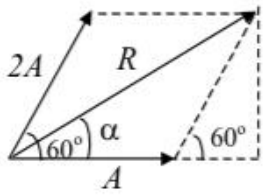
SOLUTION

1. (C)



From the figure,

$$\tan\alpha = \frac{2A\sin 60^\circ}{A + 2A\cos 60^\circ} = \frac{\sqrt{3}}{2}$$

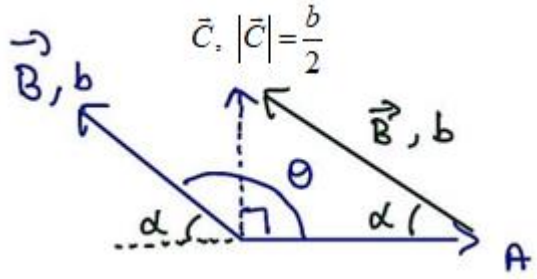
2. (C)

Resultant force, $\vec{F} = 15\hat{i} + 0\hat{j} + 25\hat{k}$

Clearly, the particle shall move in *x*-*z* plane.

3. (B)

$\vec{C} = \vec{A} + \vec{B},\ |\vec{C}| = \frac{b}{2}$



$$\sin\alpha = \frac{|\vec{C}|}{|\vec{B}|} = \frac{b/2}{b} = \frac{1}{2}$$

$\Rightarrow \quad \alpha = 30^\circ$

$\Rightarrow \quad \theta = 180 - \alpha = 150^\circ$

4. (D)

Displacement $= 4\hat{k}$

Force $= -\hat{i} + 2\hat{j} + 3\hat{k}$

Since, work *W* is the scalar product of force and displacement.

$W = \left(-\hat{i} + 2\hat{j} + 3\hat{k}\right) \cdot 4\hat{k}$

W = 12 Joule.

Hence, (D) is correct.

5. (B)

$\vec{A} \times \vec{B}$ is a vector $\perp$ to both $\vec{A}$ and $\vec{B}$

Now, $\vec{A} \times \vec{B} = \left(\vec{i} - 2\hat{j} + \hat{k}\right) \times \left(3\vec{i} + \hat{j} - 2\hat{k}\right) = 3\vec{i} + 5\hat{j} + 7\hat{k}$

Now, $\hat{n} = \dfrac{\vec{A} \times \vec{B}}{\left|\vec{A} \times \vec{B}\right|}$

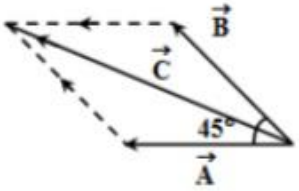
$= \dfrac{3\vec{i} + 5\hat{j} + 7\hat{k}}{\sqrt{3^2 + 5^2 + 7^2}} = \dfrac{3\vec{i} + 5\hat{j} + 7\hat{k}}{\sqrt{83}}$

Hence, (B) is correct.

6. (D)

$\vec{A}.\vec{B} = AB\cos\theta \qquad .....(1)$

$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta\hat{n} \qquad .....(2)$



Again given $\vec{C} = \vec{A} + \vec{B}$

$\therefore \quad \left|\vec{C}\right| = \left(A^2 + B^2 + 2AB\cos 45^\circ\right)^{1/2}$

$= \left(A^2 + B^2 + \sqrt{2}AB\right)^{1/2}$

Hence, (D) is correct.

7. (C)

$x = 4 + 4\cos 60^\circ = 6\,\text{m}, \quad y = 4\sin 60^\circ = 2\sqrt{3}\,\text{m}$

$r = \sqrt{x^2 + y^2} = \sqrt{36 + 12} = \sqrt{48}\,\text{m} = 6.9\,\text{m}$

8. (B)

$\vec{A} \perp^r \vec{B}$

$\vec{A}.\vec{B} = 0 = \left(5\hat{i} + 7\hat{j} + 3\hat{k}\right)\left(2\hat{i} + 2\hat{j} - a\hat{k}\right) = 10 + 14 - 3a$

$\therefore \quad 3a = 24 \Rightarrow a = 8$

Hence, (B) is correct.

9. (C)

P = 2 dyne, Q = 3 dune, R = 4 dyne, $\theta = ?$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$4^2 = 2^2 + 3^2 + 2 \times 2 \times 3 \times \cos\theta$

$\cos\theta = \dfrac{1}{4}$

$\theta = \cos^{-1}(0.25)$

10. (A)

$F\sqrt{10} = \sqrt{(2F)^2 + \left(F\sqrt{2}\right)^2 + 2(2F)F\sqrt{2}\cos\theta}$

On squaring both sides

$\Rightarrow \quad 10F^2 = 4F^2 + 2F^2 + 4F^2\sqrt{2}\cos\theta$

$4F^2 = 4F^2\sqrt{2}\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$

11. (C)

$\overrightarrow{AB} = \hat{i} + \hat{j} + \hat{k},\ \overrightarrow{CD} = -3\hat{i} - 3\hat{j} - 3\hat{k}$

$\Rightarrow \quad \overrightarrow{CD} = -3\overrightarrow{AB}$

i.e., $\overrightarrow{AB}$ is anti-parallal to $\overrightarrow{CD}$.

12. (B)

$\vec{A} = \hat{i} - 3\hat{j} + 2\hat{k},\ \vec{B} = 3\hat{i} + 3\hat{j} - 7\hat{k}$

$\vec{A} + \vec{B} + \vec{C} = \hat{j}$

$\Rightarrow \quad \vec{C} = \hat{j} - \left(\vec{A} + \vec{B}\right)$

$\vec{C} = \hat{j} - \left(4\hat{i} - 5\hat{k}\right)$

$\vec{C} = -4\hat{i} + \hat{j} + 5\hat{k}$

13. (D)

As $\vec{A}\cdot\vec{B} = 0$ and $\vec{A}\cdot\vec{C} = 0$

$\Rightarrow \quad \vec{A}$ is perpendicular to both $\vec{B}$ and $\vec{C}$.

Also, $\vec{B}\times\vec{C}$ gives a vector perpendicular to both $\vec{B}$ and $\vec{C}$.

$\Rightarrow \quad \vec{A}$ is parallel to $\vec{B}\times\vec{C}$.

14. (B)

$\vec{A} = 2\hat{i} + \hat{j},\ \vec{B} = 3\hat{j} - \hat{k},\ \vec{C} = 6\hat{i} - 2\hat{k}$

$\Rightarrow \quad \vec{A} - 2\vec{B} + 3\vec{C} = 20\hat{i} - 5\hat{j} - 4\hat{k}$

15. (A)

$\left|\vec{A} - \vec{B}\right| = \left|\vec{A}\right| = \left|\vec{B}\right|$

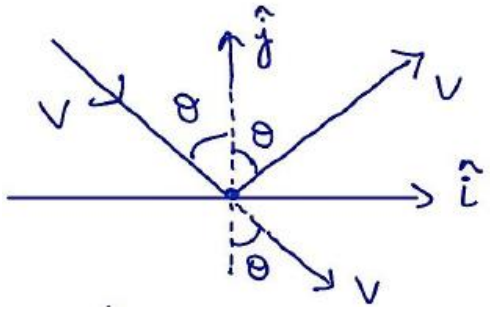
As $\left|\vec{A} - \vec{B}\right| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$A = \sqrt{A^2 + A^2 - 2(A)(A)\cos\theta}$

$A^2 = 2A^2 - 2A^2\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

16. (A)



$\vec{u} = v\sin\theta\hat{i} - v\cos\theta\hat{j}$

$\vec{v} = v\sin\theta\hat{i} + v\cos\theta\hat{j}$

$\Delta\vec{v} = \vec{v} - \vec{u} = 2v\cos\theta\hat{j}$

$|\Delta\vec{v}| = 2v\cos\theta$

17. (D)

Vector division is not allowed.

18. (A)

$\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$

$\Rightarrow \quad |\vec{F}| = \sqrt{(6)^2 + (-8)^2 + (10)^2} = 10\sqrt{2}\ \text{N}$

$a = 1\ \text{m/s}^2$

Mass $(m) = \dfrac{F}{a} = 10\sqrt{2}\ \text{kg}$

19. (A)

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

As $\theta_x = \theta_y = \theta_z = \theta$ Let.

$\Rightarrow \quad 3\cos^2\theta = 1 \quad \Rightarrow \quad \cos\theta = \pm\dfrac{1}{\sqrt{3}}$

* $A_x = A\cos\theta_x = \pm\dfrac{A}{\sqrt{3}}$

20. (A)

$\vec{A} = 2\hat{i} + 3\hat{j},\ \vec{B} = \hat{i} + \hat{j}$

Component of $\vec{A}$ along $\vec{B} = |\vec{A}|\cos\theta$

$= |\vec{A}|\dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

$= \dfrac{2+3}{\sqrt{1^2+1^2}} = \dfrac{5}{\sqrt{2}}$

SOLUTION

21. (C)

Wt. of 1 gm atom of nitrogen = 14 gm

Wt. of 11.2 L N_2 at 1 atm and 273 K $= 28 \times \frac{1}{2}$

$\Rightarrow$ 14 gm

22. (B)

$\frac{8 \times 10^{-18}}{3.2 \times 10^{5}} \times 6.02 \times 10^{23}$

23. (A)

(A) $\frac{4}{12} N_A$ (B) $\frac{12}{40} N_A$ (C) $\frac{6.35}{63.5} N_A$ (D) $\frac{22.4}{112} N_A$

24. (C)

$\frac{3.1 \times 10^{-3}}{62} \times 6.023 \times 10^{23} \times 32$

25. (C)

$\frac{0.9}{180} \times 12$

(A) $\frac{0.048}{32} \times 4$ (B) $\frac{0.17}{17} \times 3$ (C) $\frac{0.30}{30} \times 6$ (D) $\frac{0.03}{2} \times 2$

26. (B)

$\frac{M_A}{M_B} = 2$

$\Rightarrow M_A = 2M_B$ or $D_A = 2D_B$

27. (A)

$M_{avg} = \frac{x \cdot M_{NO_2} + (100 - x) M_{NO}}{100}$

$34 = \frac{x \times 46 + (100 - x) 30}{100}$

$x = 25\%$

28. (C)

$3A + 5B \rightarrow 7C + 9D$

$\frac{5}{3} \quad \frac{7}{5}$

(L.R.) $\frac{49}{5} \quad \frac{63}{5}$

29. (B)

$n_{HCl} = n_{NaOH} = \frac{7.3}{36.5}$

$\% \text{ NaCl} = \frac{20 - \left[\frac{7.3}{36.5} \times 40\right]}{20} \times 100$

30. (A)

$I_2 + 10HNO_3 \rightarrow 2HIO_3 + 10NO_2 + 4H_2O$

$\frac{5}{254} \times 10 \times 63 = 12.4 \text{ g}$

31. (D)

$\frac{2}{82} \times 2 \times 22.4$

32. (D)

$A_2SO_4 \cdot xH_2O \rightarrow A_2SO_4 + x\,H_2O$

$\frac{18x}{110 + 18x} \times 100 = 45 \Rightarrow x = 5$

33. (D)

$2X + 3Y \rightarrow 3Z$

$n = \quad 2 \quad 2$

(L.R.) $\frac{1.75}{2} \times 100 = 87.5\ \%$ yield

34. (B)

$\frac{21.2}{106} \times 100 \text{ kg}$

35. (C)

$$\left.\begin{aligned} n_C &= \frac{7.2}{12} = 0.6 \\ n_H &= \frac{0.6}{1} = 0.6 \\ n_O &= \frac{1.6}{16} = 0.1 \end{aligned}\right\} C_6H_6O$$

36. (C)

$$n_c = \frac{88}{44}; \quad n_h = \frac{36}{18} \times 2$$

$= 2$ $\quad\quad$ $= 4$

Only in (C) option C : H : : 1 : 2

37. (A)

$$\frac{10}{M} = \frac{11.21}{233.4} (\text{POAC on Ba})$$

$$M = 208.2\ (BaCl_2)$$

38. (C)

$$\frac{70}{36.5} \times 10$$

39. (D)

$$V \times 0.2 = 16 \times 0.5$$

$$V = 40$$

Vol. required = 40 – 16 = 24 mL

40. (C)

$$M = \frac{1}{30} \times 3$$

SOLUTION

41. (A)

$4n\alpha = \pi \Rightarrow 2n\alpha = \frac{\pi}{2}$

$\cot\alpha.\cot 2\alpha.\cot 3\alpha \ldots\ldots \cot(2n-1)\alpha$

$= \cot\alpha.\cot 2\alpha.\cot 3\alpha \ldots\ldots \cot\left(\frac{\pi}{2}-3\alpha\right).\cot\left(\frac{\pi}{2}-\alpha\right).\cot\left(\frac{\pi}{2}-\alpha\right)$

$= 1$

Trick : Put $n = 1 \Rightarrow$ only option (A) is possible.

42. (A)

$\tan 7.5^\circ = \sqrt{\frac{1-\cos 15^\circ}{1+\cos 15^\circ}}$

$= \sqrt{\frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}}$

$= \sqrt{\frac{2\sqrt{2}-\sqrt{3}-1}{2\sqrt{2}+\sqrt{3}+1}}$

$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}+1}$

43. (C)

Clearly, $\cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$

$= \frac{1-a}{1+a}$ is rational.

44. (A)

$2\tan 18^\circ + 3\sec 18^\circ - 4\cos 18^\circ$

$= \frac{2\sin 18^\circ + 3 - 4\cos^2 18^\circ}{\cos 18^\circ}$

$$= \frac{2\sin 18^o + 3 - 4 + 4\sin^2 18^o}{\cos 18^o}$$

$$= \frac{4\sin^2 18^o + 2\sin 18^o - 1}{\cos 18^o}$$

$$= \frac{4 \times \frac{\left(\sqrt{5}-1\right)^2}{16} + 2.\frac{\sqrt{5}-1}{4} - 1}{\cos 18^o}$$

$$= \frac{6 - 2\sqrt{5} + 2\sqrt{5} - 2 - 4}{4\cos 18^o} = 0$$

45. (C)

$$\tan^2\alpha - 2\tan^2\beta = 1$$

$$\Rightarrow \quad \tan^2\alpha = 1 + 2\tan^2\alpha$$

$$\Rightarrow \quad 1 + \tan^2\alpha = 2 + 2\tan^2\beta$$

$$\Rightarrow \quad \sec^2\alpha = 2\sec^2\beta$$

$$\Rightarrow \quad \cos^2\beta = 2\cos^2\alpha$$

$$\Rightarrow \quad 2\cos^2\beta = 4\sin^2\alpha$$

$$\Rightarrow \quad 1 + \cos 2\beta = 2\left(1 + \cos 2\alpha\right)$$

$$\Rightarrow \quad 2\cos 2\alpha - \cos 2\beta = -1$$

46. (B)

$$\tan 30^o = \frac{1}{\sqrt{3}}$$

47. (B)

$$2\sin 2^o + 4\sin 4^o + 6\sin 6^o + ... + 178\sin 178^o + 180\sin 180^o$$

$$= \left(2+178\right)\sin 2^o + \left(4+176\right)\sin 4^o + \left(6+174\right)\sin 6^o + ... + \sin 88^o$$

$$= 180^o\left(\sin 2^o + \sin 4^o + \sin 6^o + ...\right) + 90\sin 90^o$$

$$= 180 \times \frac{\sin\left(44 \cdot \frac{2}{2}\right)}{\sin\left(\frac{2}{2}\right)} \times \sin\left(2 + \left(44-1\right)\cdot\frac{2}{2}\right) + 90^o$$

$$= 180 \times \frac{\sin 44^o}{\sin 1^o} \times \sin 45^o + 90$$

$$= \frac{180}{\sqrt{2}} \times \frac{\sin\left(45^o - 1^o\right)}{\sin 1^o} + 90$$

$$= \frac{180}{\sqrt{2}}\left[\frac{\frac{1}{\sqrt{2}}\cos 1^o - \frac{1}{\sqrt{2}}\sin 1^o}{\sin 1^o}\right] + 90$$

$$= \frac{180^\circ}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\cot 1^\circ - \frac{1}{\sqrt{2}}\right] + 90^\circ$$

$$= 90^\circ \cot 1^\circ - \frac{180}{2} + 90$$

$$= 90\cot 1^\circ$$

48. (A)

$$\cos 2x + 2\cos x = 1$$

$$2\cos x = 1 - \cos(2x)2$$

$$2\cos x = 2\sin^2 x$$

$$\cos x = \sin^2 x$$

$$\therefore \quad \sin^2 x\left(2 - \cos^2 x\right)$$

$$= \sin^2 x\left(1 + \sin^2 x\right)$$

$$= \sin^2 x + \sin^4 x$$

$$= \sin^2 x + \cos^2 x$$

$$= 1$$

49. (A)

$$\tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta).....\left(1+\sec 2^n\theta\right)$$

$$= \tan\frac{\theta}{2} \times \frac{1+\tan\theta}{\cos\theta} \times \frac{1+\tan 2\theta}{\cos 2\theta} \times \frac{1+\tan 4\theta}{\cos 4\theta}$$

$$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} \times \frac{2\cos^2\frac{\theta}{2}}{\cos\theta} \times \frac{2\cos^2\theta}{\cos 2\theta} \times \frac{2\cos^2 2\theta}{\cos 4\theta}$$

$$= 2^3 \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} \cdot \frac{\cos\theta\cos 2\theta}{\cos 4\theta}$$

$$= 2^2 \cdot \sin\theta \cdot \cos\theta \cdot \frac{\cos 2\theta}{\cos 4\theta}$$

$$= 2\frac{\sin 2\theta\cos 2\theta}{\cos 4\theta}$$

$$= \tan 4\theta$$

$$= \tan 2^2\theta$$

$$\therefore \ \tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta).....\left(1+\sec 2^{2n}\theta\right)$$

$$= \tan 2^n\theta$$

50. (B)

$$\therefore \quad A = \sum_{n=1}^{44} \cos n^\circ$$

$$= \cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos 44^\circ$$

$$B = \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + ... + \sin 44^\circ$$

$$x=\frac{A}{B}=\frac{\cos 1^{o}+\cos 2^{o}+\cos 3^{o}+.....+\cos 44^{o}}{\sin 1^{o}+\sin 2^{o}+\sin 3^{o}+.....+\sin 44^{o}}$$

$$=\frac{\sin 89^{o}+\sin 88^{o}+\sin 87^{o}+.....+\sin 46^{o}}{\sin 1^{o}+\sin 2^{o}+\sin 3^{o}+.....+\sin 44^{o}}$$

$$=\frac{\frac{\sin\left(44.\frac{1}{2}\right)}{\sin\left(\frac{1}{2}\right)}\times\cos\left(1+\left(44-1\right)\cdot\frac{1}{2}\right)}{\frac{\sin\left(44.\frac{1}{2}\right)}{\sin\frac{1}{2}}\times\sin\left(1+\left(44-1\right)\frac{1}{2}\right)}$$

$$=\frac{\cos\left(22+\frac{1}{2}\right)}{\sin\left(22+\frac{1}{2}\right)}=\cot 22\frac{1}{2}^{o}$$

$$=\sqrt{2}+1$$

$$\therefore\quad\left[100\left(\sqrt{2}+1-1\right)\right]=\left[100\times 1.4\right]=141$$

51. (B)

Trick : $\alpha=\frac{7}{3}$

$$\Rightarrow\quad\beta=\gamma=\frac{\pi}{6}$$

$\therefore$ Option (B)

52. (A)

$$f\left(\alpha,\beta\right)=\cos^{2}\alpha+\sin^{2}\alpha\cdot\cos^{2}\beta$$

$$=1-\sin^{2}\alpha+\sin^{2}\alpha\cdot\cos^{2}\beta$$

$$=1-\left(1-\cos 2\beta\right)\sin^{2}\,ga$$

$$=1-\left(1-\cos^{2}\beta\right)\sin^{2}\alpha$$

$$=1-2\sin^{2}\alpha\sin^{2}\beta$$

$$f\left(\frac{\pi}{12},\frac{\pi}{3}\right)=1-2\sin^{2}\frac{\pi}{12}\sin^{2}\frac{\pi}{3}$$

$$=1-2\cdot\frac{\left(\sqrt{3}-1\right)^{2}}{8}\times\frac{3}{4}$$

$$=1-\frac{3\left(4-2\sqrt{3}\right)}{16}$$

$$=1-\frac{3\left(2-\sqrt{3}\right)}{8}=\frac{8-6+3\sqrt{3}}{8}=\frac{2+3\sqrt{3}}{8}$$

53. (A)

Trick : Put $x = 0$

$\Rightarrow \quad f(x) = 8$

$g(x) = 8$

$\therefore$ Option (A)

54. (B)

$$\sum_{r=1}^{n}\left(\frac{1}{\cos\theta + \cos(2r+1)\theta}\right)$$

$$= \frac{1}{\cos\theta + \cos 3\theta} + \frac{1}{\cos\theta + \cos 5\theta} + ... + \frac{1}{\cos\theta + \cos(2n+1)\theta}$$

$$= \frac{1}{2\cos 2\theta \cdot \cos\theta} + \frac{1}{2\cos 3\theta \cdot \cos 2\theta} + ... + \frac{1}{2\cos(n+1)\theta \cdot \cos n\theta}$$

$$= \frac{1}{2\sin\theta}\left[\frac{\sin(2\theta-\theta)}{\cos 2\theta \cdot \cos\theta} + \frac{\sin(3\theta-2\theta)}{\cos 3\theta \cos 2\theta} + ...\right]$$

$$= \frac{1}{2\sin\theta}\left[\frac{\sin 2\theta\cos\theta - \cos 2\theta\sin\theta}{\cos 2\theta \cdot \cos\theta} + \frac{\sin 3\theta\cos 2\theta - \cos 3\theta\sin 2\theta}{\cos 3\theta \cdot \cos 2\theta}\right]$$

$$= \frac{1}{2\sin\theta}\left[\tan 2\theta - \tan\theta + \tan 3\theta - \tan 2\theta + ... \tan(n+1)\theta - \tan n\theta\right]$$

$$= \frac{1}{2\sin\theta}\left[\tan(n+1)\theta - \tan\theta\right]$$

$$= \frac{1}{2\sin\theta} \times \frac{\sin(n+1-1)\theta}{\cos(n+1)\theta\cos\theta}$$

$$= \frac{\sin n\theta}{\cos(n+1)\theta \cdot \sin 2\theta}$$

55. (C)

Let $y = 27^{\cos 3x}.81^{\sin 3x}$

$= 3^{3\cos 3x}.3^{4\sin 3x}$

$= 3^{3\cos 3x + 4\sin 3x}$

Then minimum value of y is

$3^{-5} = \frac{1}{243}$

56. (C)

$$\sum_{k=1}^{35}\sin 5k = \tan\frac{m^{\circ}}{n^{\circ}}$$

$$\Rightarrow \quad \sin 5 + \sin 10 + ... + \sin 5.35 = \tan\frac{m^{\circ}}{n^{\circ}}$$

$$\Rightarrow \quad \frac{\sin\left(35\cdot\frac{5}{2}\right)}{\sin\left(\frac{5}{2}\right)}\times\sin\left(5+(35-1)\times\frac{5}{2}\right)=\tan\frac{m^{\circ}}{n^{\circ}}$$

$$\Rightarrow \quad \frac{\sin\left(\frac{175^{\circ}}{2}\right)}{\sin\left(\frac{5^{\circ}}{2}\right)}\times\sin\left(\frac{175+5}{2}\right)=\tan\frac{m^{\circ}}{n^{\circ}}$$

$$\Rightarrow \quad \frac{\sin\left(\frac{175^{\circ}}{2}\right)}{\sin\left(\frac{5^{\circ}}{2}\right)}=\tan\frac{m}{n}$$

$$\Rightarrow \quad \frac{\sin\left(\frac{175}{2}\right)}{\cos\left(\frac{175}{2}\right)}=\frac{\sin\left(\frac{m}{n}\right)}{\cos\left(\frac{m}{n}\right)}$$

$$\therefore \quad \frac{m}{n}=\frac{175}{2}$$

$$\therefore \quad m+n=175+2=177$$

57. (A)

$$\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}$$

$$=\frac{\sin\theta(1+2\cos\theta)}{\cos\theta(1+2\cos\theta)}$$

$$=\tan\theta;\ \tan\theta\in(-\infty,\infty)$$

58. (A)

$$A+B+C=\pi$$

$$\sin\left(A+\frac{C}{2}\right)=K\sin\frac{C}{2}$$

$$\Rightarrow \quad \frac{\sin\left(A+\frac{C}{2}\right)}{\sin\frac{C}{2}}=\frac{K}{1}$$

$$\Rightarrow \quad \frac{\sin\left(A+\frac{C}{2}\right)+\sin\frac{C}{2}}{\sin\left(A+\frac{C}{2}\right)-\sin\frac{C}{2}}=\frac{K+1}{K-1}$$

$$\Rightarrow \quad \frac{2\sin\left(\frac{A+C}{2}\right)\cos\frac{A}{2}}{2\cos\left(\frac{A+C}{2}\right)\sin\frac{A}{2}}=\frac{K+1}{K-1}$$

$$\Rightarrow \quad \frac{\cos\frac{B}{2}}{\sin\frac{B}{2}}\cdot\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}=\frac{K+1}{K-1}$$

$$\Rightarrow \quad \sin\frac{A}{2}\cdot\sin\frac{B}{2}=\frac{K-1}{K+1}$$

59. (C)

$A+B+C=\pi$

$\cos A=\cos B\cdot\cos C$

$\therefore \quad \cos(B+C)=-\cos B\cos C$

$\Rightarrow \quad \cos B\cos C-\sin B\sin C=-\cos B\cos C$

$\Rightarrow \quad \sin B\sin C=2\cos B\cos C$

$\Rightarrow \quad \tan B\tan C=2$

60. (C)

$$\frac{\sin(3x-2x)}{\sin 2x\cdot\sin 3x}+\frac{\sin(4x-3x)}{\sin 3x\cdot\sin 4x}+\frac{\sin(5x-4x)}{\sin 4x\cdot\sin 5x}$$

$$=\frac{\sin 3x\cdot\cos 2x-\cos 3x\cdot\sin 2x}{\sin 2x\cdot\sin 3x}+\frac{\sin 4x\cdot\cos 3x-\cos 4x\cdot\sin 3x}{\sin 3x\cdot\sin 4x}$$

$$=(\cot 2x-\cot 3x)+(\cot 3x-\cot 4x)+.....+\cot(n+1)x-\cot(n+2)x$$

$$=\cot 2x-\cot(n+2)x$$