IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 10/06/23**

TOPIC: FLUID MECHANICS

SOLUTIONS

1. (A)
We know that fluids move from higher pressure to lower pressure and in a fluid pressure increases with depth; so pressure at to p $(= P_0)$ is lesser than at the bottom $(P_0 + h\rho g)$ and so the air bubble will move from bottom to top (it cannot move sideways as the pressure at same level in a fluid is same). Furthermore, in coming from bottom to top, pressure decreases; so in accordance with Boyles's law i.e., PV = constant, volume will increases, i.e. bubble will grow in size.

2. (C)
Suppose volume and density of the body be V and ρ respectively. Then according to law of floatation in water
Weight = upthrust
$V\rho g = \frac{2}{3}V\rho_w g$ …….(i)
In liquid, $V\rho g = \frac{1}{4}V_{\rho Lg}$ ……(ii)
From eqns (i) and (ii) $\frac{2}{3}V\rho_w g = \frac{1}{4}V\rho_L g$
Or $\frac{\rho_L}{\rho_g} = \frac{2/3}{1/4} = \frac{8}{3}$
or $\rho_L = \frac{8}{3}\rho_w = \frac{8}{3}\times 1g/cc$

3. (A)
When the substances are mixed in equal volumes, then
$V\rho_1 + V\rho_2 = 2V\times 4$ ...(i)
When the two substances are mixed in equal masses, then
$\frac{m}{\rho_1} + \frac{m}{\rho_2} = \frac{2m}{3}$ ...(ii)
From eqn. (i) $\rho_1 + \rho_2 = 8$ ...(iii)
From eqn. (ii) $\frac{1}{\rho_1} + \frac{1}{\rho_2} = \frac{2}{3}$
or $\frac{\rho_1 + \rho_2}{\rho_1\,\rho_2} = \frac{2}{3}$
or $\frac{8}{\rho_1\,\rho_2} = \frac{2}{3}$ or $\rho_1\,\rho_2 = 12$ ...(iv)

Now, $\rho_1 - \rho_2 = \left[(\rho_1+\rho_2)^2 - 4\rho_1\rho_2\right]^{1/2}$

$= [64-48]^{1/2} = 4$ ...(v)

Solving eqns. (iii) and (v), we get;

$\rho_1 = 6$ and $\rho_2 = 2$.

4. (B)

Velocity of ball just before entering the surface of water $= \sqrt{2gh} = \sqrt{2\times 980\times 9}cm/\sec$

This velocity is retarded by the upthrust acting on the ball due to water. The retardation

$$\frac{V\rho g - V\sigma g}{V\sigma} = \left(\frac{\rho-\sigma}{\sigma}\right)g = \left(\frac{0.4-1}{0.4}\right)g = -\frac{3}{2}g$$

Hence, depth of penetration of ball into water is given by

$$O^2 - 2\times 980\times 9 = 2\left(-\frac{3}{2}\right)\times 980\times h$$

$\therefore h = 6cm$

5 (A)

Base area of boat $= 3\times 2m^2$

When man sits in boat, it sinks by 1cm. Therefore, increase in volume of water displaced.

$= 3\times 2\times 1\times 10^{-2}m^3$

Weight of additional water displaced

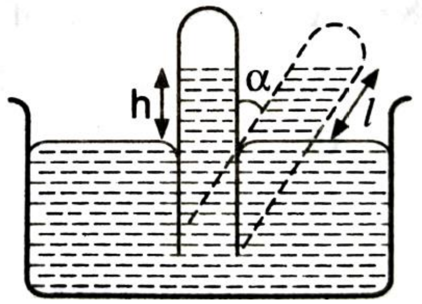
$= 6\times 10^{-2}\times 1000g$

This must be equal to weight of man

$\therefore Mg = 6\times 10^{-2}\times 1000g$

Hence, M = 60kg

6. (A)



The vertical height of mercury level in a barometer does not change

$$\frac{h}{l} = \cos\alpha$$

Or $l = \dfrac{h}{\cos\alpha}$

7. (D)

According to Archimede's principle

Weight of the body= Weight of the liquid displaced

Let V be volume of the block

In water

$$V_{block}\, g = \left(\frac{4}{5}V\right)\rho_{water}\, g$$

$\rho_{block} = \frac{4}{5}\rho_{water}$

In liquid

$V\rho_{block}\, g = V\rho_{liquid}\, g$

$\rho_{block}\, g = \rho_{liquid}\, g$

From equation (i) and (ii) we get

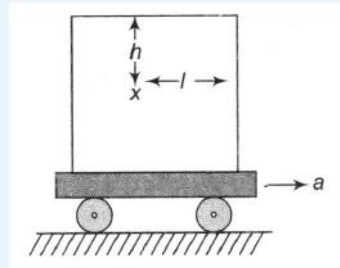
$\rho_{liquid} = \frac{4}{5}\rho_{water} = \frac{4}{5}\times 10^3 kgm^{-3} = 800 kgm^{-3}$

8. (C)

The pressure P at point x is the sum of pressure P_1 *and* P_2 where P_1 is the pressure due to gravity and P_2 is the pressure necessary to impart an acceleration *a* to the column of water of length *l*.

Now, pressure $P_1 = hdg$

Pressure P_2 acting on a column of length *l* and area of cross-section A gives it an acceleration a in the horizontal direction



$P_2 A = Alda$

$P_2 = lda$

As pressure is a scalar quantity to

$P = P_1 + P_2 = hdg + Ida = d(hg + la)$

9. (C)

Consider a small portion of mass m of the liquid mg is the weight acting downward and ma is the frictious force acting against the direction of motion. The resultant of these two forces will make the surface of liquid inclined in such a way that the h resultant is normal to the surface for which

$\tan\theta = \frac{ma}{mg} = \frac{a}{g}$

Or $\theta = \tan^{-1}(a/g)$

In the present problem, $\tan\theta = \frac{h}{l}$

$So, \frac{h}{l} = \frac{a}{g}$ or $h = \frac{al}{g}$

(where h is the difference in the heights in the two limbs)

10 (A)

The cube is in equilibrium under the following three forces:

(a) Spring force Kx, where x = elongation of the spring
(b) Gravitational force
W = Weight of the cube = mg
(c) Buoyant force F_B (or upward thrust) imparted by the liquid on the cube given as

$F_B = V\,dg$

(where V = volume of immersed portion of the cube)
For complete immersion
V= Volume of cube
For equilibrium of the cube

$Kx + F_B = mg$

$\therefore x = \frac{mg - F_B}{K} = \frac{mg - Vdg}{K}$

Where V = m/D

$\therefore x = \frac{mg}{K}\left[1 - \frac{d}{D}\right]$

11. (D)
As C.S areas of both the tubes A and C are same and tube is horizontal, hence according to equation of continuity $\upsilon_A = \upsilon_C$ and therefore, according to Bernoullis equations

$= P + \frac{1}{2}p\upsilon^2 =$constant

$P_A = P_C$

i.e height of liquid is same in both the tubes A and C.

12. (C)
The pressure at the free surface of the liquid and also at outside of point P is atmospheric pressure. Hence, there will be no effect of atmospheric pressure on the flow of liquid from hole P. The liquid on the free surface has no kinetic energy but only potential energy. On the other hand the liquid coming out of the hole has both kinetic and potential energies. Let υ be the velocity of efflux of the liquid coming out from the hole. According to Bernoullis theorem,

$P + 0 + \rho gH = P + \rho g(H - D) + \frac{1}{2}p\upsilon^2$

$\upsilon = \sqrt{2gD}$

After coming from the hole the liquid adopts a parabolic path. If it takes t sec in falling through a vertical distance (H – D) then

$(H - D) = \frac{1}{2}gt^2 \text{ or } t = \sqrt{[2(H-D)/g]}$ ……(ii)

From equation (i) and (ii)

$x = \upsilon t = 2\sqrt{[D(H-D)]}$

13 (D)
Let h be the height of liquid surface in the vessel. The velocity of efflux is given by

$\upsilon_{eff.} = \sqrt{(2gh)}$

If H be the height of table, then

$H = \frac{1}{2}gt^2 \text{ or } t = \sqrt{(2H/g)}$

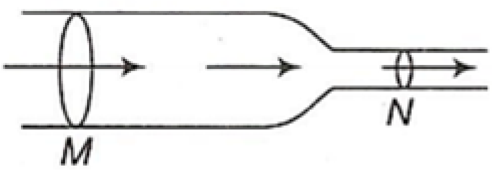
$\therefore R = \upsilon_{eff} \times t = \sqrt{2gh}\,\sqrt{2H/g}$

$R^2 = 4hH \text{ or } h = \frac{R^2}{4H}$

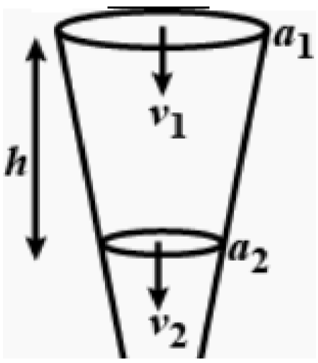
14. (C)
Volume flowing per sec in left tube
= volume flowing per sec in 1st branch tube + volume flowing per sec in 2nd branch tube
$A\upsilon_1 = A\upsilon_2 + 1.5A\upsilon$
$A \times 3 = A \times 1.5 + 1.5A\upsilon$
$\upsilon = 1m/s$

15. (D)
$\upsilon_1 = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$
From Bernoulli's theorem
$\rho gh + 2\rho g\left(\frac{h}{2}\right) = \frac{1}{2}(2p)\upsilon_2^2$
$\therefore \upsilon_2 = \sqrt{2gh}$
$\therefore \frac{\upsilon_1}{\upsilon_2} = \frac{1}{\sqrt{2}}$

16. (D)
According to principle of continuity, for a streamline flow of fluid through a tube of non-uniform cross-section the rate of flow of fluid (Q) is same at every point in the tube.
i.e. $A\upsilon =$ constant or $A_1\upsilon_1 = A_2\upsilon_2$
Therefore, the rate of flow of fluid is same at M and N.



17. (D)



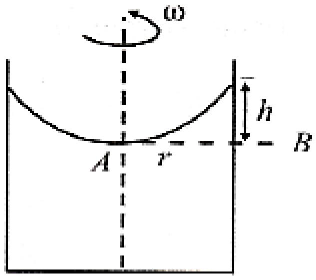
Here, $d_1 = 8 \times 10^{-3} m$
$\upsilon_1 = 0.4ms^{-1}$
h = 0.2 m
According to equation of motion
$\upsilon_2 = \sqrt{\upsilon_1^2 + 2gh} = \sqrt{(0.4)^2 + 2 \times 10 \times 0.2}$
$= 2ms^{-1}$
$\therefore$ According to equation of continuity
$a_1\upsilon_1 = a_2\upsilon_2$

$$\pi \times \left(\frac{8 \times 10^{-3}}{2} \right) \times 0.4 = \pi \times \left(\frac{d_2^2}{2} \right) \times 2$$

$d_2 = 3.6 \times 10^{-3} m$

18. (B)

From Bernoulli's theorem



$P_A + \frac{1}{2} d\upsilon_A^2 + dgh_A = P_B + \frac{1}{2} d\upsilon_B^2 + dgh_B$

Here $h_A = h_C$

$\therefore P_A + \frac{1}{2} d\upsilon_A^2 = P_C + \frac{1}{2} d\upsilon_B^2$

$P_A - P_B = \frac{1}{2} d[\upsilon_B^2 - \upsilon_A^2]$

Now, $\upsilon_A = 0, \upsilon_B = r\omega$ and $P_A - P_B = hdg$

$\therefore \quad hdg = \frac{1}{2} dr^2 \omega^2$ or $h = \frac{r^2 \omega^2}{2g}$.

19. (A)

The height of water in the tank becomes maximum when the volume of water flowing into the tank per second becomes equal to the volume flowing out per sec. Volume of water flowing out per second $= A\upsilon = A\sqrt{2gh}$

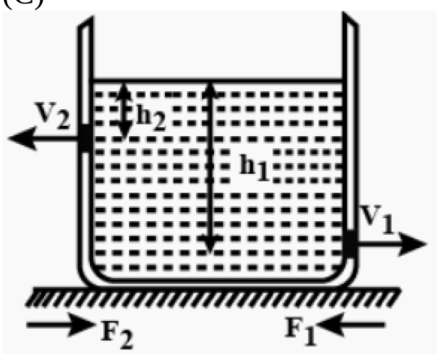
Volume of water flowing in one second = $70 cm^3 / \text{sec}$

$A\sqrt{2gh} = 70, 1 \times \sqrt{2gh} = 70$

Or $2 \times 980 \times h = 4900$

$\therefore h = 2.5 cm$

20. (C)



Thrust force

$F = F_1 - F_2$

$= \rho a \upsilon_1^2 - \rho a \upsilon_2^2$

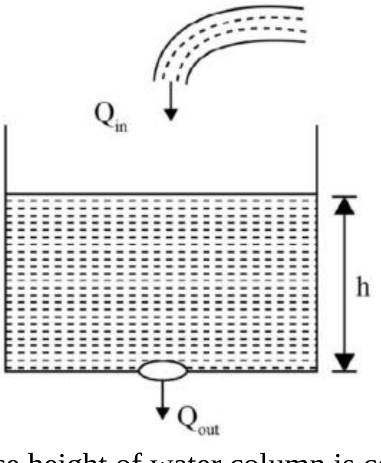
$\rho a (2gh_1) - \rho a (2gh_2)$

$2\rho ag(h_1 - h_2)$
$= 2\rho agh$

21. (6)
$h_w + 8cm + h_0 = 22cm + 22cm$
$\Rightarrow h_w + h_0 = 36cm$
$P_C = P_B \Rightarrow h_0\rho_0 g = h_w\rho_w g \Rightarrow h_0 \times 0.8 = h_w \times 1$
$\Rightarrow h_0 = \frac{h_w}{0.8} = 1.25h_w \Rightarrow h\ w + 1.25h_w = 36$
$\Rightarrow hw = \frac{36}{2.25} = 16cm \Rightarrow BE = 22 - 16 = 6cm$

22. (10)
Using Bernoulli's theorem, we have
$P_B = P_{atm} + \frac{1}{2}\rho v^2$ …(i)
And $P_B - P_{atm} = \int_{l-h}^{l} dx.\rho\omega^2 x = \frac{\rho\omega^2}{2}[l^2 - (l-h)^2]$
$P_B - P_{atm} = \frac{1}{2}\rho\omega^2(2lh - h^2)$ … (ii)
From equs (i) and (ii)
$\frac{1}{2}\rho\omega^2(2l-h)h = \frac{1}{2}\rho v^2$
$\Rightarrow v = \omega h\sqrt{\left(\frac{2l}{h} - 1\right)}$
$\Rightarrow 5 \times 1\sqrt{\left(\frac{2 \times 2.5}{1} - 1\right)} = 10m/s$

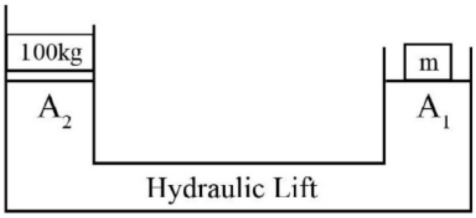
23 (5)



Since height of water column is constant therefore, water inflow rate $(Q_{in}) = 10^{-4} m^3 s^{-1}$
$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$
$\therefore 10^{-4} = 10^{-4} \times \sqrt{20 \times h}$
$\therefore h = \frac{1}{20} m = 5cm$

24. (25600)

Using Pascals law



$$\frac{100\times g}{A_2}=\frac{mg}{A_1} \qquad \ldots\ldots(1)$$

Let m mass can lift M_0 in second case then

$$\frac{M_0 g}{16A_2}=\frac{mg}{A_1/16} \qquad \ldots\ldots(2)$$

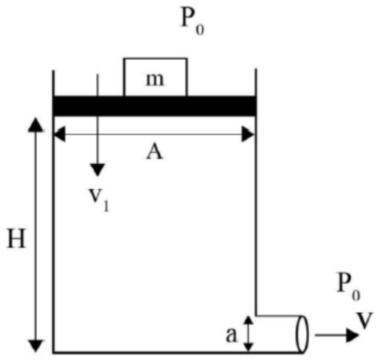
$$\left\{\sin ce\ A=\frac{\pi d^2}{4}\right\}$$

From equation (1) and (2) we get

$$\frac{M_0}{16\times 100}=16$$

$$\Rightarrow M_0=25600kg$$

25. (3)



m = 24g

A = 0.4 m^2

$a=1cm^2$

$H=40cm$

Using Bernoulli's equation

$$\Rightarrow\left(P_0+\frac{mg}{A}\right)+\rho gh+\frac{1}{2}\rho v_1^2$$

$$=P_0+0+\frac{1}{2}\rho\upsilon^2 \qquad \ldots\ldots(1)$$

$\Rightarrow$Neglecting v_1

$$\Rightarrow v=\sqrt{2gH+\frac{2mg}{A\rho}}$$

$$\Rightarrow v=\sqrt{8+1.2}$$

$$\Rightarrow v=3.033m/s$$

$$\Rightarrow v=3m/s$$

26 (2)

From equation of continuity(Av = constant)

$$\frac{\pi}{4}(8)^2(0.25) = \frac{\pi}{4}(2)^2(v) \quad \text{.......(i)}$$

Here, v is the velocity of water with which water comes out of the syrings (Horizontally)

Solving Eq (i), we get

$v = 4m/s$

The path of water after leaving the syring will be a parabola. Substituting proper values in equation of trajectory.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos 2\theta}$$

According to question, we have

$$-1.25 = R\tan 0^0 - \frac{(10)(R^2)}{(2)(4)^2\cos^2 0^0}$$

(R = horizontal range)

Solving this equation we get R = 2m

27. (50)

$$h_{oil} = \frac{S_{water}}{S_{oil}} h_{water}$$

$$h_0 = \frac{S_w}{S_0} h_w$$

$$= \frac{800}{1000} \times (h_0 - 0.1)$$

$$h_0 = 0.8h_0 - 0.08$$

$$h_0 = 0.4$$

Depth = 0.4 + 0.1 = 0.5m

= 50 cm

28. (485)

Using equation of continuity

$$A_1V_1 = A_2V_2$$

$$1 \times 10 = 0.5 \times \upsilon_B$$

$$\upsilon_B = 20cm/s$$

Using Bernoullis Equation at end A and B

$$P_A + \frac{1}{2}p\upsilon_A^2 + pgh_1 + P_B + \frac{1}{2}P\upsilon_B^2 + pgh_2$$

$$P_B - P_A = pg(h_1 - h_2) + \frac{1}{2}p(\upsilon_A^2 - \upsilon_B^2)$$

$$= 1000 \times 10 \times 0.05 + \frac{1}{2} \times 1000[0.01 - 0.04] = 500 - 15 = 485N/m^2$$

29. (303 to 300)

$$P_1 = P0 + \rho g d_1$$

$$P_2 = P0 + \rho d g_2$$

$$\Delta P = P_2 - P_1 = \rho g \Delta d$$

$$(8.08 \times 10^{-6} - 5.05 \times 10^6) = 10^3 \times 10 \times \Delta d$$

$$\Delta d = 303m \approx 300m$$

300m

30. (2)

The linear speed of the liquid at the sides is $r\omega$. So, the difference in height is given as follows

$2gh = \omega^2 r^2$

$h = \omega^2 r^2 / 2g$

Here $\omega = 2\pi f$

Therefore $h = [(2\times 2\pi)^2 (5\times 10^{-2})^2] / (2\times 10) = 2\text{cm}$

Answer 2.0

Answer Key

31. (B)
32. (A)
33. (A)
34. (B)
35. (C)
36. (C)
37. (A)
38. (C)
39. (D)
40. (B)
41. (B)
42. (B)
43. (C)
44. (D)
45. (C)
46. (B)
47. (B)
48. (B)
49. (B)
50. (D)
51. (4)
52. (2)
53. (5)
54. (4)
55. (4)
56. (9)
57. (5)
58. (6)
59. (2)
60. (3)

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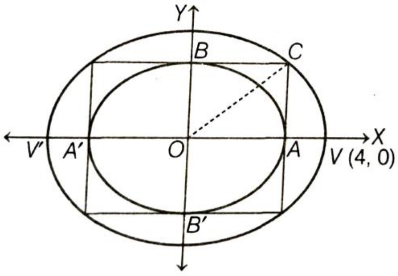
TOPIC: ELLIPSE

SOLUTIONS

61. (B)

Given that $x^2+4y^2=4$



$\Rightarrow \frac{x^2}{4}+\frac{y^2}{1}=1$

$\therefore a^2=4 \text{ and } b^2=1$

A = 2 and b = 1

Length of minor axes is 1 unit

$\because$ B = (0, 1) and A = (2, 0)

$\therefore$ Point C is (2, 1)

$\because$ Given that vertex (4, 0) is required ellipse

$\therefore$ Equation of ellipse is $\frac{x^2}{4^2}+\frac{y^2}{b^2}=1$

$\because$ It passes through C (2, 1) we get

$\frac{2^2}{16}+\frac{1}{b^2}=1$

$\Rightarrow b^2=\frac{1}{1-\frac{1}{4}}$

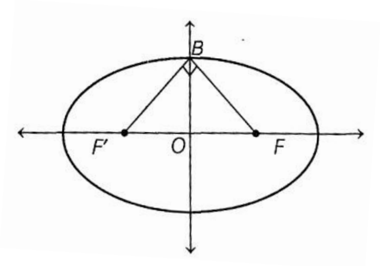
$\Rightarrow b^2=\frac{4}{3}$

$\therefore$ Equation is $\frac{x^2}{16}+\frac{3y^2}{4}=1$

$\Rightarrow x^2+12y^2=16$

62. (C)

$\because$ Given $\angle FBF'=90^0 \Rightarrow FB \perp F'B$



$\because$ Slope of $F'B \times$ slope of FB =-1

$\because F \text{ and } F'$ are (ae, 0) and (-ae, 0) and B = (0, b)

$\therefore \frac{b-0}{a+ae} \times \frac{b-0}{0-ae} = -1$

$\Rightarrow \frac{b^2}{a^2e^2} = 1 \Rightarrow b^2 = a^2e^2$

$\Rightarrow e^2 = \frac{b^2}{a^2}$

$\Rightarrow e = \sqrt{1-\frac{b^2}{a^2}} \Rightarrow e = \sqrt{1-e^2}$

$\Rightarrow e^2 = 1-e^2$

$\Rightarrow 2e^2 = 1 \Rightarrow e^2 = \frac{1}{2}$

$\Rightarrow e = \pm\frac{1}{\sqrt{2}}$

$\because e > 0 \Rightarrow e = \frac{1}{\sqrt{2}}$

63. (B)

$\because$ Given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\because$ There are exactly two points in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose distance from centre is same, the points would be either end points of the major axis or of the minor axis.

But $\frac{\sqrt{a^2+2b^2}}{2} < a$

So, the points are the vertices of minor axis,

Hence, $b = \frac{\sqrt{a^2+2b^2}}{2}$

$a^2 = 2b^2$

$\therefore \frac{b^2}{a^2} = \frac{1}{2}$

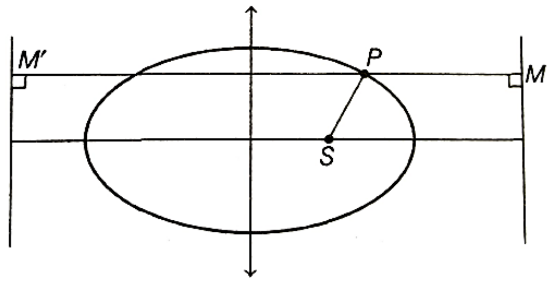
$\because e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}$

64. (C)

Given ellipse $x^2 + 2y^2 = 2$

$\Rightarrow \frac{x^2}{2} + \frac{y^2}{1} = 1$

$a^2 = 2 \, and \; b^2 = 1$



$\therefore e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}$

Given $PS = \sqrt{2} = ePM$

$\Rightarrow \sqrt{2} = \frac{1}{\sqrt{2}} PM \Rightarrow PM = 2$

But $MM' = \frac{2a}{e} = \frac{2\sqrt{2}}{\frac{1}{\sqrt{2}}} = 4$

$\therefore PM' = 2$

$\therefore PM : Pm' = 2 : 2 = 1 : 1$

65. (D)

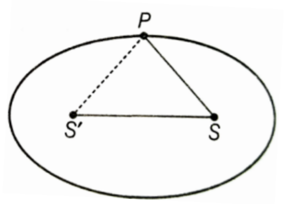
$\because$ Gives axes are 6 cm and 4 cm

$\therefore 2a = 6 \, and \; 2b = 4$

$a = 3 \; and \; b = 2$

Distance between two point = $SS' = 2ae$

$\therefore e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{4}{9}} = \frac{\sqrt{5}}{3}$



$\therefore 2ae = 2 \times 3 \times \frac{\sqrt{5}}{3} = 2\sqrt{5}$

$\therefore$ Length of string = $SPSS' = SP + PS + SS'$

$= 2a + 2ae$

$= \left(6 + 2\sqrt{5}\right)$ units

66. (A)

One of the focus of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is on Y-axis $\left(0, 5\sqrt{3}\right)$

$\Rightarrow be = 5\sqrt{3}$ …..(i) {where e is eccentricity of ellipse}

According to the question

$2b - 2a = 10$

$\Rightarrow b - a = 5$

$\Rightarrow b > a$

On squaring Eq. (i) both the sides, we get

$b^2e^2 = 75$

$\Rightarrow b^2\left(1-\frac{a^2}{b^2}\right)=75$

$\Rightarrow b^2-a^2=75$

$\Rightarrow (b-a)(b+a)=75$

$\Rightarrow 5(b+a)=75$ [From eq (iii)]

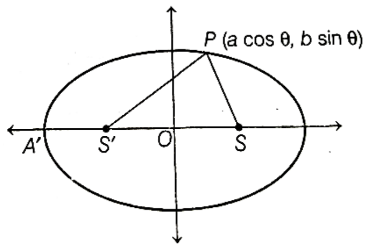
$\Rightarrow b+a=15$ ....(iii)

By solving eqs (ii) and (iii) we get

$b=10 \ and \ a=5$

$\therefore$ Length of latusrectum = $\frac{2a^2}{b}=\frac{2\times 25}{10}=5units$

67. (B)



$\because$ It is given P be any point on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$

$\frac{x^2}{25}+\frac{y^2}{16}=1$

$\because a=5 \ and \ b=4$

$\therefore P(a\cos\theta, b\sin\theta)=P(5\cos\theta, 4\sin\theta)$

Now, $S \ and \ S'$ are foci of ellipse

$\because S=(ae,0) \ and \ S'=(-ae,0)$

$\therefore e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

$\therefore S=(3,0) \ and \ S'=(-3,0)$

$\because$ We know that $SP=a+ex=a+e(5\cos\theta)$

$=a+5e\cos\theta=5+3\cos\theta$

And $S'P=a-ex=a-5e\cos\theta-5-3\cos\theta$

$\therefore SP.S'P=(5+3\cos\theta)(5-3\cos\theta)$

$=25-9\cos^2\theta$

$\therefore$ Maximum value = 25 – 0 = 25

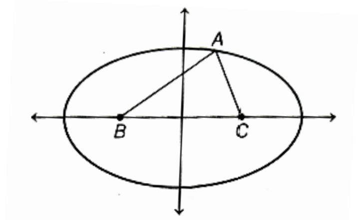
Minimum value = 25-9=16

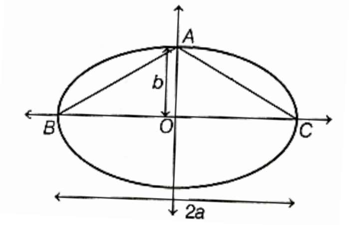
$\because$ Difference = 25 – 16 = 9

68. (C)

$\because$ Let A, B, C are the three vertices of a triangle whose two vertices (B and C) are on major axis and A be the moving point.

$\therefore$ For maximum area of ΔABC





First base will be maximum and latitudes should also be maximum
So, it clearly see that axis is maximum when B and C are vertices and A be on minor axis as per given in figure

$\therefore (\Delta ABC)_{\max} = \frac{1}{2} \times BC \times AO$

$$= \frac{1}{2} \times 8 \times 3 = 12 \begin{cases} \because \frac{x^2}{16} + \frac{y2}{9} = 1 \\ a^2 = 16 \text{ and } b^2 = 9 \\ a = 4 \text{ and } b = 3 \\ 2a = 8 \end{cases}$$

69. (C)
Let point R on the ellipse is R $(a\cos\theta, b\sin\theta)$

$\because$ Ellipse is $x^2 + 2y^2 = 2$

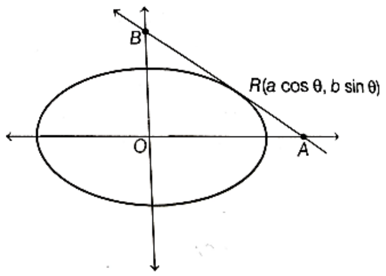
$\frac{x^2}{2} + \frac{y^2}{1} = 1$

$\Rightarrow a^2 = 2 \, and \; b^2 = 1$

$\therefore R\left(\sqrt{2}\cos\theta \sin\theta\right)$

$\therefore$ Equation of tangent at R.

$\Rightarrow \frac{x\left(\sqrt{2}\cos\theta\right)}{2} + y(\sin\theta) = 1$



$\therefore$ *For* A put y = 0 $\Rightarrow x = \frac{\sqrt{2}}{\cos\theta}$

$A \Rightarrow \left(\sqrt{2}\sec\theta, 0\right)$

For B put x = 0 $\Rightarrow y = \frac{1}{\sin\theta}$

$B \Rightarrow (0 \operatorname{cosec}\theta)$

$\because Q(h,k)$ be the middle point of AB

$\therefore h=\frac{\cos ec\theta+0}{2}$

$\Rightarrow \sqrt{2}h=\sec\theta, \sin\theta=\frac{1}{2k}$

And $\cos\theta=\frac{1}{\sqrt{2}h}$

$\because \sin^2\theta+\cos^2\theta=1$

$\Rightarrow \frac{1}{2h^2}+\frac{1}{4k^2}=1$

$\therefore$ Locus is $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

70. (C)

Given ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$

$\because a^2=25 \; and \; b^2=16$

Let any point P on the ellipse be $(a\cos\theta, b\sin\theta)\Rightarrow(5\cos\theta, 4\sin\theta)$

$\therefore$ Equation of tangent at P

$\frac{x\cos\theta}{5}+\frac{y\sin\theta}{4}=1$

It meets the line x =0 at $Q(0, 4\cos ec\theta)$

Image of Q in the line $y=x$ is R ($4\cos ec\;\theta, 0)$

$\because$ Equation of circle is

$x(x-4\cos c\theta)+y(y-4\cos ec\theta)=0$

$\Rightarrow x^2-4x\cos ec\theta+y^2-4y\cos ec\theta=0$

$\Rightarrow (x^2+y^2)-\cos ec\theta(4x+4y)=0$

It represents a family of circles passing through the intersection of $x^2+y^2=0 \; and \; x+y=0$.

Now, solving them, we get x = 0 and y =0

$\therefore$ Fixed point is (0,0)

71. (B)

Given point $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ $and\;\; ellipse\; \frac{x^2}{27}+y^2=1$

$\therefore$ Equation of tangent is $\frac{x}{3\sqrt{3}}\cos\theta+y\sin\theta=1$

$\therefore$ Intercept are $\left(0, \frac{1}{\sin\theta}\right)\left(\frac{3\sqrt{3}}{\cos\theta}, 0\right)$

Sum of intercept (S) = $\frac{1}{\sin\theta}+\frac{3\sqrt{3}}{\cos\theta}$

$=3\sqrt{3}\sec\theta+\cos ec\theta$

For minimum of (S)

$\frac{dS}{d\theta}\Rightarrow 3\sqrt{3}\sec\theta\tan\theta-\cos ec\theta\cot\theta=0$

$\Rightarrow \tan^2\theta=\frac{1}{3\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$

At $\theta = \frac{\pi}{6} \Rightarrow \frac{d^2S}{d\theta^2} > 0$

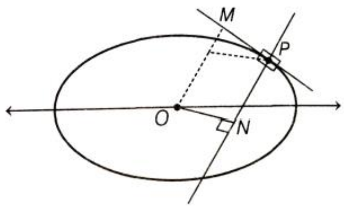
$\therefore S$ will be minimum at $\theta = \frac{\pi}{6}$

72. (D)

Let P $(2\cos\theta, \sin\theta)$

Equation of normal at P $2x\sec\theta - y\operatorname{cosec}\theta = 3$

$\therefore OM \perp PM$



$\therefore$ Equation of tangent at P

$\frac{x\cos\theta}{2} + y\sin\theta = 1$

$\because ON = \left|\frac{3}{\sqrt{4\sec^2\theta + \operatorname{cosec}^2\theta}}\right|$

$$OM = \left|\frac{1}{\sqrt{\left(\frac{\cos\theta}{2}\right)^2 + (\sin\theta)^2}}\right|$$

$\because$ ONPM is rectangle

$\therefore$ Area of $\Delta OPN = \frac{1}{2}$ area of rectangle ONPQ

$$= \frac{1}{2}\left[\frac{3}{\sqrt{4\sec^2\theta + \operatorname{cosec}^2\theta}} \times \frac{1}{\sqrt{\frac{\cos^2\theta}{4} + \sin^2\theta}}\right]$$

$$= \frac{3}{2\sqrt{1 + \frac{1}{4}\cot^2\theta + 4\tan^2\theta + 1}}$$

$$= \frac{3}{2\sqrt{\left(4\tan^2\theta\right) + 2 + \frac{1}{4}\cot^2\theta}}$$

$$= \frac{3}{2\sqrt{\left(2\tan\theta + \frac{1}{2}\cot\theta\right)^2}}$$

$$= \frac{3}{2\left|2\tan\theta + \frac{1}{2}\cot\theta\right|}$$

$\because$ Area will be maximum, when $2\tan\theta = \frac{1}{2}\cot\theta$ using application of derivatives

$\Rightarrow \tan^2\theta = \frac{1}{4} \Rightarrow \tan\theta = \pm\frac{1}{2}$

$\tan\theta = \frac{1}{2} \Rightarrow P = \left(\frac{4}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$ or $\left(\frac{-4}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$

$\tan\theta = \frac{-1}{2} \Rightarrow P = \left(\frac{-4}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$ or $\left(\frac{4}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$

73. (D)

Equation of QR is $\Rightarrow$T=0 ((chord of contact)

$\frac{8x}{4} + \frac{27y}{9} = 1$

$\Rightarrow 2x + 3y = 1$

Now, equation of the pair of line passing through origin and points (Q, R) is given by

$\left(\frac{x^2}{4} + \frac{y^2}{9}\right) = (2x+3y)^2$ (using homogenisation)

$\Rightarrow 9x^2 + 4y^2 = 36(4x^2 + 9y^2 + 12xy)$

$\Rightarrow 135x^2 + 432xy + 320y^2 = 0$

$h = 216, a = 135$ *and* $b = 320$

$\Rightarrow \tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$

$\Rightarrow \tan\theta = \frac{2\sqrt{(216)^2 - 135\times 320}}{455}$

$\Rightarrow \theta = \tan^{-1}\frac{8\sqrt{2916 - 2700}}{455} = \tan^{-1}\frac{48\sqrt{6}}{455}$

74. (C)

Given m . n = m + n and ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\Rightarrow mn - m = n \Rightarrow m(n-1) = n$

$\because$ m and n are positive integer

$\therefore$ n is divisible by n – 1

$\Rightarrow n = 1$ *and* $m = \frac{2}{1} = 2$

Now, point is (2, 2)

$\therefore$ Equation of chord of contact w.r.t. (2, 2) is T = 0

$\Rightarrow \frac{2x}{9} + \frac{2y}{4} = 1 \Rightarrow 4x + 9y = 18$

75. (B)

Given equation of ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Let P(h, k) be the pole, then the equation of the polar is

$T = 0$

$$\Rightarrow \frac{hx}{16}+\frac{ky}{9}=1 \Rightarrow \frac{ky}{9}=-\frac{hx}{16}+1$$

$$\Rightarrow y=-\left(\frac{9h}{16k}\right)x+\frac{9}{k}$$

The line touches the ellipse

$$\frac{x^2}{2}+\frac{y^2}{1}=1$$

$$\therefore c^2=a^2m^2+b^2$$

$$\Rightarrow \left(\frac{9}{k}\right)^2=2\left(\frac{9h}{16k}\right)^2+1$$

$$\Rightarrow \frac{81}{k^2}=2\times\frac{81h^2}{256k^2}+1$$

$$\Rightarrow 81=\frac{81h^2}{128}+k^2$$

$$\Rightarrow 1=\frac{81h^2}{81\times 128}+\frac{k^2}{81}$$

$$\therefore \frac{h^2}{128}+\frac{k^2}{81}=1$$

$\therefore$ Locus is $\frac{x^2}{128}+\frac{y^2}{81}=1$

76. (C)

Let the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\therefore y=3x$ *and* $2y=-5x$ is a pair of conjugate diameter therefore

$$\Rightarrow m_1m_2=-\frac{b^2}{a^2}\left\{m_1=3 \text{ and } m_2=\frac{-5}{2}\right\}$$

$$\therefore 3\times\left(\frac{-5}{2}\right)=\frac{-b^2}{a^2}\Rightarrow\frac{b^2}{a^2}=\frac{15}{2}$$

$\therefore$ 'e' of the ellipse is the between (0, 1)

$$\therefore e=\sqrt{1-\frac{a^2}{b^2}}=\sqrt{1-\frac{2}{15}}=\sqrt{\frac{13}{15}}$$

77. (A)

Given, ellipse is $\frac{x^2}{16}+\frac{y^2}{9}=1$

Now, point P $\left(\frac{\pi}{4}\right)=\left(a\cos\frac{\pi}{4},b\sin\frac{\pi}{4}\right)=\left(4\times\frac{1}{\sqrt{2}}.9\times\frac{1}{\sqrt{2}}\right)=\left(\frac{4}{\sqrt{2}},\frac{9}{\sqrt{2}}\right)$

Now equation of tangent at P $\left(\frac{\pi}{4}\right)$

$$\Rightarrow \frac{x}{16}\times\frac{4}{\sqrt{2}}+\frac{y}{9}\left(\frac{9}{\sqrt{2}}\right)=1$$

$$\Rightarrow \frac{x}{4\sqrt{2}}+\frac{y}{\sqrt{2}}=1$$

$\Rightarrow y=-\frac{\sqrt{2}x}{4\sqrt{2}}+\sqrt{2}=\frac{-1}{4}x+\sqrt{2}$

$\therefore m=-\frac{1}{4}\Rightarrow \tan\phi=-\frac{1}{4}$

$\therefore$ Length of subtangent $=|y_1\cot\phi|$

$=\left|\frac{9}{\sqrt{2}}\times 4\right|=\frac{36}{\sqrt{2}}$

$\therefore$ Length of subnormal $=|y_1\tan\phi|$

$=\left|\frac{9}{\sqrt{2}}\times\frac{1}{4}\right|=\frac{9}{4\sqrt{2}}$

Now, $\frac{length\ of\ sub\tan gent}{length\ of\ subnormal}=\frac{36}{\frac{9}{4}}=16$

78. (D)

Given ellipse are $\Rightarrow \frac{x^2}{4}+\frac{y^2}{1}=1\ and\ \frac{x^2}{1}+\frac{y^2}{a^2}=1$

$\Rightarrow a_1^2=4, b_1^2=1\ and\ a_2^2=1, b_2^2=a^2$

$a^2=b^2-5b+7$ for the two ellipse to intersect at four distinct point.

$b_2>1$

$b_2^2>1$

$\Rightarrow b^2-5b+7>1$

$\Rightarrow b^2-5b+6>0$

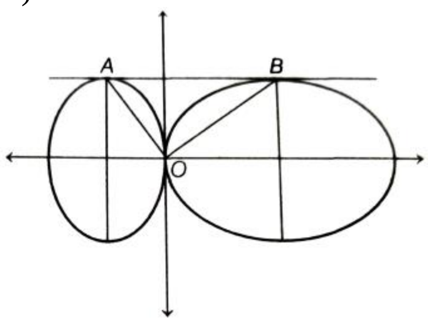
$\Rightarrow b^2-5b+6>0$

$\Rightarrow (b-2)(b-3)>0$

$\therefore b\in(-\infty,2)\cup(3,\infty)$

$\therefore$ b does not lies in [2, 3]

79. (A)



Given ellipse

$\frac{(x-4)^2}{25}+\frac{y^2}{4}=1\ and\ \frac{(x+1)^2}{1}+\frac{y^2}{4}=1$

$\because$ B(4, 2) is one end of the minor axis of the eliipse

$\frac{(x-4)^2}{25}+\frac{y^2}{4}=1$

And A (-1, 2) is one end of the major axis of the second ellipse, therefore

$AB=5, OB=\sqrt{16+4}=\sqrt{20}$

And $OA=\sqrt{1+4}=\sqrt{5}$

$\because OA^2+OB^2=20+5=25=AB^2$

$\because$ OAB is right angled triangle $\angle AOB = \frac{\pi}{2}$

80. (B)

$\because$ Equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ having slope 'm' is $y = mx + \sqrt{a^2m^2+b^2}$

$\therefore$ The line $y = mx + \sqrt{4m^2+1}$ $\left\{\because S : \frac{x^2}{4}+y^2=1\right\}$ .....(i)

$\because$ The line touches the circle $x^2+y^2=3$

$\therefore$ Radius = distance between centre (0,0) to lien (i)

$$\sqrt{3} = \left|\frac{\sqrt{4m^2+1}}{\sqrt{1+m^2}}\right|$$

On squaring both sides, we get

$$3 = \frac{4m^2+1}{m^2+1}$$

$\Rightarrow 3m^2+3 = 4m^2+1$

$\Rightarrow m^2 = 2 \Rightarrow m = \pm\sqrt{2}$

$\therefore$ Product of slope is -2

81. (4)

Given ellipse is $\frac{x^2}{16}+\frac{y^2}{9}=1$

$\Rightarrow a^2 = 16$

and $b^2 = 9$

$\therefore e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

$\therefore$ Focii $= (\pm ae,\ 0) = (\pm\sqrt{7}, 0)$

And radius of circle is = $\sqrt{7+9} = \sqrt{16} = 4$ units

82. (0.36)

Let (h, k) be the point of intersection of tangents θ_1 *and* θ_2

$$\therefore h = \frac{a\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}$$

$$\text{And } k = \frac{b\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1\theta_2}{2}\right)}$$

$$\frac{h}{a} = \frac{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \quad ......(i)$$

$$\frac{k}{b}=\frac{\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \qquad \ldots\ldots(ii)$$

Squaring and adding both equation we get

$$\frac{h^2}{a^2}+\frac{k^2}{b^2}=\frac{1}{\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)} \qquad \ldots\ldots(iii)$$

It is given that $b(\sin\theta_1+\sin\theta_2)=3$

$\Rightarrow \sin\theta_1+\sin\theta_2=1 \qquad [\because b=3]$

$$\Rightarrow 2\sin\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)=1$$

$$\Rightarrow \frac{k}{b}=\frac{\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}=\frac{1}{2\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)} \qquad \ldots.(iv)$$

From Eqs. (iii) and (iv), we get

$$\frac{h^2}{a^2}+\frac{k^2}{b^2}=\frac{2k}{b}$$

$$\Rightarrow \frac{h^2}{25}+\frac{k^2}{9}=\frac{2k}{3}$$

$\therefore$ Locus is $\frac{x^2}{25}+\frac{y^2}{9}=\frac{2y}{3}$

$\Rightarrow 9x^2+25y^2=150y$

$\Rightarrow 9x^2+25y^2-150y=0$

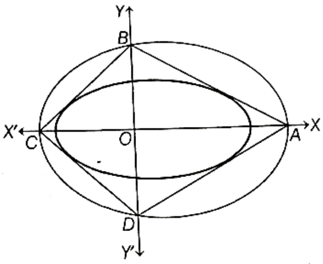
$\therefore$ compare with $ax^2+by^2+2gx+2fy+c=0$ we get

$a=9,\ b=25,\ g=0,\ f=-75\ and\ c=0$

$\because$ Required value $\frac{a}{b}=\frac{9}{25}=0.36$

83. (7.070)

Given ellipse is $\frac{x^2}{16}+\frac{y^2}{9}=1$



Clearly the vertices of the square parallel on the director circle of the given ellipse

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$\because$ The equation of director circle is

$x^2+y^2=a^2+b^2$

$x^2+y^2=16+9=25=r^2$

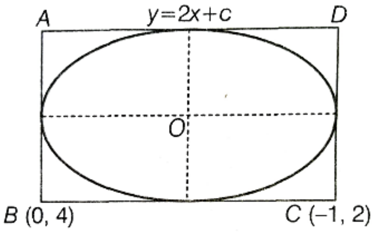
$r = 5$

Thus, the length of AC = (diagram) $= 2 \times 5 = 10$

$\because a\sqrt{2} = 10$ (where a is side of square)

$\Rightarrow a = \frac{10}{\sqrt{2}} = 5\sqrt{2} = 5 \times 1.414 = 7.070$

84. (17)



Let $AD \Rightarrow y = 2x + c$

So, $BC \Rightarrow y = 2x + 4$

$AB = x + 2y = 8$

And $DC = x + 2y - 3 = 0$

Let $BC = \sqrt{5}$ *and* $AB = 2b$

Clearly $2a = \sqrt{5} \Rightarrow a = \frac{\sqrt{5}}{2}$

It is given that area of ellipse = $\frac{5\pi}{2}$

$\Rightarrow \pi ab = \frac{5\pi}{2}$

$\Rightarrow \pi \frac{\sqrt{5}}{2} b = \frac{5\pi}{2} \Rightarrow b = \sqrt{5}$

Also, $b = \left| \frac{c-4}{2\sqrt{5}} \right| = \sqrt{5}$

$\Rightarrow \frac{c-4}{2\sqrt{5}} = \pm 5$

$\Rightarrow c - 4 = \pm 10$

$\Rightarrow c = 4 \pm 10$

$c = 14, -6$

When c = 14

On solving the equation

$AB : x + 2y = 8$ A

And $AD : y = 2x + 14,$ we get

A = (-4, 6) and D = (-5,4)

When c = -6 we get

$A = (3, 2)$ *and* $D = (3, 0)$

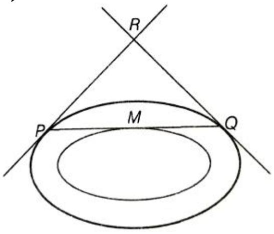
$\therefore$ Required value

$= 14 + (-6) + (-4) + 6 + (-5) + 4 + (3) + 2 + 3$

$= 14 - 10 + 6 - 5 + 4 + 3 + 2 + 3$

$= 4 + 1 + 1 + 3 + 2 + 3 = 17$

85. (2)



Equation of ellipse $x^2+4y^2=4$

$\frac{x^2}{4}+y^2=1$

Equation of tangent at $M(2\cos\theta, \sin\theta)is$

$\frac{x(2\cos\theta)}{4}+y\sin\theta=1$

$\frac{x\cos\theta}{2}+y\sin\theta=1$

Equation of chord of contact of R(h, k) w.r.t.

$\frac{x^2}{6}+\frac{y^2}{3}=1\ is\ \frac{hx}{6}+\frac{ky}{3}=1$

Eqs (i) and (ii) are identical

$\frac{\frac{h}{3}}{\cos\theta}=\frac{\frac{k}{3}}{\sin\theta}=1$

$\Rightarrow h^2+k^2=9$

$\Rightarrow (h,k)$ lies on director circle of ellipse

$\frac{x^2}{6}+\frac{y^2}{3}=1$

$\therefore$ Tangent will intersect at right angle $=\frac{\pi}{2}$

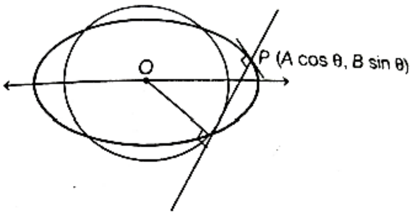
$\therefore k=2$

86. (2)

Equation of ellipse is

$\frac{x^2}{(a^2+2a+2)^2}+\frac{y^2}{(a^2+1)^2}=1$

$\Rightarrow \frac{x^2}{A^2}+\frac{y^2}{B^2}=1$



Equation of normal at P

$Ax\sec\theta - By\operatorname{cosec}\theta = A^2-B^2$

$\because$ Radius of circle = perpendicular distance from centre to normal at P

$=\left|\frac{A^2-B^2}{\sqrt{A^2\sec^2\theta+B^2\operatorname{cosec}^2\theta}}\right|$

$$= \left| \frac{A^2 - B^2}{\sqrt{A^2 + B^2 + A^2 \tan^2 \theta + B^2 \cot^2 \theta}} \right| \le \frac{|A^2 - B^2|}{\sqrt{A^2 + B^2 + 2AB}}$$

$\therefore r_{\max} = |A - B| = a^2 + 2a + 2 - (a^2 + 1)$

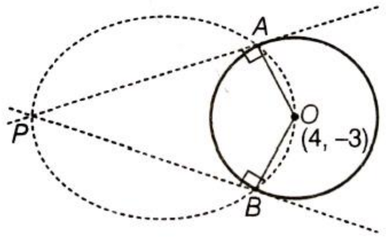
$= 2a + 1$

$5 = 2a + 1 \Rightarrow a = 2$

87. (9)

Given, circle is $x^2 + y^2 - 8x+6y+1=0$

Centre O(4, -3)



The circumcircle of ΔPAB will circumscribe the quadrilateral PBOA. Also, hence one of the diameter must be OP.

$\therefore$ Equation of circumcircle of ΔPAB will be

$(x-2)(x-4) + (y-3)(y+3) = 0$

$\Rightarrow x^2 + y^2 - 6x - 1 = 0$ .....(i)

Director circle of given ellipse will be

$(x+5)^2 + (y-3)^2 = 9 + b^2$

$\Rightarrow x^2 + y^2 + 10 - 6y + 25 - b^2 = 0$ ....(ii)

$\because$ From Eqs(i) and (ii) by applying condition of orthogonality, we get

$2(g_1 g_2 + f_1 f_2) = c_1 + c_2$

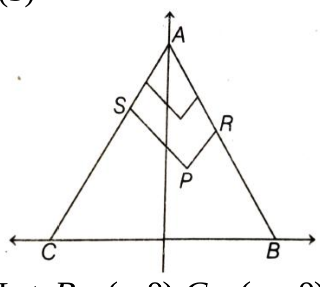
$\Rightarrow 2[-3(5) + 0(-3)] = -1 + 25 - b^2$

$\Rightarrow -30 = 24 - b^2$

$\because b^2 = 54$

$\therefore \frac{b^2}{6} = \frac{54}{6} = 9$

88. (3)



Let $B = (a,0), C = (-a,0)$ *and* $A = (0,a)$

Equation of line BA is

$x + y = a$ and that of CA is y = x +a

Let $P = (x, y)$ distance from $BC = y$

and area of PRAS = $PR \times PS$

$= \left(\frac{x + y - a}{\sqrt{2}} \right) \times \left(\frac{x - y + a}{\sqrt{2}} \right)$

According to the question

$$y^2 = \pm\frac{1}{2}\left\{\frac{x^2-(y-a)^2}{2}\right\}$$

$$\pm 4y^2 = x^2 - y^2 - 2ay - a^2$$

When it is $+4y^2 \Rightarrow$ It forms a Hyperbola

When it is $-4y^2 \Rightarrow$ It forms an ellipse

$$\because -4y^2 = x^2 - y^2 - 2ay - a^2 = 0$$

$$\Rightarrow x^2 + 3y^2 - 2ay - a^2 = 0$$

$$\Rightarrow x^2 + 2\left(y - \frac{1}{3}\right)^2 = \frac{4a^2}{3}$$

$$\Rightarrow \frac{x^2}{\frac{4a^2}{3}} + \frac{\left(y-\frac{1}{3}\right)^2}{\frac{4a^2}{9}} = 1$$

$$\therefore e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{\frac{4a^2}{9}}{\frac{4a^2}{3}} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\Rightarrow 5e^2 = \frac{10}{3} \Rightarrow [5e^2] = \left(\frac{10}{3}\right) = 3$$

89. (0.8)

Given $x^2 + 9y^2 - 4x + 6y + 4 = 0$

$$\Rightarrow x^2 - 4x + 9y^2 + 6y + 4 = 0$$

$$\Rightarrow (x-2)^2 + (3y+1)^2 = 1$$

$$\Rightarrow (x-2)^2 + (3y+1)^2 = 1$$

$$\Rightarrow (x-2)^2 + 9\left(y + \frac{1}{3}\right)^2 = 1$$

$$\Rightarrow \frac{(x-2)^2}{1} + \frac{\left(y+\frac{1}{3}\right)^2}{\frac{1}{9}} = 1$$

Which is equation of ellipse having centre at $\left(2, \frac{-1}{3}\right)$.

General point on ellipse

$$p(4,9) = \left(2 + a\cos\theta, \frac{-1}{3} + b\sin\theta\right)\left(2 + \cos\theta, \frac{-1}{3} + \frac{1}{3}\sin\theta\right)$$

$$\therefore x = 2 + \cos\theta \ \textit{and} \ y = \frac{-1}{3}(1 - \sin\theta)$$

$$\because 4x - 9y = 8 + 4\cos\theta + 3 - 3\sin\theta$$

$$f(x,y) = 4x - 9y$$

$$\therefore f_{\max} = 11 + \sqrt{4^2 + 3^2} = 11 + 5 = 16$$

$$\left[\because a\sin\theta \pm b\cos\theta \ \textit{lies between} \ -\sqrt{a^2+b^2} \ \textit{to} \ \sqrt{a^2+b^2}\right]$$

$\therefore$ Required answer = $= \frac{16}{20} = \frac{4}{5} = 0.8$

90. (6)

Given line is x – y – 2 = 0

Image of (h,k) on ellipse about x – y – 2 = 0 is say $(h'k')$

$\Rightarrow \frac{h'-h}{1} = \frac{k'-k}{-1} = -2\frac{(h-k-2)}{1^2+1^2}$

$\Rightarrow h'-h = k'-k = -h+k+2$

$\because h' = h-h+k+2$

$and\ k' = h-k+k-2$

$\Rightarrow h' = k+2\ and\ k' = h-2$

$\therefore (h,k) \equiv (k'+2, h'-2)$

$\therefore$ Equation of ellipse

$\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$

$\Rightarrow \frac{(h-4)2}{16} + \frac{(h'-2-3)^2}{9} = 1$

$\Rightarrow \frac{(k'+2-4)^2}{16} + \frac{(h'-2-3)}{9} = 1$

$\Rightarrow 16h'^2 + 9k'^2 - 36k' - 160h' + 292 = 0$

$\therefore$ Locus is $16x^2 + 9y^2 - 36y - 160x + 292 = 0$

$\because \frac{k_1+k_2}{22} = \frac{292-160}{22} = 6$