IIT – JEE: 2024 **TW TEST (3 YRS.)** **DATE: 05/11/22**

TOPIC: CIRCULAR MOTION, WPE

SOLUTION

1. **(D)**

Acceleration $a = \frac{mg - K\Delta x}{m}$

Velocity will be maximum when $a = 0$

$\therefore \quad \Delta x = \frac{mg}{K}$

So, height $h = l - \frac{mg}{K}$

2. **(A)**

Let normal reaction makes an angle θ from vertical, then

$v^2 = 2gr(1-\cos\theta)$ and $\frac{mv^2}{r} = mg\cos\theta$

$\Rightarrow$ height from ground $h = \frac{2r}{3}$

3. **(B)**

Work done by centripetal remains zero.

4. **(C)**

$Mg - T = \frac{Mg}{4} \Rightarrow T = \frac{3Mg}{4}$.

So, $W = \frac{3Mg}{4} d\cos 180$

5. **(A)**

By work energy theorem

$0 = W_T + W_F + W_{mg} = 0 + W_F - mg\ell(1-\cos\theta)$

$\Rightarrow \quad W_F = mg\ell(1-\cos\theta)$

6. **(D)**

Only centripetal acceleration exist.

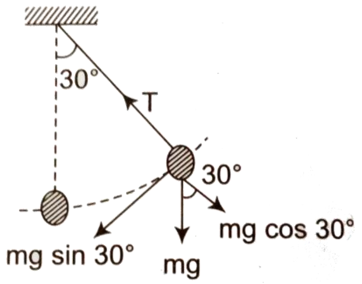
7. **(B)**

Rate of change of speed

$\frac{dv}{dt} =$ tangential acceleration

$= \dfrac{\text{tangential force}}{\text{mass}}$

$= \dfrac{mg\sin 30^{\circ}}{m}$



$= 10\left(\dfrac{1}{2}\right) m/s^2 = 5\, m/s^2$

8. **(C)**

As tangential acceleration is constant in magnitude but angular acceleration is constant in magnitude and direction both.

9. **(C)**

$\omega_A - \omega_B = -\dfrac{4\pi}{3}$

So, $\Delta T = \dfrac{2\pi}{|\Delta\omega|} = 1.5 \text{ min}$

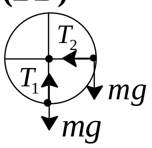
10. **(D)**

a_t can have any value in comparison to a_c

11. **(D)**

$N\sin\theta = \dfrac{mv^2}{R}$ and $N\cos\theta = mg$

12. **(BD)**



13. **(ABC)**

By work energy theorem

$W_{nc} + W_c = \Delta k$ and $\Delta U = -W_c$

14. **(BC)**

Kinetic energy never be negative

15. **(BCD)**

16. **(4.00)**

Using $k_1\ell_1 = k_2\ell_2 = k\ell$

$$k_1 = \frac{k.\ell}{3\ell/4} = \frac{4}{3}k$$

$$k_2 = \frac{k.\ell}{\ell/4} = 4k$$

Maximum speed is attained when block is passing through the position when springs acquire natural length.

i.e. $\sum F_{\text{block}} = 0$

Applying W/E theorem

$$\frac{1}{2}mv^2 - 0 = \frac{1}{2}\cdot\frac{4}{3}k.x^2 + \frac{1}{2}\cdot 4k.x^2$$

$$= \frac{2}{3}kx^2 + 2kx^2$$

$$\frac{1}{2}mv^2 = \frac{8}{3}kx^2$$

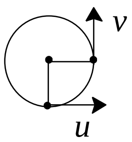
$$\Rightarrow \quad v^2 = \frac{16}{3}\frac{kx^2}{m}$$

$$v_{\max} = 4x\sqrt{\frac{k}{3m}}$$

17. **(37.5)**

$$\Delta K = \frac{1}{2}M\left[V_2^2 - V_1^2\right] = 37.5\text{ J}$$

18. **(4.00)**



$$\Delta v = \sqrt{u^2 + v^2} = \sqrt{u^2 + u^2 - 2Lg} = \sqrt{2u^2 - 2Lg}$$

19. **(2.00)**

As safe speed is given as $v = \sqrt{\mu rg} \Rightarrow \mu = \dfrac{v^2}{rg}$

20. **(0.5)**

$$\frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} \Rightarrow \frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{1}{2}$$

$$\frac{v_1^2/r_1}{v_2^2/r_2} = \left(\frac{v_1}{v_2}\right)^2\cdot\left(\frac{r_2}{r_1}\right) = \frac{1}{4}\cdot 2 = \frac{1}{2} = 0.5$$

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TOPIC: ENERGETICS

SOLUTION

21. **(A)**

During isothermal process, temperature remains constant. While, during adiabatic compression, final temp↑, because $q=-w, w<0 \Rightarrow q>0$.

Irreversible work > reversible work.

22. **(C)**

$\left(\frac{\partial H}{\partial T}\right)_P = C_P;\ \left(\frac{\partial U}{\partial T}\right) = C_V;\ C_P > C_V$

$\frac{C_P}{C_V} = \gamma$

23. **(D)**

$q = 0$

ΔU = -5x(40-50) = 50 L-bar

ΔH = ΔU + Δ(PV) = 50 + (40x5 – 50x3) = 100 L-bar

24. **(C)**

25. **(B)**

$A(s) \rightarrow B(s);\ \Delta G^{\circ} = 20\text{ kJ}$

$dG = Vdp$

$d(\Delta G) = (\Delta V)dP$

26. **(A)**

$3A + B \rightarrow 2C + 4D(l)$

$\Delta H^{\circ} = 50 - \frac{2\times2\times200}{1000} = 49.2\text{ kcal/mole}$

$\Delta G^{\circ} = 49.2 + \frac{200\times400}{1000} = 129.2\text{ kcal/mole}$

27. **(D)**

$\Delta G^{\circ}_{Rx^n} = -394.4 + 137.2 = -257.2\text{ kJ/mol}$

$\Delta H^{\circ} = -257.2 - 300\times0.094 = -285.4\text{ kJ/mol}$

28. **(D)**

$\Delta S < 0 \Rightarrow \Delta G < 0$ if T is low.

29. **(A)**

$H^+ + OH^- \rightarrow H_2O$

$-57.3 = -285.5 - x$

$\Rightarrow x = -228.5$

30. **(B)**

$$\frac{1}{2}N_2(g) + \frac{3}{2}H_2(g) \rightarrow NH_3(g)$$

$$\Delta H_f = \frac{x_1}{2} + \frac{3x_2}{2} - 3x_3$$

31. **(ABCD)**

32. **(BCD)**

33. **(ABC)**

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{300}{600} = \frac{1}{2}$$

$$\Delta S_{AB} = 20\ J/K = \frac{q_{\text{reversible}}}{600}$$

$$\Rightarrow q_{AB} = 12000\ \text{J}$$

$$W_{AB} = -12000\ \text{J}$$

$$q_{BC} = q_{DA} = 0;\ q_{CD} = -6000\ \text{J}$$

$$\therefore W_{CD} = +6000\ \text{J}$$

$$|W_{BC}| = |W_{DA}|$$

34. **(AC)**

For CA :

$$V = -\frac{T}{300} + 3$$

$$\therefore P = 300R\left(\frac{3}{V} - 1\right)$$

$$W_{CA} = -\int_2^1 300R\left(\frac{3}{V} - 1\right)dV = 660\ \text{cal}$$

$$W_{AB} = -840\ \text{cal} = -1 \times 2 \times 600 \ln\left(\frac{2}{1}\right)$$

$$W_{\text{overall}} = W_{AB} + W_{BC} + W_{CA}$$

$$= -840 + 0 + 660 = -180\ \text{cal}$$

$$\Delta H_{AB} = 0$$

$$\Delta U_{CA} = \frac{1 \times 5}{2} \times 2[300] = 1500\ \text{cal}$$

$$q_{BC} = \frac{1 \times 5}{2} \times 2[-300] = -1500 \text{ cal}$$

35. **(ABC)**

$\Delta S_{\text{surr}} = 0 \Rightarrow$ Adiabatic process.

36. **(100.00)**

37. **(130.00)**

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\frac{273}{3^{2/3}} = T_2$$

38. **(5.20)**

$$W = -\int P dV = -\int_1^3 6V^2 dV = 2 \times [27 - 1]$$

= 52 bar-m^3 or 5200 kJ

39. **(66.70)**

$$\Delta U = Q + w$$

$$= n_{C_{P_m}} \Delta T + n\Delta H_{\text{vap}} + w$$

$$= \frac{100}{20} \times \frac{60}{1000} \times 50 + \frac{40}{20} \times 30 - \Delta n_g RT$$

$$= 15 + 60 - \frac{2 \times 8.3 \times 500}{1000}$$

$$= 66.7 \text{ kJ} \Rightarrow 6670 \text{ J}$$

40. **(148.80)**

TOPIC: BINOMIAL THEOREM

SOLUTION

41. (D)

$$\frac{10!}{r!s!(10-r-s)!} \quad 2^{r/2}\, 3^{s/3} \quad 5^{\frac{10-r-s}{6}}$$

(r, s) ≡ (4,0), (10,0) (4,6) for rational terms.

42. (B)

$$a_r + a_{r+1} = {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$$

$$\sum_{r=0}^{n-1}\left({}^{n+1}C_{r+1}\right)^2 = {}^{2n+2}C_{n+1} - {}^{n+1}C_0 - {}^{n+1}C_{n+1}$$

$$= {}^{2n+2}C_{n+1} - 2$$

43. (A)

$$(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_rx^r + ... \qquad .....(i)$$

$$\left(1+\frac{1}{x}\right)^n = C_0 + C_1\frac{1}{x} + C_2\frac{1}{x^2} + ... + C_r\frac{1}{x^r} + ... \qquad .....(ii)$$

Multiplying both sides and equating coefficient of x^r in $\frac{1}{x^n}(1+x)^{2n}$ or the coefficient of x^{n+r} in $(1+x)^{2n}$ we get the value of required expression

$$= {}^{2n}C_{n+r} = \frac{(2n)!}{(n-r)!(n+r)!}$$

44. (D)

$(1 + n)^{m+1} - n^{m+1} = 1 + {}^{m+1}C_1.n + {}^{m+1}C_2.\, n^2 + ...+....+ {}^{m+1}C_m \,.\, n^m$

put n = 1, 2, 3,..... n and add

45. (B)

Let y + z = a

$(x + a)^{2007} + (x - a)^{2007}$

$= 2[x^{2007} + {}^{2007}C_2\, x^{2005}a^2 + {}^{2007}C_4x^{2003}.\, a^4 + ...]$

No. of terms

$= 1 + 3 + 5 + ...+2007 = (1004)^2$

46. (BD)

$2^{4n} - 2^n(7n + 1) = 2^n \,.\, 8^n - 2^n(7n + 1) = 2^n[(1 + 7)^n - 7n - 1] = 2^n[{}^nC_2 \,.\, 7^2 + {}^nC_3 \,.\, 7^3 + ...]$

Clearly multiple of 49.

47. (BD)

$$S_n = \text{coeff of } x^n \text{ in } (1+x)^n + \frac{(1+x)^{n+1}}{2} + \frac{(1+x)^{n+2}}{2} + ...+ \frac{(1+x)^{2n}}{2^n}$$

$$S_n = \text{coeff of } x^n \text{ in } (1+x)^n \times \left[\left(\frac{1+x}{2}\right)^{n+1} - 1\right] \div \left[\frac{1+x}{2} - 1\right]$$

$$= \text{coeff of } x^n \text{ in } \frac{1}{2^n} \times \left[\frac{(1+x)^{2n+1} - 2^{n+1}(1+x)^n}{x-1}\right]$$

$$= \text{coeff of } x^n \text{ in } \frac{1}{2^n} \times \left[2^{n+1}(1+x)^n - (1+x)^{2n+1}\right]\left[1 + x + x^2 + ...+\infty\right]$$

$$= \frac{1}{2^n} \times \left[2^{n+1}\left({}^nC_0 + {}^nC_1 + {}^nC_2 + ...+ {}^nC_n\right) - \left({}^{2n+1}C_0 + {}^{2n+1}C_1 + ...+ {}^{2n+1}C_n\right)\right]$$

$$= \frac{1}{2^n} \times \left[2^{n+1}.\, 2^n - \frac{2^{2n+1}}{2}\right] = 2^{n+1} - 2^n = 2^n$$

$\Rightarrow S_n = 2^n$

48. (ABC)

$(13^n - 3^n) + 7^n = 10k + 7^n$

Last digit will be 3 if n = 4k + 3; $k \in W$

49. (ABC)

$f(x) = (1 + x^2 - x^3)^{1000} = \sum A_r \,.\, x^r$

Coeff. of x^{20} in f(x) and f(-x) are same $\Rightarrow$ a = d

$g(x) = (1 - x^2 + x^3)^{1000} = \sum B_r \,.\, x^r$

Coeff. of x^{20} in g(x) and g(-x) are same $\Rightarrow$ b = c

Coeff. of x^{20} in $(1 + x^2 + x^3)^{1000}$ largest compared to b, c since it involves sums of all positive terms while b and c are made by combination +ve and –ve terms.

50. (AC)

Let $f' = \left(5\sqrt{3} - 8\right)^{2n+1}$

[x] + {x} - f′ = even

$\Rightarrow$ {x} - f′ = 0 , [x] even

$\Rightarrow$x . {x} = x. f′ = $(11)^{2n+1}$

51. (ABCD)

$(1 + i)^n = P_n + i\, Q_n$; $(1 - i)^n = P_n - i\, Q_n$

by multiplying $(1+1)^n = P_n^2 + Q_n^2 \Rightarrow P_n^2 + Q_n^2 = 2^n$

$(1+i)^8 = ((1+i)^2)^4 = (2i)^4 = 16 = P_8 + iQ_8$

$P_8 = 16,\; Q_8 = 0$

$(1+i)^{10} = (2i)^5 = 32i = P_{10} + iQ_{10}$

$Q_{10} = 32,\; P_{10} = 0$

52. (BD)

53. (BC)

54. **(BCD)**

(A) $^{n}C_3 \left(\frac{x}{a}\right)^{n-3} \left(\frac{1}{x}\right)^{3} = \frac{5}{2} \Rightarrow n = 6$

$a^3 = 8 \Rightarrow a = 2$

(B) $\sum_{p=1}^{p=4} {}^4C_r \cdot {}^rC_p = \sum_{p=1}^{p=4}\sum_{r=p}^{r=4} {}^4C_p \cdot {}^{4-p}C_{r-p}$

$= \sum_{p=1}^{p=4} {}^4C_p \cdot 2^{4-p} = 3^4 - 2^4 = 65$

(C) $(1-x)^5 \frac{(1-x^4)^4}{(1-x)^4} = (1-x)(1-x^4)^4 \Rightarrow$ coeff of x^{13} = $-(-{}^4C_3) = 4$

(D) $\sum_{r=0}^{4} {}^4C_r (r-2)^2 = 4+4+4+4=16$

55. **(ABCD)**

(A) $\left(1+1+1+1\right)^{10} = \sum \frac{10!}{a!b!c!d}$

(B) $\left(2-3+5+7\right)^{10} = \sum \frac{2^a(-3)^b5^c7^d}{a!b!c!d} \times 10!$

(C) $\left(2+3+4+5\right)^{10} + (2+3+4-5)^{10} = 2\sum \frac{10!\ 2^a3^b5^d4^c}{a!b!c!d}$ where d is even

(D) $\left(2+3+4+5\right)^{10} - (3+4+5)^{10} = \sum \frac{2^a3^b4^c5^d}{a!b!c!d} \times 10!$where a is natural

56. **(337.00)**

x = $\left(\sqrt{3}+1\right)^{2018}$, $N = [x]+1$

Let y = $\left(\sqrt{3}-1\right)^{2018}$

$x + y = [x] + f + y = \left(\sqrt{3}+1\right)^{2018} + \left(\sqrt{3}-1\right)^{2018} = 2^{1009}\left[\left(2+\sqrt{3}\right)^{1009} + \left(2-\sqrt{3}\right)^{1009}\right]$

$[x] + 1 = 2^{1009}.2\ [{}^{1009}C_0 \cdot 2^{1009} + {}^{1009}C_2 \cdot 2^{1007} \cdot 3 + ...+......+ {}^{1009}C_{1008} \cdot 2^1\ 3^{504}]$

$[x] + 1 = 2^{1009} .2^1. 2^1\ [{}^{1009}C_0.2^{1008} + {}^{1009}C_2.2^{1006} .3 + ...+....+ {}^{1009}C_{1006} \cdot 2^2 \cdot 3^{503} + {}^{1009}C_{1008}. 3^{504}]$

$N = 2^{1011}$. ODD

$N = (2^3)^{337}$. ODD

P = 337

57. **(21.00)**

$(7-1)^{2007} + (7+1)^{2007} = 49K + 2007 \times 7 \times 2 = 49\lambda + 21$

58. **(10.00)**

Coeff. of x^{10} in ${}^{10}C_0(1+x)^{20} - {}^{10}C_1(1+x)^{18} + {}^{10}C_2(1+x)^{16} - {}^{10}C_3(1+x)^{14} + ...+{}^{10}C_{10} (1+x)^0$

= coeff. of x^{10} in $((1+x)^2 - 1)^{10}$

= coeff. of x^{10} in $(2+x)^{10} .x^{10} = 2^{10}$

59. (21.00)

$7-2r=3 \Rightarrow r=2$

$\therefore$ The coefficient is ${}^7C_2=21$.

60. (1.00)

$$\frac{(18+7)^3}{(3+2)^6}=\frac{25^3}{5^6}=1$$