

PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DUBAI / DHULE

IIT – JEE: 2024

TW TEST (FLAGSHIP – 1 & 2)

DATE: 05/11/22

TOPIC: KINEMATICS – I & II

SOLUTION

1. (A)

Speed of car at $t = 3\text{ s}$ is

$$u_1 = at = 2 \times 3 = 6\text{ ms}^{-1}$$

= initial velocity of first coin.

$$\therefore \text{range of first coin, } R_1 = u_1 T_1 = 6 \times 1 = 6\text{ m}.$$

Speed of car at $t = 4\text{ s}$ is

$$u_2 = at = 2 \times 4 = 8\text{ ms}^{-1}$$

= initial velocity of second coin.

$$\therefore \text{range of second coin,}$$

$$R_2 = u_2 T_2 = 8 \times 1 = 8\text{ m}.$$

Distance between two position of the car where the coins are released is

$$s = u_1 \times 1 + \frac{1}{2} a \times 1^2$$

$$= 6 \times 1 + \frac{1}{2} \times 2 \times 1 = 7\text{ m}$$

$$\therefore \text{required answer is } = 7 + 8 - 6 = 9$$

2. (A)

$$\text{Time of flight of A is } T = \sqrt{\frac{2h}{g}}$$

$$\text{For collision: } x_A + x_B = d$$

$$\Rightarrow uT + vT = d$$

$$\Rightarrow v = \frac{d}{T} - u = d\sqrt{\frac{g}{2h}} - u$$

3. (B)

$$v_x = \frac{dx}{dt} = a; \quad v_y = \frac{dy}{dt} = a - 2abt$$

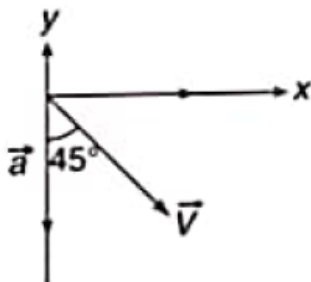
$$a_x = \frac{dv_x}{dt} = 0; \quad a_y = \frac{dv_y}{dt} = -2ab$$

$$\vec{v} = a\hat{i} + (a - 2abt)\hat{j} = a\hat{i} - (2abt - a)\hat{j}$$

$$\vec{a} = 0\hat{i} - 2a\hat{j}$$

Angle between \vec{a} and \vec{v} is $\frac{\pi}{4}$.

It means \vec{v} is directed as shown. Note that x component of \vec{v} is positive



$$\left| \frac{V_y}{V_x} \right| = \tan 45^\circ = 1$$

$$\frac{2abt - a}{a} = 1 \Rightarrow t = \frac{1}{b}.$$

4. (C)

$$u_x = 10 \cos 45^\circ = 5\sqrt{2} \text{ ms}^{-1}$$

$$a_x = 0$$

$$u_y = -10 \sin 45^\circ = -5\sqrt{2} \text{ ms}^{-1};$$

$$a_y = g \sin 45^\circ = \frac{g}{\sqrt{2}} = 5\sqrt{2} \text{ ms}^{-2}$$

$$\text{At } t = 2\text{s}$$

$$v_x = u_x = 5\sqrt{2} \text{ ms}^{-1}.$$

$$v_y = u_y + a_y t = -5\sqrt{2} + 5\sqrt{2} \times 2 = 5\sqrt{2}$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{(5\sqrt{2})^2 + (5\sqrt{2})^2} = 10 \text{ ms}^{-1}$$

5. (A)

Initially, at origin $v = v_0 \hat{j}$

As the y coordinate increases, y component of velocity remains constant and x component increases in negative x direction.

\therefore Correct path is given in (a).

6. (A)

Range $\geq X$

$$\Rightarrow \frac{v_0^2 2 \sin \theta \cos \theta}{g} \geq X$$

$$[\tan \theta = \frac{H}{X}]$$

$$v_0^2 \frac{2}{g} \cdot \frac{H}{\sqrt{H^2 + X^2}} \cdot \frac{X}{\sqrt{H^2 + X^2}} \geq X$$

$$\Rightarrow v_0 \geq \sqrt{\frac{g(H^2 + X^2)}{2H}}$$

7. (C)

$$u_x = u_y = u$$

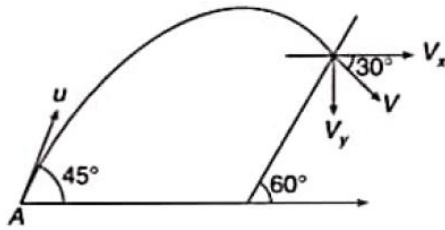
At the time of the hit

$$\tan 30^\circ = \frac{|V_y|}{v_x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{|V_y|}{u}$$

$$\Rightarrow v_y = \frac{u}{\sqrt{3}} (\downarrow)$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{u^2}{3}} = \frac{2}{\sqrt{3}} u.$$



8. (C)

As long as the two stones are in air, separation between them increases at a constant rate equal to their relative velocity. After one of the stones hits the ground, the separation between them changes with the rate equal to speed of the other stone.

As the second stone is also about to land on the ground, its speed is highest (i.e., slope of graph is highest). This is represented in (c).

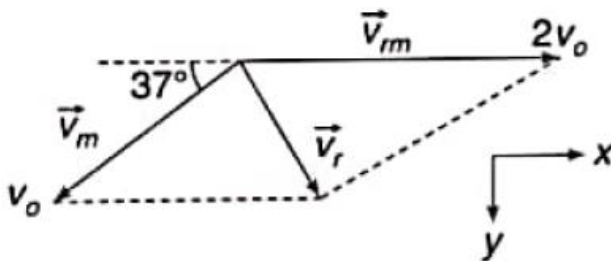
9. (A)

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

$$\Rightarrow \vec{v}_r = \vec{v}_{rm} + \vec{v}_m$$

$$= 2v_0 \hat{i} + (-v_0 \cos 37^\circ \hat{i} + v_0 \sin 37^\circ \hat{j})$$

$$= \frac{6}{5} v_0 \hat{i} + \frac{3}{5} v_0 \hat{j}$$



After man increases his velocity to $2v_0$

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

$$\begin{aligned}
 &= \left(\frac{6}{5} v_0 \hat{i} + \frac{3}{5} v_0 \hat{j} \right) - \left(-2v_0 \cos 37^\circ \hat{i} + 2v_0 \sin 37^\circ \hat{j} \right) \\
 &= \frac{14}{5} v_0 \hat{i} - \frac{3}{5} v_0 \hat{j} \\
 \therefore \quad \vec{v}_m &= \frac{v_0}{5} \sqrt{14^2 + 3^2} = \frac{v_0}{5} \sqrt{205} = \sqrt{\frac{41}{5}} v_0
 \end{aligned}$$

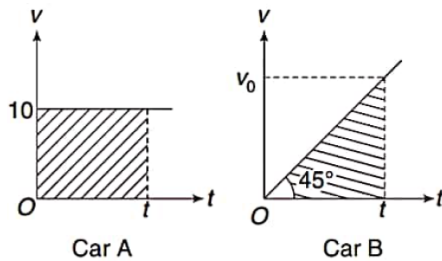
10. (A)

Displacement = Area under $v-t$ graph. Car B catches A if

$$x_B = x_A + 10.5$$

$$\Rightarrow \frac{1}{2} t(v_0) = 10t + 10.5$$

$$\Rightarrow \frac{1}{2} t(t) = 10t + 10.5 \quad [\because \tan 45^\circ = \frac{v_0}{t}]$$



$$\Rightarrow t^2 - 20t - 21 = 0$$

Solving $t = 21$ s.

11. (A)

$$v \frac{dv}{dx} = a$$

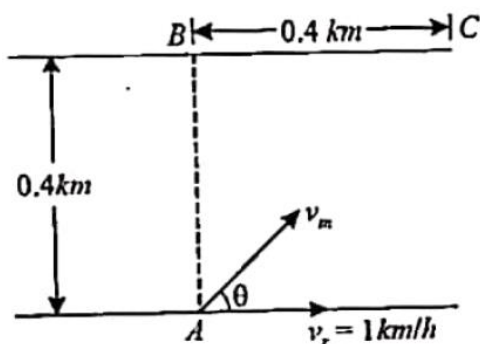
$$\Rightarrow \int_u^v v dv = \int_0^x a dx$$

$$\Rightarrow \frac{v^2}{2} - \frac{u^2}{2} = \text{Area under } a-x \text{ graph}$$

$$\Rightarrow \frac{v^2}{2} - 0 = \frac{1}{2} \times 1 \times 1$$

$$\Rightarrow v = 1 \text{ ms}^{-1}$$

12. (C)



$$v_m \sin \theta = \frac{0.4}{t}$$

$$v_m \cos \theta + v_r = \frac{0.4}{t}$$

From (1) and (2),

$$v_m \sin \theta = v_m \cos \theta + v_r$$

$$5 \sin \theta = 5 \cos \theta + 1$$

$$5\sqrt{1 - \cos^2 \theta} = 5 \cos \theta + 1$$

$$25(2 - \cos^2 \theta) = 25 \cos^2 \theta + 1 + 10 \cos \theta$$

$$50 \cos^2 \theta + 10 \cos \theta - 24$$

$$\cos \theta = \frac{3}{5}, -\frac{4}{5}$$

$$\theta = 53^\circ$$

13. (B)

From figure

$$\tan \theta = \frac{6t}{2t} = 3$$

$$x = \frac{vt}{3\sqrt{2}}$$

And $y = 2t$

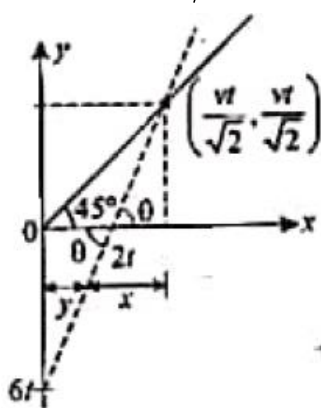
$$x + y = \frac{vt}{\sqrt{2}}$$

$$\frac{vt}{3\sqrt{2}} + 2t = \frac{vt}{\sqrt{2}}$$

$$v \left(\frac{1}{\sqrt{2}} - \frac{1}{3\sqrt{2}} \right) = 2$$

$$v \left(\frac{2}{3\sqrt{2}} \right) = 2$$

$$v = 3\sqrt{2} \text{ m/s}$$



14. (C)

Let speed of bullet is v

Velocity of bullet relative to car along x-axis

$$= (v \cos \theta - 13)$$

Along y-axis $= v \sin \theta$

$$(v \cos \theta - 13) = \frac{2}{t} \quad \dots\dots\dots (1)$$

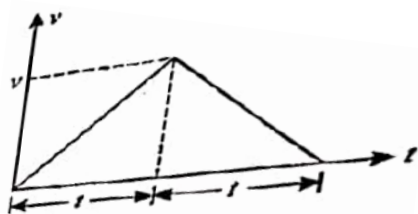
$$\Rightarrow v \sin \theta = \frac{3}{t} \quad \dots\dots\dots (2)$$

From (1) and (2),

$$t = \frac{1}{13} \left[\frac{3}{\tan \theta} - 2 \right]$$

$$\Rightarrow t = \frac{1}{13} (2) = 0.15s$$

15. (D)



$$s = \frac{1}{2} (2t) v = vt$$

$$s = at^2$$

$$t = \sqrt{\frac{s}{a}}$$

Thus, time to go to another station,

$$2t = 2\sqrt{\frac{s}{a}}$$

16. (B)

Object accelerated so the speed increases linearly with time. But the distance fallen increases as t^2 . So the average speed occurs at half of time taken to pass the window, which is before it has covered half of the height of the window.

17. (A)

$$a = -\frac{dv}{dt}$$

$$kv^2 = -\frac{dv}{dt}, \int_0^t k dt = \int_{v_0}^y -\frac{dv}{v^2}$$

$$kt + \frac{1}{v_0} = \frac{1}{v}$$

$$v = \frac{v_0}{1 + kv_0 t}$$

18. (D)

We have,

$$a = 32 - 4v$$

$$\frac{dv}{dt} = 32 - 4v$$

$$\frac{dv}{32 - 4v} = dt$$

Integrating both sides, we get

$$\int_4^v \frac{dv}{32 - 4v} = \int_0^{\ln 2} dt$$

$$-\frac{1}{4} [\ln(32 - 4v)]_4^v = \ln 2 - 0$$

$$\ln\left(\frac{32 - 4v}{16}\right) = \ln\left(\frac{1}{16}\right)$$

$$32 - 4v = 1$$

$$4v = 31$$

$$v = \frac{31}{4}$$

19. (B)

As both particles have same acceleration, their relative acceleration is zero hence the path of motion of one with respect to other will be a straight line and the line is vertical because their relative velocity along horizontal is zero.

20. (B)

With respect to wedge, the particle moves at 30° from the plane of the incline at speed of $10\sqrt{3}\text{m/s}$.

Along normal to the inclines $S = ut + \frac{1}{2}at^2$

$$0 = 10\sqrt{3} \sin 30^\circ t - \frac{1}{2}g \cos 30^\circ t^2$$

$$\Rightarrow 0 = 5\sqrt{3} - \frac{5\sqrt{3}}{2}t$$

$$\Rightarrow t = 2\text{s}$$

SOLUTION

21. (B)

Vander Waals equation

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT \text{ for } n \text{ mols}$$

$$\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT \text{ for } 1 \text{ mol}$$

$$\Rightarrow P = \frac{RT}{V_m - b} - \frac{a}{V_m^2}$$

Multiplying by $\frac{V_m}{RT}$

$$\frac{PV_m}{RT} = \frac{V_m}{V_m - b} - \frac{a}{V_m RT} = Z$$

Equation (B) is not the same expression (In fact, it is the Berthelot's equation of state).

22. (A)

$$z = 1 + \frac{Pb}{RT} \text{ (high pressure)} \quad y = c + mx$$

23. (B)

$$\text{rate of effusion} = \frac{PA}{\sqrt{2\pi RTM}}$$

24. (B)

$$\begin{aligned} \frac{r_{O_2}}{r_{CH_4}} &= \frac{n_{O_2}}{n_{CH_4}} \times \sqrt{\frac{M_{CH_4}}{M_{O_2}}} \\ &= \frac{3}{2} \times \frac{16}{32} \times \sqrt{\frac{16}{32}} = \frac{3}{4\sqrt{2}} \end{aligned}$$

25. (C)

$$Z = \frac{PV}{nRT}$$

$$0.9 = \frac{9 \times V}{1 \times 0.082 \times 273} \Rightarrow V = \frac{0.082 \times 273 \times 0.9}{9} = 2.24 \text{ L / mole}$$

∴ For 1 millimole, the volume is 2.24 mL

26. (B)

$$v_1 = \sqrt{\frac{3RT}{M}}$$

$$T_2 = 2T, M_2 = M/2$$

$$v_2 = \sqrt{\frac{3R \times 2T}{M/2}} = 2v_1$$

27. (A)

$$\frac{r_{N_2}}{r_{XeF_n}} = \frac{P_{N_2}}{P_{XeF_n}} \times \sqrt{\frac{M_{XeF_n}}{M_{N_2}}}$$

$$\frac{r_{N_2}}{r_{XeF_n}} = \frac{\frac{\text{Molar volume}}{t_{F_2}}}{\frac{\text{Molar volume}}{t_{XeF_n}}} = \frac{t_{XeF_n}}{t_{N_2}} = \frac{57}{38}$$

$$\therefore \frac{57}{38} = \frac{0.8}{1.6} \sqrt{\frac{M_{XeF_n}}{28}}$$

$$M_{XeF_n} = \left(\frac{57}{38}\right)^2 \times \left(\frac{1.6}{0.8}\right)^2 \times 28 = 252$$

$$Xe + nF = 252$$

$$131 + n \times 19 = 252 =$$

$$n = 6$$

Molecular formula = XeF₆

28. (D)

$$v = \sqrt{\frac{8RT}{\pi M}}$$

$$V_{H_2} = \sqrt{\frac{8RT_1}{\pi M_1}}; V_{C_2H_6} = \sqrt{\frac{8RT_2}{\pi M_2}}$$

$$\frac{V_{H_2}}{V_{C_2H_6}} = \sqrt{\frac{T_1 M_2}{T_2 M_1}} = \sqrt{\frac{300 \times 30}{900 \times 2}} = \sqrt{\frac{5}{1}} = 2.237 : 1$$

i.e., H₂ molecules in A will move 2.24 times faster than C₂H₆ molecules in B.

29. (D)

$$\text{Slope} = \frac{1}{nRT}$$

$$\therefore \frac{n_A}{n_B} = \frac{\tan 60}{\tan 45} = \frac{\sqrt{3}}{1}$$

30. (A)

Initial temperature TK & pressure is 830 mm Hg

Pdry gas = total pressure – vapour pressure of H₂O

$$= 830 - 30$$

$$= 800 \text{ mm Hg}$$

$$\text{Temperature} = T - \frac{T}{100} = 0.99T$$

At T temperature, pressure of dry gas 800 mm Hg

At 0.99 T temperature, pressure is = 800 × 0.99

$$= 792 \text{ mm Hg of dry gs}$$

Total pressure = Pdry gas + vapour pressure of H₂O

$$= 792 + 25 = 817 \text{ mm of Hg}$$

31. (B)

Initial volume of mixture = 100ml containing 72 ml CH₄ and 28 ml gas x. Thus $\frac{P_{\text{CH}_4}}{P_x} = \frac{72}{28}$

Also due to crack, total volume left = 100 – 21 = 79ml

$$\therefore V_{\text{CH}_4} \text{ left after diffusion} = \frac{68.35 \times 79}{100} = 54.0 \text{ ml}$$

$$V_{\text{CH}_4} \text{ diffused} = 72 - 54 = 18 \text{ ml}$$

$$V_x \text{ diffused} = 21 - 18 = 3 \text{ ml}$$

$$\frac{V_{\text{CH}_4}}{V_x} = \sqrt{\frac{M_x}{M_{\text{CH}_4}}} \times \frac{72}{28}$$

$$\frac{18}{3} = \sqrt{\frac{M_x}{16}} \times \frac{72}{28}$$

$$M_x = 87.1$$

32. (C)

$$\frac{P_C V_C}{RT_C} \times \frac{P_r V_r}{T_r} = \frac{3}{8} \times 2.21 \quad \text{or} \quad V_m = \left(\frac{3 \times 2.2}{8} \right) \frac{RT}{P}$$

$$V_m = 136.0 \text{ cm}^3$$

$$\text{Volume of 2 mol of N}_2 = 272.0 \text{ cm}^3$$

33. (B)

$$Z = \frac{PV}{nRT} \quad d = \frac{PM}{RT}$$

$$= \frac{PM}{dRT} = \frac{1 \times 27}{0.3 \times 0.082 \times 750} = 1.46$$

$\therefore Z > 1$ forces are repulsive.

34. (A)

$$\left(P + \frac{a}{V^2}\right)V = RT$$

$$\frac{PV}{RT} = 1 - \frac{a}{RTV}$$

35. (B)

$$\frac{PV}{nRT} = 1 + \beta P$$

$$\Rightarrow \frac{P \times M_0}{w/v RT} = 1 + \beta P \Rightarrow \frac{P}{d} \times \frac{M_0}{RT} = 1 + \beta P$$

$$\frac{0.25}{0.2} \times \frac{M_0}{RT} = 1 + \beta \times 0.25 \quad \dots(i)$$

$$\frac{0.50}{0.40} \times \frac{M_0}{RT} = 1 + \beta \times 0.5 \quad \dots(ii)$$

$$(i) \times (ii) - (ii) \Rightarrow \frac{0.50}{0.2} \times \frac{M_0}{RT} - \frac{0.50}{0.40} \frac{M_0}{RT} = 1$$

$$\Rightarrow \frac{0.50 M_0}{RT} \left[\frac{0.4 - 0.2}{0.2 \times 0.4} \right] = 1$$

$$M_0 = \frac{0.2 \times 0.4 \times 0.0821 \times 300}{0.2 \times 0.5} \Rightarrow 19.7$$

36. (C)

Conceptual

37. (A)

Lesser is the value of Z at low pressure, easy will be liquefaction of the gas. $Z < 1$, shows the domination of attractive forces.

38. (B)

Suppose initially, pressure of $C_6H_6(g) = p_1$ mm and that of $H_2(g) = p_2$ mm

$$\therefore p_1 + p_2 = 80 \text{ mm} \quad \dots(i)$$

After the reaction,

pressure of $C_6H_6(g) = 0$ (as all has reacted)

pressure of $H_2(g) = p_2 - 3p_1$

pressure of $C_6H_{12}(g) = p_1$

Total pressure = $p_2 - 3p_1 + p_1 = 40$ mm

$$\text{or } p_2 - 2p_1 = 40 \text{ mm} \quad \dots(ii)$$

Solving (i) and (ii), we get & applying

Fraction of C_6H_6 by volume = fraction of moles

$$\text{Fraction of pressure} = \frac{40}{3 \times 80} = \frac{1}{6}.$$

39. (D)

At a particular temperature the fraction of molecules having a particular speed remains constant.

40. (A)

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}} \Rightarrow 1.33 = \sqrt{\frac{32}{M_1}} \quad \therefore \quad M_1 = 18$$

$$V_m = \frac{M}{d} = 50 \text{ L}$$

$$Z = \frac{PV_m}{RT} = 1.22$$

Solution

41. (D)

$$T_p = \frac{1}{q} T_p = \frac{1}{q} T_p = \frac{1}{q} \quad T_q = \frac{1}{p} T_q = \frac{1}{p}$$

$$\Rightarrow a + (p-1)d = \frac{1}{q} \quad (i)$$

$$\Rightarrow a + (q-1)d = \frac{1}{p} \quad (ii)$$

by (i) – (ii)

$$(p-q)d = \frac{1}{q} - \frac{1}{p}$$

$$d = \frac{p-q}{pq \cdot p-q}$$

$$d = \frac{1}{pq} \quad (iii)$$

Putting the value of d in (i)

$$a = \frac{1}{pq} \quad (iv)$$

From (iii) & (iv)

$$a - d = \frac{1}{pq} - \frac{1}{pq} = 0$$

42. (A)

As A.M. \geq G.M.

$$\frac{1}{2}[(a+b) + (c+d)] \geq [(a+b)(c+d)]^{1/2}$$

$$\Rightarrow \frac{1}{2}(2) \geq (M)^{1/2} \Rightarrow M \leq 1$$

Also $(a+b)(c+d) > 0$

$$\therefore 0 < M \leq 1$$

43. (B)

$$S_{2n} = \frac{a(1-r^{2n})}{1-r}$$

$$t_1 + t_3 + t_5 + \dots + n^{\text{th}} \text{ term} = \frac{a(1-(r^2)^n)}{1-r^2}$$

$$\begin{aligned}\therefore \frac{a(1-r^{2n})}{1-r} &= 10 \frac{a(1-r^{2n})}{1-r^2} \\ \Rightarrow 1 &= \frac{10}{1+r} \\ \Rightarrow 1+r &= 10 \\ r &= 9\end{aligned}$$

44. (B)

Let the two numbers be a and b .

$$A = \frac{a+b}{2}$$

Also, a, p, q, b are in G.P.

$$\begin{aligned}\Rightarrow \frac{p}{a} &= \frac{q}{p} = \frac{b}{q} \Rightarrow p^2 = aq \Rightarrow \frac{p^2}{q} + \frac{q^2}{p} = a+b \\ \Rightarrow \frac{p^2}{q} + \frac{q^2}{p} &= a+b = 2A\end{aligned}$$

45. (D)

$\therefore p, q, r$ are in A.P.

$$\therefore 2q = p + r \quad (i)$$

Also $p+1, q, r$ are in G.P.

$$\therefore q^2 = (p+1)r \quad (ii)$$

Also p, q and $r+2$ are in G.P.

$$\therefore q^2 = p(r+2) \quad (iii)$$

From (ii) and (iii)

$$pr + r = pr + 2p$$

$$\Rightarrow r = 2p \quad (iv)$$

By putting this in (i):

$$2q = p + 2p$$

$$= 3p$$

$$\therefore q = \frac{3}{2}p \quad (v)$$

By putting values from (v) and (iv) in (ii),

$$\left(\frac{3}{2}p\right)^2 = (p+1)2p$$

$$\Rightarrow \frac{9}{4}p^2 = 2p^2 + 2p$$

$$\Rightarrow 9p^2 = 8p^2 + 8p$$

$$\Rightarrow p^2 = 8p$$

$$\Rightarrow p = 8 \quad \because p \neq 0$$

$$\therefore q = \frac{3}{2} \times 8 = 12$$

46. (D)

$$\begin{aligned}\sum_{K=1}^n K(K+4) &= \sum_{K=1}^n K^2 + 4 \sum_{K=1}^n K \\ &= \frac{n(n+1)(2n+1)}{6} + \frac{4(n)(n+1)}{2}\end{aligned}$$

$$\begin{aligned}
&= \frac{n(n+1)(2n+1) + 12(n)(n+1)}{6} \\
&= \frac{n(n+1)(2n+1+12)}{6} \\
&= \frac{n(n+1)(2n+13)}{6}
\end{aligned}$$

47. (B)

Let three numbers be $\frac{a}{r}, a, ar$

Then according to the question

$$\frac{a}{r} \times a \times ar = 3375$$

$$a^3 = 3375$$

$$a = 15 \quad (i)$$

$$\frac{a}{r} + a + ar = 65$$

$$a \left(\frac{1+r+r^2}{r} \right) = 65$$

$$\frac{r^2 + r + 1}{r} = \frac{65}{15} = \frac{13}{3}$$

$$\Rightarrow 3r^2 + 3r + 3 = 13r$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$\Rightarrow 3r^2 - 9r - r + 3 = 0$$

$$\Rightarrow (3r-1)(r-3) = 0$$

$$r = \frac{1}{3} \text{ or } 3$$

48. (A)

$$S_n = \frac{1}{6} n(2n^2 + 9n + 13)$$

$$\frac{1}{6} n(n+1)(2n+1) + 2 \left(\frac{n(n+1)}{2} \right) + n$$

$$\therefore T_n = n^2 + 2n + 1$$

$$T_n = (n+1)^2$$

$$\sum_{r=1}^{\infty} \frac{1}{V(T_r)^{1/2}} = \sum_{r=1}^{\infty} \frac{1}{r(r+1)} = \sum_{r=1}^{\infty} \frac{1}{r} - \frac{1}{r+1} = \frac{1}{1}$$

49. (A)

$$x = \frac{2ab}{a+b}$$

y and z can be assumed as the two geometric means of a and b

$$\Rightarrow y = a^{\frac{2}{3}} b^{\frac{1}{3}}; z = a^{\frac{1}{3}} b^{\frac{2}{3}} \Rightarrow yz = ab \quad \text{and} \quad y^3 + z^3 = a^2 b + ab^2 = ab(a+b)$$

$$\Rightarrow \frac{yz}{x(y^3 + z^3)} = \frac{ab}{\frac{2ab}{a+b} \cdot ab(a+b)} = \frac{1}{2ab}$$

50. (B)

$$\begin{aligned} & 1 + \frac{3}{2} + \frac{5}{3} + \dots + \left(\frac{2n-1}{n} \right) \\ &= (2-1) + \left(2 - \frac{1}{2} \right) + \left(2 - \frac{1}{3} \right) + \dots + \left(2 - \frac{1}{n} \right) \\ &= 2n - \left(1 + \frac{1}{2} + \dots + \frac{1}{n} \right) = 2n - H_n \end{aligned}$$

51. (D)

$$\begin{aligned} & \text{nth term} = (n-1)^{\text{th}} \text{ term} + (n-2)^{\text{th}} \text{ term} \\ & \Rightarrow ar^{n-1} = ar^{n-2} + ar^{n-3} \\ & \Rightarrow r^{n-1} = r^{n-2} + r^{n-3} \\ & \Rightarrow \frac{r^n}{r} = \frac{r^n}{r^2} + \frac{r^n}{r^3} \\ & \Rightarrow \frac{1}{r} = \frac{1}{r^2} + \frac{1}{r^3} \\ & \Rightarrow 1 = \frac{1}{r} + \frac{1}{r^2} \\ & \Rightarrow r^2 = r + 1 \\ & \Rightarrow r^2 - r - 1 = 0 \\ & r = \frac{-(-1) \pm \sqrt{(-1)^2 + 4}}{2} \\ & = \frac{1 \pm \sqrt{5}}{2} \\ & \because r > 0, \therefore r = \frac{1 + \sqrt{5}}{2} \end{aligned}$$

52. (C)

In an A.P.

$$\begin{aligned} & a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = a_4 + a_{n-3} \dots \\ & \therefore a_1 + a_{28} = a_6 + a_{23} \quad \text{(i)} \\ & \therefore a_1 + a_6 + a_{23} + a_{28} = 460 \\ & \therefore (a_1 + a_{28}) + (a_6 + a_{23}) = 460 \\ & \Rightarrow 2(a_1 + a_{28}) = 460 \text{ from (i)} \\ & a_1 + a_{28} = 230 \quad \text{(ii)} \\ & \therefore S_{28} = \frac{28}{2}(a_1 + a_{28}) \text{ from (ii)} \\ & = 14 \times 230 = 3220 \end{aligned}$$

53. (C)

$$\begin{aligned} & n^{\text{th}} \text{ term from the beginning} + n^{\text{th}} \text{ term from end} \\ &= [a + (n-1)d] + [l - (n-1)d] \\ &= a + l \end{aligned}$$

54. (C)
 $S_3 = 24$
 $\Rightarrow \frac{a(r^3 - 1)}{r - 1} = 24 \quad (i)$

Also $S_6 - S_3 = 192$

$\Rightarrow S_6 = 192 + S_3$

$\Rightarrow \frac{a(r^6 - 1)}{r - 1} = 192 + 24 = 216 \quad (ii)$

Dividing (ii) by (i)

$$\frac{r^6 - 1}{r^3 - 1} = \frac{216}{24}$$

$$\Rightarrow \frac{(r^3)^2 - (1)^2}{r^3 - 1} = 9$$

$$\Rightarrow \frac{(r^3 - 1)(r^3 + 1)}{r^3 - 1} = 9$$

$$r^3 + 1 = 9$$

$$r^3 = 8$$

$$r = 2$$

Putting this in (i)

$$\frac{a(8 - 1)}{2 - 1} = 24 \Rightarrow a = \frac{24}{7}$$

$$\therefore S_n = \frac{24}{7} \frac{2^n - 1}{2 - 1} = \frac{24}{7} (2^n - 1)$$

55. (D)
 $\sum_{k=1}^n (4 + 7^k) = (4 + 7^1) + (4 + 7^2) + (4 + 7^3) + \dots + (4 + 7^n)$
 $= 4 \times n + (7^1 + 7^2 + 7^3 + \dots + 7^n)$
 $= 4n + \frac{7(7^n - 1)}{7 - 1} \left(\text{using } S_n : \frac{a(r^n - 1)}{r - 1} \right) \text{ for } r > 1$
 $= 4n + \frac{7}{6} (7^n - 1)$

56. (C)
 $a_m = a + (m - 1)d$
 $a_n = a + (n - 1)d$
 A.M. between m^{th} and n^{th} term
 $= A.M._1 = \frac{a + (m - 1)d + a + (n - 1)d}{2}$
 $= \frac{2a + (m + n - 2)d}{2}$
 $a_p = a + (p - 1)d$
 $a_q = a + (q - 1)d$
 A.M. between p^{th} and q^{th} term
 $= A.M._2 = \frac{2a + (p + q - 2)d}{2}$

$$\begin{aligned}
&\therefore A.M_1 = A.M_2 \\
&\therefore \frac{2a + (m+n-2)d}{2} = \frac{2a + (p+q-2)d}{2} \\
&\Rightarrow 2a + (m+n-2)d = 2a + (p+q-2)d \\
&\Rightarrow (m+n-2)d = (p+q-2)d \\
&\Rightarrow m+n-2 = p+q-2 \\
&\Rightarrow m+n = p+q
\end{aligned}$$

57. (A)

$$\therefore d = 3$$

$$\therefore a_9 = a + 8d = a + 8 \times 3 = a + 24 \quad (1)$$

$$\text{Middle term of A.P.} = a_5 = a + 4d = a + 4 \times 3$$

$$= a + 12 \quad (2)$$

Last 9 terms are in G.P. with $r = 2$ and first term $= b = a_9 = a + 24$

Using (1)

$$\text{Middle term of G.P. } t_5 = br^4 = b(2)^4$$

$$= (a+24)16$$

Now, according to the question,

$$a+12=16(a+24)$$

$$\Rightarrow a + 12 = 16a + 384$$

$$\Rightarrow 12 - 384 = 15a$$

$$\Rightarrow \frac{-372}{15} = a$$

$$\Rightarrow a = \frac{-124}{5}$$

$$\therefore b = a_9 = \frac{-124}{5} + 24 = \frac{-124 + 120}{5} = \frac{-4}{5}$$

58. (B)

$0.6 + 0.66 + 0.666 \dots$ upto 30 terms

$$= 6 \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \dots \text{upto 30 terms} \right)$$

$$= \frac{6}{9} \left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots \text{upto 30 terms} \right)$$

$$= \frac{2}{3} \left(\left(1 - \frac{1}{10} \right) + \left(1 - \frac{1}{10^2} \right) + \left(1 - \frac{1}{10^3} \right) + \dots \text{upto 30 terms} \right)$$

$$= \frac{2}{3} \left(1 \times 30 - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} \dots \text{upto 30 terms} \right) \right)$$

$$= \frac{2}{3} \left[30 - \frac{1 \left(1 - \left(\frac{1}{10} \right)^{30} \right)}{1 - \frac{1}{10}} \right]$$

$$= \frac{2}{3} \left[30 - \frac{\frac{1}{10} \left(1 - \left(\frac{1}{10} \right)^{30} \right)}{\frac{9}{10}} \right]$$

$$\begin{aligned}
&= \frac{2}{3} \left[30 - \frac{1}{9} \left(1 - \left(\frac{1}{10} \right)^{30} \right) \right] \\
&= \frac{2}{3} \left[30 - \frac{1}{9} + \frac{1}{9} \left(\frac{1}{10} \right)^{30} \right] \\
&= \frac{2}{3} \left[\frac{269}{9} + \frac{1}{9} \left(\frac{1}{10} \right)^{30} \right] \\
&= \frac{2}{27} 269 + 10^{-30}
\end{aligned}$$

59. (A)

Let $T_n = an^2 + bn + c$

Taking $n = 1, 2, 3$, we get

$$a + b + c = 2 \quad (i)$$

$$4a + 2b + c = 3 \quad (ii)$$

$$9a + 3b + c = 6 \quad (iii)$$

by (ii) – (i) we get, $3a + b = 1$

$$\Rightarrow 9a + 3b = 3 \quad (iv)$$

by (iii) – (iv) we get $c = 3$

putting this in (i) we get,

$$a + b = 2 - 3$$

$$\Rightarrow a + b = -1$$

$$\Rightarrow 3a + 3b = -3 \quad (v)$$

by (iv) – (v)

$$6a = 6 \Rightarrow a = 1$$

by putting value of a and c in equation (1)

$$a + b + c = 2 \Rightarrow 1 + b + 3 = 2$$

$$b = 2 - 4 = -2$$

$$\text{i.e. } a = 1, b = -2, c = 3$$

$$\therefore T_n = n^2 - 2n + 3$$

$$\therefore S_n = \sum_{K=1}^n (K^2 - 2K + 3)$$

$$= \sum_{K=1}^n K^2 - 2 \sum_{K=1}^n K + 3n$$

$$= \frac{n(n+1)(2n+1)}{6} - \frac{2n(n+1)}{2} + 3n$$

$$= n(n+1) \left(\frac{2n+1-6}{6} \right) + 3n$$

$$= \frac{n(n+1)(2n-5)}{6} + 3n$$

$$= \frac{n(2n^2 - 3n - 5) + 18n}{6}$$

$$= \frac{2n^3 - 3n^2 - 5n + 18n}{6} = \frac{2n^3 - 3n^2 + 13n}{6}$$

$$= \frac{n(2n^2 - 3n + 13)}{6}$$

60. (B)

$$\underbrace{666\dots6}_{n \text{ digits}} = 6 + 6(10) + 6(10)^2 + \dots + 6(10)^{n-1} = \frac{6[(10)^n - 1]}{10 - 1} = \frac{2}{3}(10^n - 1)$$

$$\text{Similarly, } \underbrace{888\dots8}_{n \text{ digits}} = \frac{8}{9}(10^n - 1)$$

$$\begin{aligned} \therefore \quad & \underbrace{(666\dots6)^2}_{n \text{ digits}} + \underbrace{(888\dots8)}_{n \text{ digits}} = \frac{4}{9}(10^n - 1)^2 + \frac{8}{9}(10^n - 1) \\ & = \frac{4}{9}[10^{2n} - 2(10^n) + 1 + 2(10^n) - 2] = \frac{4}{9}[10^{2n} - 1] \end{aligned}$$