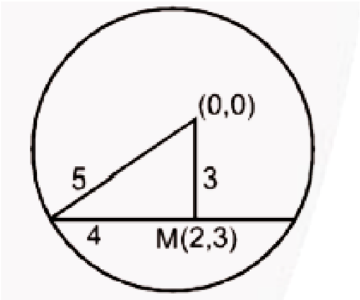
64. (B)

Let slope of required line is m

$y-3=m(x-2) \quad \Rightarrow mx-y+(3-2m)=0$

length of $\perp$ from origin $=3$

$\Rightarrow 9+4m^2-12m=9m^2 \Rightarrow 5m^2+12m=0 \Rightarrow m=0,-\frac{12}{5}$



Hence lines are $y-3=0 \Rightarrow y=3$

$y-2=-\frac{12}{5}(x-2)$

$\Rightarrow 5y-15=-12x+24$

$\Rightarrow 12x+5y=39$

65. (A)

Let any point on the circle $x^2+y^2+2gx+2fy+p=0\,(\alpha,\beta)$

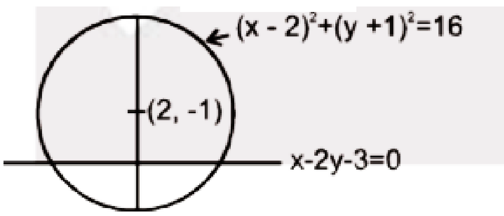
This point satisfies $\alpha^2+\beta^2+2g\alpha+2f\beta+p=0$

Length of tangent from this point to circle $x^2+y^2+2gx+2fy+q=0$

Length $=\sqrt{S_1}=\sqrt{\alpha^2+\beta^2+2g\alpha+q}=\sqrt{q-p}$

66. (B)

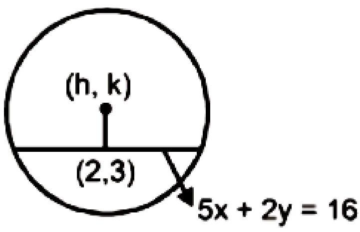
Required diameter is $\perp$ to given line



Hence $y+1=-(x-2)$

$\Rightarrow 2x+y-3=0$

67. (A)



$\left(\frac{k-3}{h-2}\right)\left(-\frac{5}{2}\right)=-1 \Rightarrow 2x-5y+11=0$

68. (A)

Let mid-point be $(h,k) \Rightarrow hx+ky=h^2+k^2$

Subtends right angle $\Rightarrow x^2-2(x+y)\left(\frac{hx+ky}{h^2+k^2}\right)=0$

$\Rightarrow (h^2+k^2)x^2-2(x+y)(hx+ky)=0$

Since angle 90°, Coefficient of x^2+ Coefficient of $y^2=0 \Rightarrow h^2+k^2-2h-2k=0$

$\Rightarrow$ Locus $x^2+y^2-2x-2y=0$

69. (C)

Length of intercepts on x-axis $=2\sqrt{g^2-c}=2\sqrt{\frac{25}{4}+14}=2\sqrt{\frac{81}{4}}=9$

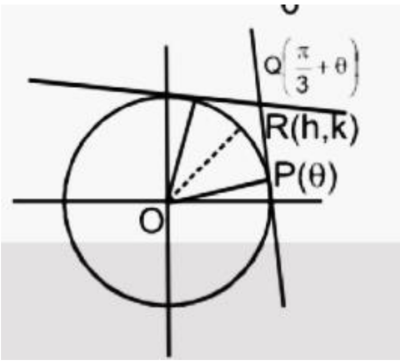
On y-axis $=2\sqrt{f^2-c}=2\sqrt{\left(\frac{13}{2}\right)^2+14}=2\sqrt{\frac{169+56}{2}}=2\sqrt{\frac{225}{4}}=15$

70. (C)

$(0,0)$ lies on director circle of given circle, hence angle $=\frac{\pi}{2}$

71. (A)

$\angle POQ=\frac{\pi}{3}$ and $\angle POR=\frac{\pi}{6}$



$OP = OR\cos 30^\circ$

$a=\sqrt{h^2+k^2}\,\frac{\sqrt{3}}{2}$

$\Rightarrow x^2+y^2=\frac{4a^2}{3}$

72. (A)

Normal to the circle $x^2+y^2-4x+4y-17=0$ also passes through centre.

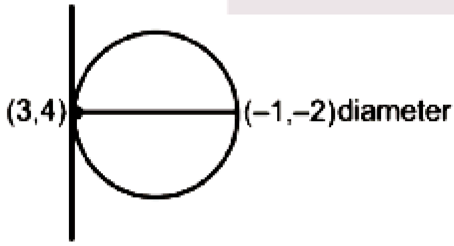
Hence its equation is line joining $(2,-2)$ and $(1,1)$

$(y-1)=\frac{1+2}{1-2}(x-1)$

$\Rightarrow y-1=-3x+3$

$\Rightarrow 3x+y-4=0$

73. (B)



$(x-3)(x+1)+(y-4)(y+2)=0$

Equation $x^2+y^2-2x-2y-11=0$

74. (A)

$S_1-S_2 \quad \Rightarrow \quad 7x-8y+16=0$

$S_2-S_3 \quad \Rightarrow \quad 2x-4y+20=0$

$S_3-S_1=0 \quad \Rightarrow \quad 9x-12y+36=0$

On solving centre $(8,9)$

Length of tangent $=\sqrt{S_1}=\sqrt{64+81-16+27-7}=\sqrt{149} \Rightarrow (x-8)^2+(y-9)^2=149$

$=x^2+y^2-16x-18y-4=0$

75. (A)

as we know $L_{int}=\sqrt{d^2-(r_1+r_2)^2}=7$

$\Rightarrow L_{ext}=\sqrt{d^2-(r_1-r_2)^2}=11$

squaring & subtract $r_1r_2=18$

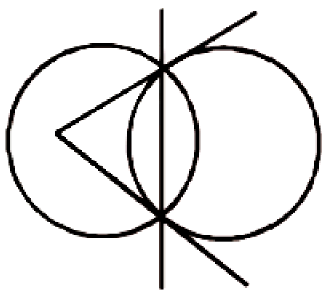
76. (B)

Equation of common chord is $S_1-S_2=0$

$\Rightarrow 5x-3y-10=0$

This chord is also chord of contact.

Let point of intersection is $p(h,k)$



Then $hx+ky-12=0$ compare both equations

$\frac{h}{5}=\frac{k}{-3}=\frac{-12}{-10}$

$\Rightarrow (h,k)\equiv\left(6,-\frac{18}{5}\right)$

77. (A)

$x^2+y^2-10x+\lambda(2x-y)=0 \qquad \ldots\ldots(i)$

$x^2+y^2+2x(\lambda-5)-\lambda y=0$

Centre $\left(-(\lambda-5),\lambda/2\right)$

Using on $y=2x \Rightarrow \frac{\lambda}{2}=-2(\lambda-5)\Rightarrow \frac{5\lambda}{2}=10$

Putting $\lambda=4 \Rightarrow x^2+y^2-2x-4y=0$

78. (B)

$y=m_1x+c_1$ and $y=m_2x+c_2$ coordinate axes at $A\left(\frac{-c_1}{m_1},0\right),B\left(\frac{-c_2}{m_2}.0\right),C(0,c_1)$ and $D(0,c_2)$.

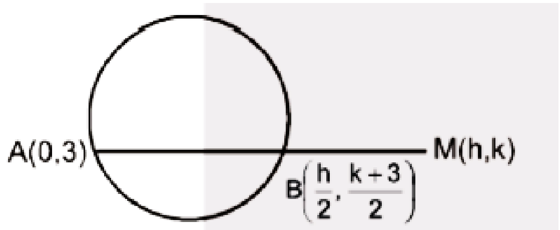
These points can be concyclic only when $x_1x_2=y_1y_2$

$\Rightarrow \frac{c_1\ c_2}{m_1\ m_2}=c_1c_2$

$\Rightarrow m_1\ m_2=1$

$\frac{b}{a}=1$

79. (B)



B lies on circle $\left(\frac{h}{2}\right)^2+4\left(\frac{h}{2}\right)+\left(\frac{k+3}{2}-3\right)^2=0$

$\Rightarrow \frac{h^2}{4}+2h+\frac{(k-3)^2}{4}=0$

Hence locus of (h,k) $x^2+8x+(y-3)^2=0$

80. (B)

If two circles touch each other, then

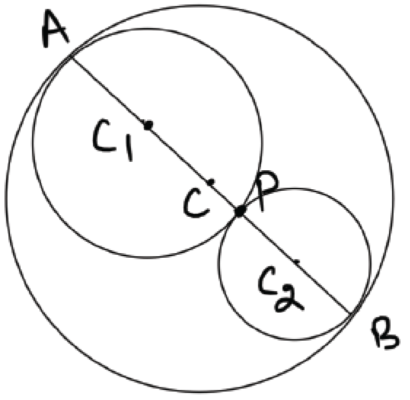
$C_1C_2=r_1+r_2$

$\sqrt{(-g_1+g_2)^2+(-f_1+f_2)^2}=\sqrt{g_1^2+f_1^2}+\sqrt{g_2^2+f_2^2}$ squaring both sides

$-2g_1g_2-2f_1f_2=2\sqrt{(g_1^2+f_2^2)(g_2^2+f_2^2)}$

$\Rightarrow (g_1f_2)^2+(g_2f_1)^2-2g_1g_2f_1f_2=0\Rightarrow \frac{g_1}{g_2}=\frac{f_1}{f_2}$

81. (2)

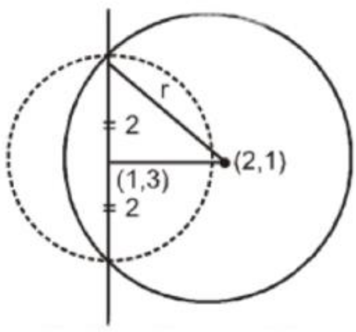


$2r_1 + 2r_2 = 2R$

$\therefore r_1 + r_2 = R = \sqrt{2+2} = 2$

82. (3)

Clearly from the figure the radius of bigger circle



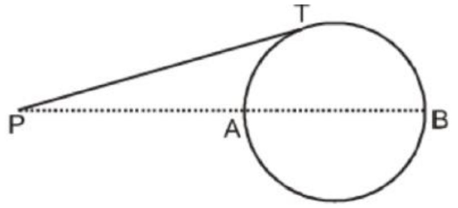
$x^2 = 2^2 + \left\{(2-1)^2 + (1-3)^2\right\}$

$r^2 = 9$ or $r = 3$

83. (12)

As we know

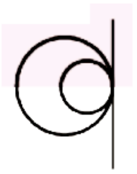
$PA.PB = PT^2 = (\text{Length of tangent})^2$



Length of tangent $= \sqrt{16 \times 9} = 12$

84. (1)

$C_1C_2 = 5$, $r_1 = 7_1 r_2 = 2$



$C_1C_2 = |r_1 - r_2|$ one common tangent

85. (2)

$S_1 - S_2$ is the required common chord i.e. $2x = a$

Make homogenous , we get $x^2 + y^2 - 8.4\frac{x^2}{a^2} = 0$

As pair of lines subtending angle of 90° at origin

$\therefore$ Coefficient of $x^2 +$ coefficient of $y^2 = 0$

As pair of lines substending angle of 90° at origin

$\therefore$ coefficient of $x^2 +$coefficient of $y^2 = 0$

$\therefore a = \pm 4$

$\Rightarrow$ Two values of a

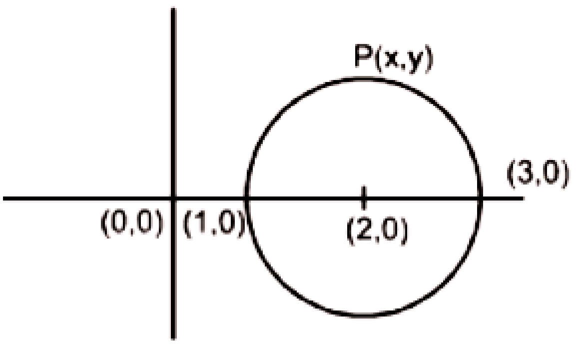
86. (0)

Parallel and distinct lines

87. (10)

$x^2 + y^2 - 4x + 3 = 0$

$\sqrt{x^2 + y^2}$ represents distance of p from origin



Hence $M = 3^2 + 0^2$

$M = 1^2 + 0^2$

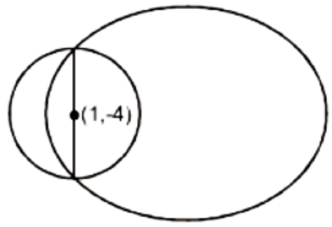
$M + m = 10$

88. (10)

Common chord of given circle

$6x + 4y + (p + q) = 0$

This is diameter of $x^2 + y^2 - 2x + 8y - q = 0$



centre $(1, -4)$

$6 - 16 + (p + q) = 0 \Rightarrow p + q = 10$

89. (5)

Tangent at $(1,-1)$ is $x(1)+y(-1)+2(x+1)-3(y-1)-12=0$

$\Rightarrow 3x-4y=7$

Required circle is

$(x-1)^2+(y+1)^2+\lambda(3x-4y-7)=0$

It pass through $(4,0)$

$\Rightarrow 9+1+\lambda(12-7)=0 \Rightarrow \lambda=-2$

$\Rightarrow$ required circle is $x^2+y^2-8x+10y+16=0$

$\Rightarrow$ Radius $=\sqrt{16+25-16}=5$

90. (1)

Point $\left(t,\frac{1}{t}\right)$ lies on $x^2+y^2=16$

$\Rightarrow t^2+\frac{1}{t^2}=16$

$\Rightarrow t^4-16t^2+1=0$ ……(i)

If roots are t_1,t_2,t_3,t_4 then

$t_1t_2t_3t_4$ ……..(ii)