IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 04/03/23**

TOPIC: CENTRE OF MASS & COLLISION

Solutions

1. (D)
The final orientation of the system will be such that the centre of mass will be vertically below the point of suspension.

2. (B)
MgL = (M+m)gh

3. (D)
By the conservation of linear momentum, Velocity of the hoop is

$$V_D = \frac{2V_0}{3}$$

By conservation of Energy

$$\frac{1}{2}mV_0^{\,2} + \frac{1}{2}mV_0^{\,2} = \frac{1}{2}m\left(\frac{2V_0}{3}\right)^2 + \frac{1}{2}mX^2 + \frac{1}{2}mX^2$$

4. (C)

$$P_{avg} = \frac{W_{total}}{T} = \frac{W_1 + W_2}{T} \qquad ...(1)$$

For W_1,

$$dw = Fdx = \lambda x\, gdx$$

$$W_1 = \int dw = \int_0^L \lambda x\, gdx = \frac{\lambda g L^2}{2} \quad \left(\because \lambda = \frac{m}{L}\right)$$

From work energy theorem

$$W = \Delta K.E = \frac{1}{2}Mv^2 - 0 = \frac{1}{2}Mv^2$$

Put the value of W_1 and W_2 in equation (1)

$$P_{avg} = \frac{\frac{\lambda g \ell^2}{2} + \frac{1}{2}\lambda L v^2}{\frac{L}{v}} \left(T = \frac{L}{v}\right)$$

$$= \frac{\lambda v g L}{2} + \frac{1}{2}\lambda v^3$$

5. (A)
Line $3x - 4y = 0$, $3x - 4y = 0$ has slope

$\tan\theta_1 = \frac{3}{4}$

Line $12y + 5x = 0$, $12y + 5x = 0$ has slope

$\tan\theta_2 = -\frac{5}{12}$

For A, $y_A = (10\sin\theta_1)t$

For B, $y_B = (v\sin\theta_2)t$

Now according to question,

$$y_{cm} = \frac{m_A y_A + m_B y_B}{m_A + m_B} = 0$$

$$\Rightarrow \ (1)(10\sin\theta_1)t - (2)(v\sin\theta_2)t = 0$$

$$\Rightarrow \ v = \frac{10\sin\theta_1}{2\sin\theta_2} = (5)\left(\frac{3}{5}\times\frac{13}{5}\right) = \frac{39}{5}$$

6. (C)

Position of Center of Gravity of first slab is at a height $= \frac{b}{2}$ from the bottom edge of the slab.

Weight of each slab = volume × density × g = $b^3\rho g$

Now, for the column of N no. of slabs, the CG will be at a height $$

Center of gravity of column of slabs $= \frac{\text{Total height of N slabs}}{2} = \frac{Nb}{2}$ from the ground.

So, $PE_{final} = \text{Total weight of N slabs} \times \text{height of center of mass} = N \times b^2\rho g \times \frac{Nb}{2} = \frac{N^2 b^4 \rho g}{2}$

And, PE of all the slabs staying on ground, before arranging those $= PE_{\text{initial}} = Nb^3\rho g \times \frac{b}{2} = \frac{Nb^4\rho g}{2}$

So, change in potential energy,

$$\Delta PE = PE_{final} - PE_{initial} = Work\ done = \frac{N^2 b^4\rho g}{2} - \frac{N^2 b^4 \rho g}{2} = \frac{1}{2}(N^2 - N)b^4\rho g$$

7. (C)

Given :

Mass of bullet = a

Velocity of bullet = b

Mass of wood = c

The final velocity = ?

Initial momentum = ab

Final momentum = $(a + c)v$ [v is the final velocity, and it combines with mass $(a + c)$]

Equating this, we get

$$v = \frac{ab}{a+c}$$

8. (A)

$$\because \ x_{CM} = \frac{mL + 2m\times 2L + ... + nm\times nL}{m + 2m + ... + nm}$$

$$= \frac{L\left(1^2 + 2^2 + ... + n^2\right)}{\left(1 + 2 + ... + n\right)}$$

$$= L\frac{\Sigma n^2}{\Sigma n} = L\frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}}$$

$$= \frac{(2n+1)L}{3}$$

9. (C)

Given: A *T*-shaped object with dimensions shown in the figure is lying on a smooth floor. A force $\vec{F}$ is applied at the point *P* parallel to *AB*, such that the object has only the translational motion without rotation.

To find the location of *P* with respect to *C*.

Solution:

To have the required motion, the force $\vec{F}$ has to be applied at the centre of mass.

The point *P* must be the centre of mass of the *T*-shaped object since the force *F* does not produce any rotational motion of the object.

The horizontal part of the *T*-shaped object has length *L*. If the mass of the horizontal portion is '*m*', the mass of the vertical portion of the *T*-shaped object is 2*m* since its length is 2*L*.

Therefore, the *T*-shaped object reduces to two point masses *m* and 2*m* at distances 2*L* and *L* respectively from the point *C*. The distance '*r*' of the centre of mass of the system from the point *C* is given by

$$r = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$\Rightarrow \quad r = \frac{m \times 2L + 2m \times L}{m + 2m}$$

$\Rightarrow \quad r = \frac{4L}{3}$ is the location of P with respect to *C*.

10. (B)

CM will be at rest, $m(50 - x) = 99m(x)$

$$\Rightarrow \quad x = \frac{1}{2}m$$

11. (D)

$\vec{R} = (x, y)$

$$\frac{\int \vec{R}.dm}{\int dm} = \frac{\int \frac{\alpha_0 xy}{ab}(dx\,dy)(x\hat{i} + y\hat{j})}{\int \frac{\alpha_0 xy}{ab}dx.dy} \frac{\int \frac{\alpha_0 xy}{ab} \times dx.dy\hat{i}}{\int \frac{\alpha_0 xy}{ab}dx.yd} + \frac{\int \frac{\alpha_0 xy}{ab} \times dx.dy(y)\hat{j}}{\int \frac{\alpha_0 xy}{ab}dx.dy}$$

$y = COM \qquad x =$

$dy = \qquad dx =$

$$\frac{\int_0^a x^2dx}{\int_0^a xdx}\hat{i}+\frac{\int_0^b y^2dy}{\int_0^b ydx}$$

$$\frac{2a}{3}\hat{i}+\frac{2b}{3}\hat{j}$$

12. (C)

$$\vec{R}_{\text{cm}}=\frac{m_1\vec{r}_1\ m_2\vec{r}_2}{m_1+m_2}=-2\hat{i}-\hat{j}+\hat{k}$$

13. (D)

$$V_{cm}=\frac{m_1v_1+m_2v_2}{m_1+m_2}$$

14. (D)

Velocity of CoM will remain unchanged as explosion was an internal force.

Ratio of velocities will be inverse of ratio of masses i.e., Speed of smaller fragment will be twice that of largest fragment.

K.E. will increase as both fragments have their K.E. where as initial mass was on rest.

Fragments have equal magnitude of momentum in ground frame and centre of mass frame as CoM is also not moving.

15. (A)

$$v=u\ln\left(\frac{m_i}{m_f}\right)-gt=500\ln\left(\frac{50000}{40000}\right)-100=111.5-100=11.5$$

16. (B)

We are given that the mass of the big particle is 4m and velocity of two fragments are mutually perpendicular to each other. If p_1 and p_2 are the momentum of the fragments then momentum of third fragment is

$$p_3=\sqrt{p_1^2+p_2^2}=\sqrt{(mv)^2+(mv)^2}$$
$$=\sqrt{2}\,mv$$

Velocity of third fragment $=\frac{v}{\sqrt{2}}$

Total energy released in the explosion $=\frac{1}{2}mv^2+\frac{1}{2}mv^2+\frac{1}{2}\times 2m\left(\frac{v}{\sqrt{2}}\right)^2=\frac{3}{2}mv^2$

Thus, the total energy released in the explosion is $\frac{3}{2}mv^2$

17. (C)

Component of initial momentum along the wall is

$$p_{iy}=mv_0\sin 37^{\circ}\,\hat{j}$$

Component of final momentum along the wall is

$$p_{fy} = \frac{mv_0}{2}\sin 53^o \hat{j}$$

So, component of impulse along the wall is $J_y = p_{fy} - p_{iy}$

$$= \frac{mv_0}{2}\times\frac{4}{5} - \frac{mv_0}{1}\times\frac{3}{5}\hat{j} = -\frac{mv_0}{5}\hat{j}$$

18. (A)

Velocity of light ball, when it is descended by height $2l$

$$v = \sqrt{2g\times 2l} = 2\sqrt{gl}$$

From conservation of linear momentum.

$$m\times 2\sqrt{gl} = 2mv' + mv'$$

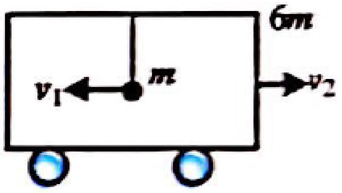
$$2m\sqrt{gl} = 3mv'$$

$$\frac{2}{3}\sqrt{gl} = v'$$

19. (C)

When the rod becomes vertical

$$mv_1 = 6mv_2 \Rightarrow v_1 = 6v_2 \qquad ...(i)$$



$$\frac{1}{2}mv_1^2 + \frac{1}{2}6mv_2^2 = mgl \qquad ...(ii)$$

Solve (i) and (ii) to get

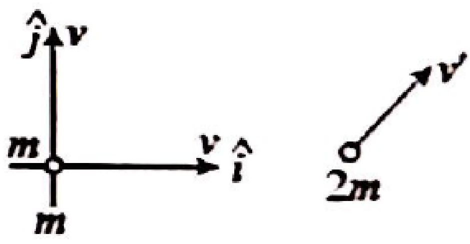
$$v_{\text{rel}} = v_1 + v_2 = \sqrt{\frac{7}{3}gl}$$

20. (B)

Conservation of linear momentum yields

$$mv\hat{i} + mv\hat{j} = (2m)\vec{v}'$$

$$,mv^2 \Rightarrow \vec{v}' = \frac{v}{2}(i+\hat{j}) \Rightarrow v' = \frac{v}{\sqrt{2}}$$



$$\eta = 1 - \frac{KF_f}{KE_i} = 1 - \frac{\left(\frac{1}{2}\right)(2m)v'^2}{\left(\frac{1}{2}\right)mv^2 + \left(\frac{1}{2}\right)mv^2}$$

$$\Rightarrow \quad \eta = 1 - \left(\frac{v'}{v}\right)^2 = 1 - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

21. (2)

Find the area under the graph from 0 to 2.

22. (4)

$$s = 4t = 2t + 5t^2$$

$$5t^2 = 2t$$

$t = 0.4$
$v = 2 + gt = 6\ \text{m/s}$
$4 \times 4 \times 4 \times 6 = 8v$
$v = 5$
$J = 4(v - 4) = 4$

23. (5)

Given: $P_1 = 20$ Ns

Let the minimum momentum of other part be P_2

Applying conservation of momentum in *y* direction :

$P_1 \sin 30 = P_2 \sin\theta$

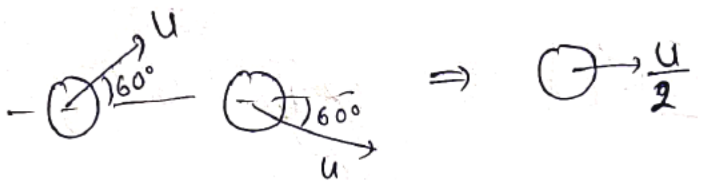
$20 \times \frac{1}{2} = P_2 \sin\theta$

$\Rightarrow\ P_2 \sin\theta = 10$ Ns

Now for P_2 to be minimum, $\sin\theta$ must be maximum i.e. $\sin\theta = 1$

$\Rightarrow\ P_2 = 10$ Ns

24. (120)



Angle between velocity is 120°.

25. (3)

Lets take a small element at a distance *x* of lengths *dx*.

The velocity with which element will strike the floor,

$v = \sqrt{2gx}$

Momentum transferred to floor is,

$dP = dmv = \left[\frac{M}{L} x dx \sqrt{2gx}\right]$

$F_1 = \frac{dp}{dt}$ and $v = \frac{dx}{dt} = \sqrt{2gx}$

$F_1 = \frac{M\sqrt{2gx} \times \sqrt{2gx}}{L}$

$F_1 = \frac{2mgx}{L}$

Now, the force entered by length *x* due to its own weight is,

$w = \frac{mgx}{L}$

Therefore,

Total force $= \frac{mgx}{L} + \frac{2mgx}{L}$

$F = \frac{3mgx}{L}$

26. (3)

Apply momentum conservation.

27. (8)

Thrust is a reaction force. When a system accelerates a mass in a particular direction, the accelerated mass will cause a force of equal magnitude and opposite direction on that system. Thrust is defined as,

$$F_{\text{thrust}} = u\frac{dm}{dt}$$
$$= \left(5\times10^4\right)(40)$$
$$= 2\times10^6 \text{ N}$$

28. (2)

Horizontal Distance moved by the block wrt wedge = 12m.

For COM to not move Wedge must move 2m in the opposite direction.

29. (5)

It's velocity is 60 m/s downwards and 80 m/s forward. Apply momentum conservation to get this.

30. (4)

$$S_{avg} = \frac{\int_0^{\sqrt{\frac{2h}{g}}} \frac{1}{2}gt^2 dt}{\int_0^{\sqrt{\frac{2h}{g}}} dt} = \frac{g}{6}.\frac{2h}{g} = \frac{h}{3}$$

$$H = h - \frac{h}{3} = \frac{2h}{3}$$

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TOPIC: GOC

Solutions

31. (C)
It is more contributing to resonance hybrid (Explains aromatic character).

32. (D)
Due to loss of planarity.

33. (B)
Maximum +M effect.

34. (B)
Due to resonance

35. (B)
Resonance and Hyperconjugation.

36. (C)
More branching

37. (C)
No empty orbital in N.

38. (D)
$-CH_3$ is a positive inductive effect group.

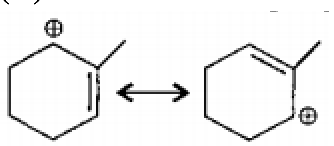
39. (C)
$-NH_2$ is a negative inductive effect group.

40. (B)
FCH_2COOH

41. (B)
–CN is electron withdrawing group via resonance.

42. (C)
–OH is electron releasing group via resonance.

43. (A)



44. (C)
–NO can show both +M and –M effect.

45. (B)
Octet complete.

46. (A)

$$CH_2 = CH - \overset{\oplus}{C}H - \overset{\ominus}{C}H - O - CH_3$$

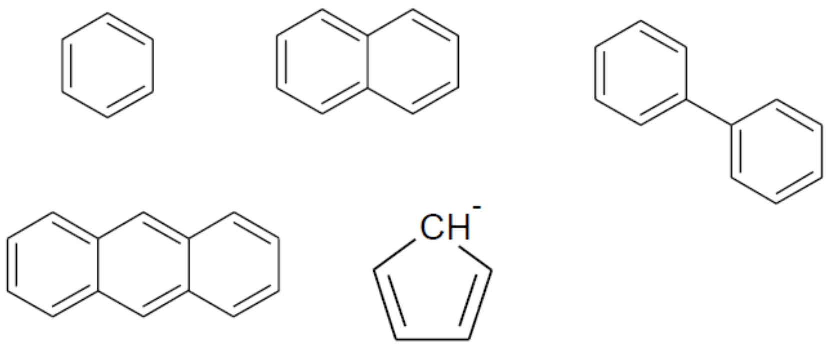
47. (D)
On the basis of hyperconjugation.

48. (D)
I effect is distance dependent.

49. (A)
$-NO_2 > -Cl > -Br > -I$

50. (A)
Only electrons are allowed to move, positions of nuclei don't change.

51. (5)



52. (5)
$-SH$, $-NH_2$, $-NO_2$, $-CN$, $-CHO$

53. (5)

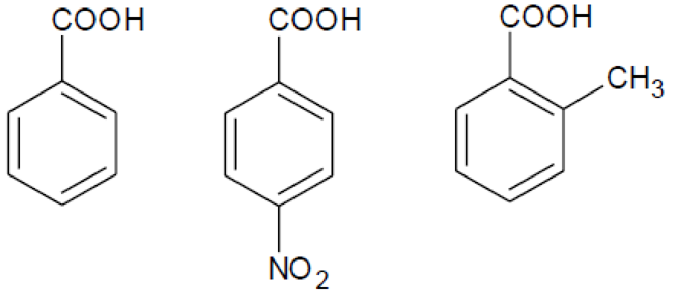
54. (3)
$-CH_3$, $-CH_2CH_3$, $-COO^-$

55. (7)

56. (6)

$-O^-$, $-OH$, $-NH_2$, $-NHCOCH_3$, $-OCOCH_3$, $-NHCOC_2H_5$

57. (3)



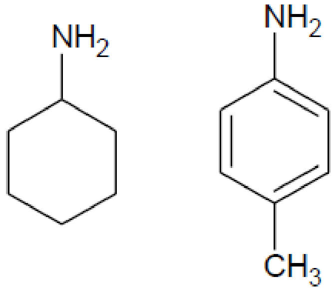
58. (3)

$-CHO$, $-NO_2$, $-CN$

59. (2)

$-CH_2-\overset{\oplus}{C}H_2,\ -CH_2-\overline{C}H_2$

60. (2)



Solutions

61. (B)

$f(x)=\tan\left(\frac{\pi}{4}\sin^2 x\right)$

$\sin x \in [-1,1]$

$\therefore\ \sin x \in [0,1]$

$\Rightarrow\ \frac{\pi}{4}\sin^2 x \in \left[0, \frac{\pi}{4}\right]$

$\therefore\ f(x) \in \tan\left[0, \frac{\pi}{4}\right]$

$f(x) \in \left[\tan 0, \tan\frac{\pi}{4}\right]$

$\therefore\ f(x) \in [0,1]$

62. (A)

Range of $f(x)$ is $(1,2)\cup[5,\infty)$

Which is not equal to its codomain.

But $f(x)$ is monotonic increasing function

So, $f(x)$ is one-one but not onto

63. (B)

All bijective function of $f: A \to A$ are

$\left[(1,1),(2,2),(3,3)\right], \left[(1,1),(2,3),(3,2)\right], \left[(1,2),(2,1),(3,3)\right]$

$\left[(1,2),(2,3),(3,1)\right], \left[(1,3),(2,1),(3,2)\right], \left[(1,3),(2,2),(3,1)\right]$

But $f(1)\neq 3$, $f(2)\neq 1$ and $f(3)\neq 2$

$\therefore$ Number of bijective functions are $\left[(1,1),(2,2),(3,3)\right]$ and $\left[(1,2),(2,3),(3,1)\right]$

$\therefore$ 2

64. (B)

$f: R \to R \qquad f(x)=ax^3+bx^2+cx+d\ (a\neq 0)$

For f to be one-one

$f(x_1) = f(x_2)$

i.e., $x_1 = x_2$

for f to be onto :

range = co-domain

$f(x)$ is a (i) strictly increasing function, i.e. $f'(x) > 0$

or (ii) strictly decreasing function, i.e. $f'(x) < 0$

$f'(x) = 3ax^2 + 2bx + c$

$\therefore \; 3ax^2 + 2bx + c > 0 \qquad ...(\text{i})$

or $\; 3ax^2 + 2bx + c < 0 \qquad ...(\text{ii})$

$\therefore \; D < 0$

$(2b)^2 - 4(3a)(c) < 0$

or $\; b^2 - 3ac < 0$

65. (A)

$x^{\sqrt{x}} = (\sqrt{x})^x$

Take log both sides

$\Rightarrow \sqrt{x}\log x = x\log\sqrt{x}$

$\Rightarrow \sqrt{x}\log x = \frac{x}{2}\log x$

$\Rightarrow \left(\sqrt{x} - \frac{x}{2}\right)\log x = 0$

$\therefore \log x = 0$ or $\sqrt{x} = \frac{x}{2}$

$\therefore \; x = 1$ or $4x = x^2$

$x = 0$ (not possible) or $x = 4$

$\therefore \; x = 1$ or $x = 4$.

Hence, one of p & q is composite number.

66. (D)

Let $y = \sqrt{(ab) + (a-b)\sqrt{ab + (a-b)\sqrt{ab + (a-b)\sqrt{ab + ...}}}}$

$\therefore \; y = \sqrt{ab + (a-b)y}$

Squaring both sides

$y^2 = ab + (a-b)y$

$y^2 - (a-b)y - ab = 0$

$\therefore \; y = \frac{(a-b) \pm \sqrt{(a-b)^2 + 4ab}}{2}$

$y = \frac{(a-b) \pm (a+b)}{2}$

$\therefore\ y=\frac{(a-b)+(a+b)}{2}$ or $y=\frac{(a-b)-(a+b)}{2}$

$y=a$ or $y=-b$

$\therefore$ independent of none of a & b.

67. (D)

$$f(x)=\frac{e^{|x|}-e^{-x}}{e^{x}+e^{-x}}$$

f is not one-one as $f(x)=0$ and $f(-1)=0$

f is also not onto as for $y = 1$, there is no $x\in R$

Such that $f(x)=1$. If there is such an $x\in R$, then $e^{|x|}-e^{-x}=e^{x}+e^{-x}$. Clearly $x\neq 0$.

For $x>0$, the equation gives $e^{-x}=0$ (not possible) and for $x<0$, $\frac{e^{2x}+1}{e^{x}}=0$ which is also not possible.

$\therefore$ f is neither one-one nor onto.

68. (C)

$f(x)=3x-4$

$\Rightarrow f^{-1}(x)=\frac{x+4}{3}$

$g(x)=2+3x$

$\Rightarrow g^{-1}(x)=\frac{x-2}{3}$

$f^{-1}(5)=\frac{5+4}{3}=3$

$(g^{-1}of^{-1})(5)=g^{-1}\left[f^{-1}(5)\right]g^{-1}(3)=\frac{3-2}{3}=\frac{1}{3}$

69. (C)

$\log_e\left(x+\sqrt{x^2-1}\right)$

$f(x)=\frac{e^{x}+e^{-x}}{2}=y$

$x=f^{-1}(y)$

$e^{x}+e^{-x}=2y$

$e^{2x}-2ye^{x}+1=0$

$e^{x}=\frac{2y\pm\sqrt{4y^2-4}}{2}$

$e^{x}=\frac{y\pm\sqrt{y^2-1}}{1}$

Range of f is $(-\infty,\infty)\Rightarrow e^{x}=\frac{y\pm\sqrt{y^2-1}}{1}$

As, if we take $e^x = \frac{y-\sqrt{y^2-1}}{1}$, which is always small

$\therefore\ x = \log_e\left(y+\sqrt{y^2-1}\right)$

$\therefore\ f^{-1}(x) = \log_e\left(x+\sqrt{x^2-1}\right)$

70. (D)

Substituting the values, we get

$\Rightarrow\ \cos\left(\ln\frac{1}{x}\right)\cdot\cos\left(\ln\frac{1}{y}\right)-\frac{1}{2}\left[\cos\left(\ln\left(\frac{x}{y}\right)\right)+\cos\left(\ln(xy)\right)\right]$

$\Rightarrow\ \cos\left(\ln x^{-1}\right)\cos\left(\ln y^{-1}\right)-\frac{1}{2}\left[\cos\left(\ln x-\ln y\right)+\cos\left(\ln x+\ln y\right)\right]$

$\Rightarrow\ -\cos(\ln x)\cdot-\cos(\ln y)-\frac{1}{2}\left[2\cos(\ln x)\cos(\ln y)\right]$

$\Rightarrow\ \cos(\ln x)\cos(\ln y)-\cos(\ln x)\cos(\ln y)$

$=0$

71. (B)

$\frac{|\sin x|}{\cos x}$ period of N^r is π and D^r is 2π

$\therefore$ period of $\frac{|\sin x|}{\cos x}$ is LCM of π and $2\pi \Rightarrow$ i.e. 2π

Similarly $\frac{\sin x}{|\cos x|}$ period of N^r is 2π and D^r is π.

$\therefore$ period of $\frac{\sin x}{|\cos x|}$ is LCM of π and $2\pi\ \Rightarrow$ i.e. 2π

Hence the period of the given function is LCM of 2π & 2π. i.e. 2π.

72. (A)

For (B) $f(x) = \sin\frac{1}{x}$

$= (goh)x$, where $g(x)=\sin x, h(x)=\frac{1}{x}$

$\therefore\ f(x)$ is not periodic.

For (C) $f(x) = x\cos x = g(x)\cdot h(x)$, where $g(x)=x, h(x)=\cos x$

$\therefore\ f(x)$ is not periodic.

For (D) $f(x)=\sin\sqrt{x}=(goh)x$, where $g(x)=\sin x, h(x)=\sqrt{x}$

$\therefore\ f$ is not periodic.

For (A) $f(x) = x-[x]$

$=\{x\}$

$\therefore\ \{\ \}$ is a periodic functions with period 1.

$\therefore$ f is periodic.

73. (B)

$$f(x)=\ln\left(\frac{x^2+e}{x^2+1}\right)=\ln\left(\frac{x^2+1-1+e}{x^2+1}\right)=\ln\left(1+\frac{e-1}{x^2+1}\right)$$

$\therefore$ clearly range is (0, 1]

74. (A)

$$f(x)=\frac{\sin(\pi[x])}{x^2+1}$$

$\because$ $[x]$ always gives integer values and $\sin n\pi=0$

$\therefore$ $f(x)=0$

75. (A)

$$\frac{\tan x}{\tan x}=1$$

Which doesn't repeat after sometime and is constant.

76. (B)

Put $x=y=0$

$\therefore$ $f(0)+f(0)=2f^2(0)$

$\because$ $f(0)\neq 0$

$\therefore$ $f(0)=1$

Put $x=2$ and $y=1$

$\therefore$ $f(3)+f(1)=2f(2)f(1)$

$\therefore$ $f(3)-2f(1)f(2)=-f(1)$

$\because$ $f(1)=K$

$\therefore$ $f(3)-2k\ f(2)=-K$

77. (A)

Put $y=0$

$\therefore$ $f(x)=f(0)$

$\because$ $f(2005)=2005,\ \therefore\ f(0)=2005$

$\therefore$ $f(-2005)=f(0)=2005$

78. (D)

Since $f(x)$ is an odd function

$$\left[\frac{x^2}{a}\right]=0 \text{ for all } x\in[-10,10]\Rightarrow 0\leq\frac{x^2}{a}<1\ \forall\ x\in[-10,10]$$

$\Rightarrow a>100$

79. (B)

$$f(x)=\ln\left(\frac{1+x}{1-x}\right),\ g(x)=\frac{3x+x^3}{1+3x^3}$$

$$\therefore\ f(g(x))=\ln\left[\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right]$$

$$\Rightarrow\ f(g(x))=\ln\left[\frac{(1+x)^3}{(1-x)^3}\right]$$

$$=3\ln\left(\frac{1+x}{1-x}\right)$$

$$=3f(x)$$

80. (D)

$$(hogof)(x)=h(g(f(x)))$$

$$=h(g(x)) \qquad \because f(x)=x$$

$$=h(2x^2+1)$$

$$=2x^2+1+1$$

$$=2(x^2+1)$$

$$\therefore\ (hogof)(x)=2(x^2+1)$$

81. (2)

$$f(1)+2f(2)+3f(3)+....+n\ f(n)=n(n+1)f(n)$$

$$\Rightarrow\ f(1)+2f(2)+3f(3)+....+nf(n)+(n+1)f(n+1)=(n+1)(n+2)f(n+1)$$

$$\Rightarrow\ n(n+1)f(n)=(n+1)^2 f(n+1)$$

$$\Rightarrow\ nf(n)=(n+1)f(n+1)$$

$$\therefore\ 2f(2)=3f(3)=......=nf(n)$$

$$\therefore\ f(n)=\frac{1}{n}$$

$$\therefore\ 2126f(1063)=2126\times\frac{1}{1063}=2$$

82. (9)

$$f(x)=x^3+\frac{1}{x^3}-4\left(x^2+\frac{1}{x^2}\right)+13$$

We know that

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)$$

$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$

For $x=2+\sqrt{3},\ x+\frac{1}{x}=2+\sqrt{3}+\frac{1}{2+\sqrt{3}}$

$=2+\sqrt{3}+\frac{2-\sqrt{3}}{1}$

$=4$

$\because\ f(x)=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)-4\left[\left(x+\frac{1}{x}\right)^2-2\right]+13$

$f\left(2+\sqrt{3}\right)=4^3-3\times4-4\left[(4)^2-2\right]+13$

$=64-12-56+13$

$=9$

83. (7)

$f(2a-x)=f(x)$

$\Rightarrow\ f(2a-x)=-f(x)$

$\therefore\ f$ is odd $\Rightarrow\ f(x+4a)=f(x)$

$\Rightarrow\ f$ is periodic with period $4a$

$\Rightarrow\ f(1+4r)=f(1)$

Now, $\sum_{r=0}^{\infty}\left[f(1)\right]^r=8\Rightarrow\frac{1}{1-f(1)}=8$

$\Rightarrow\ f(1)=\frac{7}{8}$

$\therefore\ 8f(1)=7$

84. (2)

Given $f\left(x-f(y)\right)=f\left(f(y)\right)+xf(y)+f(x)-1$

Putting $x=f(y)=0$, then $f(0)=f(0)+0+f(0)-1$

$\therefore\ f(0)=1$

Now, Put $x=f(y)=\lambda$ in equation, then

$f(0)=f(\lambda)+\lambda^2+f(\lambda)-1$

$\Rightarrow\ 1=2f(\lambda)+\lambda^2-1$

$\therefore\ f(\lambda)=1-\frac{\lambda^2}{2}$

$\therefore\ f(x)=1-\frac{x^2}{2}$

$\therefore\ \left|\left(f(16)\right)\right|-125=\left|1-\frac{16^2}{2}\right|-125$

$=127-125=2$

85. (1)

If $f^3(x)-3f^2(x)+3f(x)-1=x^6$

$\Rightarrow \left(f(x)-1\right)^3=x^6$

$\Rightarrow f(x)-1=x^2$

$\Rightarrow f(x)=x^2+1$

$\therefore f(0)=1$

86. (2)

$f(x)=\frac{1}{x},\ g(x)=\sqrt{x}$

$4\left(go\sqrt{f}\right)(16)=4g\left(\sqrt{f(16)}\right)$

$=4g\left(\sqrt{\frac{1}{16}}\right)$

$=4g\left(\frac{1}{4}\right)$

$=4\times\sqrt{\frac{1}{4}}$

$=2$

87. (2)

Whenever, $f(x)+f\left(\frac{1}{x}\right)=f(x)\cdot f\left(\frac{1}{x}\right)$, then $f(x)=1+x^n$ or $f(x)=1-x^n$ $(n>0)$

Here, $f(2)=-7$

$\therefore f(x)=1-x^n \qquad f(2)=1-2^n$

$\Rightarrow 2^n=8$

$\Rightarrow x=3$

$\therefore f(x)=1-x^3$

$\therefore \frac{f(3)+30}{2}=\frac{1-3^3+30}{2}=2$

88. (1)

$G(n)=n-(-1)^{n-1}(n-1)$

$GoG(n)=G\left(n-(-1)^{n-1}(n-1)\right)$

$=n-(-1)^{n-1}(n-1)-(-1)^{n-1}\left((n-1)-(-1)^{n-1}(n-1)\right)$

$=n-(n-1)$

$=1$

89. (4)

$g\left(f\left(f(2)\right)\right)=K$

$g\left(f\left(f(2)\right)\right)=g\left[f\left(f(2)\right)\right]$

$=g\left[f(5)\right] \quad \because \; f(x)=2x+1$

$=g[11] \quad \because \; f(x)=2x+1$

$=11^2+1 \quad \because \; g(x)=x^2+1$

$\therefore \; K=122$

No. of divisors of 122 is 4 (1, 2, 61 and 122)

90. (1)

$f(x)=\left(100^5-x^{10}\right)^{\frac{1}{10}}$

$\Rightarrow \left(f(x)\right)^{10}=100^5-x^{10}$

$f\left(f(x)\right)=\left(100^5-f(x)^{10}\right)^{\frac{1}{10}}$

$=\left[100^5-\left(100^5-x^{10}\right)\right]^{\frac{1}{10}}$

$=x$

$\therefore \; \frac{1}{2^{10}} f\left(f(1024)\right)=\frac{1}{2^{10}}\times 1024=1$