IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 22/04/23**

TOPIC: SHM

SOLUTIONS

1. (A)
In $y = A\sin\omega t + B$, the oscillating part is $A\sin\omega t$, so amplitude of motion is A.

2. (C)
Given, $y_1 = 0.1\sin(100\pi t + \pi/3) \Rightarrow v_1 = \frac{dy_1}{dt} = 10\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$

And $y_2 = 0.1\cos(100\pi t) \Rightarrow v_2 = \frac{dy_2}{dt} = 10\pi\sin(100\pi t)$

$= 10\pi\cos\left(100\pi t + \frac{\pi}{2}\right)$

So, $\phi = \phi_1 - \phi_2 = \pi/3 - \pi/2 = -\pi/6$

3. (B)
$a = -bx$, on comparing with $a = -\omega^2 x$
We get , $\omega = \sqrt{b}$.
$\therefore \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{b}}$

4. (C)
At $T = 2\pi\sqrt{\frac{\ell}{g}}$, so time period becomes double when length becomes four times.

5. (B)
When some mercury is drained off, the centre of gravity of the bob moves down and so length of the pendulum increases, which result increase in time period.

6. (B)
$g = \frac{GM}{R^2}$. On the planet $g' = \frac{G(2M)}{(2R)^2} = \frac{g}{2}$.

$T = 2\pi\sqrt{\ell/g}$ and $T' = 2\pi\sqrt{\ell/(g/2)} = \sqrt{2}T$

$= \sqrt{2}\times 2 = 2\sqrt{2}$.

7. (C)

$$T = 2\pi\sqrt{\frac{\ell}{g}},$$

$$T' = 2\pi\sqrt{\frac{\ell}{g+a}} = 2\pi\sqrt{\frac{\ell}{\left(g+\frac{g}{4}\right)}} = \frac{2}{\sqrt{5}}T.$$

8. (D)

$$\omega_1 A_1 = \omega_2 A_2$$

Or $\sqrt{\frac{k_1}{m}}A_1 = \sqrt{\frac{k_2}{m}}A_2$

Or $\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$.

9. (A)

$T_1 = \frac{T}{12}$ and $T_2 = \frac{T}{6}$

Clearly, $T_2 = 2T_1$

10. (A)

$x = \sin\omega t - \cos\omega t$

$\therefore \quad \frac{d^2x}{dt^2} = -\omega^2(\sin\omega t - \cos\omega t)$

$= \quad -\omega^2 x$. So it represents SHM.

11. (C)

$$(\omega_1 + \omega_2)t = \frac{6\omega_1 t}{5} = \pi$$

12. (A)

Let 'x' is the displacement of block from its equilibrium position.

$$m\frac{d^2x}{dt^2} = -kx - \frac{kx}{21}$$

$$\frac{d^2x}{dt^2} = -\left(\frac{22k}{21m}\right)x$$

$$T = 2\pi\sqrt{\frac{21m}{22k}}$$

13. (B)

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

So $\frac{T_P}{T_q} = \sqrt{\frac{I_P}{d_P} \cdot \frac{d_q}{I_q}}$

$\frac{6}{10} = \sqrt{\frac{16}{25} \cdot \frac{d_q}{d_p}}$

$\frac{d_q}{d_p} = \frac{9}{16}$ …….. (1)

$d_q + d_p = 20$ ………… (2)

$\Rightarrow d_p = \frac{64}{5} = 12.80$

14. (C)

The velocity of a particle executing S.H.M. at a distance X from its mean position is given by

$$V = \omega\sqrt{A^2 - X^2}$$

Where ω is its angular velocity and A, its amplitude

$\therefore \quad V_1 = \omega\sqrt{A^2 - X_1^2}$ ………. (i)

And $V_2 = \omega\sqrt{A^2 - X_2^2}$ …….. (ii)

Squaring (i) and (ii), we get

$V_1^2 = \omega^2\left(A^2 - X_1^2\right)$ …………. (iii)

$V_2^2 = \omega^2\left(A^2 - X_2^2\right)$ ………….. (iv)

Subtracting (iv) from (iii), we get

$$V_1^2 - V_2^2 = \omega^2\left(X_2^2 - X_1^2\right)$$

$$\omega^2 = \frac{V_1^2 - V_2^2}{X_2^2 - X_1^2} \text{ or } \omega = \sqrt{\frac{V_1^2 - V_2^2}{X_2^2 - X_1^2}}$$

Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{X_2^2 - X_1^2}{V_1^2 - V_2^2}}$

15. (A)

Angular frequency of the system,

$$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

Maximum acceleration of the system will be, $\omega^2 A$ or $\frac{kA}{2m}$

This acceleration to the lower block is provided by friction.

Hence, $f_{max} = ma_{max}$

$$= m\omega^2 A = m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$$

16. (D)

$x_1 + x_2 = A$ and $k_1x_1 = k_2x_2$

Or $\frac{x_1}{x_2} = \frac{k_2}{k_1}$

Solving these equations, we get

$$x_1 = \left(\frac{k_2}{k_1 + k_2}\right)A$$

17. (D)

From the principle of conservation of momentum,

total momentum before collision = total momentum after collision, i.e.

$$m_1 u = (m_1 + m_2)V$$

Where V is the common velocity of the bullet-block system.

Putting $m_1 = m$ and $m_2 = 80m$, we get

$$V = \frac{u}{81}$$

The kinetic energy of the system $= \frac{1}{2}(m_1 + m_2)V^2$

$$= \frac{1}{2}(m + 80m)\frac{u^2}{(81)^2}$$

$$= \frac{1}{2}m\frac{u^2}{81}$$

This is the total energy of the system since the string is unstretched before the collision.

If A is the maximum compression of the spring (which is the amplitude of motion), the potential energy is $\frac{1}{2}kA^2$.

Now, loss in K.E. = gain in P.E.

Or $\frac{1}{2}m\frac{u^2}{81} = \frac{1}{2}kA^2$

$\Rightarrow$ $A = \frac{u}{9}\sqrt{\frac{m}{k}}$, which is choice (D).

18. (D)

Equation can be written as

$x_1 = A\cos \omega t$

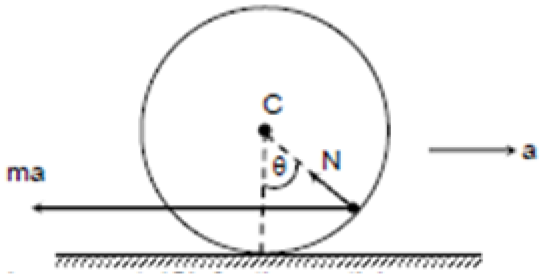
and $x_2 = A\sin(\omega t - \pi/6)$

Here $\omega = \frac{2\pi}{T}$

Equating $x_1 = x_2$ we get $t = T/6$

19 (C)

Centre of ring has acceleration 'a'

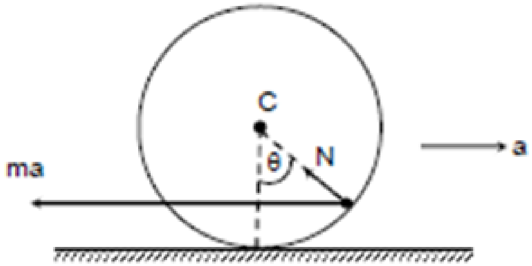


a:acceleration ring w.r.t. ground

20. (C)

Solution for Que. No. 19 & 20

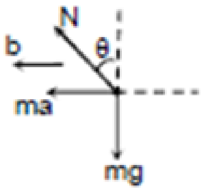
Centre of ring has are ‘a’



a : acceleration ring w.r.t. ground

b : acceleration of particles w.r.t. ring

Hence w.r.t. ‘C’, for the particle



$N\cos\theta = mg$ ……… (i)

$N\sin\theta + ma = mb$ …….. (ii)

And for the ring

$\Rightarrow \quad N\sin\theta - f = Ma$ ……. (iii)

$f.R = MR^2.a/R$

$\Rightarrow \quad f = Ma$ …….. (iv)

$\Rightarrow \quad N\sin\theta = 2Ma$

$N = mg$

$mg.\theta = 2Ma$

$\Rightarrow \quad a = \frac{mg}{2M}\theta$

From (ii)

$N\sin\theta + ma = mb$

$\Rightarrow \quad 2Ma + ma = mb \Rightarrow a(2M+m) = mb$

$\Rightarrow \quad \frac{mg}{2M}(2M+m)\theta = m.b$

Put $\theta = \frac{x}{R}$ [x: displacement of particle w.r.t. ring]

$\Rightarrow \quad \frac{g}{2M}(2M+m)\frac{x}{R} = b \quad \Rightarrow \quad b = \frac{g}{R}\left(1+\frac{m}{2M}\right).x$

$$\omega = \sqrt{\frac{g}{R}\left(1+\frac{m}{2m}\right)}$$

Part – 2

$$a = mb/\left(m+2M\right)$$

$\Rightarrow$ $$y = \frac{mx}{m+2M}$$

[y: displacement of ring w.r.t. ground]

$\Rightarrow$ $$y = \frac{mR\theta}{m+2M}$$

21. (4)

Time taken by particle to travel from mean position to given position $= 4-2 = 2\,\text{sec}$.

$$x = A\sin\omega t$$

$$= A\sin\frac{2\pi}{16}\times 2$$

$$= \frac{A}{\sqrt{2}}$$

$\therefore$ $$V = \omega\sqrt{A^2 - x^2}$$

$$= \frac{\omega A}{\sqrt{2}} = \frac{2\pi}{16}\times\frac{32\sqrt{2}}{\pi}\times\frac{1}{\sqrt{2}} = 4\,\text{m/s}.$$

22. (5)

$$KE = \frac{1}{2}m\omega^2 A^2$$

$\Rightarrow$ $$18 = \frac{1}{2}\times 1\times\omega^2\times 36\times 10^{-4}$$

$\Rightarrow$ $$\omega = \sqrt{10000}$$

$$= 100\,\text{rad/s}$$

23. (2)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2\,\text{sec}.$$

In 2.5 sec, it completes $\frac{5}{4}$ oscillation

Total distance $4a + a$

$$= 5a$$

24. (0)

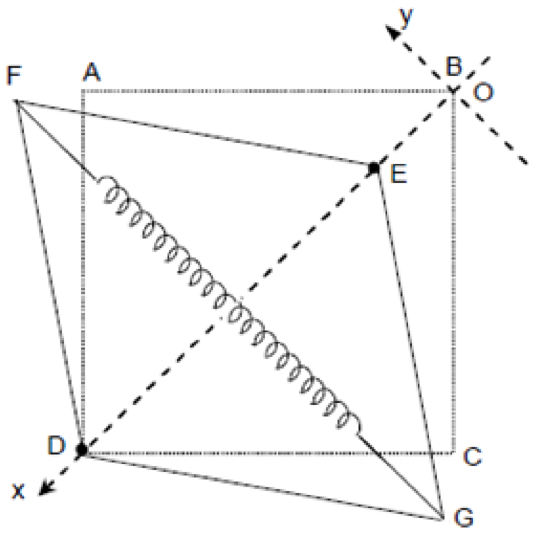
During one line period, change in velocity becomes zero.

$\therefore$ Average acceleration is zero.

25. (4)

Time period of simple pendulum is independent of amplitude.

26. (4)



$OE = x$

$FG = \sqrt{2}a + 2y$

From the property of rhombus, we can write

$$\left(\frac{a}{\sqrt{2}} + y\right)^2 + \left(\frac{a}{\sqrt{2}} - \frac{x}{2}\right)^2 = a^2$$

Neglecting x^2 and y^2, we have $y = \frac{x}{2}$

Using COME, we can write

$$\frac{mv^2}{2} + \frac{k}{2}(x)^2 = E = \text{constant}$$

$$\Rightarrow T = \pi\sqrt{\frac{4m}{k}}$$

27. (5)

$$\frac{m\ell^2}{12}\alpha = -\left\{\frac{2k\theta}{12} + \frac{k\theta}{4}\right\}\ell^2 \Rightarrow \alpha = -\left(\frac{5k}{m}\right)\theta$$

28. (2)

$x = 0.2\cos 5\pi t$

Velocity $= \frac{dx}{dt} = -\pi\sin 5\pi t$

Speed $= \pi\left|\sin 5\pi t\right|$

$$v_{avg} = \frac{\pi\int_0^{0.7}\left|\sin 5\pi t\right| dt}{0.7}$$

$$= \frac{\pi}{0.7} \times 7 \times \int_0^{0.1} \sin 5\pi t\, dt = \frac{10\pi}{5\pi}\left[-\cos 5\pi t\right]_0^{0.1}$$

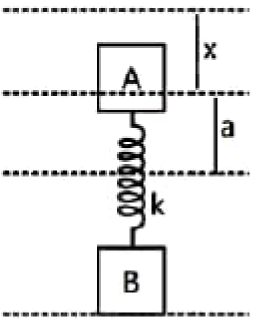
$v_{avg} = 2\ m/s$

29. (60)

$m_1 = 1\,kg;\ m_2 = 4\,kg$

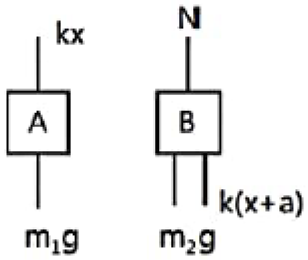
$a = 1.6\,cm$

$kx = m_1 g$



$k = \omega^2 m_1 = 25^2 \times 1 = 625\ N/m$

$N_{max} = m_2 g + k(x+a)$



$= (m_1 + m_2)g + ka = 50 + \frac{625 \times 1.6}{100}$

$N_{max} = 60\,N$

30. (4.83 to 4.89)

$x = 1\,m \qquad v = 3\,ms^{-1}$

$x = 2\ mv = 2\ ms^{-1}$

$v = 3 = \omega\sqrt{A^2 - 1}$

$3 = \omega\sqrt{A^2 - 1}$

$2 = \omega\sqrt{A^2 - 4}$

$\Rightarrow \frac{9}{4} = \frac{A^2 - 1}{A^2 - 4} \Rightarrow 9A^2 - 36 = 4A^2 - 4$

$\Rightarrow 5A^2 = 32 \Rightarrow A = \sqrt{\frac{32}{5}} = \sqrt{6.4}$

$\Rightarrow A = 2.53\,m$

$3 = \omega\sqrt{6.4 - 1}$

$w = \frac{3}{\sqrt{5.4}} = 1.29\ rad/s$

Period of motion: $T = \frac{2\pi}{\omega} = 4.86\ s$

IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 22/04/23**

TOPIC: VOLUMETRIC ANALYSIS

SOLUTIONS

31. (D)
$2(+2)+x+6(-2)=0 \Rightarrow x=+8$

32. (A)
The oxidation state of Fe in $Fe(CO)_5, Fe_2O_3, K_4[Fe(CN)_6]$ and $FeSO_4.(NH_4)_2SO_4.6H_2O$ are $0, +3, +2$ and $+2$, respectively.

33. (B)
$H_2SO_4 + BaO_2 \rightarrow BaSO_4^{-2} + H_2O_2^{-1}$

34. (D)
Oxidation state of Cr in K_3CrO_8 is $+5$. In CrO_5, it is $+6, K_3CrO_8$ has four peroxide (O_2^{2-}) linkage while CrO_5 has only two peroxide linkage.

35. (C)
$H_2S + SO_2 \rightarrow S + H_2O$
Same element but belonging to different molecule undergoes oxidation and reduction and hence, it is not disproportionation. In fact, it is comproportionation

36. (C)
$2NO_3^- + 10H^+ + 8e^- \rightarrow N_2O + 5H_2O$

37. (B)
Sulphurated hydrogen (g) $+ Sn(S) \rightarrow$ Tin Sulphide $(s) + H_2(g)$
x mole x mole
$= x \times 34 g$ $= x \times 2 g$
No change in volume means to change in moles of gases.
Mass of sulphur combined with 2x gm hydrogen
$= 34x - 2x = 32x\,g$
$\therefore E_{sulphur} = \frac{32x}{2x} \times 1 = 16$

38. (A)

$NaHC_2O_4$ is behaving as monobasic acid and hence, $R = \frac{M}{1} = \frac{23+1+2\times12+4\times16}{1} = 112$.

39. (C)

$$\overset{+3\,-2}{As_2S_3} + 20H_2O + 28e^- \rightarrow 2\overset{+5}{As}O_4^{3-} + 3\overset{+6}{S}O_4^{2-} + 40H^+$$

(As: $-3\times8=24$; S: $-2\times2e=4$)

$n_f = 24+4 = 28$

40. (C)

Reaction is balanced on the loss or gain of $10e^-$ $\left(n_f = \frac{10}{3} \qquad Eq.Wt. = \frac{3M}{10}\right)$

41. (A)

Reaction is balanced by the loss or gain of $2e^-$

$n_f = \frac{2}{2} = 1$; Eq. Wt. = M. wt.

42. (D)

$x\times5 = y\times6 \Rightarrow x:y = 6:5$

43. (D)

Valency $= \frac{2\times V.D.}{E+35.5} = \frac{2\times77}{3+35.5} = 4$

$\therefore$ Atomic mass $= 3\times4 = 12$

44. (B)

$n_{eq}KMnO_4 = n_{eq}Na_2S_2O_3$

or $\frac{V\times0.1}{1000}\times3 = \frac{0.158}{158}\times8 \Rightarrow V = 26.67ml$

45. (A)

$n_{eq}NH_3 = n_{eq}H_2SO_4$

Or $\frac{V}{22400}\times1 = \frac{30\times(1-0.2)}{1000} \Rightarrow V = 537.6ml$

46. (A)

$n_{eq}KH_2PO_4 = n_{eq}OH^-$

Or $\frac{w}{136}\times1 = \frac{25\times0.1}{1000} \Rightarrow w = 0.34g$

$\therefore$ % Purity $= \frac{0.34}{0.5}\times100 = 68\%$

47. (A)

$n_{eq} SeO_2 = n_{eq} Cr^{2+}$

Or $\frac{12.5 \times 0.05}{1000} \times (4 - x) = \frac{25 \times 0.1}{1000} \times 1 \Rightarrow x = 0$

48. (C)

$n_{eq} KMnO_4 = n_{eq} H_2C_2O_4$

Or $\frac{w}{158} \times 5 = \frac{50 \times 0.2}{1000} \times 2 \Rightarrow w = 0.632\,g$

49. (D)

$n_{eq} H_2O_2 = n_{eq} KMnO_4$

Or $\frac{1 \times \frac{x}{100}}{34} \times 2 = \frac{x \times N}{1000} \Rightarrow N = 0.588$

50. (C)

$n_{eq} S_2O_3^{2-} = n_{eq} K_2S_2O_8$

Or $\frac{V \times 0.25}{1000} \times 1 = \frac{1}{270} \times 2 \Rightarrow v = 29.63\,ml$

51. (0)

Oxidation state of Cr in both compounds is +6.

52. (4)

$3x + 2(-2) = 0 \Rightarrow x = +\frac{4}{3}$

53. (5)

$2(+2) + 2x + 7(-2) = 0 \Rightarrow x = +5$

54. (3)

$\frac{N_{Fe}}{N_O} = \frac{\frac{70}{56}}{\frac{30}{16}} = \frac{2}{3} \Rightarrow \text{Formula} = Fe_2O_3$

$\therefore$ Oxidation state of $Fe = +3$

55. (75)

$3X_2O_3 + 4Cr_2O_7^{2-} + 26H^+ \rightarrow 6XO_4^- + 8Cr^{3+} + 13H_2O$

Method II: (After equivalent concept)

n_{eq} of $X_2O_3 = n_{eq} Cr_2O_7^{2-}$

Or $n \times 8 = 1 \times 6 \Rightarrow n_{X_2O_3} = \frac{3}{4}$

56. (48)

$4N_A$ electrons means 4 equivalents . Hence, mass of Mg needed $= 4 \times 12 = 48\,gm$

57. (112)

$NaHC_2O_4$ is behaving as base and hence, $E = \frac{M}{I}$

58. (17)

H_2O_2 is acting as reductant. Its equivalent weight is $\frac{34}{2} = 17$

59. (4)

$n_{eq}NH_3 = n_{eq}O_2 = 1 \times 4 = 4$

60. (51)

$n_{eq}H_2S = n_{eq}KMnO_4$

Or $\frac{w}{34} \times 8 = \frac{6.32 \times 3}{158} \Rightarrow w = 0.51g$

TOPIC: CONTINUITY

SOLUTIONS

61. (C)

We have, $f(x)=\begin{cases} x+2, & x<1 \\ 4x-1, & 1\le x\le 3 \\ x^2+5, & x>3 \end{cases}$

$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-} x+2=1+2=3$ ...(i)

$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+} 4x-1=4-1=3$ ...(ii)

$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-} 4x-1=12-1=11$ ...(iii)

$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+} x^2+5=9+5=14$ ...(iv)

From Eqs. (i) and (ii), $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)=f(1)=3$

$\therefore$ $f(x)$ is continuous at $x=1$.

From Eqs. (iii) and (iv), $\lim_{x\to 3^-} f(x)\ne\lim_{x\to 3^+} f(x)$

$\therefore$ $f(x)$ discontinuous at $x=3$.

62. (C)

We have, $f(x)=\begin{cases} \dfrac{1}{e^{4x}+1}, & x\ne 0 \\ 0, & x=0 \end{cases}$

$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\dfrac{1}{e^{4x}+1}$

$=\lim_{x\to(0-h)}\dfrac{1}{e^{-4h}+1}$

$=\lim_{h\to 0}\dfrac{1}{e^{-4h}+1}=\dfrac{1}{e^0+1}=\dfrac{1}{2}$

$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\dfrac{1}{e^{4h}+1}$

$=\lim_{x\to(0+h)}\dfrac{1}{e^{4h}+1}$

$=\dfrac{1}{e^0+1}=\dfrac{1}{2}$

$\therefore$ $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)\ne f(0)=0$

Hence, $f(x)$ is discontinuous.

63. (A)

We have, $f(x)=\begin{cases}\dfrac{x^2-(a+2)x+2a}{x-2}, & x\neq 2\\ 2, & x=2\end{cases}$

$f(x)$ is continuous at $x=2$.

$\therefore \lim\limits_{x\to 2}\dfrac{x^2-(a+2)x+2a}{x-2}=2 \qquad [\because f(2)=2]$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{(x-a)(x-2)}{x-2}=2$

$\Rightarrow 2-a=2$

$\therefore a=0$

64. (A)

We have, $f(x)=\begin{cases}\dfrac{\log(1+2ax)-\log(1-bx)}{x}, & x\neq 0\\ k, & x=0\end{cases}$

$f(x)$ is continuous at $x=0$.

$\therefore \lim\limits_{x\to 0} f(x)=f(0)$

$\Rightarrow \lim\limits_{x\to 0}\dfrac{\log(1+2ax)-\log(1-bx)}{x}=k \qquad [\because f(0=k)]$

$\Rightarrow \lim\limits_{x\to 0}\dfrac{\log(1+2ax)\cdot 2a}{2ax}-\lim\limits_{x\to 0}\dfrac{\log(1-bx)\cdot(-b)}{-bx}=k$

$\Rightarrow 2a+b=k \qquad \left[\because \lim\limits_{x\to 0}\dfrac{\log(1+x)}{x}=1\right]$

65. (A)

We have, $f(x)=\begin{cases}[x]+[-x], & x\neq 2\\ \lambda, & x=2\end{cases}$

$f(x)$ is continuous at $x=2$.

$\therefore \lim\limits_{x\to 0} f(x)=f(2)$

$\Rightarrow \lim\limits_{x\to 0}[x]+[-x]=\lambda \qquad [\because f(2)=\lambda]$

$\Rightarrow -1=\lambda \qquad [\because [x]+[-x]=0,\ x\in$ Integer and $[x]+[-x]=-1,\ x\notin$ Integer]

$\therefore \lambda=-1$

66. (A)

We have, $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2}, & \text{if } x<0\\ a, & \text{if } x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{if } x>0\end{cases}$

For $f(x)$ is continuous at $x=0$,

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x)=f(0)$

Now, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1-\cos 4x}{x^2}$

$= \lim_{h\to 0} \frac{1-\cos 4h}{h^2}$

$= \lim_{h\to 0} \frac{4\times 2\sin^2 2h}{(2h)^2} = 8$

and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$

$= \lim_{h\to 0} \frac{\sqrt{h}}{\sqrt{16+\sqrt{h}}-4}$

$= \lim_{h\to 0} \frac{\sqrt{h}\left(\sqrt{16+\sqrt{h}}+4\right)}{16+\sqrt{h}-16}$

$= \lim_{x\to 0} \sqrt{16+\sqrt{h}}+4 = 8$

$\therefore \quad \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\therefore \quad 8 = 8 = a \qquad [\because f(0) = a]$

Hence, $f(x)$ is continuous at $x = 0$, and the value of a is 8.

67. (C)

We have, $f(x) = \frac{1}{\log|x|}$

$f(x)$ is not defined at $x = 0, \pm 1$.

$\therefore$ $f(x)$ is discontinuous at three points.

68. (B)

(A) $\tan x$ is discontinuous at $x = \frac{n\pi}{2}, n \in I$.

(B) $\frac{|x|}{x}$ is discontinuous at $x = \{-1, 0, 1\}$.

(C) $x + [x]$ is discontinuous at $x \in$ integer.

(D) $\sin[\pi x]$ is discontinuous at $x \in n$ or $\left(n + \frac{1}{2}\right)$.

Hence, $\frac{|x|}{x}$ has finite number of points of discontinuity.

69. (A)

We have, $f(x) = \frac{1-\tan x}{4x-\pi}, x \neq \frac{\pi}{4}$

$f(x)$ is continuous at $x = \frac{\pi}{4}$.

$\therefore \lim_{x\to \frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$

$$f\left(\frac{\pi}{4}\right)=\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{4x-\pi}$$

$$=\lim_{x\to\frac{\pi}{4}+h}\frac{1-\tan\left(\frac{\pi}{4}+h\right)}{4\left(\frac{\pi}{4}+h\right)-\pi}$$

$$=\lim_{h\to 0}\frac{1-\frac{1+\tan h}{1-\tan h}}{4h}=\lim_{h\to 0}\frac{-2\tan h}{4h(1-\tan h)}=\frac{-1}{2}$$

70. (B)

We have, $f(x)=\dfrac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$

$f(x)$ is continuous at $x=0$.

$\therefore \lim_{x\to 0} f(x)=f(0)$

$$\Rightarrow f(0)=\lim_{x\to 0}\frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$$

$$=\lim_{x\to 0}\frac{\left[\left(a^2-ax+x^2\right)-\left(a^2+ax+x^2\right)\right]\left(\sqrt{a+x}+\sqrt{a-x}\right)}{(a+x)-(a-x)\left(\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2}\right)}$$

$$=\lim_{x\to 0}\frac{-2ax\left(\sqrt{a+x}+\sqrt{a-x}\right)}{2x\left(\sqrt{a^2-ax+x^2}+\sqrt{a^2+ax+x^2}\right)}$$

$$=\frac{-2a\sqrt{a}}{2a}=-\sqrt{a}$$

71. (D)

We have, $f(x)=\dfrac{x-e^x+\cos 2x}{x^2}$, $x\neq 0$

$f(x)$ is continuous at $x=0$.

$\therefore \quad \lim_{x\to 0} f(x)=f(0)$

$$\Rightarrow \quad f(0)=\lim_{x\to 0}\frac{x-e^x+\cos 2x}{x^2}$$

$$\Rightarrow \quad f(0)=\lim_{x\to 0}\frac{1-e^x+2\sin 2x}{2x}$$ [applying L' Hospital rule]

$$\Rightarrow \quad f(0)=\lim_{x\to 0}\frac{-e^x-4\cos 2x}{2}$$

$$\Rightarrow \quad f(0)=\frac{-5}{2}$$

$\therefore \quad [f(0)]=\left[\frac{-5}{2}\right]=-3$ and $\{f(0)\}=\left\{\frac{-5}{2}\right\}=0.5$

$\therefore \quad [f(0)]\{f(0)\}=-3\times 0.5=-1.5$

72. (C)

We have, $f(x)=\dfrac{x(1+a\cos x)-b\sin x}{x^3}$, $x\neq 0$ and $f(0)=1$

$f(x)$ is continuous at $x=0$.

$\Rightarrow \quad f(0)=\lim\limits_{x\to 0}\dfrac{x\cdot(1+a\cos x)-b\sin x}{x^3}$

$=\lim\limits_{x\to 0}\dfrac{x(-a\sin x)+(1+a\cos x)-b\cos x}{3x^2}$ [using L' Hospital rule]

$\Rightarrow \quad 1+a-b=0$ [for limit to exists] ….. (i)

Again, using L' Hospital rule

$=\lim\limits_{x\to 0}\dfrac{-a\sin x-ax\cos x-a\sin x+b\sin x}{6x}$

$=\lim\limits_{x\to 0}\dfrac{(b-2a)\sin x}{6x}-\dfrac{ax\cos x}{6x}$

$f(0)=\dfrac{b-2a}{6}-\dfrac{a}{6}$

$\Rightarrow \quad 1=\dfrac{b-3a}{6}$

$\Rightarrow \quad b-3a=6$ ...(ii)

From Eqs. (i) and (ii), we get

$a=-\dfrac{5}{2}$ and $b=-\dfrac{3}{2}$.

73. (D)

$\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{2}{e^{2x}-1}\right)=\lim\limits_{x\to 0}\dfrac{e^{2x}-1-2x}{x\left(e^{2x}-1\right)}$

$=\lim\limits_{x\to 0}\dfrac{2e^{2x}-2}{\left(e^{2x}-1\right)+2xe^{2x}}$ [by L' Hospital's rule]

$=\lim\limits_{x\to 0}\dfrac{4e^{2x}}{4e^{2x}+4xe^{2x}}$ [by L' Hospital's rule]

$=1$

So, $f(x)$ is continuous at $x=0$.

$\therefore \quad \lim\limits_{x\to 0} f(x)=f(0)$

$\Rightarrow \quad f(0)=1$

74. (A)

Here, $f(x)=\begin{cases}\dfrac{\sin(p+1)x+\sin x}{x}, & x<0\\ q, & x=0\\ \dfrac{x+x^2-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$

Since, $f(x)$ is continuous for $x\in R$.

So, $f(x)$ is continuous at $x=0$.

RHL at $x=0$,

$$\lim_{h\to 0}\frac{\sqrt{h+h^2}-\sqrt{h}}{h^{3/2}}=\lim_{h\to 0}\frac{\sqrt{h}\left\{\sqrt{h+1}-1\right\}}{h\left(\sqrt{h}\right)}$$

$$=\lim_{h\to 0}\left(\frac{\sqrt{h+1}-1}{h}\times\frac{\sqrt{h+1}+1}{\sqrt{h+1}+1}\right)$$

$$=\lim_{h\to 0}\frac{(h+1)-1}{h\left\{\sqrt{h+1}+1\right\}}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{h+1}+1}=\frac{1}{2} \qquad \text{..... (i)}$$

LHL at $x=0$,

$$\lim_{h\to 0}\frac{\sin(p+1)(-h)+\sin(-h)}{-h}$$

$$=\lim_{h\to 0}\frac{\sin(p+1)h}{h}+\frac{\sin h}{h}$$

$$\Rightarrow \quad p+1+1=p+2 \qquad ...(\text{ii})$$

$$f(0)=q \qquad ...(\text{iii})$$

From Eqs. (i), (ii) and (iii)

$$q=\frac{1}{2} \text{ and } p=-\frac{3}{2}$$

75. (D)

Given, $f(x)=\dfrac{4-x^2}{4x-x^3} \Rightarrow f(x)=\dfrac{4-x^2}{x\left(4-x^2\right)}$

Clearly, $f(x)$ is not defined at $x=0,\pm 2$.

$\therefore$ $f(x)$ is discontinuous at $x=0,\pm 2$.

Also, domain of f is $R-\{-2, 0, 2\}$.

Hence, $f(x)$ is continuous everywhere in its domain.

76. (A)

$$\text{Given, } f(x)=\begin{cases}-4\sin x+\cos x, & \text{for } x\le -\frac{\pi}{2}\\ a\sin x+b, & \text{for } -\frac{\pi}{2}<x<\frac{\pi}{2}\\ \cos x+2, & \text{for } \frac{\pi}{2}\le x\end{cases}$$

LHL at $x=\dfrac{-\pi}{2}$,

$$\lim_{x\to -\frac{\pi}{2}^-} f(x)=\lim_{h\to 0}-4\sin\left(\frac{-\pi}{2}-h\right)+\cos\left(\frac{-\pi}{2}-h\right)$$

$$=\lim_{h\to 0}4\cos h-\sin h=4$$

RHL at $x=\dfrac{-\pi}{2}$,

$$\lim_{x \to -\frac{\pi^-}{2}} f(x) = \lim_{x \to 0} a \sin\left(\frac{-\pi}{2} - h\right) + b$$

$$= \lim_{x \to 0} (-a \cos \text{h} + b) - a + b$$

$\because$ $f(x)$ is continuous at $x = \frac{-\pi}{2}$.

$\therefore$ $-a + b = 4$ ...(i)

Now, LHL at $x = \frac{\pi}{2}$,

$$\lim_{x \to \frac{\pi^-}{2}} f(x) = \lim_{h \to 0} a \sin\left(\frac{\pi}{2} - h\right) + b = a + b$$

RHL at $x = \frac{\pi}{2}$,

$$\lim_{x \to \frac{\pi^+}{2}} f(x) = \lim_{h \to 0} \cos\left(\frac{\pi}{2} + h\right) + 2 = 2 \Rightarrow a + b = 2 \quad ...(ii)$$

On solving eqs. (i) and (ii), we get

$a = -1, b = 3$

77. (D)

Given, $f(x) = [x^2 + 1] = [x^2] + 1$

Now, x^2 is monotonic in the range of [1, 3].

Hence, $[x^2]$ is discontinuous when x^2 is integer.

$\therefore$ $x^2 = 2, 3, 4, ..., 9.$

or $x = \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}$

Note it is right continuous at $x = 1$ but not left continuous at $x = 3$.

$\therefore$ $\lim_{x \to 1^+} [x^2 + 1] = 2 = f(1)$

and $\lim_{x \to 3^-} [x^2 + 1] = 9 \neq 10 = f(3)$

hence, $f(x)$ is continuous for all x except eight points.

78. (B)

Given, $f(x) = [2x^3 - 5] = [2x^3] - 5$

$(2x^3)$ is monotonic in the range [1, 2].

$\therefore (2x^3)$ is discontinuous at integral point in (1, 2).

$\therefore$ $2x^3 = 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15$.

Hence , $f(x)$ is discontinuous at 13 points.

79. (B)

Given, $f(x) = \begin{cases} |x+1|, & \text{if } x < -2 \\ 2x+3, & \text{if } -2 \le x < 0 \\ x^2+3, & \text{if } 0 \le x < 3 \\ x^3 - 15, & \text{if } 3 \le x \end{cases} \Rightarrow \begin{cases} -(x+1), & \text{if } x < -2 \\ 2x+3, & \text{if } -2 \le x < 0 \\ x^2+3, & \text{if } 0 \le x < 3 \\ x^3 - 15, & \text{if } 3 \le x \end{cases}$

At $x = -2$,

$f\left(-2^-\right)=\lim_{h\to0}-\left(-2-h+1\right)=1$

$f\left(-2^+\right)=\lim_{h\to0}-\left(-2+h\right)+3=-1$

$f\left(x\right)$ is discontinuous at $x=-2$.

Now, at $x=0$,

$f\left(0^-\right)=f\left(0^+\right)=f\left(0\right)=3$

$\therefore$ $f\left(x\right)$ is continuous at $x=0$.

At $x=3$,

$f\left(3^-\right)=f\left(3^+\right)=f\left(3\right)=12$

$f\left(x\right)$ is continuous at $x=3$.

Hence, only point of discontinuity is $x=-2$.

80. (C)

As, $f\left(x\right)$ is continuous for all $x\in R$ and $f\left(x\right)=5, \forall \in$ irrational number

$\Rightarrow$ $f\left(x\right)$ must be constant.

$\therefore$ $f\left(x\right)=f\left(3\right)=5$

81. (7)

$\because$ f(x) is continuous at x =2

$\therefore$ f(2–0) = f(2+0) = f(2) = k

But f(2+0)

$= \lim_{h\to0}\frac{(2+h)^3+(2+h)^2-16(2+h)+20}{(2+h-2)^2}$

$= \lim_{h\to0}\frac{h^3+7h^2}{h^2} = 7$

82. (8)

Since f(x) is continuous at x = 2

$\therefore$ f(2) = $\lim_{x\to2^+}$ f(x)

$\Rightarrow$ 1 = $\lim_{x\to2^+}$ (ax+ b)

$\therefore$ 1= 2a + b ...(1)

Again f(x) is continuous at x = 4,

$\therefore$ f(4) = $\lim_{x\to4^-}$ f(x)

$\Rightarrow$ 7 = $\lim_{x\to4}$ (ax+ b)

$\therefore$ 7 = 4a + b ...(2)

Solving (1) and (2), we get a= 3, b = –5.

83. (1)

Let us first examine continuity at x = 0.

f (0) = 0 ($\because$ 0 $\in$ Q)

= f (0–0) = $\lim_{h\to0}$ f(0- h) = $\lim_{h\to0}$ f(–h)

= $\lim_{h\to0}$ {–h or h according as – h $\in$ Q or –h $\notin$ Q)

= 0

f(0+0) = $\lim_{h\to 0}$ f(0+h) = $\lim_{h\to 0}$ f(h)

= $\lim_{h\to 0}$ {h or –h} = 0

f(0) = f(0 – 0) = f(0 + 0)

⇒ f(x) is continuous at x = 0.

Now let a ∈ R, a ≠ 0, then

f(a–0) = $\lim_{h\to 0}$ f(a–h)

= $\lim_{h\to 0}$ {(a–h) or – (a–h)}

= a or –a, which is not unique.

⇒ f(a–0) does not exist

⇒ f(x) is not continuous at a ∈ R_0.

Hence f(x) is continuous only at x = 0.

84. (4.5)

$$f\left(\frac{\pi}{2}-0\right) = \lim_{h\to 0}\frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)}$$

$$= \lim_{h\to 0}\frac{1-\cos^3 h}{3\sin^2 h}$$

$$= \lim_{h\to 0}\frac{(1-\cos h)(1+\cos h+\cos^2 h)}{3(1-\cos h)(1+\cos h)}$$

= 1/2

$$f\left(\frac{\pi}{2}+0\right) = \lim_{h\to 0}\frac{b\left[1-\sin\left(\frac{\pi}{2}+h\right)\right]}{\left[\pi-2\left(\frac{\pi}{2}+h\right)\right]}$$

$$= \lim_{h\to 0}\frac{b(1-\cos h)}{4h^2}$$

$$= \lim_{h\to 0}\frac{2b\sin^2 h/2}{4h^2} = \frac{b}{8}$$

Now f(x) is continuous at x = $\frac{\pi}{2}$

$$\Rightarrow f\left(\frac{\pi}{2}-0\right) = f\left(\frac{\pi}{2}+0\right) = f\left(\frac{\pi}{2}\right) \Rightarrow \frac{1}{2} = \frac{b}{8} = a$$

∴ a = 1/2, b = 4

a + b = 4.5

85. (0)

Obviously the function f(x) is continuous at x = 1 and 3. Therefore $\lim_{x\to 1^+}$ f(x) = f(1)

⇒ a + b = 2 ...(1)

and $\lim_{x\to 3^-}$ f(x) = f(3)

⇒ 3a + b = 6 ...(2)

Solving (1) and (2) , we get a = 2, b = 0.

86. (5)

$$f(x)=\begin{cases}(1+3x)^{\frac{5}{3x}}, & x\neq 0\\ k, & x=0\end{cases}$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^-} f(x)=\lim_{x\to 0}(1+3x)^{\frac{5}{3x}}=e^{\lim_{x\to 0}3x\times\frac{5}{3x}}=e^5$$

$$\lim_{x\to 0} f(x)=k$$

$\therefore$ $f(x)$ is continuous at $x=0$ $\therefore e^k=e^5 \Rightarrow k=5$

87. (0)

$$f(x)=\begin{cases}\dfrac{x^2-(A+2)x+2A}{x-2}, & x\neq 2\\ 2, & x=2\end{cases}$$

Continuous at $x=2$

$$\lim_{h\to 0} f(2+h)=\lim_{h\to 0} f(2-h)=f(2)$$

$$\lim_{h\to 0} f(2+h)=\frac{(2+h)^2-(A+2)(2+h)+2A}{h}$$

$$\Rightarrow \frac{4+4h+h^2-2A-4-Ah-2h+2A}{h}$$

$$=\frac{2h+h^2-Ah}{h}$$

$$f(2)=2$$

So $\dfrac{h^2+2h-Ah}{h}=2$

$$\lim_{h\to 0} h^2+2h-Ah=2h$$

$\boxed{A=0}$

88. (0)

89. (5)

90. (2)