IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 15/04/23**

TOPIC: ROTATIONAL DYNAMICS

SOLUTIONS

1. **(d)**

2. **(a)**

3. **(b)**

$$T = KS^2$$

$$\frac{1}{2}mv^2 = KS^2$$

$$v = \sqrt{\frac{2K}{m}}S \qquad ...(i)$$

Differentiating w.r.t. time t :

$$a_t = \frac{dv}{dt} = \sqrt{\frac{2K}{m}}\frac{dS}{dt} = \sqrt{\frac{2K}{m}}v$$

$$= \sqrt{\frac{2K}{m}} \cdot \sqrt{\frac{2K}{m}}S = \frac{2KS}{m}$$

4. **(a)**

5. **(b)**

6. **(a)**

7. **(a)**

By theorem of parallel axes,

$$I = I_C + Mh^2$$

$$I = \left(\frac{mL^2}{12} + mR^2\right)$$

$$\therefore\ (K.E)_{rot.} = \frac{1}{2}I\omega^2 = \frac{1}{2}m\left(R^2 + \frac{L^2}{12}\right)\omega^2$$

8. **(d)**

9. **(a)**

$$\frac{1}{2}mv^2\left(1+\frac{2}{5}\right)=mg(3R)$$

$$v=\sqrt{\frac{30gR}{7}}$$

10. **(d)**

$$I=I_O+I_A+I_B=0=ma^2+m\left(\frac{a}{2}\right)^2=\frac{5ma^2}{4}$$

11. **(c)**

12. **(a)**

$$\alpha=\frac{\tau}{I}=\frac{Mg\left(\frac{\ell}{2}\cos\theta\right)}{M\ell^2/3}=\frac{3g}{2\ell}\cos\theta$$

13. **(d)**

14. **(b)**

15. **(b)**

16. **(c)**

Two point masses and bar taken together as a system the angular momentum about centre of bar is

$$(2mv)(l)+(2mv)(2l)=\left[\frac{8m\times(6l)^2}{12}+2ml^2+m4l^2\right]\omega$$

$$\Rightarrow\quad 30l\omega=6v\Rightarrow\omega=\frac{v}{5l}\Rightarrow v_c=0\,;$$

$$E=\frac{1}{2}I\omega^2=\frac{1}{2}\times30ml^2\times\left(\frac{v}{5l}\right)^2=\frac{3mv^2}{5}$$

17. **(a)**

18. **(b)**

19. **(d)**

20. **(c)**

$$\frac{E_{rot.}}{E_{trans}}=\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2}=\frac{\frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{v^2}{R^2}}{\frac{1}{2}Mv^2}=\frac{2}{5}$$

21. **(1)**

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + m_4r_4^2$$
$$= 2\times(0.4)^2 + 5(0.2)^2 + 5\times(0.2)^2 + 2\times(0.4)^2$$
$$= 1.04\,kgm^2 = 1\,kgm^2$$

22. **(2)**

23. **(3)**

24. **(3)**

25. **(3)**

26. **(2)**

27. **(6)**

$$(K.E.)_{rot.} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{v^2}{R^2} = \frac{1}{5}Mv^2$$
$$= \frac{1}{5}\times 2\times(4)^2 = 6.4\ J$$

28. **(5)**

29. **(5)**

30. **(2)**

Angular momentum is given by

$$L = I\omega = (MK^2)\omega$$
$$\therefore\ K = \sqrt{\frac{L}{M\omega}} = \sqrt{\frac{1.8}{1.5\times 0.3}} = 2\ m$$

ANSWER KEY

31. (3)

32. (3)

33. (4)

34. (4)

35. (4)

36. (3)

37. (4)

38. (1)

39. (3)

40. (1)

41. (4)

42. (2)

43. (2)

44. (1)

45. (1)

46. (2)

47. (3)

48. (4)

49. (1)

50. (1)

51. (6)

52. (5)

53. (8)

54. (3)

55. (12)

56. (5)

57. (2)

55. (8)

59. (9)

60. (4)

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TOPIC: LIMITS

SOLUTIONS

61. **(c)**
Using L-Hospital rule

62. **(a)**

$$\lim_{x\to1}\frac{\sqrt{x^2-1}+\sqrt{x-1}}{\sqrt{x^2-1}}=\lim_{x\to1}\frac{\sqrt{(x-1)[x+1]}+\sqrt{x-1}}{\sqrt{(x-1)(x+1)}}$$

$$=\frac{\sqrt{2}+1}{\sqrt{2}}$$

63. **(b)**

On rationalizing given limit $=\lim_{x\to0}\frac{x^3}{\left[\sin^{-1}\left(x^3\right)\right]}\cdot\frac{1}{1+\sqrt{1-x^2}}$

64. **(a)**

$$\lim_{x\to1}\left[\sec\left(\frac{\pi x}{2}\right)\log x\right]=\lim_{x\to1}\frac{\log x}{\cos\left(\frac{\pi x}{2}\right)}=\left(\frac{0}{0}\right)$$

$$=\lim_{x\to1}\frac{\frac{1}{x}}{-\sin\left(\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}}=-\frac{1}{\frac{\pi}{2}\sin\frac{\pi}{2}}=-\frac{2}{\pi}$$

65. **(b)**

$$\lim_{x\to0}\frac{e^{\sin x}\left(e^{x-\sin x}-1\right)}{2\left(x-\sin x\right)}=\frac{1}{2}$$

66. **(b)**
By L-Hospital rule

67. **(b)**

$$\lim_{x\to0}\frac{3\sin x-\left(3\sin x-4\sin^3 x\right)}{x^3}=4$$

68. **(c)**

Given limit is $\sum \frac{n^2+n}{2n^3}$

69. **(b)**

$\lim_{x\to 1}(2-x)^{\tan\left(\frac{\pi x}{2}\right)} = e^{\lim_{x\to 1}\tan\left(\frac{\pi x}{2}\right)(2-x-1)} = e^{\lim_{x\to 1}\frac{(1-x)}{\cot\left(\frac{\pi x}{2}\right)}}$ and using L hospital rule.

70. **(c)**

$\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

71. **(b)**

$\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$

It is a $(\infty, -\infty)$ form.

We have to convert it into either $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$ form.

Now, $\lim_{x\to 1}\left[\frac{1-x^3-3(1-x)}{(1-x)(1-x)^3}\right]$

$$= \lim_{x\to 1}\left[\frac{1-x^3-3+3x}{1-x^3-x+x^4}\right]$$

$$= \lim_{x\to 1}\left[\frac{-x^3+3x-2}{x^4-x^3-x+1}\right]$$

At x = 1, it is a $\left(\frac{0}{0}\right)$ form.

On applying L' Hospital rule, we get

$$\lim_{x\to 1}\left[\frac{-3x^2+3}{4x^3-3x^2-1}\right]$$

Still at $x \to 1$, it is a $\left(\frac{0}{0}\right)$ form.

$$\therefore \quad \lim_{x\to 1}\left[\frac{-6x}{12x^2-6x}\right] = \lim_{x\to 1}\left[\frac{-6}{12x-6}\right]$$

$$= \frac{-6}{12-6} = \frac{-6}{6} = -1$$

72. **(c)**

Using L-Hospital rule given limit is

$$\lim_{x\to a}\frac{\frac{1}{2\sqrt{a+2x}}(2)-\frac{1}{2\sqrt{3x}}(3)}{\frac{1}{2\sqrt{3a+x}}-2\frac{1}{2\sqrt{x}}} = \frac{2}{3\sqrt{3}}$$

73. **(b)**

$$\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=f'(x)$$

74. **(d)**

$$\lim_{x\to 0}\sqrt{\frac{1-\cos x}{2}}=\underset{x\to 0}{\text{Lt}}\frac{\sqrt{\sin^2 x/2}}{x}=\underset{x\to 0}{\text{Lt}}\frac{|\sin x/2|}{x}$$ does not exist.

75. **(d)**

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}=e^{\lim_{x\to 0}\left(\frac{\sin x}{x}-1\right)\frac{\sin x}{x-\sin x}}=1/e$$

76. **(b)**

Given limit $=\lim_{\theta\to 0}\frac{1}{\sin\left[\frac{\pi}{\sqrt{\theta+4}+2}\right]}=\frac{1}{\sin\frac{\pi}{4}}=\sqrt{2}$

77. **(a)**

Given limit is $\frac{2\sin^2 x\sin 5x}{x^2\sin 3x}=\frac{10}{3}$

78. **(c)**

Given limit is

$$\lim_{(x-5)\to 0}\frac{\sin^2(x-5)}{(x-5)^2}\lim_{x\to 5}\frac{\tan(x-5)}{(x+5)}=0$$

79. **(d)**

$$\lim_{x\to\pi}(1-4\tan x)^{\cot x}=\lim_{e^{x\to\pi}}\cot x(1-4\tan x-1)=e^{-4}$$

80. **(c)**

$$\lim_{x\to\sqrt{2}}\left(\frac{x^4-4}{x^2+3x\sqrt{2}-8}\right)$$

At $x=\sqrt{2}$, it is an indeterminate $\left(\frac{0}{0}\right)$ form.

On applying L' Hospital's rule, we get $\lim_{x\to\sqrt{2}}\left(\frac{4x^3}{2x+3\sqrt{2}}\right)=\frac{4(2\sqrt{2})}{2\sqrt{2}+3\sqrt{2}}=\frac{8}{5}$

81. **(4)**

Using $\lim_{x\to a}\frac{x^p-a^p}{x^q-a^q}=\frac{p}{q}a^{p-q}$

82. **(0)**

Definition of step function

83. **(0)**

Divide with x

84. **(100)**

Divide with x^{10}

85. **(4)**

$$\lim_{x\to 9}\frac{\frac{1}{2\sqrt{f(x)}}\cdot f'(x)}{\frac{1}{2\sqrt{x}}}=\frac{f'(9)}{\sqrt{f(9)}}\times\sqrt{9}=\frac{4}{3}\times 3=4$$

86. **(5)**

$f(a)=2,\ f'(a)=1,\ g(a)=-1,\ g'(a)=2$

$$\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}$$

By L – H Rule

$=g'(a)f(a)-g(a)f'(a)$

$=(2)(2)-(-1)(1)=5$

87. **(1)**

Given limit is $\lim_{x\to\frac{\pi}{4}}\frac{\tan\left(\frac{\pi}{4}-x\right)}{\frac{\pi}{4}-x}=1$

88. **(0)**

89. **(3)**

$f(x)=x^2+x+1\Rightarrow\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=\left[f'(x)\right]_{x=1}=3$

90. **(1)**

$e^{\lim_{x\to 0}\frac{\cos x-1}{\sin x}}\ e^{\lim_{x\to 0}\frac{-\sin x}{\cos x}}$