IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 17/06/23**

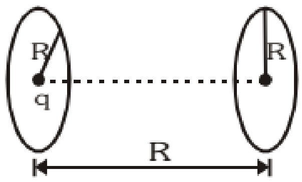
TOPIC: ELECTROSTATICS

SOLUTIONS

1. (B)
Charge moves $\perp^r$ to the field lines. So the work done will be zero.

2. (B)

$$U_i = \frac{KQ_1 q}{R} + \frac{KQ_2 q}{\sqrt{R^2 + R^2}}$$



$$U_f = \frac{KQ_1 q}{\sqrt{R^2 + R^2}} + \frac{KQ_2 q}{R}$$

Work done by external force : $\Delta U = U_f - U_i$

3. (B)
By mechanical energy conservation

$$(PE + KE)_i = (PE) + (KE)_f$$

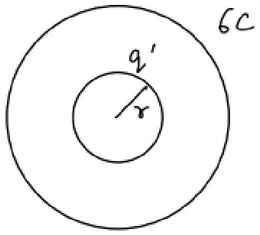
$$0 + \frac{1}{2}mv^2 + 0 = \frac{KQ^2}{d} + \frac{1}{2}m\left(\frac{v}{2}\right)^2 \times 2$$

($\because$ from momentum conservation at closet approach, both particle will move with a common speed $v/2$))

4. (B)

$$V = (n-1)\frac{KQ}{r},\ E = \frac{KQ}{r^2} \Rightarrow \frac{V}{E} = (n-1r)$$

5. (B)



$$\frac{kq'}{r} + \frac{k6}{3r} = 0$$

$$\Rightarrow \frac{kq'}{r} = \frac{-k(6)}{3r}$$

$$\boxed{q' = -2}$$

6. (A)

$F = QE \Rightarrow 300 = 3 \quad E \Rightarrow \quad E = 100$ N/C

$$E = \frac{dV}{dx} \Rightarrow \Delta V = 100 \times \frac{1}{10} = 10 \text{ V}$$

Slope from x axis

$y = 3 + x$; $\tan\theta = 1 \Rightarrow \theta = 45$

$\therefore$ Electric field in vector from $\vec{E} = 100\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$$\therefore V = -\int_{(3,1)}^{(1,3)} \vec{E}\cdot d\vec{r}$$

$$V = = \frac{-100}{\sqrt{2}}\left[\int_3^1 dx + \int_1^3 dy\right] = 50 \sqrt{2} [-2 + 2] = 0$$

7. (D)

Slope from x axis

$y = 3 + x$; $\tan\theta = 1 \Rightarrow \theta = 45$

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Alternate solution : The direction of electric field and the slope of line A (3,1) & (1,3) is $\perp$ to each others so the dot product $\vec{E}.d\vec{r}$ becomes zero.

8. (D)

Equipotential lines are always $\perp^r$ to the electric field strength lines .

$\because$ slope of equipotential lines =2

$\therefore$ Slope of electric field must be = $-1/2$

$\Rightarrow$ Electric field strength vector $= -8\hat{i} + 4\hat{j}$

9. (B)

$$E = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) = -k\left[4x\hat{i} - 2y\hat{j} + 2z\hat{k}\right]$$

$$= -k\left[\sqrt{16+4+4}\right] = 2k\sqrt{6}$$

10. (C)

Slope of equipotential lines will be = 1/2

∴ Slope of electric lines must be = – 2

OR

$$E_x = -\left(\frac{\Delta V}{\Delta x}\right)_{y=\text{constant}} = -\frac{(4-2)V}{(4-2)cm} = -100\ V/m$$

$$E_y = -\left(\frac{\Delta V}{\Delta y}\right)_{x=\text{constant}} = -\frac{(2-4)V}{(1-0)cm} = 200\ V/m$$

11. (A)

$-\int_{\infty}^{\ell=0} \vec{E}.\vec{d\ell}$ is the potential at the centre of the ring

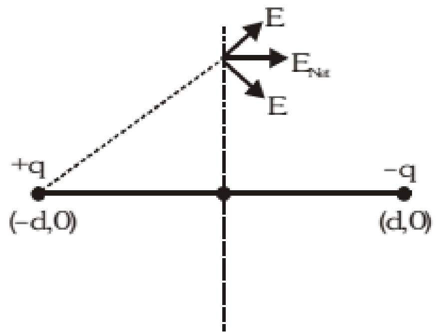
which is

$$V = \int dV = \int \frac{Kdq}{0.5} = \frac{Kq}{0.5}$$

$$V = \frac{9\times10^{9}\times1.11\times10^{-10}\times2}{1} = 2\ \text{volt}$$

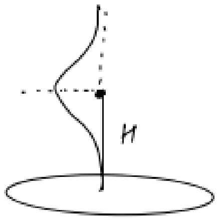
12. (A)

In figure (–d,0) to (d,0) on x–axis the direction of $\vec{E}$ in +ve x–axis and left side of (–d,0) the direction in –ve x–axis, but on y–axis, at any point the net electric field along the x–axis.



13. (B)

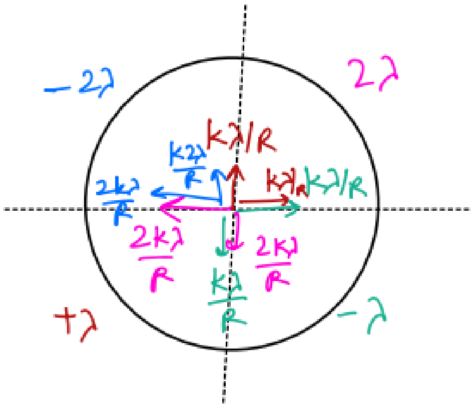
13.

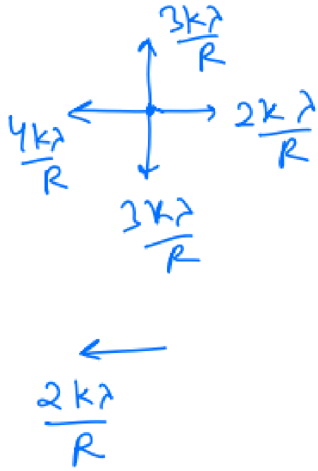


Electric field is maximum at
x = R/√2

Cleary when $h > \frac{R}{\sqrt{2}}$, object is moved upwards E decreases and object will come down and vice - e -versa

14. (A)

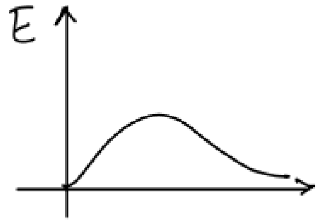




15. (0)

$$\text{Work done} = q\,\Delta V = q(0) = 0$$

16. (B)

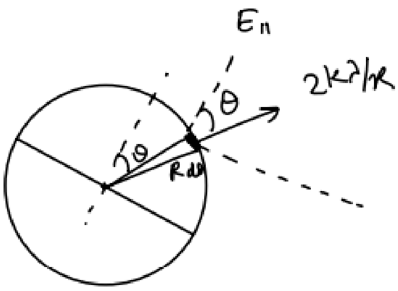


$$E = \frac{kQx}{(R^2+x^2)^{3/2}}$$

E is max at $x = R/\sqrt{2}$

17. (B)

$$E = \frac{2k\lambda}{R}$$



$$F_{\parallel} = \int_{-\pi/2}^{+\pi/2} (dq)\,\frac{2k\lambda}{R}\cos\theta$$

$$= \int (R\,d\theta)\,\frac{q}{\pi R} \times \frac{2k\lambda}{R}\cos\theta$$

$$= \frac{2k\lambda}{\pi R}\,q \int_{-\pi/2}^{+\pi/2} \cos\theta \, d\theta$$

$$= \frac{4k\lambda}{\pi R}\,q = \frac{\lambda q}{\pi^2 \epsilon_0 R}$$

18. (A)

$$\text{distance} = \sqrt{(4-1)^2 + (7-3)^2 + (2-2)^2}$$

$$= \sqrt{9+16} = 5$$

$$V = \frac{kq}{r} = \frac{9\times10^{9}\times10^{-8}}{5} = 1.8\times10 = 18\ V$$

19. (B)

To fix Y, external force has to be applied

But work done by that force is zero

since displacement is zero (0).

$\Rightarrow$ E is conserved.

But not momentum

20. (A)

X $\qquad\qquad\qquad$ Y

$m, +q_1$ $\qquad\qquad\qquad$ $m, +q_2$

Apply conservation of energy, momentum conservation and $e = 1$.

21. (1.25)

$$\frac{kQ_1}{R_1} = \frac{kQ_2}{R_2}$$

$$\frac{k\,\sigma_1\,4\pi R_1^2}{R_1} = \frac{k\,\sigma_2\,4\pi R_2^2}{R_2}$$

$$\sigma_1 R_1 = \sigma_2 R_2$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} = \frac{5}{4}$$

22. (8)

$$E \times 5 = 2000$$

$$\Rightarrow E = 400 \text{ N/C}$$

$$\Delta V = Ed = 400 \times 0.02$$

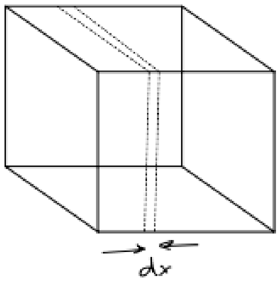
$$= 8 \text{ V}$$

23. (1200)

$$W = q\,(V_B - V_A)$$

$$\Rightarrow V_B - V_A = \frac{12}{0.01} = 1200$$

24. (0.75)



$dq = \rho\, dV$

$q = \int \rho A\, dx$

$= A \int \rho\, dx$

$= A \times$ area under ρ-x curve

$\Rightarrow q_{enc} = \frac{1}{2} \times \left(\frac{1}{2}+1\right) \times \rho_0$

$= \frac{3}{4} \rho_0$

$\text{flux} = \frac{Q_{enc}}{\epsilon_0} = \frac{3\rho_0}{4\epsilon_0}$

$= \frac{3}{4} \times \frac{8.85 \times 10^{-12}}{8.85 \times 10^{-12}} = 0.75$

25. (20)

$E = -\frac{\partial U}{\partial r} = -(10x+10)$

$= -(10+10) = -20.$

26. (4.5)

Force between two line charges On a unit length

$$= \frac{2K\lambda}{r} \times \lambda = \frac{2 \times 9 \times 10^9 \times \left(5 \times 10^{-6}\right)^2}{0.1} = 4.5 \text{ N/m}$$

27. (0)

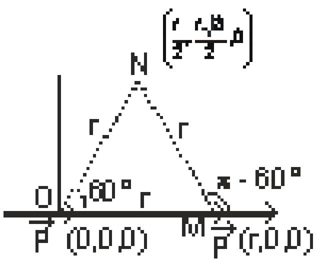
As ON = MN = OM = r

So it is equilateral triangle :

$\therefore$ Potential at N due to two dipoles ;

$V = V_1 + V_2$

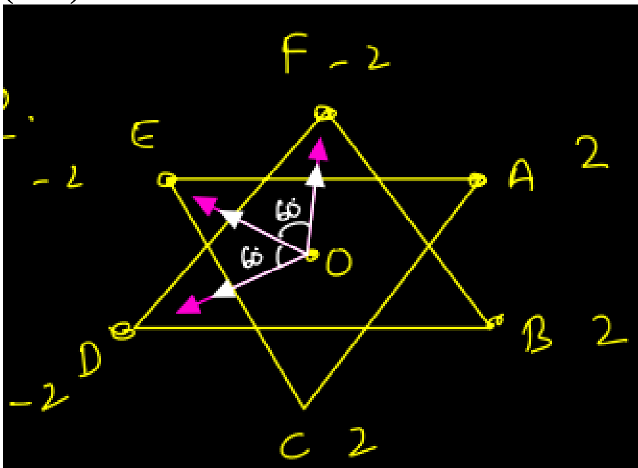
$$= \frac{K_p \cos 60^\circ}{r^2} + \frac{K_p \cos(\pi - 60^\circ)}{r^2} = 0$$



28. (0.1)

$$\Delta V = \frac{2J}{20} = 0.1$$

29. (43.2)



$$OF = 10 \sin 60^\circ \times \frac{2}{3}$$

$$= \frac{10\sqrt{3}}{2} \times \frac{2}{3}$$

$$= \left(\frac{10}{\sqrt{3}}\right)$$

Force along OF

$$\frac{k(2)(2)}{\left(\frac{0.1}{\sqrt{3}}\right)^2} \times 2$$

Net force

$$= \frac{k(2)(2)}{\left(\frac{0.1}{\sqrt{3}}\right)^2} \times 2 + \left(\frac{k(2)(2)}{\left(\frac{0.1}{\sqrt{3}}\right)^2} \times 2\right) \times 2 \times \cos 60^\circ$$

30. (400)

$$W = q\,\Delta V$$

$$V_B = \frac{k(5)}{0.01} \qquad V_A = \frac{k(5)}{0.05}$$

$$\Delta V = \frac{k(5)}{0.05} - \frac{k(5)}{0.01}$$

$$= \frac{5k - 25k}{0.05} = -\frac{20k}{0.05}$$

$$= -400k = -400 \times \frac{1}{4\pi \epsilon_0}$$

$$\Rightarrow N = 400$$

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TOPIC: ELECTROCHEMISTRY

SOLUTIONS

31. (B)
In galvanic cell, e^- flows from anode (Zn) to cathode (Ag).

32. (C)
$E^\circ = -0.83 - 1.82 = -1.65 < 0$ (non-spontaneous)

33. (D)
Mg and Zn get oxidised in preference to Fe.

34. (C)

35. (B)
$6 \times E^\circ = 4 \times 0.54 + 2 \times 0.76 \Rightarrow E^\circ = 0.613V$

36. (B)

37. (B)
$-0.25 = -E^\circ_{Cl^-|AgCl\|Ag} + \frac{0.059}{1}\log\left[Cl^-\right]$
Now , $0.191 = +0.799 + 0.059\log K_{sp}$
$\Rightarrow K_{sp} = 5 \times 10^{-11}$

38. (A)
$\Delta G^\circ_{MnO_4^-|MnO_2} = \Delta G^\circ_1 - \Delta G^\circ_2$
$\Rightarrow -3 \times F \times E^\circ = -5 \times 1.51 \times F + 2 \times 1.23 \times F$
$\Rightarrow E^\circ = \frac{-2 \times 1.23 + 5 \times 1.51}{3} \simeq 1.70V$

39. (C)
$E_{R.P} = -\frac{0.06}{2}\log\left\{\frac{P_{H_2}}{\left[H^+\right]^2}\right\} < 0$ when $P_{H_2} > \left[H^+\right]^2$

40. (C)
$E = E^\circ - \frac{0.06}{2}\log\left[\frac{\left[Zn^{2+}\right]P_{H_2}}{\left[H^+\right]^2}\right]$

As $\left[H^+\right]\uparrow; E\uparrow$ and equilibrium shifts to right.

41. (D)

$$E^\circ_{I^-|AgI|Ag} = E^\circ_{Ag^+|Ag} + 0.059\log\left(K_{sp}\right)$$

$$= +0.8 + 0.059\log\left(8.3\times10^{-17}\right) = -0.15V$$

42. (C)

$$H_2 \longrightarrow 2H^+ + 2e^-$$

$$2\times\begin{cases} AgI \to Ag^+ + I^- \\ Ag^+ + e^- \to Ag \end{cases}$$

$$H_2 + 2AgI \to 2H^+ + 2Ag + 2I^-$$

$$0 = -0.151 - \frac{0.059}{2}\log\left[H^+\right]^2$$

$$\Rightarrow pH = \frac{0.151}{0.059} = 2.56$$

43. (B)

$$E_{cell} = -\frac{0.059}{2}\log\frac{\left[H^+\right]_A^2}{\left[H^+\right]_C^2}$$

$$= -\frac{0.059}{2}\log\frac{10^{-5}}{10^{-3}} = +0.059V$$

44. (B)

Net cell reaction

$A: -2Ag + CO_3^{2-} \to Ag_2CO_3 + 2e^-$

$C: AgBr + e^- \to Ag + Br^-] \times 2$

$$Q = \frac{[Br^-]^2}{[CO_3^{2-}]} \Leftarrow 2AgBr + CO_3^{2-} \to Ag_2CO_3 + 2Br^-$$

At equil; $\sqrt{Q} = \sqrt{K} = \dfrac{4\times10^{-13}}{\sqrt{8\times10^{-12}}} = \sqrt{2}\times10^{-7}$

45. (A)

Net cell reaction: $Fe^{2+} + Ag^+ \to Fe^{3+} + Ag$

$$0 = 0.7991 - 0.771 - \frac{0.0591}{1}\log\frac{1}{\left[Ag^+\right]}$$

$$\Rightarrow \left[Ag^+\right] = 0.335M$$

46. (C)

$$E = E^\circ - \frac{0.0591}{2}\log Q$$

$$0.199 = 0 + 0.76 - \frac{0.0591}{2}\log\frac{\left[Zn^{2+}\right]\left(p_{H_2}\right)}{\left[H^+\right]^2}$$

$$\frac{0.561\times 2}{0.0591} = \log\frac{0.1\times 10}{\left[H^+\right]^2}$$

$$18.98 = -1 - 2\log\left[H^+\right]$$

$$199.98 = 2pH$$

$$pH = 10$$

47. (C)

$$Cu + Zn^{2+} \longrightarrow Cu^{2+} + Zn, \Delta G_1^\circ$$

$$Zn \longrightarrow Zn^{2+} + 2e^-, \Delta G_2^\circ$$

$$Cu^{2+} + 4NH_3 \longrightarrow \left[Cu\left(NH_3\right)_4\right]^{2+}, \Delta G_3^\circ$$

$$Cu + 4NH_3 \longrightarrow \left[Cu\left(NH_3\right)_4\right]^{2+}_{+2e^-}, \Delta G_4^\circ$$

$$\Delta G_4^\circ = \Delta G_1^\circ + \Delta G_2^\circ + \Delta G_3^\circ$$

$$-2\times F\times E^\circ = -2\times F\times\left(-1.1\right)$$

$$-2\times F\times 0.76 - 2.303RT\log\left(10^{20}\right)$$

$$\Rightarrow E^\circ = -1.1 + 0.76 + \frac{0.06}{2}\times 20 = +0.26V$$

48. (C)

Reaction quotient $\propto \frac{1}{\left[OH^-\right]^4}$ for P

Reaction quotient $\propto \left[H^+\right]^{12}$ for Q

In both above cases $pH\uparrow$ favours forward reaction.

49. (A)

$$E = E^\circ - \frac{0.059}{4}\log\left\{\frac{1}{\left[H^+\right]^4}\right\}$$

$$= E^\circ - 0.059pH; \text{As } pH\uparrow, E\downarrow$$

50. (D)

$$E = 1.33 - \frac{0.060}{6}\log\frac{1}{\left[H^+\right]^{14}}$$

$$= 1.33 - 0.14pH$$

$$= +0.77V > 0.54V$$

Hence, only I^- can be oxidised.

51. (65.6)

$$\Delta S = nF\left(\frac{dE_{cell}}{dT}\right)$$

$$H_2 + Hg_2Cl_2 \longrightarrow 2H^+ + 2Cl^- + 2Hg; (n = 2)$$

52. (40)

53. (4)

$$A:\ H_2 \longrightarrow 2H^+ + e^- \times 2$$

$$C:\ O_2 + 2H_2O + 4e^- \longrightarrow 4OH^-$$

$$2H_2 + O_2 \longrightarrow 2H_2O$$

$$\Delta G^\circ_{R\times n} = 2\times(-237.2) + 2\times 80$$

$$= -4\times 96500 \times E^\circ$$

$$\Rightarrow E^\circ = +0.40V$$

54. (9)

$$0.283 = \frac{0.059}{2}\log K \Rightarrow K = 3.9\times 10^9$$

55. (8)

$$E = 0.80 - \frac{0.059}{1}\log\left(\frac{10^{-2}}{10^{-2}}\right)$$

56. (2)

$$E = 2.00 - \frac{0.059}{6}\log\left[\frac{(0.001)^2}{(0.1)^3}\right] = 2.03V$$

$$\left[2Al + 3Cu^{2+} \longrightarrow 2Al^{3+} + 3Cu\right]$$

57. (23)

$$\left[Ag^+\right] = \frac{1.8\times 10^{-10}}{0.8} = 2.25\times 10^{-10}$$

$$E_{RP} = +0.799 - \frac{0.059}{1}\log\left(\frac{1}{2.25\times 10^{-10}}\right) = -0.23V$$

Hence, O.P is $+0.23V$

58. (2)

$$E = 0.743 - \frac{0.059}{5}\log\left[\frac{1\times 1^3}{2.3\times 10^{-2}\times 10^{-20}\times 10^{-24}}\right]$$

$$= 0.20V$$

59. (358)

$$0.192 = +0.402 - \frac{0.059}{2}\log\frac{0.9\times 0.975}{\left[H^+\right]^2}$$

$$\Rightarrow pH = 3.58$$

60. (2)

$$E^\circ_{cell} = E^\circ_{Al|Al^{3+}} + E^\circ_{Cu^{2+}|Cu} = +1.66 + 0.34 = +2.00V$$

SOLUTIONS

61. **(A)**

$y=\left(1+x^{1/2}\right)\left(1-x^{1/2}\right)=1-x$

$\therefore \quad \frac{dy}{dx}=-1$ **Ans.[A]**

62. **(C)**

$\frac{dx}{d\theta}=a\,(1+\cos\theta),\ \frac{dy}{d\theta}=\text{a}\sin\theta$

$\therefore\ \frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{a\sin\theta}{a(1+\cos\theta)}=\tan\frac{1}{2}\theta$

63. **(A)**

$y=\log e^x-\log(e^x+1)$

$=x-\log(e^x+1)$

$\therefore\ \frac{dy}{dx}=1-\frac{e^x}{e^x+1}=\frac{1}{e^x+1}$

64. **(C)**

$\frac{dy}{dx}=\frac{-2x}{(x^2-a^2)^2}\ \Rightarrow\ \frac{d^2y}{dx^2}$

$=-\frac{(x^2-a^2)^2.2-2x.2(x^2-a^2).2x}{(x^2-a^2)^4}$

$=\frac{2(3x^2+a^2)}{(x^2-a^2)^3}$

65. **(B)**

$y=\frac{\sec x-\tan x}{\sec x+\tan x}\cdot\frac{\sec x-\tan x}{\sec x-\tan x}$

$=\frac{\left(\sec x-\tan x\right)^2}{1}$

$\therefore\ \frac{dy}{dx}=2\left(\sec x-\tan x\right)\left(\sec x\tan x-\sec^2 x\right)$

$=-2\sec x\,(\sec x-\tan x)^2$

66. **(B)**

Let us first express y in terms of *x* because all alternatives are in terms of *x*.

So, $x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow \quad x^2(1+y) = y^2(1+x)$

$\Rightarrow \quad x^2 - y^2 + x^2y - y^2x = 0$

$\Rightarrow \quad (x-y)(x+y+xy) = 0$

$\Rightarrow \quad x + y + xy = 0 \qquad (\because x \neq y)$

$\Rightarrow \quad y = -\dfrac{x}{1+x}$

$\therefore \quad \dfrac{dy}{dx} = -\dfrac{(1+x)1 - x.1}{(1+x)^2} = -\dfrac{1}{(1+x)^2}$

67. **(C)**

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\sin x}} \cdot \frac{1}{2\sqrt{\sin x}} . \cos x$$

$$= \frac{\sqrt{1+\sin x}}{2\sqrt{\sin x}} = \frac{1}{2}\sqrt{1+\text{cosec } x}$$

68. **(C)**

$$y = \log_x 10 = \frac{\log_e 10}{\log_e x}$$

$$\therefore \quad \frac{dy}{dx} = \log_e 10 \left\{ -\frac{1}{(\log_e x)^2} . \frac{1}{x} \right\}$$

$$= -\frac{1}{x\log_e 10} . \frac{(\log_e 10)^2}{(\log_e x)^2}$$

$$= -\frac{(\log_e 10)^2}{x\log_e 10}$$

69. **(D)**

$\because \cos(xy) - x = 0$

$$\therefore \quad \frac{dy}{dx} = -\frac{-y\sin(xy) - 1}{-x\sin(xy)} = -\frac{y + \text{cosec}(xy)}{x}$$

70. **(A)**

Let $f(x, y) = x^2e^y + 2xye^x + 13$

$$\therefore \quad \frac{dy}{dx} = -\frac{\partial f}{\partial x} / \frac{\partial f}{\partial y}$$

$$= -\frac{2xe^y + 2ye^x + 2xye^x}{x^2e^y + 2xe^x}$$

Dividing Num^r and Den^r by e^x

$$\frac{dy}{dx} = -\frac{2xe^{y-x} + 2y(x+1)}{x(xe^{y-x} + 2)}$$

71. **(D)**

Taking log on both sides, we have

$y \log x + x \log y = 0$

Now using partial derivatives, we have

$$\frac{dy}{dx} = -\frac{y/x + \log y}{\log x + x/y} = -\frac{y(y + x\log y)}{x(x + y\log x)}$$

72. **(B)**

$x = e^{\tan^{-1}\left(\frac{y - x^2}{x^2}\right)}$

Taking logarithm of both the sides, we get

$\log x = \tan^{-1}\left(\frac{y - x^2}{x^2}\right)$

$\Rightarrow \quad y = x^2 + x^2 \tan(\log x)$

$\frac{dy}{dx} = 2x + 2x\tan(\log x) + x^2 \sec^2(\log x).\ \frac{1}{x}$

$= 2x\,[1 + \tan(\log x)] + x\sec^2(\log x).$

73. **(C)**

$y = \tan^{-1}\frac{3x - x^3}{1 - 3x^2} = 3\tan^{-1} x$

$\therefore \quad \frac{dy}{dx} = \frac{3}{1 + x^2}$

74. **(B)**

$y = 2\tan^{-1} x$

$\therefore \quad \frac{dy}{dx} = \frac{2}{1 + x^2}$

75. **(C)**

$y = \tan^{-1}\left(\frac{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}\right)$, where

$x^2 = \cos\theta$

$= \tan^{-1}\left(\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2}\right)$

$= \tan^{-1}\left(\frac{1 + \tan\theta/2}{1 - \tan\theta/2}\right)$

$= \tan^{-1}[\tan(\pi/4 + \theta/2)] = \pi/4 + \theta/2$

$= \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

$\therefore \quad \frac{dy}{dx} = -\frac{1}{2}\frac{1}{\sqrt{1 - x^4}}.2x = -\frac{x}{\sqrt{1 - x^4}}$

76. **(B)**

Substituting $x = \sin\theta$ and $y = \sin\phi$ in the given equation, we get

$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$\Rightarrow 2\cos\frac{\theta + \varphi}{2}.\cos\frac{\theta - \varphi}{2} = 2a\cos\frac{\theta + \varphi}{2}.\sin\frac{\theta - \varphi}{2}$

$\Rightarrow \cot\frac{\theta - \varphi}{2} = a \Rightarrow \theta - \phi = 2\cot^{-1} a$

$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$

Differentiating with respect to x, we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

77. **(C)**

$y = \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1} x$

$\Rightarrow \frac{dy}{dx} = \frac{2x}{1+x^2}$ and z = $\tan^{-1} x \Rightarrow \frac{dz}{dx} = \frac{1}{1+x^2}$

$\therefore \quad \frac{dz}{dx} = \frac{dy/dx}{dz/dx} = \frac{2}{1+x^2}.\frac{1+x^2}{1} = 2$

78. **(B)**

Here $y = \sqrt{\sin x + y} \quad \Rightarrow y^2 = \sin x + y+$

$\therefore 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$

79. **(A)**

Here $y = \frac{x}{a+\frac{x}{b+y}} = \frac{x(b+y)}{a(b+y)+x}$

$\Rightarrow \quad aby + ay^2 + xy = bx + xy$

$\Rightarrow \quad ay^2 + aby = bx$

$\Rightarrow \quad 2ay\frac{dy}{dx} + ab\frac{dy}{dx} = b$

$\Rightarrow \quad \frac{dy}{dx} = \frac{b}{a(b+2y)}$

80. **(C)**

$y = e^{x+y}$

$\Rightarrow \quad \log y = x + y \quad \Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{1-y}$

81. **(1)**

$\frac{dy}{dx} = \left(\frac{dy}{d\theta}\right)/\left(\frac{dx}{d\theta}\right)$

$= \frac{3a\sin^2\theta.\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$

$\therefore$ exp. = $\sqrt{1+\tan^2\theta} = \sec\theta$

82. **(6)**

From the given equation, we have

$y^2(1 - x^2) = (\sin^{-1} x)^2$

$\Rightarrow \quad (1-x^2)\, 2y \dfrac{dy}{dx} - 2xy^2 = 2 \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$

$\Rightarrow \quad 2(1-x^2)\, y \dfrac{dy}{dx} - 2xy^2 = 2y$

$\Rightarrow \quad (1-x^2) \dfrac{dy}{dx} = 1 + xy = 6$

83. **(1)**

$\because\ g(x) = f[f(x)]$

$= f\{|x-2|\}$

$= ||x-2|-2|$

But $x > 20 \Rightarrow |x-2| = x-2$

$\Rightarrow \quad g(x) = |x-2-2| = x-4$

$\therefore \quad g'(x) = 1$

84. **(11)**

$h'(x) = 2f(x) f'(x) + 2g(x) g'(x)$

$= 2f(x) g(x) + 2g(x) f''(x)$

$= 2f(x) g(x) - 2f(x) g(x)$

$= 0 \qquad [\because\ f''(x) = -f(x)]$

$\Rightarrow \quad h(x) = c$

$\Rightarrow \quad h(10) = h(5) = 11$

85. **(1)**

$\because\ \lambda\,\text{n}\, x = \log_e x$, so

$f(x) = \log_x (\log_e x) = \dfrac{\log(\log x)}{\log x}$

$\Rightarrow f'(x) = \dfrac{\log x\left(\dfrac{1}{x\log x}\right) - \log(\log x)\dfrac{1}{x}}{(\log x)^2}$

$\therefore \quad ef'(e) = e\dfrac{1/e - 0}{(1)^2} = \dfrac{1}{e}\ e = 1$

86. **(0)**

When $1 < x \le 3$,

$f(x) = (x-1) - (x-3) = 2$

$\Rightarrow\ f'(2-0) = 0,\ f'(2+0) = 0$

$\therefore\ f'(2) = 0$

87. **(1)**

Let $y = \sin 2x \cos 2x \cos 3x + \log_2 2^{x+3}$

$= \dfrac{1}{2} \sin 4x \cos 3x + (x+3)\log_2 2$

$= \dfrac{1}{4}[\sin 7x + \sin x] + x + 3$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{1}{4}\ [7\cos 7x + \cos x] + 1$

$\therefore \quad \left(\dfrac{dy}{dx}\right)_{x=\pi} = \dfrac{1}{4}\ [7\cos 7\pi + \cos \pi] + 1$

$= \frac{1}{4}\ [-8] + 1 = -1$

88. **(3)**

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3}{2t}$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{1}{2}} = 3$$

89. **(1)**

$$y = e^{x^2 \ln x}$$

$$\left.\frac{dy}{dx}\right| = e^{x^2 \ln x}\left(2x\ln x + x^2 \cdot \frac{1}{x}\right)$$

$$= 1 \text{ When } x = 1$$

90. **(3)**

$$y = \frac{\pi}{2} + 3x$$

$$\frac{dy}{dx} = 3 \text{ for all } x$$