IIT – JEE: 2023 **TW TEST (MAIN)** **DATE: 11/03/23**

TOPIC: HEAT & THERMODYNAMICS

SOLUTIONS

1. (A)
As the two bodies are of same mass and dropped from same height, the final Kinetic energy is also same for both the bodies. When they hit the ground and this is converted to heat, the body with lower specific heat capacity will have higher rise in temperature
$Q = ms\Delta T$
As copper has lower specific heat capacity, its temperature will be higher

2. (D)
Thermal capacity $=$ mass $\times$ specific heat
Mass= volume $\times$density
For a sphere, volume is directly proportional to its cube of radius
$\therefore TC \propto r^3.\rho.C$
$$\Rightarrow \frac{TC_1}{TC_2} = \frac{r_1^3.\rho_1.C_1}{r_2^3.\rho_2.C_2}$$
$$\frac{TC_1}{TC_2} = \left(\frac{1}{2}\right)^3.\left(\frac{2}{3}\right).\left(\frac{3}{4}\right)$$
$$\frac{TC_1}{TC_2} = \left(\frac{1}{2}\right)^3.\left(\frac{2}{3}\right).\left(\frac{3}{4}\right)$$
$$\frac{TC_1}{TC_2} = \frac{1}{16}$$

3. (C)
Loss in temperature of one liquid is equal to the gain in temperature of another liquid.
$m_1.c_1.\Delta T_1 = m_2.c_2.\Delta T_2$
Given the masses are same
$c_1.(30-26) = c_2.(26-20)$
$$\frac{c_1}{c_2} = \frac{6}{4} = 3/2$$

4. (A)
Density of water is maximum at 4°C, this is because of anomalous expansion of water.
Let volume of the sphere be V and ρ be its density, then buoyant force,
$F = V\rho G$ (g = acceleration due to gravity)
$\Rightarrow F \propto \rho$ ($\because$ V and g are almost constant)
$$\Rightarrow \frac{F_{4^\circ C}}{F_{0^\circ C}} = \frac{\rho_{4^\circ C}}{\rho_{0^\circ C}} > 1 \qquad \left(\because \rho_{4^\circ C} > \rho_{0^\circ C}\right)$$

$\Rightarrow \quad F_{4^\circ C} > F_{0^\circ C}$

Hence, buoyancy will be less in water at 0°C than that in water at 4°C.

5. (C)

The scale is calibrated at $20^\circ C$

At $40^\circ C$, the measuring gaps will be more. So, it will measure lesser than actual.

Using , $l_2 = l_1(1+\alpha dt)$ where $\alpha = 10^{-5}, dt =$

20 and $l_1 = 5$, we get $l_2 = 5.001m$

6. (B)

When length of the liquid column remains constant, then the level of liquid moves down with respect to the container, thus γ must be less than 3α.

Now, we can write $V = V_0(1+\gamma\Delta T)$

Since, $V = Al_0 = [A_0(1+2\alpha\Delta T)]l_0 = V_0(1+2\alpha\Delta T)$

Hence, $V_0(1+\gamma\Delta T) = V_0(1+2\alpha\Delta T) \Rightarrow \gamma = 2\alpha$.

7. (C)

Since in the region AB, temperature is constant, at this temperature, phase of the material changes from solid to liquid and $(H_2 - H_1)$ heat will be absorbed by the material. This heat is known as the heat of melting of the solid.

Similarly in the region CD, temperature is constant. Therefore at this temperature, phase of the material changes from liquid to gas and $(H_4 - H_3)$ heat will be absorbed by the material. This heat is known as the heat of vaporisation of the liquid.

8. (C)

Energy $= \frac{1}{2}mv^2 = mc\Delta\theta$

$\Rightarrow \quad \Delta\theta \propto v^2$

Temperature does not depend upon the mass of the balls.

$\therefore \quad \frac{mL}{t} = P$

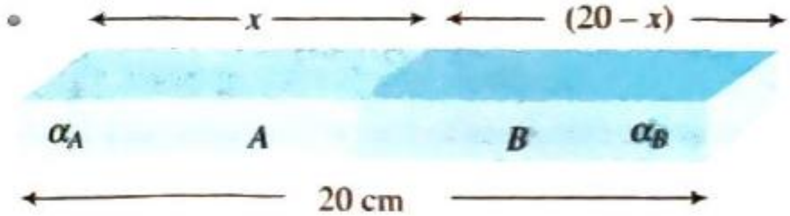
or $L = \frac{Pt}{m}$

9. (B)

$\Delta L = L_0 \alpha \Delta\theta$

Rod A: $0.075 = 20 \times \alpha_A \times 100 \Rightarrow \alpha_A = \frac{75}{2} \times 10^{-6} / {}^\circ C$

Rod B : $0.045 = 20 \times \alpha_B \times 100 \Rightarrow \alpha_B = \frac{45}{2} \times 10^{-6} / {}^\circ C$



For composite rod : Taking *x* cm of A and (20 – *x*) cm of B, we have

$0.060 = x\alpha_A \times 100 + (20-x)\alpha_B \times 100$

$$= x\left[\frac{75}{2}\times10^{-6}\times100+(20-x)\times\frac{45}{2}\times10^{-6}\times100\right]$$

On solving, we get x = 10 cm

10. (D)

Coefficient of volume expansion

$$\gamma=\frac{\Delta\rho}{\rho\Delta T}=\frac{(\rho_1-\rho_2)}{\rho(\Delta\theta)}=\frac{(10-9.7)}{10\times(100-0)}=3\times10^{-4}$$

Hence, coefficient of linear expansion

$$\alpha=\frac{\gamma}{3}=10^{-4}\ /\ ^{o}C$$

11. (D)

Since tension in the two rods will be same,

$$A_1Y_1\alpha_1\Delta\theta=A_2Y_2\alpha_2\Delta\theta$$

$$A_1Y_1\alpha_1=A_2Y_2\alpha_2$$

12. (A)

Let m gram of steam get condensed into water (by heat loss).
This happens in following two steps:

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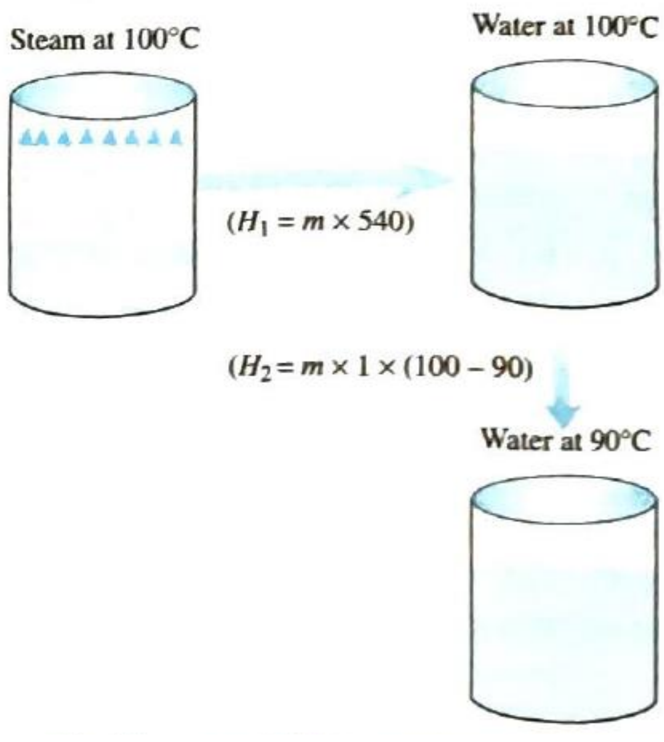


Heat gained by water (20°C) to raise its temperature upto 90° = 22 × 1 × (90 – 20)

Hence, in equilibrium, Heat lost = heat gain

$\Rightarrow\ m\times540+m\times1\times(100-90)=22\times1\times(90-20)$

$\Rightarrow\ m=2.8\,g$

Net mass of the water present in the mixture = 22 +2.8 = 24.8 g

13. (C)

Efficiency $=\frac{0.54\times746}{500}=0.80$ or 80%

(0.5 kW = 500 W and 0.54 hp = 0.54 × 746 W)

$\therefore$ 80% of the electrical energy is converted to mechanical energy and the rest 20% is converted to heat energy.

$\therefore$ $\frac{20}{100}\times 500 = 100\text{ W}$ of power is converted to heat

$\therefore$ Heat produced in 1 h (or 3600 s) = 100 × 3600 = 36 × 10^4 J

$= \frac{36\times 10^4}{4.18}\text{ cal} = 8.6\times 10^4\text{ cal}$

14. (D)

Since at 20 degree temperature steel tape measures correct measurement of wood. Now at 0 degree length is 25 cm therefore, when temperature is shifted the scale reading 25 cm will increase and therefore the length which is 25 cm at 0 degree will be less than 25 cm at 20 degree.

15. (C)

Heat given by water, $Q_1 = 10\times 10 = 100\text{ cal}$

Heat taken by ice to melt, $Q_2 = 10\times 0.5\times\left[0-(-20)\right]+10\times 8 = 900\text{ cal}$

As $Q_1 < Q_2$, so ice will not completely melt and final temperature = 0°C.

As heat given by water in cooling up to 0°C is only just sufficient to increase the temperature of ice from –20°C to 0°C.

Hence mixture in equilibrium will consist of 10 g ice and 10 g water at 0°C.

16. (C)

From given curve, Melting point for A = 60°C and melting point for B = 20°C

Time taken by A for fusion = (6 – 2) = 4 minutes

Time taken by B for fusion = (6.5 – 4) = 2.5 minutes

$\therefore$ $\frac{H_A}{H_B} = \frac{6\times 4\times 60}{6\times 2.5\times 60} = \frac{8}{5}$

17. (D)

It is clear that at desired temperature, $T^{o}C$, the densities of the wood and benzene must be equal for the wood to just sink.

i.e., $\rho_w(T) = \rho_B(T)$

If m is the mass of wood (which is assumed to be constant) then, if $(V_0)_w$ and $(V_0)_B$ are the respective volumes at 0°C of mass m of wood and benzene,

$$(\rho_0)_w (V_0)_w = (\rho_0)_B (V_0)_B = m$$

$$(\rho_0)_w = 880\text{ kg/m}^3 \text{ and } (\rho_0)_B = 900\text{ kg/m}^3$$

Hence, $(V_0)_w = \frac{m}{880}(m^3)$

End, $(V_0)_B = \frac{m}{900}(m^3)$

We then have, $(V_T)_W = (V_0)_W\left(1+1.2\times 10^{-3}T\right)$

$$(V_T)_B = (V_0)_B\left(1+1.5\times 10^{-3}T\right)$$

Thus $\frac{(V_T)_W}{(V_T)_B} = \frac{(\rho_B)_T}{(\rho_W)_T} = 1 = \frac{(V_0)_W\,1+1.2\times 10^{-3}T}{(V_0)_B\,1+1.5\times 10^{-3}T}$

Solving for T, we have T = 83.2°C.

18. (A)

$$m_1 \times 1 \times (50-30) = m_1 \times 1 \times (80-50)$$

$$m_1 \times 20 = m_1 \times 30 \text{ or } \frac{m_1}{m_2} = \frac{3}{2}$$

Mass of water from tank $A = \frac{3}{5} \times 40 = 24 \text{ kg}$

Mass of water from tank $B = \frac{2}{5} \times 40 = 16 \text{ kg}$

19. (D)

$$V = V_0(1 + \gamma\Delta\theta)$$

$\Rightarrow$ Change in volume, $V - V_0 = \Delta V = A.\Delta l = V_0 \gamma \Delta\theta$

$$\Rightarrow \Delta l = \frac{V_0 \gamma \Delta\theta}{A} = \frac{10^{-6} \times 18 \times 10^{-5} \times (100-\theta)}{0.004 \times 10^{-4}}$$

$$= 45 \times 10^{-3} \text{m} = 4.5 \text{cm}$$

20. (C)

Heat supplied is $L_{fusion} + Mc\Delta T + ML_{\upsilon ap}$

$$Q_1 = 10 \times 336 + 10 \times 4.2 \times 100 + 10 \times 2260$$

$$Q_1 = 30160 \text{J or } 7200 \text{ cal}$$

Heat for calorimeter

$$Q_2 = 10 \times 1 \times 100 = 1000 \text{cal}$$

$$Q = Q_1 + Q_2 = 8200 \text{cal}$$

21. (150)

Since specific heat of lead is given in Joules, hence use W = Q instead of W = JQ.

So, $\frac{1}{2} \times \left(\frac{1}{2} mv^2\right) = m.c.\Delta\theta \Rightarrow \delta\theta = \frac{v^2}{4c} = \frac{(300)^2}{4 \times 150} = 150\ ^{o}C$

22. (5)

$$\Delta L = \Delta L \Delta T$$

$$\Delta(2L) = 2\alpha(2L)\Delta T$$

$$\Delta(3L) = \alpha_{composite}(3L)\Delta T$$

$$\therefore \Delta L \Delta T + 2\alpha(2L)\Delta T = \alpha_{composite}(3L)\Delta T$$

$$\Rightarrow \alpha_{composite} = \frac{5}{3}\alpha$$

Comparing with $\frac{5x}{3}, x = 5$

23. (2)

Area expansion $= 2\times$ linear expansion

Therefore surface area will increase by 2% .

24. (8)

Let power lost to surrounding be Q.

$$16 - Q = \left(\frac{dm}{dt}\right) S(10)$$

and $32 - Q = 3\left[\left(\frac{dm}{dt}\right)S(10)\right]$

$\Rightarrow \quad \frac{32 - Q}{16 - Q} = 3 \Rightarrow Q = 8\ W$

25. (3)

Energy released by water from 25°C to 0°C

= 2500 × 1 × 25 = 62500 cal

Energy to bring ice to 0°C = $2000 \times \frac{1}{2} \times 15 = 15000$ cal 2

Energy used to melt ice of m gram = m80 cal

$\therefore$ Ice melt $m = \left(\frac{62500 - 15000}{80}\right) = 593.75\ g$

So, mass of water = (2500 + 593.75) g = 3093.75 g = 3 kg

26. (6)

Energy with 5 kg of H_2O at 20°C to become water at 0°C,

$E_1 = 5000 \times 1 \times 20 = 1000000$ cal

Energy to raise the temperature of 2 kg ice form –20°C to 0°C,

$E_2 = 2000 \times 0.5 \times 20 = 20000$ cal

$(E_1 - E_2)$ = 8000 cal is available to melt ice at 0°C.

So only 1000 g or 1 kg of ice would have melt.

So, the amount of water available 1 + 5 = 6 kg

27. (4)

$(OR)^2 = (PR)^2 - (PO)^2 = l^2 - \left(\frac{l}{2}\right)^2$

$\Rightarrow \quad (OR)^2 = \left[l(1 + \alpha_2 t)\right]^2 - \left[\frac{l}{2}(1 + \alpha_1 t)\right]^2$

$\Rightarrow \quad l^2 - \frac{l^2}{4} = l^2\left(1 + \alpha_2^2 t^2 + 2\alpha_2 t\right) - \frac{l^2}{4}\left(1 + \alpha_1^2 t^2 + 2\alpha_1 t\right)$

Neglecing $\alpha_2^2 t^2$ and $\alpha_1^2 t^2$, we get

$0 = l^2(\alpha_2 t) - \frac{l^2}{4}(2\alpha_1 t) \Rightarrow 2\alpha_2 \Rightarrow \alpha_1 = 4\alpha_2$

28. (8)

Suppose m gram ice melts, then heat required for its melting

= mL = m × 80 cal

Heat available with stem for being condensed and then brought to 0°C

= 1 × 540 + 1 × 1 × (100 – 0) = 640 cal

Now, heat lost = heat taken

$\Rightarrow \quad 604 = m \times 80 \Rightarrow m = 8\ g$

29. (3)

$\gamma_{mercury} = 20\alpha_{glass} = \frac{20}{3}\gamma_{glass}$

Let the volume of mercury is $V_{mercury}$

Since the volume above mercury remains same,

$$\gamma_{mercury} V_{mercury} = \gamma_{glass} V_{glass}$$

$$\Rightarrow \frac{20}{3} \gamma_{glass} V_{mercury} = \gamma_{glass} V$$

$$\Rightarrow V_{mercury} = \frac{3V}{20}$$

$$\Rightarrow x = 3$$

30. (0.02)

$$V = V_0 \left(1 + \gamma \Delta T\right)$$

$$V = V_0 + V_0 \gamma \Delta T$$

$$\frac{V - V_0}{V_0} = \gamma \Delta T$$

Now since mass M is constant

$$\frac{\frac{M}{\rho} - \frac{M}{\rho_0}}{\frac{M}{\rho_0}} = \gamma \Delta T$$

IIT – JEE: 2023 **TW TEST (MAIN)** **DATE: 11/03/23**

TOPIC: IONIC EQUILIBRIUM

SOLUTIONS

31. (C)

$\left[H^+\right] = \sqrt{K_w} \Rightarrow P^H = -\log\left(2.5\times10^{-14}\right)^{1/2} = 6.8$

32. (C)

No. of H_3O^+ ions $= \dfrac{1\times10^{-12}}{1000}\times6.02\times10^{23} = 6.02\times10^{8}$

33. (D)

On neglecting the contribution of water , $\left[H^+\right] = 10^{-6}M$

When contribution of water is considered ,

$$H_2O \rightleftharpoons H^+ + OH^-$$

Equilibrium $\quad (x+10^{-6})M \quad xM$

Now, $\left(x+10^{-6}\right).x = 10^{-14} \Rightarrow x = 9.9\times10^{-9}$

$\therefore \left[H^+\right]_2 = \left(x+10^{-6}\right)M = 1.0099\times10^{-6}M$

Now % error in $\left[H^+\right] = \dfrac{\left[H^+\right]_2 - \left[H^+\right]}{\left[H^+\right]_2} = \left[H^+\right]\times100\%$

$= 0.98\%$

34. (A)

Smaller P_{Ka}, stronger acid, greater $\left[H^+\right]$

35. (C)

$$CH_3COOH \rightleftharpoons CH_3COOH^- + H^+$$

	CH_3COOH	CH_3COOH^-	H^+
	1M	0	0.1 M
Equilibrium	$1-x$	x	$0.1+x$
	$\simeq 1M$		$\simeq 0.1M$

Now, $K_a = \dfrac{\left[CH_3COO^-\right]\left[H^+\right]}{\left[CH_3COOH\right]} \Rightarrow 2\times10^{-5} = \dfrac{x\times0.1}{1}$

36. (D)

$HA \rightleftharpoons H^+ + A^-$

low pH $\rightleftharpoons$ High $\left[H^+\right]$

$\Rightarrow$ Equilibrium in backward direction

High pH $\Rightarrow$ low $\left[H^+\right]$

$\Rightarrow$ Equilibrium in forward direction

37. (D)

$pK_a = 5 \Rightarrow pH\,range = 4\,to\,6$

38. (A)

Sodium acetate is basic in nature.

39. (B)

	CH_3NH_2	+ H^+	$\rightleftharpoons CH_3NH_3^+$
	0.1 mole	0.08 mole	0
Final	0.02 mole	0	0.08 mole

$$pOH = pK_b + \log\frac{\left[CH_3NH_3^+\right]_0}{\left[CH_3NH_2\right]_0}$$

$$= -\log\left(5\times10^{-4}\right) + \log\frac{0.08}{0.02}$$

$$\therefore \left[OH^-\right] = \frac{5\times10^{-4}}{4} \Rightarrow \left[H^+\right] = \frac{10^{-14}}{\left[OH^-\right]} = 8\times10^{-11}\,M$$

40. (A)

Stronger the acid, smaller is pH

41. (C)

CH_3COONa = Basic, CH_3COOH = Weak acid,

CH_3COONH_4 = Neutral, NaOH = strong base, HCl = strong acid

42. (D)

$$\left[H^+\right] = \frac{75\times\frac{1}{5} - 25\times\frac{1}{5}}{100} = 0.1;\ pH = 1$$

43. (B)

$H_2O \rightleftharpoons H^+ + OH^-$; $HCl \longrightarrow H^+ + Cl^-$

$c_1 + x \quad x \qquad c_1 \qquad c_1 \quad c_1$

$c_1 = x$ (given)

$\Rightarrow \quad 10^{-14} = 2c_1^2$

$\Rightarrow \quad \sqrt{50}\times10^{-8} = c_1$

44. (D)

$[H^+]_{new} = \frac{10^{-6}}{100} = 10^{-8}$; So, now contribution of H^+ from H_2O should also be considered. Ph = 6.95.

45. (B)

$H_3B \rightleftharpoons H^+ + H_2B^-$

$H_2B^- \rightleftharpoons H^+ + HB^{2-}$

$HB^{2-} \rightleftharpoons H^+ + B^{3-}$

$HB^{2-} + H_2O \rightleftharpoons H_2B^- + OH^-$

$$K_b\left(HB^{2-}\right) = \frac{\left[H_2B^-\right]\left[OH^-\right]}{\left[HB^{2-}\right]} = \frac{K_w}{K_2}$$

46. (C)

$[H^+] = [H^+]_{HCl} + [H^+]_{AcOH}$;

$$1.69 \times 10^{-5} = \frac{0.01 \times 0.01\alpha}{0.01}$$

$\Rightarrow \ \alpha = 1.69 \times 10^{-3}$

47. (B)

$$\underset{(NaH_2PO_4)}{pH_2} = \frac{2.2 + 7.2}{2}; \qquad \underset{(Na_2HPO_4)}{pH_1} = \frac{7.2 + 12.0}{2}$$

48. (B)

$$K_a = \frac{c\alpha^2}{1-\alpha} = \frac{0.2 \times 0.09}{0.7}$$

49. (D)

$$pH = \frac{1}{2}[pk_a - \log c]$$

50. (A)

Order of basic strength :

$Ba(OH)_2 > NaOH > NH_4OH$

51. (13)

As all have 0.1 M concentration, $\left[KOH\right]_{final} = 0.1M$

$\therefore pOH = -\log\left(0.1\right) = 1.0$ and $pH = 13$

52. (6)

$$n_{OH^-} \text{ taken } = \frac{100 \times 0.5}{1000} = 0.05$$

n_{H^+} taken $= \dfrac{250 \times 0.2}{1000} = 0.05$

Hence, resulting solution is neutral $pH = -\log\left(10^{-6}\right) = 6.0$

53. (4)

$pH_{CH_3COOH} = pOH_{NH_3} = 3.2 \Rightarrow pH_{NH_3} = 14 - 3.2 = 10.8$

$2.7x = 10.8$

$x = 4$

54. (665)

$\left[H^+\right] = \sqrt{K_a.C} = \sqrt{2\times10^{-12}\times0.02} = 2\times10^{-7}\,M$

As $\left[H^+\right]$ is very small, contribution of H^+ from water must be considered.

	$HA \rightleftharpoons$	H^+	+	A^-
Equilibrium	$(0.02-x)M$	$(x+y)M$		xM
	$H_2O \rightleftharpoons$	H^+	+	OH^-
Equilibrium		$(x+y)M$		yM

Now, $K_a = \dfrac{\left[H^+\right]\left[A^-\right]}{[HA]}$

$\Rightarrow 2\times10^{-12} = \dfrac{(x+y).x}{(0.02-x)} \approx \dfrac{(x+y).x}{0.02}$

Or. $4\times10^{-14} = (x+y).x$ (1)

And $K_w = \left[H^+\right]\left[OH^-\right] \Rightarrow 10^{-14} = (x+y).y$ (2)

From (1) + (2), $(x+y) = \sqrt{5\times10^{-14}}\,M = \left[H^+\right]$

$\therefore P^H = -\log\left(5\times10^{-14}\right)^{1/2} = 6.65 = x$

$100x = 665$

55. (99)

$P^H = P^{K_a} + \log\dfrac{\left[CN^-\right]_0}{[HCN]_0}$

$= -\log\left(2.5\times10^{-10}\right) + \log\dfrac{80\times0.4/100}{20\times0.8/100}$

$= 9.9 = x$

$10x = 99$

56. (65)

$pH = 7 + \dfrac{1}{2}\left(pK_a - pK_b\right) = 7 + \dfrac{1}{2}(3.8 - 4.8) = 6.5 = x$

$10x = 65$

57. (1150)

pK_b of $CN^- = 4.70 \Rightarrow pK_a$ of HCN=9.30

Now, $pH = 7 + \frac{1}{2}(pK_a + \log C)$

$= 7 + \frac{1}{2}(9.30 + \log 0.5) = 11.5 = x$

$100x = 1150$

58. (2)

Equal volumes of both with consume and hence,

$[CH_3COONa] = \frac{0.01}{2} = 0.005$

Now, $pH = 7 + \frac{1}{2}(pK_a + \log C)$

$= 7 + \frac{1}{2}(4.7 + \log 0.005) = 8.2 = 4.1x$

$x = 2$

59. (9)

$\underset{0.1-x}{A^-} + H_2O \rightleftharpoons \underset{x}{HA} + \underset{x}{OH^-}$

$\frac{x^2}{0.1} = 10^{-9}$

$\Rightarrow 1x = 10^{-5}$ or pOH = 5

60. (5)

$14 - 8 = pK_b + \log \frac{2.5}{2.5} \Rightarrow pK_b = 6$

$pH = \frac{1}{2}[pK_w - pK_b - \log c] = \frac{1}{2}[14 - 6 - \log 0.01]$

IIT – JEE: 2023 **TW TEST (MAIN)** **DATE: 11/03/23**

TOPIC: PARABOLA

SOLUTIONS

61. (D)

62. (B)

63. (D)

64. (D)

65. (C)

66. (C)

67. (D)

68. (C)

69. (C)

70. (B)

71. (C)

72. (B)

73. (C)

74. (D)

75. (A)

76. (C)

77. (D)

78. (D)

79. (D)

80. (A)

81. (3)

For normal $C = -2am - am^3$

82. (90)

$\therefore \; m_1 m_2 = -1$

83. (3)

Focus is (1, 0) third vertex is (–1, 0).

Hence, directrix is $x + 3 = 0$.

84. (8)

By $t_1 = -t - \frac{2}{t} = -\left(t + \frac{2}{t}\right)$

Diff. w.r.t. x

$\frac{dt_1}{dt} = \frac{2}{t^2} - 1 = 0$

$t = \pm\sqrt{2}$

$t_1^2 = \frac{4}{t^2} + t^2 + 4 = 8$

$\therefore$ min. value of $t_1^2 = 8$

85. (1)

Given line and parabola are

$y = -x - 1$ and $y^2 = 4kx$

Condition for tangency

$c = \frac{a}{m} \Rightarrow -1 = \frac{k}{-1}$

$\therefore k = 1$

86. (1)

Any normal to the parabola $y^2 = 4x$ is

$y = mx - 2m - m^3$

this passes thro' (1, 0)

$\therefore 0 = m - 2m - m^3 = 0 - m - m^3 = 0$

$\Rightarrow -m(1 + m^2) = 0$

$\Rightarrow m - 0 \;\; [\because m^2 + 1 \neq 0]$

$\therefore$ Normal is y = 0

$\therefore$ there is only one normal

87. (6)

Focal distance of a point $P(x_1, y_1)$ on the parabola $y^2 = 4ax$ is $x_1 + a = x_1 + 3$ $[\because 4a = 12 \Rightarrow a = 3]$

But $y_1^2 = 12x_1 \therefore (6)^2 = 12x_1 \Rightarrow x_1 = 3$

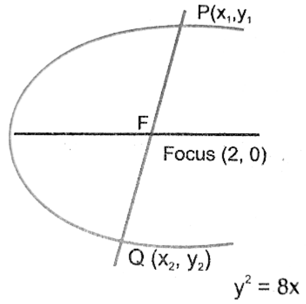
$\therefore$ focal distance = 3 + 3 = 6

88. (2)

Latus Rectum = Twice the distance of the focus from the directrix.

89. (20)

Since (x_1, y_1) and (x_2, y_2) are points on the parabola $y^2 = 8x$



$y_1^2 = 8x_1$

$y_2^2 = 8x_2$

Also $\frac{1}{FP} + \frac{1}{FQ} = \frac{2}{\ell} = \frac{2}{4} = \frac{1}{2}$ [ℓ = semi-latus rectum = $= \frac{1}{2}(8) = 4$]

$\therefore \frac{1}{x_1 + 2} + \frac{1}{x_2 + 1} = \frac{1}{2}$

$\Rightarrow (x_2 + 2 + x_1) = x_1 x_2 + 2(x_1 + x_2) + 4$

$\Rightarrow 8 = x_1 x_2 + 4 \Rightarrow x_1 x_2 = 8 - 4 = 4$

$y_1^2 y_2^2 = 64 x_1 x_2 = 64(4) = 256$

$\Rightarrow y_1 y_2 = 16$

$\therefore x_1 x_2 + y_1 y_2 = 4 + 16 = 20$

90. (2)

$y = x + a$ meets $y^2 = 4(x+1)$ if

$(x + a)^2 = 4(x+1) = 4x + 4$

$\Rightarrow x^2 + 2ax + a^2 - 4x - 4 = 0$

$\Rightarrow x^2 + (2a - 4)x + 4a^2 - 4$

For equal roots $(2a - 4)^2 = 4(a^2 - 4)$

$\Rightarrow 4x^2 - 16a + 16 = 4a^2 - 16$

$\Rightarrow 16a = 32 \Rightarrow a = 2$