TRIGO EQUATION
EXERCISE – 1 (A)

1. **(A)**

$\Rightarrow \tan\frac{2}{3}\theta = \sqrt{3}$

$\Rightarrow \frac{2\theta}{3} = n\pi + \frac{\pi}{3}$

$\Rightarrow 2\theta = 3n\pi + \pi$

$\Rightarrow \theta = 3n\frac{\pi}{2} + \frac{\pi}{2}, n \in I$

2. **(C)**

$\Rightarrow \sec\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$

3. **(C)**

$\Rightarrow \cos\left(\frac{-\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right) = 0$

$\Rightarrow \frac{\theta}{2} = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\pi$

4. **(A)**

$\Rightarrow 2\sin x + \tan x = 0$

$\Rightarrow \cos x \neq (2n+1)\pi$

$\Rightarrow 2\sin x + \frac{\sin x}{\cos x} = 0$

$\Rightarrow \sin x\left(\frac{2\cos x + 1}{\cos x}\right) = 0$

$\Rightarrow \sin x = 0$ or $2\cos x + 1 = 0$ …….$\cos x \neq 0$

$\Rightarrow x = n\pi$ $x = 2n\pi \pm 2\frac{\pi}{3}$

$\Rightarrow \therefore x = (3n \pm 1)\left(\frac{2\pi}{3}\right)$ and $n\pi$

5. **(B)**

$\Rightarrow (2\cos x - 1)(3 + 2\cos x) = 0$

$\Rightarrow 0 \leq x \leq 2\pi$

$\Rightarrow \cos x = \frac{1}{2}$ or $\cos = -\frac{3}{2}$ ….(not possible)

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3} \text{in} \{0, 2\pi\}$

$\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$

6. **(B)**

$\Rightarrow \cos^2\theta = 1$

$\Rightarrow \cos\theta = \pm 1$

$\Rightarrow \cos\theta = 1$ or $\cos\theta = -1$

$\Rightarrow \theta = 2n\pi$ $\quad \theta = (2n+1)\pi$

$\Rightarrow \therefore\ \theta = n\pi$

7. **()**

$\Rightarrow (1+\tan\theta)(1+\tan\varphi) = 2$

$\Rightarrow (1+\tan\theta\tan\varphi+\tan\theta+\tan\varphi) = 2$

$\Rightarrow \tan\theta+\tan\varphi = 1-\tan\theta\tan\varphi$

$\Rightarrow \dfrac{\tan\theta+\tan\varphi}{1-\tan\theta\tan\varphi} = 1 = \tan\dfrac{\pi}{4}$

$\Rightarrow \tan(\theta+\varphi) = \tan\dfrac{\pi}{4}$

$\Rightarrow \theta+\varphi = 45^o$

8. **(A)**

$\cos\theta + \cos 2\theta = 2$

$\cos\theta + 2\cos^2\theta - 1 = 2$

$2\cos^2\theta + \cos\theta - 3 = 0$

$\cos\theta = \dfrac{-3}{2}, 1$

$\cos\theta = 1 \Rightarrow \theta = 2n\pi, n \in I$

9. **()**

$\Rightarrow \sin^2\theta - 2\cos\theta + \dfrac{1}{4} = 0$

$\Rightarrow 1 - \cos^2 - 2\cos\theta + \dfrac{1}{4} = 0$

$\Rightarrow \dfrac{5}{4} - \cos^2\theta - 2\cos\theta = 0$

$\Rightarrow \cos^2\theta + 2\cos\theta - \dfrac{5}{4} = 0$

$\Rightarrow (\cos\theta+1)^2 - \dfrac{9}{4} = 0$

$\Rightarrow \left(\cos\theta + 1 - \dfrac{3}{2}\right)\left(\cos\theta + 1 + \dfrac{3}{2}\right) = 0$

$\Rightarrow \left(\cos\theta - \dfrac{1}{2}\right)\left(\cos\theta + \dfrac{5}{2}\right) = 0$

$\Rightarrow \cos\theta = \dfrac{1}{2};\quad \cos\theta = \dfrac{5}{2}$ (not possible)

$\Rightarrow \theta = 2n\pi \pm \dfrac{\pi}{3}$

10. **(C)**

$\Rightarrow \tan^2\theta + 2\sqrt{3}\tan\theta = 1$

$\Rightarrow \tan^2\theta + 2\sqrt{3}\tan\theta - 1 = 0$

$\Rightarrow \tan\theta = \dfrac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = \dfrac{-2\sqrt{3} \pm 2\times 2}{2}$

$\Rightarrow \tan\theta = -\sqrt{3} \pm 2$

$\Rightarrow \tan\theta = 2-\sqrt{3}$ or $\tan\theta = -2-\sqrt{3}$

$\Rightarrow \tan 15^o = 2-\sqrt{3}$ $\tan 75^o = 2+\sqrt{3}$

$\tan -75^o = -2-\sqrt{3}$

$\Rightarrow \tan\theta = \tan 15^o = \tan 2-\sqrt{3}$

$= \tan\dfrac{\pi}{12}$

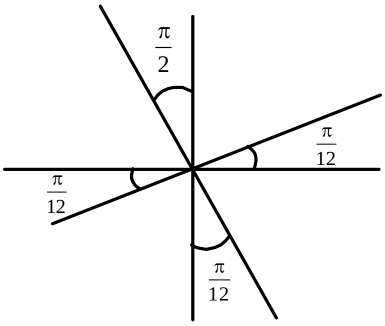
$\theta = n\pi + \dfrac{\pi}{12}$

$\Rightarrow \tan\theta = \tan\left(-2-\sqrt{3}\right)$

$= \tan -75$

$= \tan -\dfrac{5\pi}{12}$

$\Rightarrow \theta = n\pi - \dfrac{5\pi}{12}$



$\Rightarrow \theta = n\pi + \dfrac{7\pi}{12}$

$= \left(2n+1\right)\dfrac{\pi}{2} + \dfrac{\pi}{12}$

$\Rightarrow \therefore\ \theta = \left(6n+1\right)\dfrac{\pi}{12}$

11. **(B)**

$\Rightarrow 25\cos^2\theta + 5\cos\theta - 12 = 0$

$\Rightarrow \alpha$ is the root then

$\Rightarrow \cos\alpha = \dfrac{-5 \pm \sqrt{25+1200}}{50}$

$= -\dfrac{4}{5}, \dfrac{3}{5}$

$\Rightarrow \cos\alpha = -\dfrac{4}{5}$ ……..II quadrant

$\Rightarrow \sin\alpha = \dfrac{3}{5}$

$\Rightarrow \sin 2\alpha = 2\left(-\dfrac{4}{5}\right)\left(\dfrac{3}{5}\right) = -\dfrac{24}{25}$

12. **(A)**

$\Rightarrow \cos x + \sec x = 2$

We know arithmetic mean > Geometric mean

$\Rightarrow \frac{a+\frac{1}{a}}{2} \geq \sqrt{a \times \frac{1}{a}}$

$\Rightarrow a+\frac{1}{a} \geq 2$ (not possible so only equality holds)

Now for $a=\cos x=1$

$\Rightarrow n=2n\pi$

13. **(C)**

$\Rightarrow \tan\theta=\sqrt{3}$

$\Rightarrow \theta=\frac{\pi}{3},\frac{4\pi}{3}$

$\Rightarrow \operatorname{cosec}\theta=-\frac{2}{\sqrt{3}} \Rightarrow \sin\theta=\frac{-\sqrt{3}}{2}$

$\Rightarrow \theta=\frac{4\pi}{3} \text{ or } \frac{5\pi}{3}$

$\Rightarrow \therefore\ \theta=\frac{4\pi}{3}$

$\Rightarrow \theta=2n\pi+\frac{4\pi}{3}$

14. **(B)**

$\Rightarrow \sin^2\theta+\sec\theta+\sqrt{3}\tan\theta=0$

$\Rightarrow \sec\theta \neq \infty\ ;\ \ \cos\theta \neq 0$

$\Rightarrow \theta \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)+\sqrt{3}\tan\theta=0$

$\Rightarrow \tan\theta\left(\sin\theta+\sqrt{3}\right)=0$

$\Rightarrow \tan\theta=0$

$\Rightarrow \sin\theta \neq -\sqrt{3}$

$\Rightarrow \theta=n\pi$

15. **(C)**

$\Rightarrow 3\left(\sec^2\theta+\tan^2\theta\right)=5$

$\Rightarrow 3\left(1+\tan^2\theta+\tan^2\theta\right)=5$

$\Rightarrow 3+6\tan^2\theta=5$

$\Rightarrow 6\tan^2\theta=2$

$\Rightarrow \tan^2\theta=\frac{1}{3}=\tan^2\frac{\pi}{6}$

$\Rightarrow \theta=n\pm\frac{\pi}{6}$

16. **(A)**

$\Rightarrow 2\cot^2\theta=\operatorname{cosec}^2\theta$

$=1+\cot^2\theta$

$\Rightarrow \cot^2\theta=1$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$

17. **(A)**

$2\cos^2 x + 3\sin x - 3 = 0$

$2\left(1-\sin^2 x\right) + 3\sin x - 3 = 0$

$2 - 2\sin^2 x + 3\sin x - 3 = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

$\sin x = 1$ or $\sin x = \frac{1}{2}$

FQ $0 \le x \le 180^o$

$x = 90^o, 30^o, 150^o$

18. **(B)**

$2\sin^2 \theta = 3\cos\theta$

$2\left(1-\cos^2 \theta\right) = 3\cos\theta$

$2 - 2\cos^2 \theta = 3\cos\theta$

$2\cos^2 \theta + 3\cos\theta - 2 = 0$

$2\cos^2 \theta + 4\cos\theta - \cos\theta - 2 = 0$

$\left(\cos\theta + 2\right)\left(2\cos\theta - 1\right) = 0$

$\cos\theta \neq -2$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

19. **(A)**

$\Rightarrow \sin 7\theta + \sin\theta = \sin 4\theta$

$\Rightarrow 2\sin 4\theta \cos 3\theta = \sin 4\theta$

$\Rightarrow \sin 4\theta\left(2\cos 3\theta - 1\right) = 0$

$\Rightarrow \sin 4\theta = 0$ or $2\cos 3\theta - 1 = 0$

$\Rightarrow 4\theta = n\pi$ $\cos 3\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{n\pi}{4}$ $3\theta = 2m\pi \pm \frac{\pi}{3}$

$\theta = \frac{2m\pi}{3} \pm \frac{\pi}{9}$

between $0 < \theta < \frac{\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{4}$ $\theta = \frac{\pi}{9}$

$\Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

20. **(B)**

$\cot\theta = \sin 2\theta$

$\frac{\cos\theta}{\sin\theta} = 2\sin\theta\cos\theta$

$\cos\theta = 2\sin^2\theta\cos\theta$

$\cos\theta\left(2\sin^2\theta - 1\right) = 0$

$-\cos\theta\cdot\cos 2\theta = 0$

$\cos\theta = 0$ or $\cos 2\theta = 0$

$\theta = 90^o$ or $\theta = 45^o$

21. **(C)**

$\Rightarrow 2\tan^2 a = \sec^2\theta$

$= 1 + \tan^2\theta$

$\Rightarrow \tan^2\theta = 1$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$

22. **(C)**

$\Rightarrow a\sin x + b\cos x = c$

Will have solution when $|c| < \sqrt{a^2 + b^2}$

23. **(C)**

From $(\pi\cos\theta) = \cot(\pi\sin\theta)$

From $(\pi\cos\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right)$

$\pi\cos\theta = \frac{\pi}{2} - \pi\sin\theta$

$\pi(\sin\theta + \cos\theta) = \frac{\pi}{2}$

$\sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right] = \frac{1}{2}$

$\sqrt{2}\left[\sin\left(\theta + \frac{\pi}{4}\right)\right] = \frac{1}{2}$

$\sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

24. **(B)**

$\Rightarrow \cos p\theta = \cos q\theta$

$\Rightarrow p\theta = 2n\pi \pm q\theta$

$\Rightarrow p\theta \pm q\theta = 2n\pi$

$\Rightarrow \theta(p \pm q) = 2n\pi$

$\Rightarrow \theta = \frac{2n\pi}{p \pm q}$

25. **(A)**

$\Rightarrow \tan 5\theta = \cot 2\theta$

$\Rightarrow \tan 5\theta = \tan\left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow 5\theta = n\pi + \frac{\pi}{2} - 2\theta$

$\Rightarrow 7\theta = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\frac{\pi}{14}$

$\Rightarrow \theta = \frac{n\pi}{7} + \frac{\pi}{14}$

$\Rightarrow \tan 5\theta \neq \infty$

$\Rightarrow 5\theta \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta \neq (2n+1)\frac{\pi}{14}$

Hence $(2n+1) \neq 7, 21,$ etc.

26. **(B)**

$\Rightarrow \tan\theta + \cot\theta = 2$

$\Rightarrow 2\,\text{cosec}\,2\theta = 2$

$\Rightarrow \sin 2\theta = 1$

$\Rightarrow 2\theta = 2n\pi + \frac{\pi}{2}$

$\Rightarrow \theta = n\pi + \frac{\pi}{4}$

27. **(C)**

$\Rightarrow \cot\theta + \tan\theta = 2\,\text{cosec}\,\theta$

$\Rightarrow 2\,\text{cosec}\theta 2\theta = 2\,\text{cosec}\theta$

$\Rightarrow \sin\theta = \sin 2\theta$

$\Rightarrow 2\sin\theta\cos\theta = \sin\theta$

$\Rightarrow \sin\theta(2\cos\theta - 1) = 0$

$\Rightarrow \sin\theta \neq 0 \because\ \theta \neq m\pi$

$\Rightarrow \therefore \cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow \theta = 2\pi\pi \pm \frac{\pi}{3}$

$\Rightarrow \tan\theta \neq 0$

$\Rightarrow \theta = n\pi$

$\Rightarrow \cot\theta \neq \infty$

$\Rightarrow \theta \neq (2n+1)\frac{\pi}{2}$

28. **(B)**

$\Rightarrow \tan 2\theta \tan\theta = 1$

$\Rightarrow \tan 2\theta \tan\theta - 1 = 0$

$\Rightarrow \frac{\sin 2\theta \sin\theta}{\cos 2\theta \cos\theta} - 1 = 0$

$\Rightarrow \frac{\sin 2\theta \sin\theta - \cos 2\theta \cos\theta}{\cos 2\theta \cos\theta} = 0$

$\Rightarrow \frac{\cos 3\theta}{\cos\theta \cos 2\theta} = 0$

$\Rightarrow 3\theta = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\dfrac{\pi}{6}$

$\Rightarrow \theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$\Rightarrow \theta = n\pi \pm \dfrac{\pi}{6}$

$\Rightarrow \cos\theta \neq 0$

$\Rightarrow \theta \neq (2n+1)\dfrac{\pi}{2}$

$\Rightarrow \cos 2\theta \neq 0$

$\Rightarrow \theta \neq (2n+1)\dfrac{\pi}{4}$

$\Rightarrow \dfrac{\pi}{2}$ and $\dfrac{9\pi}{6}$ are ruled out.

29. **(A)**

$\Rightarrow \sin\left(\dfrac{\pi}{4}\cos\theta\right) = \cos\left(\dfrac{\pi}{4}\tan\theta\right)$

$\Rightarrow \cos\left(\dfrac{\pi}{2} - \dfrac{\pi}{4}\cot\theta\right) = \cos\left(\dfrac{\pi}{4}\tan\theta\right)$

$\Rightarrow \therefore \dfrac{\pi}{2} - \dfrac{\pi}{4}\cot\theta = 2n\pi \pm \dfrac{\pi}{4}\tan\theta$

$\Rightarrow \dfrac{\pi}{2} - 2n\pi = \dfrac{\pi}{4}\cot\theta \pm \dfrac{\pi}{4}\tan\theta$

$\Rightarrow -2n\pi$ can be taken as $2n\pi$ ($\because$ n can be negative integer)

$\Rightarrow \therefore 2n\pi + \dfrac{\pi}{2} = \dfrac{\pi}{4}(\cot\theta \pm \tan\theta)$

$\Rightarrow 8n + 2 = \cot\theta \pm \tan\theta$

$\Rightarrow 2 = \cot\theta \pm \tan\theta$

$\Rightarrow 2 = \dfrac{1}{\tan\theta} \pm \tan\theta$

$\Rightarrow 2 = \dfrac{1}{\tan\theta} + \tan\theta$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = n\pi + \dfrac{\pi}{4}$

$\Rightarrow 2 = \dfrac{1}{\tan\theta} - \tan\theta$

$\Rightarrow 2\tan\theta = 1 - \tan^2\theta$

$\Rightarrow \tan^2\theta + 2\tan\theta - 1 = 0$

$\Rightarrow \tan\theta = \dfrac{-2 \pm \sqrt{8}}{2}$

$\Rightarrow \tan\theta = -1 \pm \sqrt{2}$

30. **(C)**

$\Rightarrow 1 + \cot\theta = \operatorname{cosec}\theta$

$\Rightarrow \dfrac{\cos\theta + \sin\theta}{\sin\theta} = \dfrac{1}{\sin\theta}$

$\Rightarrow \sin\theta + \cos\theta = 1$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \sin\frac{\pi}{4} \qquad \ldots\ldots \sin\theta \neq 0; \theta \neq n\pi$$

$$\Rightarrow \theta + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$$

$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$$

$$\Rightarrow n = 0 \qquad \theta = 0$$

$$\Rightarrow n = 1 \qquad \pi - \frac{\pi}{4} - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\Rightarrow n = 2 \qquad 2\pi$$

$$\Rightarrow n = 3 \qquad 3\pi - \frac{\pi}{2} = \frac{5\pi}{2}$$

$$\Rightarrow \theta = 2n\pi + \frac{\pi}{2}$$

31. **(D)**

$$\Rightarrow \cos y \cos\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - y\right)\cos x + \sin y \cos\left(\frac{\pi}{2} - x\right) + \cos x \sin\left(\frac{\pi}{2} - y\right) = 0$$

$$\Rightarrow \cos y \sin x - \sin y \cos x + \sin y \sin x + \cos x \cos y = 0$$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin(x - y) + \frac{1}{\sqrt{2}}\cos(x - y) = 0$$

$$\Rightarrow \sin\left(x - y + \frac{\pi}{4}\right) = 0$$

$$\Rightarrow x - y + \frac{\pi}{4} = n\pi$$

$$\Rightarrow x = n\pi - \frac{\pi}{4} + y$$

32. **(C)**

$$\Rightarrow \sin^2\theta + \sin\theta - 2 = 0$$

$$\Rightarrow (\sin\theta + 2)(\sin\theta - 1) = 0$$

Not possible $\qquad \theta = n\pi + (-1)^n \frac{\pi}{2}$

$$\Rightarrow \theta = 2n\pi + \frac{\pi}{2}$$

33. **(A)**

$$\Rightarrow 2\sin^2\theta = 4 + 3\cos\theta$$

$$\Rightarrow 2 - 2\cos^2\theta = 4 + 3\cos\theta$$

$$\Rightarrow 2\cos^2\theta + 3.\cos\theta + 2 = 0$$

$$\Rightarrow 0 = b^2 - 4ac = 9 - 16 = -7$$

No real roots.

34. **(D)**

$$\Rightarrow 3\cos x + 4\sin x = 6$$

$$\Rightarrow -5 \leq 3\cos x + 4\sin x \leq 5$$

For real roots it will never equal 6.

35. **(D)**

$\Rightarrow \cos^2\theta + \sin\theta + 1 = 0$

$\Rightarrow 1 - \sin^2\theta + \sin\theta + 1 = 0$

$\Rightarrow (1-\sin\theta)(1+\sin\theta) + (1+\sin\theta) = 0$

$\Rightarrow (1+\sin\theta)(1-\sin\theta+1) = 0$

$\Rightarrow (1+\sin\theta)(2-\sin\theta) = 0$

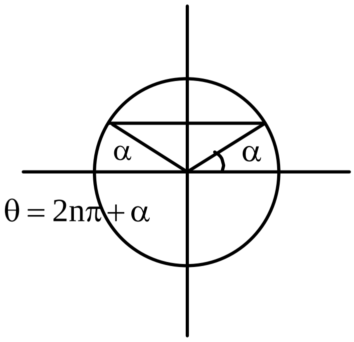
$\Rightarrow \sin\theta = -1$ or $\sin\theta = 2$ (not possible)

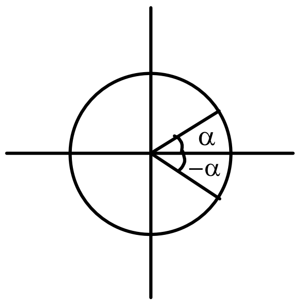
$\Rightarrow \theta = \frac{3\pi}{2}$ (principle solution)

36. **(A)**

$\Rightarrow \sin\theta = \sin\alpha$ $\quad$ $\cos\theta = \cos\alpha$

$\Rightarrow \theta = n\pi + (-1)^n \alpha$ $\quad$ $\theta = 2n\pi \pm \alpha$





37. **(A)**

$\Rightarrow \sin 2x + \sin 4x = 2\sin 3x$

$\Rightarrow 2\sin 3x \cos x = 2\sin 3x$

$\Rightarrow \sin 3x \cos x = \sin 3x$

$\Rightarrow \sin 3x(\cos x - 1) = 0$

$\Rightarrow \sin 3x = 0$ or $\cos x = 1$

$\Rightarrow x = \frac{n\pi}{3}$ $\quad$ $x = 2\pi, 4\pi \ldots\ldots\ldots.$

38. **(A)**

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = n\pi + \frac{\pi}{3}$

$\Rightarrow \frac{\pi}{3} \text{and} \frac{4\pi}{3}$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{3}$

$\Rightarrow \sec\theta = 2$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

39. **(C)**

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}$

P.S. $\frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$

P.S. $= \frac{\pi}{6}, \frac{11\pi}{6}$

Common value $= \frac{\pi}{6}$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{6}$

40. **(B)**

$\Rightarrow \sin\theta = \sqrt{3}\cos\theta$ ….$-\pi < \theta < 0$

$\Rightarrow \tan\theta = \sqrt{3}$ …..$\cos\theta \neq 0$

$\Rightarrow \theta = n\pi + \frac{\pi}{3}$ …..$\theta \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = -\pi + \frac{\pi}{3}$ …..for $n = -1, \theta \neq \frac{\pi}{2}$

$= -\frac{2\pi}{3}$

$= -\frac{4\pi}{6}$

41. **(B)**

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \theta = n\pi + \frac{5\pi}{6}$

P.S. $= \frac{5\pi}{6}, \frac{11\pi}{6}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{5\pi}{6}$

$\Rightarrow$ P.S. $= \frac{5\pi}{6}, -\frac{5\pi}{6}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}$

P.S. $= \frac{5\pi}{6}, \frac{11\pi}{6}$

Common solution $= \frac{5\pi}{6}$

42. **(C)**

$\Rightarrow \sin 5x + \sin 3x + \sin x = 0$ .....$0 \le x \le \frac{\pi}{2}$

$\Rightarrow \sin 5x + \sin x + \sin 3x = 0$

$\Rightarrow 2\sin 2x \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0$

$\Rightarrow 3x = m\pi$

$\Rightarrow x = \frac{m\pi}{3}$

P.S. $= 0, \frac{\pi}{3}, \frac{2\pi}{3}$

$\Rightarrow \cos 2x = -\frac{1}{2}$

$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = x\pi \pm \frac{\pi}{3}$

$\Rightarrow x = \frac{\pi}{3}, -\frac{\pi}{3}$

Common value between 0 and $\frac{\pi}{2}$ is $\frac{\pi}{3}$

43. **(A)**

$\Rightarrow \sin 2\theta = \cos 3\theta$

$\Rightarrow \cos 3\theta = \sin 2\theta$

$\Rightarrow \cos 3\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow 3\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right)$ or $3\theta = 2n\pi - \left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow 5\theta = 2n\pi + \frac{\pi}{2}$ $\theta = 2n\pi - \frac{\pi}{2}$

$\Rightarrow \theta = \frac{(4n+1)}{10}\pi$

$\Rightarrow \theta = \frac{\pi}{10}, \frac{5\pi}{10}, \frac{9\pi}{10}$ etc.........

$\Rightarrow \therefore$ acute angle $= \frac{\pi}{10}$

$\Rightarrow \theta = 18^o$

$\Rightarrow \sin 18^o = \frac{\sqrt{5}-1}{4}$

44. **(C)**

$\Rightarrow \sqrt{3}(\cot\theta + \tan\theta) = 4$

As, $\cot\theta + \tan\theta = 2\operatorname{cosec} 2\theta$

$\Rightarrow \tan\theta \neq \infty \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow \cot\theta \neq \infty \neq n\pi$

$\Rightarrow \therefore 2\sqrt{3}\,\mathrm{cosec}\,2\theta = 4$

$\Rightarrow \mathrm{cosec}\,2\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow 2\theta = n\pi + (-1)^n \frac{\pi}{3}$

$\Rightarrow \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{6}$

45. **(D)**

$\Rightarrow \sin x + \frac{1}{\sin x} = \frac{7}{2\sqrt{3}}$

$\Rightarrow 2\sqrt{3}\sin^2 x + 2\sqrt{3} = 7\sin x$

$\Rightarrow 2\sqrt{3}\sin^2 x - 7x + 2\sqrt{3} = 0$

$\Rightarrow \sin x = \frac{7 \pm \sqrt{49 - 4\left(2\sqrt{3}\right)^2}}{4\sqrt{3}}$

$= \frac{7 \pm \sqrt{49-48}}{4\sqrt{3}}$

$= \frac{7 \pm 1}{4\sqrt{3}} = \frac{2}{\sqrt{3}}, \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin = \frac{2}{\sqrt{3}}$ ……..(not possible)

$\Rightarrow \therefore \sin x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = 60^o$

46. **(C)**

$\cos 3x \cdot \cos 7x = \cos^2 2x$

$2\cos 7x + \cos 3x = 2\cos^2 2x$

$\cos 10x + \cos 4x = 2\cos^2 2x$

$\cos 10x + \cos 4x = 1 + \cos 4x$

$\cos 10x = 1 \Rightarrow x = \frac{x\pi}{5}, n \in I$

47. **(D)**

$6\sin^2 x + \sin x - 1 = 0$ …(1)

$6\sin^2 x + 3\sin x - 2\sin x - 1 = 0$

$(2\sin x + 1)(3\sin x - 1) = 0$

$\sin x = \frac{-1}{2}, \sin x = \frac{1}{3}$

Then sum of roots of
Equation 1 in $x \in [0, 2\pi]$
is 4π

48. **(C)**

$2\sin^2 x + 5\sin x + 2 = 0$

$2\sin^2 x + 4\sin x + \sin x + 2 = 0$

$(\sin x + 2)(2\sin x + 1) = 0$

$\sin x = -2$ or $\sin x = \frac{-1}{2}$

Then $x = x\pi + (-1)^n \left(\frac{-\pi}{6}\right), n \in I$

49. **(B)**

$\tan\theta = \cot 2\theta$

$\tan\theta = \tan\left(\frac{\pi}{2} - 2\theta\right)$

$\theta = n\pi + \frac{\pi}{2} - 2\theta$

$3\theta = n\pi + \frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{6}$, where

$n \in I, n \neq 3m+1, m \in I$.

$= \frac{\pi}{2}, \frac{3\pi}{2} \ldots\ldots.$ $\theta = (2n+1)\frac{\pi}{4}$

$\frac{\pi}{4}, \frac{3\pi}{4} \ldots\ldots\ldots$

50. **(D)**

$\tan\theta + \tan 2\theta + \tan\theta \tan 2\theta = 1$

$\tan\theta + \tan 2\theta = 1 - \tan\theta \tan 2\theta$

$\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta \tan 2\theta} = 1$

$\tan(\theta + 2\theta) = 1$

$\tan 3\theta = 1$

$3\theta = n\pi + \frac{\pi}{4}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{12}, n \in I$

Where $\frac{n\pi}{3} + \frac{\pi}{12} \neq \left\{\frac{(2n+1)\pi}{4} (2k+1)\frac{\pi}{2}\right\}$

$m, k \in I$

51. **(B)**

$a\cos x + s\sin x = 13$

For no real solution

$\sqrt{a^2+25}<13$
$a^2+25<169$
$a^2-144<0$
$(a-12)(a+12)<0$
$-12<a<12$
$a\in(-12,12)$

52. **(A)**
$\tan 6x=\tan x$
$6x=n\pi+x, n\in I$
$5x=n\pi, n\in I$
$x=\frac{n\pi}{5}, n\in I$
$x=\frac{\pi}{5},\frac{2\pi}{5},\frac{3\pi}{5},........$
is form an A.P.
then C. D is $\frac{\pi}{5}$

53. **(A)**
For $x\in\left(0,\frac{\pi}{2}\right)$
$\cos^2 x=1-\sin 2x$
$1-\sin^2 x=1-2\sin x\cos x$
$\sin x(\sin x-2\cos x)$
$\sin x=0$
$\tan x=2$
$x=\tan^{-1}2$

54. **(D)**
$|\sin x|<\frac{1}{2}$
$\frac{-1}{2}<\sin x<\frac{1}{2}$
Hence $x\in$
$\left(\frac{-\lambda}{6},\frac{\lambda}{6}\right)\cup\left(\frac{5\pi}{6},\frac{7\pi}{6}\right)$
Dig.
$\left(2n\pi-\frac{\pi}{6},2n\pi+\frac{\pi}{6}\right)\cup\left(2n\pi+\frac{5\pi}{6},2n\pi+\frac{7\pi}{6}\right)$
Where $n\in I$

55. **(B)**
$\cos^5 x=1+\left(1-\cos^2 x\right)^2$
$\cos^5 x=1+1+\cos^4 x-2\cos^2 x$
Let $\cos x=t$
$t^5-t^4+2t^2-2=0$
$(t-1)\left(t^4+2t+2\right)=0$

$t = 1 \text{ or } t^4 + 2t + 2 = 0$
$\cos x = 1$
$x = 2n\pi, n \in I$

56. **(D)**

$\tan^2\theta = 1 - \sec 2\theta$

$\tan^2\theta = 1 - \left(\dfrac{1+\tan^2\theta}{1-\tan^2\theta}\right)$

$\tan^2\theta = \dfrac{1-\tan^2\theta - 1 - \tan^2\theta}{1-\tan^2\theta}$

$\tan^2\theta\left(1-\tan^2\theta\right) = -2\tan^2\theta$

$\tan^2\theta - \tan^4\theta = -2\tan^2\theta$

$\tan^4\theta = 3\tan^2\theta$

$\tan^2\theta\left(\tan^2\theta - 3\right) = 0$

$\tan^2\theta = 0$

$\tan^2\theta = \left(\sqrt{3}\right)^2$

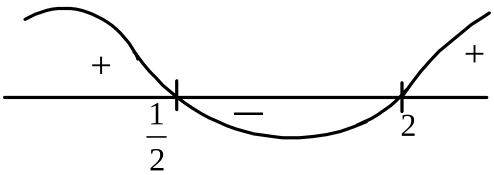
$\tan\theta = 0 \qquad \tan^2\theta = \tan^2\dfrac{\pi}{3}$

$\theta = n\pi \qquad \theta = n\pi \pm \dfrac{\pi}{3}$

Then $\theta = \dfrac{n\pi}{3}, n \in I$

57. **(D)**

$2\sin^2\theta - 5\sin\theta + 2 > 0$
$2\sin^2\theta - 4\sin\theta - \sin\theta + 2 > 0$
$2\sin\theta\left(\sin\theta - 2\right) - 1\left(\sin\theta - 2\right) > 0$
$\left(\sin\theta - 2\right)\left(2\sin\theta - 1\right) > 0$



$\sin\theta < \dfrac{1}{2} \qquad$ or $\sin\theta > 2$ (not possible)

$\theta \in \left(0, \dfrac{\pi}{6}\right) \cup \left(\dfrac{5\pi}{6}, 2\pi\right)$

58. **(B)**

$\left|\cos x\right| = \sin x$

$\Rightarrow \left|\cos x\right|$ is always positive

$\therefore$ x must lie in 1st and 2nd quadrant,
So $\sin x$ is positive

and $\left|\cos x\right| = \sin x$ at $x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$

$\Rightarrow 0 \le x \le 4\pi$ is 2 periods
So total solution is 4.

59. **(D)**

$|\sin x|^2 + |\sin x| + b = 0$

$t^2 + t + b = 0 \qquad \in [0,1]$

The root is negative, other roots must it lie is [0,1] for 2 values of x.

$f(0) \le 0 \qquad \Rightarrow \qquad b \le 0$

$f(1) > 0 \qquad\qquad b > -2$

$(-2, 0]$

60. **(D)**

$3\tan^2 x \ge 4\sin^2 x$

$3\sin^2 x \ge 4\sin^2 x \cos^2 x \qquad$ [where $x \ne \frac{\lambda}{2}$]

$\sin^2 x\left(3 - 4\cos^2 x\right) \ge 0$

$\sin x = 0$

$x = nz$

$3 - 4\cos^2 x \ge 0$

$4\cos^2 x - 3 \le 0$

$\left(\cos x = \frac{\sqrt{3}}{2}\right)\left(\cos x + \frac{\sqrt{3}}{2}\right) \le 0$

$-\frac{\sqrt{3}}{2} \le \cos x \le \frac{\sqrt{3}}{2}$

Where $\cos x \ne 0$

$x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right] - \left\{\frac{z}{2}\right\} \cup \{0, z\}$

Hence (D)

TRIGO EQUATION
EXERCISE – 1 (B)

1. **(D)**

$\frac{1}{2}(\sin 8\theta + \sin 2\theta) = \frac{1}{2}(\sin 16\theta + \sin 2\theta)$

$\therefore\ \sin 8\theta = \sin 16\theta$

$\sin 16\theta - \sin 8\theta = 0$

$2\sin(4\theta)\cos(12\theta) = 0$

$4\theta = n\pi$

$\theta = \frac{n\pi}{4}, \qquad 12\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{24}$

$0, \frac{\pi}{4}, \frac{\pi}{2}, \quad \frac{\pi}{24}, \frac{3\pi}{24}, \frac{5\pi}{24}, \frac{7\pi}{12}, \frac{9\pi}{24}, \frac{11\pi}{12}$

$= 9$

2. **(B)**

$\lambda = \frac{\sin 4x \cos 4x}{2}$

$= \frac{\sin 8x}{4} \qquad \therefore \frac{-1}{4} \le \lambda \le \frac{1}{4}$

3. **(D)**

$\tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$

$\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3}$

$\tan 3\theta = \tan\frac{\pi}{3}$

$3\theta = n\pi + \frac{\pi}{3}$

$\theta = (3n+1)\frac{\pi}{9}$

4. **(D)**

$\tan\theta + \tan 4\theta = \tan 7\theta(\tan\theta\tan 4\theta - 1)$

$\frac{\tan\theta + \tan 4\theta}{1 - \tan\theta\tan 4\theta} = -\tan 7\theta$

$\tan 5\theta = -\tan 7\theta$

$\tan 5\theta + \tan 7\theta = 0$

$\frac{\sin 12\theta}{\cos 5\theta \cos 7\theta} = 0 \qquad$ but

$\sin 12\theta \qquad \theta \ne (2n+1)\frac{\pi}{2}$

$12\theta = n\pi \qquad \ne (2n+1)\frac{\pi}{8}$

$\theta = \frac{n\pi}{12} \qquad \ne (2n+1)\frac{\pi}{14}$

$$\neq (2n+1)\frac{\pi}{10}$$

$$\therefore \theta = \frac{n\pi}{12}, \qquad n \neq 6(2k+1)$$

5. **(B)**

$$\frac{\cos 3x}{4} = \frac{1}{4}, \qquad \cos 3x = 1$$

$$3x = 2n\pi$$

$$x = 2n\frac{\pi}{3}$$

$$\left\{0 + \frac{2\pi}{3} + \frac{4\pi}{3} + \frac{6\pi}{3} + ... + \frac{18\pi}{3}\right\}$$

$$= \frac{10}{2}\left(0 + \frac{18\pi}{3}\right) = 30\pi$$

6. **(B)**

$$\sin^3 x + \sin x \cos x + \cos^3 x = 1$$

$$(\sin x + \cos x)(1 - \sin x \cos x) + (\sin x \cos x - 1) = 0$$

$$(\sin x + \cos x) = 1$$

Or

$$(\sin x \cos x) = 1$$

$$\sin x \cos x \neq 1 \qquad \therefore \sin x + \cos x = 1$$

$$2\sin\frac{\pi}{2}\cos\frac{\pi}{2} = 2\sin^2\frac{\pi}{2}$$

$$\sin\frac{\pi}{2} = 0 \text{ or } \tan\frac{\pi}{2} = 1$$

$$x = 2n\pi \qquad \text{or} \qquad \frac{x}{2} = n\pi + \frac{\pi}{4}$$

$$x = 2n\pi + \frac{\pi}{2}$$

7. **(D)**

$$5\cos 2\theta + (1 + \cos\theta) + 1 = 0$$

$$5(2\cos^2\theta - 1) + 1 + \cos\theta + 1 = 0$$

$$10\cos^2\theta + \cos\theta - 3 = 0$$

$$\cos\theta = \frac{1}{2}, -\frac{3}{5}$$

$$\theta = \frac{\pi}{3}, \quad \pi - \cos\left(\frac{3}{5}\right)$$

8. **(D)**

$$7\cos^2 x + \sin x \cos x - 3 = 0$$

$x = (2n+1)\frac{\pi}{2}$ is not a solution

Divided by $\cos^2 x$

$$7 + \tan x - 3(1 + \tan^2 x) = 0$$

$$3\tan^2 x - \tan x - 4 = 0$$
$$\tan = -1 \text{ or } \frac{4}{3}$$
$$x = n\pi + \frac{3\pi}{4} \text{ or } k\pi + \tan^{-1}\left(\frac{4}{3}\right)$$

9. **(C)**

$$4\sin^2 x + 4\sin x + a^2 - 3 = 0$$
$$(2\sin x + 1)^2 + a^2 - 4 = 0$$
$$(2\sin x + 1)^2 = 4 - a^2$$
$$-2 \le 2\sin x \le 2$$
$$-1 \le 2\sin x + 1 \le 3$$
$$0 \le (2\sin x + 1)^2 \le 9$$
$$0 \le (4 - a^2) \le 9$$
$$-9 \le a^2 - 4 \le 0$$
$$-5 \le a^2 \le 4$$
$$a^2 \le 4$$
$$-2 \le a \le 2$$

10. **(A)**

$$3\tan(\theta - 15^o) = \tan(\theta + 15^o)$$
$$\frac{\tan(\theta + 15^o)}{\tan(\theta - 15^o)} = \frac{3}{1}$$
$$\frac{\sin 2\theta}{\sin 30^o} = \frac{4}{2} = 2$$
$$\sin 2\theta = 1$$
$$2\theta = 2n\pi + \frac{\pi}{2}$$
$$\theta = n\pi + \frac{\pi}{4}$$

11. **(B)**

$$\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 4$$
$$\left(\frac{1 + \tan\theta}{1 - \tan\theta}\right) + \left(\frac{1 - \tan\theta}{1 + \tan\theta}\right) = 4$$
$$2 + 2 + a^2\theta = 4(1 - \tan^2\theta)$$
$$6 + a^2\theta = 2$$
$$\tan^2\theta = \frac{1}{3}, \ \theta = n\pi \pm \frac{\pi}{6}$$

12. **(A)**

let $t = \tan\theta$

$$t + \frac{t + (-1)}{1 - (t)(-1)} = 2$$

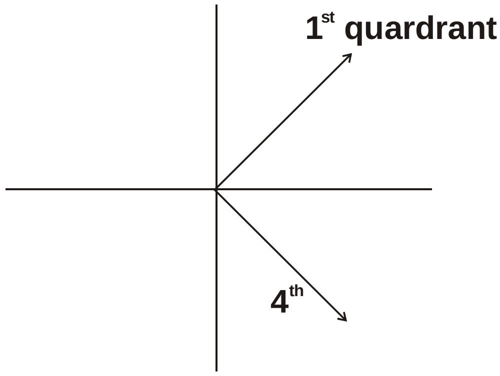
$$\frac{t(1+t)+t-1}{1+t}=2$$

$$t^2+\cancel{2t}-1=\cancel{2t}+2$$

$t^2=3$, $\quad \tan^2\theta=\tan^2\frac{\pi}{3}$

$$\theta=n\pi\pm\frac{\pi}{3}$$

13. **(D)**



$$2(\sec^2 x-1)-5\sec x=1$$

$$2\sec^2 x-5\sec x-3=0$$

$$\sec x \;=\frac{5\pm\sqrt{25+24}}{4}$$

$$=\frac{12}{4}\text{ or }\frac{-1}{2}$$

$$=3\text{ or }\frac{-1}{2}$$

seen = 3, $\quad [0,6\pi]:6$ soln.

$\left[6\pi,6\pi+\frac{3\pi}{2}\right]:$ 1 soln.

$\left[0,\frac{15\pi}{2}\right]:7$ soln.

$n_{max}=15$

14. **(C)**

$$1+\tan^2 x=\sqrt{2}(1-\tan^2 x)$$

$$\frac{1}{\sqrt{2}}=\cos^2 x$$

$$2x=2n\pi\pm\frac{\pi}{4}$$

$$x=n\pi\pm\frac{\pi}{8}$$

15. **(D)**

$6\tan^2 x-2\cos^2 x=\cos^2 x$

$6\tan^2 x-(1+\cos 2x)=\cos 2x$

$$6\left(\frac{1-\cos 2x}{1+\cos 2x}\right)=2\cos^2 x+1$$

$$6-6\cos^2 x=(1+\cos 2x)(2\cos^2 x+1)$$

$$6-6\cos^2 x=2\cos^2 2x+2\cos^2 x+1+\cos^2 x$$

$$2\cos^2 2x+9\cos^2 x-5=0$$

$$\cos^2 x=\frac{1}{2}\text{ or }-5$$

16. **(B)**

Snx – 3Sn2x + Sn3x

= cos x – 3 cos2x + cos3x
2Sn2x cos x – 3 sn 2x
= 2cos 2x cosx – 3 cos2x
Sn 2x (2 cos x – 3) – cos 2x (2cos x – 3) = 0
(Sn 2x – cos 2x) (2 cos x – 3) = 0

$\tan x = 1 \text{ or } \cos x = \frac{3}{2}$

$2x = n\pi + \frac{\pi}{4}$

$x = \frac{n\pi}{2} + \frac{\pi}{8}$

17. **(C)**
Snx + cosx = 1

$\frac{1}{\sqrt{2}} \text{Sn} x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}}$

$\text{Sn}\left(x + \frac{\pi}{4}\right) = \text{Sn}\frac{\pi}{4}$

$x + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$

$x = n\pi + (-1)^n \frac{\pi}{4}$

$-\frac{\pi}{4}$

18. **(C)**
$6 \text{Sn}\,\theta + t \cos\theta = 9$

Let t = $\tan\frac{\theta}{2}$

$6\left(\frac{2t}{1+t^2}\right) + 7\left(\frac{1-t^2}{1+t^2}\right) = 9$

$12t + 7 - 7t^2 = 9 + 9t^2$

$8t^2 - 6t + 1 = 0$

$t = \frac{6 \pm \sqrt{36-32}}{16} = \frac{6 \pm 2}{16} = \frac{1}{2} \text{or} \frac{1}{4}$

$\tan\frac{\theta}{2} = \frac{1}{2}, \qquad \tan\frac{\theta}{2} = \frac{1}{4}$

$\tan\theta = \frac{2\left(\frac{1}{2}\right)}{1 - \frac{1}{4}} = \frac{4}{3}, \qquad \tan\theta = \frac{2\left(\frac{1}{4}\right)}{1 - \frac{1}{16}}$

= $\frac{1/2}{15/16} = \frac{8}{15}$

19. **(B)**
y = Snx – cos x

$y \in \left[-\sqrt{2}, \sqrt{2}\right]$

20. **(C)**

$K\cos x = 3Snx = K+1$

$|K\cos x - 3Snx| \le \sqrt{k^2+9}$

$\cancel{k^2} + 2k + 1 \le \cancel{k^2} + 9$

$2k \le 9-1$

$k \le 4$

21. **(C)**

$\frac{Sn\,3\theta}{2\cos 2\theta + 1} = \frac{1}{2}, \quad \frac{Sn\,3\theta\ \ Sn\theta}{2Sn\theta\cos 2\theta + Sn\theta} = \frac{1}{2}$

$\theta \ne n\pi$

$\frac{Sn3\theta\ Sn\theta}{Sn3\theta - Sn\theta + Sn\theta} = \frac{1}{2}$

$Sn\theta = \frac{1}{2}$

$\theta = n\pi + (-1)^n \frac{\pi}{6}$

22. **(C)**

$a^2 - 4a + 6 = (a-2)^2 + 2 \ge 2$

So, $\min_{a \leftarrow n} \{1, a^2 - 4a + 6\}$

$Sn\,x + asx = 1$

$Sn\left(x + \frac{\pi}{4} = \frac{1}{\sqrt{2}}\right)$

$x + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$

$x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

23. **(B)**

$1 + Snx^4 = \cos^2 3x$

$Sn^2 3x + Sn^4 x = 0$

$Sn3x = 0\ \ \&\ \ Snx = 0$

$3x = n\pi \quad \&\ \ x = n\pi$

$\therefore x = n\pi$

$x = 2\pi$ greatest

24. **(A)**

$\cos x \cos 6x = -1$

Case – 1: $\cos x = 1, \qquad \cos 6x = -1$

$x = 2n\pi,$ then $6x = 12n\pi$

cos 6x = 1

Not Possible

Case – 2 : cos x = -1, & cos 6x = 1

$x = (2n+1)\pi$

Then $6x = 6(2n+1)\pi$

$$\cos 6x = 1$$
$$x = (2n+1)\pi$$

25. **(A)**

$$\text{Snx} + \text{Sny} = 2$$
$$x = \frac{\pi}{2},\ \ y = \frac{\pi}{2}$$
$$n + y = \pi$$

26. **(A)**

$$\text{Snx} + \cos x = \sqrt{y + \frac{1}{y}} \geq \sqrt{2}$$
$$\therefore \quad \text{Sn}\,x + \cos x = \sqrt{2}$$
$$x = \frac{\pi}{4},\ \&\ y = 1$$

27. **(C)**

$$\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta\ \tan 4\theta\ \tan 7\theta$$

Refer to (Q. 4) soln.

28. **(C)**

$$\cos\theta + \cos 7\theta + \cos 3\theta + \cos 5\theta = 0$$
$$2\cos 4\theta \cos 3\theta + 2\cos 4\theta \cos\theta = 0$$
$$2\cos 4\theta\left(\cos 2\theta\ \cos\theta\right) = 0$$
$$\frac{1}{8}\frac{\text{Sns}\,\theta}{\text{Sn}\,\theta} = 0, \quad \theta \neq n\pi$$
$$\text{S}\theta = n\pi$$
$$\theta = n\frac{\pi}{8}, n \neq 8k$$

29. **(B)**

$$\left(2\,\text{Sn}2x\cos x + 3\text{Sn}\,2x\right) = \left(2\cos x\cos^2 x\right) + 3\cos 2x$$
$$\text{Sn}\,2x\left(2\cos x + 3\right) - \cos 2x\left(2\cos x + 3\right) = 0$$

$\tan 2x = 1$ or 2 cos x+ 3 = 0

$$2x = n\pi + \frac{\pi}{4}$$
$$x = \frac{n\pi}{2} + \frac{\pi}{8}$$
$$\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9}{8}, \frac{13\pi}{8}$$

30. (D)

$$4\text{Sn}\theta\cos\theta - 2\cos\theta - 2\sqrt{3}\text{Sn}\,\theta + \sqrt{3} = 0$$
$$2\cos\theta\left(2\text{Sn}\,\theta - 1\right) - \sqrt{3}\left(2\,\text{Sn}\,\theta - 1\right) = 0$$
$$\left(2\cos\theta - \sqrt{3}\right)\left(2\,\text{Sn}\theta - 1\right) = 0$$
$$\text{Sn}\,\theta = \frac{1}{2}, \quad \cos\theta = \frac{\sqrt{3}}{2}$$

$\theta = \frac{\pi}{6,}\frac{5\pi}{6},\frac{11\pi}{6}$

31. **(A)**

$(\text{Sn}\,x + \cos x)(1 - \text{Sn}\,x\cos x0) + 3\sin x\cos x = 1$

$\text{Sn}\,x + \cos x = t$

$\text{Sn}\,x\cos x = \frac{t^2 - 1}{2}$

$t\left(1 - \left(\frac{t^2-1}{2}\right)\right) + 3\left(\frac{t^2-1}{2}\right) = 1$

$t\left(\frac{3-t^2}{2}\right) + \frac{3t^2}{2} - \frac{5}{2} = 0$

$\frac{t}{2} - \frac{t^3}{2} + \frac{3t^2}{2} - \frac{5}{2} = 0$

$t^3 - 3t^2 - 3t + 5 = 0$

1	1	-3	-3	5
	↓	1	-2	-5
	1	-2	-5	$\boxed{0}$

$t^2 - 2t - 5 = 0$

$t = 1 \pm \sqrt{6}$

But $\quad t \leftarrow \left[-\sqrt{2}, \sqrt{2}\right]$

So, $\quad t = 1$

Sn x + cos x = 1

$x = 2n\pi \quad \text{or} \quad 2n + \frac{\pi}{2}$

32. **(D)**

$4\text{Sn}^2x - 8\text{Snx} + 3 \geq 0$

$(2\,\text{Snx} - 1)(2\,\text{Snx} - 3) \leq 0$

$\frac{1}{2} \leq \text{Snx} \leq \frac{3}{2}$

$\text{Snx} \geq \frac{1}{2}$

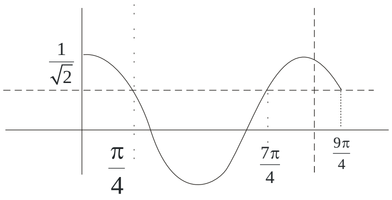
$x \leftarrow \left[\frac{\pi}{6}, \frac{5}{6}\right]$

33. **(C)**

$\cos x - \text{Sn}\,x \geq 1 \qquad x \leftarrow [0, 2\pi]$

$\cos\left(x + \frac{\pi}{4}\right) \geq \frac{1}{\sqrt{2}} \qquad \theta \leftarrow \left[\frac{\pi}{4}, \frac{9\pi}{4}\right]$

$\cos\theta \geq \frac{1}{\sqrt{2}}$



$$\frac{7\pi}{4} \le \theta \le \frac{9\pi}{4}$$

$$\frac{7\pi}{4} \le x + \frac{\pi}{4} \le \frac{9\pi}{4}$$

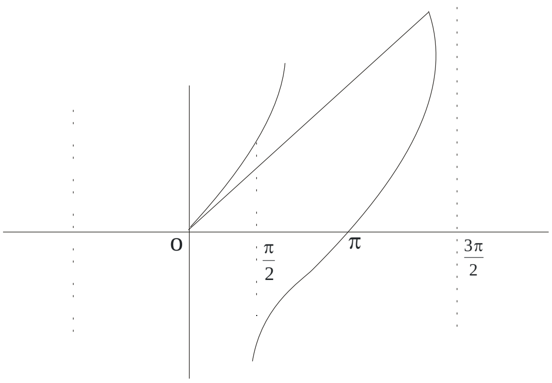
$$\frac{3\pi}{2} \le x \le 2\pi$$

x = 0 also satisfied

$$x \leftarrow \left[\frac{3\pi}{2}, 2\pi\right] \cup \{0\}$$

34. **(C)**

tan x = x



Soln. lies $\left(\pi, \frac{3\pi}{2}\right)$

35. **(B)**

$$\frac{2^{Sn\theta} + 2^{-\cos\theta}}{2} \ge \sqrt{2^{(Sn\theta - \cos\theta)}}$$

$$\ge \sqrt{2^{-\sqrt{2}}}$$

$$\ge 2^{-1/\sqrt{2}}$$

So $2^{Sn\theta} + 2^{-\cos\theta} \ge 2^{1-\frac{1}{\sqrt{2}}}$

When

$$Sn\theta = -\cos\theta = -\frac{-1}{\sqrt{2}}$$

i.e. $\theta = \frac{2n\pi + 7\pi}{4}$

36. **(A)**

$$\left(\sqrt{3}-1\right)Sn\,\theta + \left(\sqrt{3}+1\right)\cos\theta = 2$$

$$\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)Sn\,\theta + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\cos\theta = \frac{1}{\sqrt{2}}$$

$$Sn15^o\,Sn\,\theta + \cos15^o\,\cos\theta = \cos45^o$$

$$\cos\left(\theta-15^{o}\right)=\cos 45^{o}$$

$$\theta-\frac{\pi}{12}=2n\pi\pm\frac{\pi}{4}$$

$$\theta=2n\pi K\pm\frac{\pi}{4}+\frac{\pi}{12}$$

37. **(C)**

$$4Sn\theta\ Sn\,2\theta Sn\,4\theta=Sn\,3\theta$$

$$\Rightarrow 2\left(\cos\theta-\cos 3\theta\right)Sn\,4\theta-Sn\,3\theta$$

$$\Rightarrow\left(Sn\,5\theta+Sn\,3\theta-\left(Sn\,7\theta+Sn\,\theta\right)\right)=Sn\,3\theta$$

$$Sn\,\theta+Sn\,7\theta-Sn\,5\theta=0$$

$$Sn\,\theta+2Sn\,\theta\cos 6\theta=0$$

$$Sn\theta=0 \quad \text{or} \quad \cos 6\theta=\frac{-1}{2}$$

$$\theta=n\pi \qquad 6\theta=2n\pi\pm\frac{2\pi}{3}$$

$$\theta=\frac{n\pi}{3}\pm\frac{\pi}{9}$$

$$\theta=\left(3n\pm 1\right)\frac{\pi}{9}$$

38. **(C)**

$$8\cos x\cos 2x\cos 4x=\frac{Sn\,6x}{Snx}$$

$$n\neq n\pi$$

$$\frac{Sn\,8x}{Sn\,x}=\frac{Sn\,6x}{Sn\,x}$$

$$Sn\,8x=Sn\,6x$$

$$2Sn\,x\ \cos 7x=0$$

$$x=n\pi \quad \text{or} \qquad 7x=\left(2n+1\right)\frac{\pi}{2}$$

$$x=\left(2n+1\right)\frac{\pi}{14}$$

$$x=\frac{n\pi}{7}+\frac{\pi}{14}$$

39. **(B)**

$$Sn\,3\alpha=4Sn\,\alpha\left(Sn^{2}x-Sn^{2}\alpha\right)$$

$$3Sn\alpha-4Sn^{3}\alpha=\ 4Sn\alpha\ \ Sn^{2}x-4Sn^{3}\alpha$$

$$3Sn\alpha=4Sn\alpha \qquad Sn^{2}x$$

$$Sn^{2}x=\frac{3}{4}$$

$$x=n\pm\frac{\pi}{3}$$

40. **(B)**

$$\tan\left(\cot x\right)=\cot\left(\tan x\right)$$

$$= \tan\left(\frac{\pi}{2} - \tan x\right)$$

$$\cot x = \frac{\pi}{2} - \tan x + n\pi$$

$$\frac{2}{Sn\, 2x} = n\pi + \frac{\pi}{2} = (2n+1)\frac{\pi}{2}$$

$$Sn\, 2x = \frac{4}{(2n+1)\pi}$$

41. **(C)**

$$12\cos^3 x - 7\cos^2 x + 4\cos x - 9 = 0$$

1	12	-7	4	-9
	↓	12	5	0
	12	5	9-5	

$$(\cos x - 1)(12\cos^2 x + 5\cos x + 9) = 0$$

cos x = 1

$$x = (2n\pi)$$

Infinite Soln.

42. **(A)**

$$\tan 3\theta + \tan\theta = 2\tan 2\theta$$

$$\frac{Sn\, 2\theta}{\cos 3\theta \cos\theta} = \frac{2\, Sn\, 2\theta}{\cos 2\theta}$$

$\therefore$ $\cos 2\theta = 2\cos 3\theta \cos\theta$

$\cos 2\theta = \cos 4\theta + \cos 2\theta$

$\therefore$ $\cos 4\theta = 0$

$$4\theta = (2n+1)\frac{\pi}{2}$$

$$\theta \ne (2n+1)\frac{\pi}{2}$$

$$(2n+1)\frac{\pi}{6}$$

$$(2n+1)\frac{\pi}{4}$$

Either

$$Sn\, 2\theta = 0$$

$$2 = n\pi$$

$$\theta = \frac{n\pi}{2}$$

But $\theta \ne \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

$$\theta = m\pi$$

$$\theta = (2n+1)\frac{\pi}{8}$$

43. **(D)**

$$\tan\left(\frac{p\pi}{4}\right)=\tan\left(\frac{\pi}{2}-\frac{2}{4}\right)$$

$$\frac{p\pi}{4}=n\pi+\frac{\pi}{2}-\frac{9\pi}{4}$$

$$(p+q)=4n+2$$

$$=2(2n+1)$$

44. **(C)**

$$\tan(\pi\cos n)=\cot(\pi Sn\,x)$$

$$=\tan\left(\frac{\pi}{2}-\pi Snx\right)$$

$$\pi\cos x=\frac{\pi}{2}-\pi Sn\,x+n\pi$$

$$\sqrt{2}\le Snx+\cos x=n+\frac{1}{2}\le\sqrt{2}$$

$$\sqrt{2}\cos\left(\frac{\pi}{4}-x\right)=n+\frac{1}{2},\quad n=0,-1$$

$$=\pm\frac{1}{2}$$

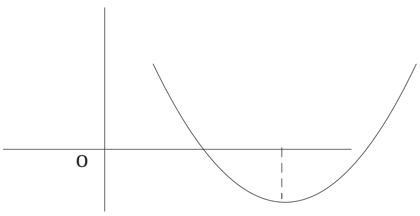
$$\cos\left(\frac{\pi}{4}-x\right)=\pm\frac{1}{2\sqrt{2}}$$

45. **(C)** $\cos^4 x+a\cos^2 x+1=0$

$D\ge 0$ $\quad a^2-4\ge 0$

$|a|>2$

Product of roots = 1



So both roots cannot lie in [0, 1]

Hence one root > 1 & one root lie $\in$ (0, 1)

So $\quad f(1)<0$

$1+a+1\le 0$

$a\le -2$

$\therefore \quad a\leftarrow(-0,-2]$

46. **(C)**

$$\tan^4 x-2\tan^2 x-2+a^2=0$$

$$\tan^4 x+2ta^2x+1$$

$$=3-a^2$$

$$\left(ta^2x-1\right)^2=3-a^2\ge 0$$

$$|a|\le\sqrt{3}$$

47. **(A)**

$x^2+4+3Sn(ax+b)-2x=0$

$(x^2-2x+1)+3(1+Sn(ax+b))=0$

$(x-1)^2+3(1+Sn(ax+b))=0$

$(x-1)=0 \quad \& \quad Sn(ax+b)=-1$

$x=1 \ \& \ Sn[a+b]=-1$

$a+b=\frac{3\pi}{2},\frac{7\pi}{2}$ etc.

$\therefore a+b=\frac{7\pi}{2}$

48. **(B)**

$3Sn\,x+4\cos ax=7$

Sn x = 1 & cos ax = 1

$x \quad =2n\pi+\frac{\pi}{2}, ax=2n\pi$

$= (4n+1)\frac{\pi}{2} \qquad \therefore \frac{a\pi}{2}(4n+1)=2m\pi$

$a=\frac{4m}{4n+1}$

$a=\frac{4m}{4n+1} \qquad\qquad m(4n+1)K$

$a=\frac{4n(4n+DK)}{4n+1}$

A = 4mk

49. **(A)**

$|Sn\,x+\cos x|=|Sn\,x|+|\cos x|$

$\therefore \quad Sn\,x\ \cos x\ge 0$

I & III Quadrant

50. **(B)**

$\frac{Sin^3\theta-\cos^3\theta}{Sn\,\theta-\cos\theta}-\frac{\cos\theta}{\sqrt{1+\cot^2\theta}} \qquad \theta\ne\frac{\pi}{4}$

$-2\tan\theta\ \cot\theta=-1$

$1+Sn\,\theta\ \cos\theta-\cos\theta|Sn\ \theta|-2=-1$

$Sn\,\theta\cos\theta=\cos\theta|Sn\,\theta|$

$|Sn\,\theta|=Sn\,\theta$

$\theta\leftarrow\left(\frac{\pi}{2},\pi\right)$

51. **(B)**

$\frac{2^{Sn\,x}+2^{Cos\,x}}{2}\ge\sqrt{2^{(Sn\,x+\cos x)}}$

$\ge\sqrt{2^{-\sqrt{2}}}$

$\geq 2^{-1/2}$

$\therefore 2^{Snx} + 2^{\cos x} \geq 2^{1-\frac{1}{2}}$

Equal if Snx = cosx = $\frac{-1}{2}$

i.e $x = \frac{5\pi}{4}$

$x = 2n\pi + \frac{5\pi}{4}$

$x = (2n+1)\pi + \frac{\pi}{4}$

52. **(C)**

$Sn(\pi(x^2 + x)) = Sn\ \pi x^2$

$\pi(x^2 + x) = n\pi + (-1)^n(\pi x^2)$

$\pi x^2 + \pi x = 2n\pi + \pi x^2$ or $\pi x^2 + \pi x = (2n+1)\pi - \pi x^2$

x = 2n

but $x \neq I$

$2x^2 + x - (2n+1) = 0$

$x = \frac{-1 \pm \sqrt{1 + 8(2n+1)}}{4}$

$x = \frac{\sqrt{1+8(2n+1)} - 1}{4}$

$x = \frac{\sqrt{odd} - 1}{} 4$

53. **(C)**

$\cos x = 1$ $\quad x = 2n\pi$

$\cos 2\lambda x = 1$ $\quad 2\lambda x = 2m\pi$

$4\lambda n\pi = 2m\pi$

$\lambda = \frac{m}{2n}$ will have

Infinite soln. if λ is rational

If λ is Inatiael the

$\lambda = 0$ is the only solution

$\therefore$ λ is inativel

54. **(D)**

$2^{(1-2Sn^2x)} - 3(2^{-2Sn^2x}) + 1 = 0$

$2(2^{-\sin^2 x})^2 - 3(2^{-Sn^2x}) + 1 = 0$

$2t^2 - 3t + 1 = 0$

t = 1, $\frac{1}{2}$

$2^{Sn^2x} = 1$ or 1

$x = n\pi,\ \ n\pi \pm \frac{\pi}{2}$

55. **(C)**

$\sqrt{3}\cos\theta - 3\text{Sn}\,\theta = 4\text{Sn}\,2\theta\cos 3\theta$

$= 2(\text{Sn}\,5\theta) - 2\text{Sn}\,\theta$

$\sqrt{3}\cos\theta - \text{Sn}\,\theta = 2\text{Sn}\,5\theta$

$2\sin\left(\frac{\pi}{3} - \theta\right) = 2\text{Sn}\,5\theta$

$\text{Sn}\left(\frac{\pi}{3} - \theta\right) = \text{m}\,5\theta$

$\theta = 2\text{Sn}\left(3\theta - \frac{\pi}{6}\right)\cos\left(2\theta + \frac{\pi}{6}\right)$

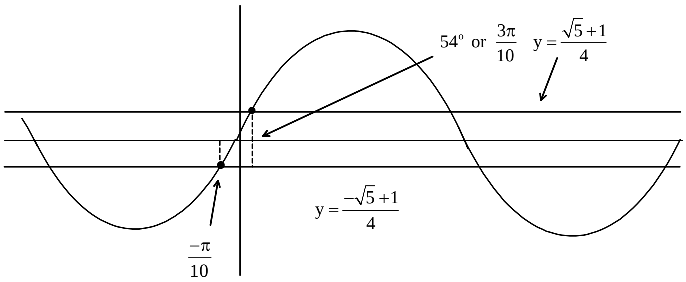
$3\theta - \frac{\pi}{6} = n\pi$ & $2\theta + \frac{\pi}{6} = (2n+1)\frac{\pi}{2}$

$3\theta = n\pi + \frac{\pi}{6}$, $\theta = n\pi + \frac{\pi}{3}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{18}$

56. **(A)**

$\frac{-\sqrt{5}+1}{4} < \text{Sn}\,x < \frac{\sqrt{5}+1}{4}$

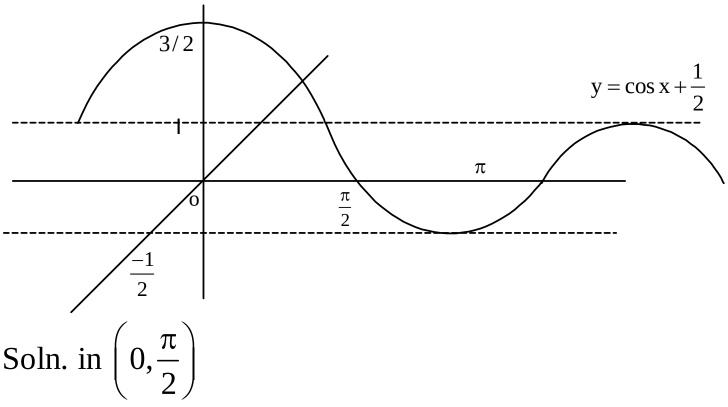


$x \leftarrow \left(\frac{-\pi}{10}, \frac{3\pi}{10}\right)$

57. **(A)**

$\cos x - x + \frac{1}{2} = 0$

$x = \cos x + \frac{1}{2}$ $y = x$



Soln. in $\left(0, \frac{\pi}{2}\right)$

58. **(B)**

$a\,\text{Sn}\,x + 1 - 2\text{Sn}^2x = 2a - 7$

$2Sn^2x - a\,Sn\,x + 2a - 7 - 1 = 0$

$2Sn^2x - a\,Sn\,x + 2(a-4) = 0$

At least one root $\in [-1,1]$

$D = a^2 - 16(a-4) \ge 0$

$a^2 - 16a + 6 \ge 0$

$(a-8)^2 \ge 0$

Snx= $\frac{a \pm (a-8)}{4}$

$= \frac{8}{4}$ or $\frac{2a-8}{4}$

= 2 or $\frac{a-4}{2}$

$-a \le \frac{a-4}{2} \le 1$

$-2 \le a - 4 \le 2$

$2 \le a \le 6$

59. **(D)**

$Sn\,x + \cos x = y^2 - y + a$

$y^2 - y = \left(y = \frac{1}{2}\right)^2 + \frac{3}{4} \ge \frac{3}{4}$

$\therefore \quad y^2 - y + a \ge \frac{3}{4} + a$

$\sin x + \cos x \le \sqrt{2}$

$\therefore$ if $\frac{3}{4} + a > \sqrt{2}$ then no soln.

$a > \sqrt{2} - \frac{3}{4}$

$a > 1.414 - 0.75$

$a \in \left(\sqrt{3}, \infty\right)$

60. **(A)**

$4Sn^2x + \tan^2 x + \cos ec^2x + \cot^2 x - 6 = 0$

$(2Snx - \cos ec)^2 + (\tan x - \cot x)^2 = 0$

$2Sn\,x - \frac{1}{Sn\,x} = 0$ & $\tan - \cot x = 0$

$Sn^2x = \frac{1}{2}$ & $\tan^2 x = 1$

$x = n\pi k \pm \frac{\pi}{4}$

TRIGO EQUATION
EXERCISE – 2A

1. (AD)

cos x = tan x

$\Rightarrow \cos^{-2} x = \sin x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$$\sin x = \frac{-1 \pm \sqrt{5}}{2}$$

$$= \frac{\sqrt{5} - 1}{2} \simeq 0.62$$

So, $x \in \left(30^0, 45^0\right)$ or $\left(135^0, 150^0\right)$

2. (AB)

$\sin^3 A + \cos^2 B = 2$

$\sin^2 A \le 1$ & $\cos^2 B \le 1$

So, $\sin A = \pm 1$, $\cos B = \pm 1$

$A = (2n+1)\frac{\pi}{2}$, $B = n\pi$

3. (AB)

$2\sin^2 x + 5\sin x \cos x + \cos^2 x + 1 = 0$

Multiple & dividing by $\cos^2 x$

$\Rightarrow 2\tan^2 x + 5\tan x + 1 + 1 + \tan^2 x = 0$

$\Rightarrow 3\tan^2 x + 5\tan x + 2 = 0$

$\Rightarrow (3\tan x + 2)(\tan x + 1) = 0$

tan = –1 or $-2/3$

4. (BD)

sin x. cos 3x – cos x . sin 3x > 0

$\Rightarrow \sin(-2x) > 0$

$\Rightarrow \sin 2x < 0$

$2x \in (\pi, 2\pi) \cup (3\pi, 4\pi)$

$x \in \left(\frac{\pi}{2}, \pi\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

5.

$|x| + |y| = 10$

(A) sin (x + y) = 0

$\Rightarrow x \in y = n\pi$; n = 0, 1, 2, 3, –1, –2, –3

7 line 2 points = 14

(B) sin2x = sin2y

$\Rightarrow 2\sin(x - y)\cos(x + y) = 0$

$\Rightarrow x - y = n\pi$ or $x + y = (2m+1)\pi/2$

n = 0, 1, 2, 3, –1, –2, –3 14 points

m = 0, $\pm 1, \pm 2$ m = 3 12 points

= 26

(C) sin2x . sin2y = 0

$2x = n\pi, 2y = n\pi$

$n = 0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6$ 26 points

$m = 0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6$ 26 points

Hence 52

(D) $|\sin x| = |\sin y|$

$\Rightarrow x = n\pi \pm y_2$

$\Rightarrow x \pm y = n\pi$

$n = 0, \pm1, \pm2, \pm3$ 28 points

6. (AB)

$\Rightarrow \cos(\theta - \alpha) = a \qquad \Rightarrow \sin(\theta - \alpha) = \sqrt{1 - a^2}$

$\Rightarrow \sin(\theta - \beta) = b \qquad \Rightarrow \sin(\theta - \beta) = \sqrt{1 - b^2}$

Now, $\sin(\alpha - \beta) = \sin((\theta - \beta) - (\theta - \alpha))$

$= \sin(\theta - \beta)\cos(\theta - \alpha) - \cos(\theta - \beta)\sin(\theta - \alpha)$

$= ab - \sqrt{1 - b^2}\sqrt{1 - \alpha^2}$

$= ab - \sqrt{1 - a^2 - b^2 + a^2 b^2}$ …(A)

And $\cos(\alpha - \beta) = \cos((\theta - \beta) - (\theta - \alpha))$

$= \cos(\theta - \beta)\cos(\theta - \alpha) - \sin(\theta - \beta)\sin(\theta - \alpha)$

$= \sqrt{1 - b^2} \times a + b \times \sqrt{1 - a^2}$

$= a\sqrt{1 - b^2} + b\sqrt{1 - a^2}$ … (B)

7. (ABCD)

$\Rightarrow \sin 5\theta = a\sin^5\theta + b\sin^3\theta + c\sin\theta + d$

$\Rightarrow \sin 5\theta = \sin(3\theta + 2\theta)$

$\Rightarrow \sin 3\theta \cos 2\theta + \cos 3\theta \sin 2\theta$

$= (3\sin\theta + 4\sin^3\theta)(\cos^2\theta - \sin^2\theta) + (4\cos^3\theta - 3\cos\theta)(2\sin\theta\cos\theta)$

$= (3\sin\theta + 4\sin^3\theta)(\cos^2\theta - \sin^2\theta) + (4\cos^3\theta - 3\cos\theta)(2\sin\theta\cos\theta)$

$= (3\sin\theta + 4\sin^3\theta)(1 - 2\sin^2\theta) + 2(4\cos^2\theta - 3)\sin\theta(1 - \sin^2\theta)$

Let $\sin\theta = t$

Then, $\sin 5\theta = 16t^5 - 20t^3 + 5t$

Therefore a = 16

$\Rightarrow$ b = –20

$\Rightarrow$ c = 5

$\Rightarrow$ d = 0

$\Rightarrow$ a + b + c + d = 10 – 20 + 5 + 0 = 1 … (A)

$\Rightarrow$ a + b + c = 16 – 20 + 5 =1 … (B)

$\Rightarrow$ 5a + 4b = 80 – 80 = 0 … (C)

$\Rightarrow$ b + 4c = –20 + 20 = 0 … (D)

8. (BC)

$\Rightarrow x + y = \frac{\pi}{4}$

$\Rightarrow$ tan x + tan y = 1

$\Rightarrow$ tan x = 1 – tan y

$\Rightarrow \tan\left(\frac{\pi}{4} - y\right) = 1 - \tan y$

$\Rightarrow \frac{1-\tan y}{1+\tan y} = 1 - \tan y$

$\Rightarrow (1-\tan y)\left(\frac{1}{1+\tan y} - 1\right) = 0$

$\Rightarrow (1-\tan y)(1-\tan y) = 0$

$\Rightarrow$ 1 – tan y = 0 or tan y = 0

$\Rightarrow$ tan y = 1   tan x = 1

$\Rightarrow y = n\pi + \frac{\pi}{4}$ $x = n\pi + \frac{\pi}{4}$

But $x + y = \frac{\pi}{4}$ $x + y = \frac{\pi}{4}$

$\Rightarrow \therefore x = -n\pi$ $\therefore y = -n\pi$

(C)   (B)

9. (BD)

$\Rightarrow \frac{4\sin^2 x\cos^2 x + 4\sin^4 x - 4\sin^2 x\cos^2 x}{4 - 4\sin^2 x\cos^2 x - 4\sin^2 x} = \frac{1}{9}$

$\Rightarrow \frac{4\sin^4 x}{4(1-\sin^2 x) - \sin^2 x\cos^2 x} = \frac{1}{9}$

$\Rightarrow \frac{\sin^4 x}{\cos^2 x - \sin^2 x\cos^2 x} = \frac{1}{9}$

$\Rightarrow \frac{\sin^4 x}{\cos^2 x(1-\sin^2 x)} = \frac{1}{9}$

$\Rightarrow \frac{\sin^4 x}{\cos^4} = \frac{1}{9}$

$\Rightarrow \tan^4 x = \frac{1}{9}$

$\Rightarrow \tan^2 x = \pm\frac{1}{3}$

$\Rightarrow \tan^2 x = \frac{1}{3}$ or $\tan^2 x = -\frac{1}{3}$

$\Rightarrow \tan x = \pm\frac{1}{\sqrt{3}}$ or   not possible

$\Rightarrow \tan x = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow \tan x = \frac{1}{\sqrt{3}}$ or $\tan x = -\frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

(B) (D)

10. (AB)

$\Rightarrow 4\sin^4 x + \cos^4 x = 1$

$\Rightarrow 4\sin^4 x = 1 - \cos^4 x$

$\Rightarrow 4\sin^4 x = \left(1-\cos^2 x\right)\left(1+\cos^2 x\right)$

$\Rightarrow 4\sin^4 x - \sin^2 x\left(1+\cos^2 x\right) = 0$

$\Rightarrow \sin^2 x - \left(4\sin^2 x - 1 - \cos^2 x\right) = 0$

$\Rightarrow \sin^2 x\left(4\sin^2 x - 1 - \cos^2 x\right) = 0$

$\Rightarrow \sin^2 x\left(4\sin^2 x - 1 - 1 + \sin^2 x\right) = 0$

$\Rightarrow \sin^2 x\left(5\sin^2 x - 2\right) = 0$

$\Rightarrow \sin^2 x = 0$ or $5\sin^2 x = 2$

$\Rightarrow x = n\pi$ $\sin^2 x = \frac{2}{5}$

(A) $\sin x = \pm\sqrt{\frac{2}{5}}$

$x = n\pi \pm \sin^{-1}\sqrt{\frac{2}{5}}$

(B)

11. (AC)

$\Rightarrow \tan^2\theta + \cos 2\theta = 1$

$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} + \cos^2\theta - \sin^2\theta = 1$

$\Rightarrow \sin^2\theta + \cos^4\theta - \sin^2\theta\cos^2\theta = \cos^2\theta$

$\Rightarrow \sin^2\theta + \left(1-\cos^2\theta\right) = \cos^2\theta - \cos^4\theta$

$\Rightarrow \sin^4\theta = \cos^2\theta\left(1-\cos^2\theta\right)$

$\Rightarrow \sin^4\theta - \sin^2\theta\cos^2\theta = 0$

$\Rightarrow \sin^2\theta\left(\sin^2\theta - \cos^2\theta\right) = 0$

$\Rightarrow \sin^2\theta = 0$ $\sin^2\theta - \cos^2\theta = 0$

$\Rightarrow \theta = n\pi$ $\tan^2\theta = 0$

$\theta = n\pi \pm \frac{\pi}{4}$

12. (BD)

$\Rightarrow \sin\theta + \sqrt{3}\cos\theta = 6x - x^2 - 11$ $\theta \in [0, 4\pi]$

$\Rightarrow \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta = \frac{6x - x^2 - 11}{2}$

$\Rightarrow \sin\theta\cos\frac{\pi}{3} + \cos\theta\sin\frac{\pi}{3} = \frac{6x - x^2 - 11}{2}$

$\Rightarrow \sin\left(\theta+\frac{\pi}{3}\right)=\frac{6x-x^2-11}{2}$

L.H.S. $=\sin\left(\theta+\frac{\pi}{3}\right)$

$\Rightarrow -1\le \sin\left(\theta+\frac{\pi}{3}\right)\le 1$

R.H.S. $=\frac{6x-x^2-11}{2}$

$\Rightarrow 6x - x^2 - 11$ max value will ne $\frac{-D}{4a}$

$\Rightarrow \frac{-D}{4a}=\frac{-(36-44)}{4\times(-1)}=-2$ at $\frac{-b}{2a}=\frac{-6}{-2}$

So max value of $\frac{6x-x^2-11}{2}$ is –1

Therefore, L.H.S. and R.H.S. are equal at –1 for x = 3

$\Rightarrow \sin\left(\theta+\frac{\pi}{3}\right)=-1$

$\Rightarrow \theta=(2n+1)\pi-\frac{\pi}{3}$

$\Rightarrow \theta=\pi-\frac{\pi}{3}$ and $\theta=3\pi-\frac{\pi}{3}$

$\Rightarrow \theta=\frac{2\pi}{3}$ and $\Rightarrow \frac{8\pi}{3}$

For x = 3

$\Rightarrow \theta=\frac{2\pi}{3}$ and $\frac{8\pi}{3}$

13. (ABD)

$\Rightarrow \cos\left(x+\frac{\pi}{3}\right)+\cos x=a$

$\Rightarrow \cos x\cos\frac{\pi}{3}-\sin x\sin\frac{\pi}{3}+\cos x=a$

$\Rightarrow \frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\cos x=a$

$\Rightarrow \frac{3}{2}\cos x-\frac{\sqrt{3}}{2}\sin x=a$

$\Rightarrow \therefore |a|\le\sqrt{\left(\frac{3}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{9}{4}+\frac{3}{4}}=\sqrt{3}$

Integral solution of $|a|\le\sqrt{3}$ is 0, –1, 1 (A)

Sum of integral values of a is = 0 + 1 + (–1) = 0 (B)

For a = 1

$\Rightarrow \frac{3}{2}\cos x-\frac{\sqrt{3}}{2}\sin x=1$

$$\Rightarrow \sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right) = 1$$

$$\Rightarrow \cos\left(\frac{\pi}{6} + x\right) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\pi}{6} + x = 2n\pi \pm \cos^{-1}\frac{1}{\sqrt{3}}$$

$$\Rightarrow x = 2nm \pm \cos^{-1}\frac{1}{\sqrt{3}} + \frac{\pi}{6}$$

No solution in $[0, 2\pi]$ (D)

14. (AC)

$$\Rightarrow x + y = \frac{2\pi}{3} \qquad \frac{\sin x}{\sin y} = 2$$

$$\Rightarrow y = \frac{2\pi}{3} - x \qquad \sin x = 2 \sin y$$

$$\Rightarrow \therefore \sin x = 2\sin\left(\frac{2\pi}{3} - x\right)$$

$$= 2\left(\sin\frac{2\pi}{3}\cos x - \cos\frac{2\pi}{3}\sin x\right)$$

$$= 2\left(\cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x\right)$$

$$= 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right)$$

$$= \sqrt{3}\cos x + \sin x$$

$$\Rightarrow \sqrt{3}\cos x = 0$$

$$\Rightarrow \cos x = 0$$

$$\Rightarrow x = (2n+1)\frac{\pi}{2}$$

$\therefore$ 2 solutions in $[0, 2\pi]$

And 4 solutions in $[0, 4\pi]$

Now, $x + y = \frac{2\pi}{3}$

$$\Rightarrow y = \frac{2\pi}{3} - x = \frac{2\pi}{3} - (2n+1)\frac{\pi}{2}$$

$$= \frac{2\pi}{3} - n\pi - \frac{\pi}{2}$$

$$= \frac{\pi}{3} - n\pi$$

$$= m\pi + \frac{\pi}{3}$$

$\therefore$ 2 solutions in $[0, 2\pi]$

And 4 solutions in $[0, 4\pi]$

15. (BD)

$\Rightarrow |\cos x| = \cos x - 2\sin x$

Case – I

$\Rightarrow \cos x \geq 0$ $\Rightarrow$ cos x = cos x – 2 sin x

$\Rightarrow$ 2 sin x = 0 $\Rightarrow$ sin x = 0

$\Rightarrow x = n\pi$

For $n \to$ even for $n \to$ odd

$\Rightarrow$ cos x = 1 cos x = –1

But cos $\geq$ x

$\Rightarrow \therefore x = 2n\pi$ (B)

Case – II

$\Rightarrow$ cos x < 0 $\Rightarrow$ – cos x = cos x – 2 sin x

$\Rightarrow$–2cos x = –2 sin x $\Rightarrow$ cos x = sin x

$\Rightarrow$ tan x = 1 $\Rightarrow x = n\pi + \frac{\pi}{4}$

$\Rightarrow$ n $\to$ even n $\to$ odd

$\Rightarrow \cos x = \frac{1}{\sqrt{2}}$ $\cos x = -\frac{1}{\sqrt{2}}$

As coc x < 0

$\Rightarrow \therefore x = (2n+1)\pi + \frac{\pi}{4}$ (D)

16. (AC)

$\Rightarrow \cos\left(\pi\sqrt{x-4}\right)\cos\left(\pi\sqrt{x}\right) = 1$

Case – 1

$\Rightarrow \cos\left(\pi\sqrt{x-4}\right) = \cos\left(\pi\sqrt{x}\right) = 1$

$\Rightarrow \cos\pi\sqrt{x-4} = 1$ and $\cos\pi\sqrt{x} = 1$

$\Rightarrow \pi\sqrt{x-r} = 2n\pi$ $\pi\sqrt{x} = 2m\pi$

$\Rightarrow \sqrt{x-4} = 2n$ $\sqrt{x} = 2m$

$\Rightarrow \therefore x = 4n^2 + 4 = 4m^2$

$\Rightarrow n^2 - m^2 = 1$

$\Rightarrow n = \pm 1$ and m = 0 as $x = \pm 1$ and n = 0

$\Rightarrow \therefore$ x = 4

Case – 2

$\Rightarrow \cos\pi\sqrt{x-4} = \cos\pi\sqrt{x} = -1$

$\Rightarrow \cos\pi\sqrt{x-4} = -1$ and $\cos\pi\sqrt{x} = -1$

$\Rightarrow \pi\sqrt{x-4}(2n+1)\pi$ $\pi\sqrt{x} = (2m+1)\pi$

$\Rightarrow \sqrt{x-4} = (2n+1)$ $\sqrt{x} = (2m+1)\pi$

$\Rightarrow x = (2n+1)^2 + 4$ $x = (2m + 1)^2$

$\Rightarrow \therefore (2n + 1)^2 + 4 = (2m + 1)^2$

$\Rightarrow (2m + 1)^2 - (2n + 1)^2 = 4$

$\Rightarrow$(2m +1 + 2n + 1) (2m + 1 – 2n – 1)= 4

$\Rightarrow$ 2(m + n + 1) 2 (m – n) = 4
$\Rightarrow$ (m + n + 1) (m – n) = 1
$\Rightarrow$ m + n + 1 = 1 m + n + 1 = –1
m – n = 1 m – n = –1
Hence, 1 solution

17. (BD)
y = 2 sin x $y \in [-2, 2]$
y = 5x² + 2x + 3, $y \in \left[-\frac{(40-60)}{20}, \infty \right)$
$y \in \left[\frac{56}{20}, \infty \right)$
Hence no solution

18. (BC)
$x^3 + x^2 + 4x + 2 \sin x = 0$
x = 0 is a solution
sin x < 0, x > π
When for, $x > \pi$, $x^3 + x^2 + 4x > 1$
Hence, no solution in $x \in (\pi, 2\pi)$
Hence, x =0 is the only solution

19. (AD)
$\tan^2(x+y) + \cot^2(x+y) \ge 2$
$1 - 2x + x^2 = 2 - (1+x)^2 \le 2$
Equations hold at x = –1 and $x + y = \frac{n\pi}{2} + \frac{\pi}{4}$
$y = \frac{n\pi}{2} + \frac{n\pi}{4} + 1$

20. (ACD)
$|\cos x|^{\sin^2 x - \frac{3}{2}\sin x + \frac{1}{2}} = 1$
$\Rightarrow$ either $|\cos x| = 1 \Rightarrow x = n\pi$
or, $\sin^2 x - \frac{3}{2}\sin x + \frac{1}{2} = 0$
$\Rightarrow$ sin x = 1, $\sin x = \frac{1}{2}$
But at sin x = 1, cos x = 0 (not possible)
So, $\sin x = 1/2$
$x = n\pi + (-1)^n \pi/6$

21. (ACD)
$\cos^2(\pi x) - \sin^2(xy) = 1/2$
$\Rightarrow 1 + \cos \pi x - 1 + \cos 2\pi y = 1$

$\Rightarrow 2\cos(\pi(x+y)).\cos(\pi(x-y))=1$

$\Rightarrow \cos\pi(x+4)=1$

$\Rightarrow \pi(x+y)=2n\pi$

$\Rightarrow x+y=2n$

$x-y=1/3$

$x=n+\frac{1}{6}, y=n-\frac{1}{6}$

$\left(\frac{7}{6},\frac{5}{6}\right),\left(\frac{-5}{6},\frac{-7}{6}\right)$

$\left(\frac{13}{6},\frac{11}{6}\right)$

22. (ABC)

$\sqrt{\cos 2x}+\sqrt{1+\sin 2x}=2\sqrt{\cos x+\sin x}$

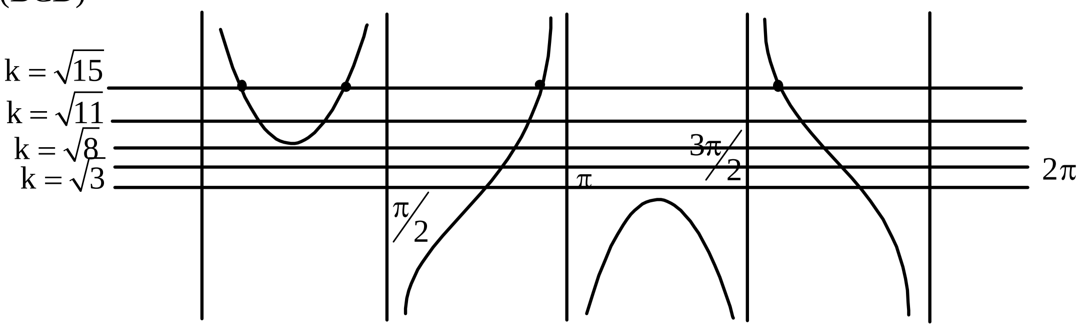
$\Rightarrow \sqrt{\cos^2 x-\sin^2 x}+\sqrt{(\cos x+\sin x)^2}-2\sqrt{\cos x+\sin x}=0$

$\Rightarrow \cos x+\sin x=0$ $\qquad$ $\sqrt{\cos x-\sin x}+\sqrt{\cos x+\sin x}=2$

$\Rightarrow \tan=-1$ $\qquad$ $\Rightarrow \cos x=1$

$\Rightarrow x=n\pi-\frac{\pi}{4}; n\in I$ $\qquad$ $\Rightarrow x=2\pi; n\in I$

23. (BCD)



4 solution $k=\sqrt{15},\sqrt{11}$

3 solution $k=\sqrt{8}$

2 solution $k=\sqrt{3}$

24. (ABC)

2 (sin x + sin y) – 2 cos (x – y) = 3

$\Rightarrow 4\sin\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)-4\cos^2\left(\frac{x-y}{2}\right)+2=3$

$\Rightarrow 4\cos^2\left(\frac{x-y}{2}\right)-4\sin\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)+1=0$

$\Rightarrow\because D=16\sin\left(\frac{x+y}{2}\right)^2-16$

For $D\geq 0, \sin\left(\frac{x+y}{2}\right)=\pm 1$

For smallest positive x & y

$$\sin\left(\frac{x+y}{2}\right)=1 \Rightarrow \frac{x+y}{2}=\frac{\pi}{2}$$

$$\cos\left(\frac{x-y}{2}\right)=1/2$$

$$\frac{x-y}{2}=\frac{\pi}{3} \quad \text{or} \quad \frac{y-x}{2}=\frac{\pi}{3}$$

$$\left(x=\frac{5\pi}{6}, y=\frac{\pi}{6}\right) \text{ or } \left(x=\frac{\pi}{6}, y=\frac{5\pi}{6}\right)$$

2 solutions

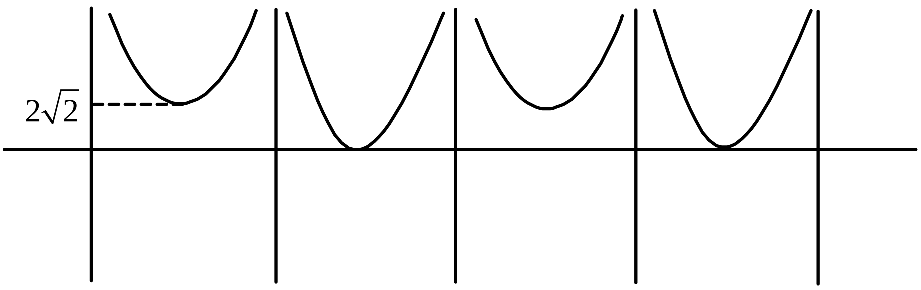
PASSAGE – I

25. (B)
Roots are x = 1, x = cos x, x = sin x
$x_1^L + x_L^2 + x_3^2 = 2$

26. (C)
For two roots equal
Either cos = 1, or $\sin\theta = 1$ or $\sin\theta = \cos\theta$
So, $\theta = 0, 2\pi, \theta = \frac{\pi}{4}, \theta = \frac{\pi}{2}, \theta = \frac{5\pi}{4}$
5 values

27. (A)
$\left|\max(\sin\theta - 1)\right| = 2$
$\left|\max(\cos\theta - 1)\right| = 2$
$\left|\max(\sin\theta - \cos\theta)\right| = \sqrt{2}$

PASSAGE – 2



28. (A)
For 8 solution, $a > 2\sqrt{2}$

29. (C)
For 6 solution, $a = 2\sqrt{2}$

30. (D)
For 2 solution, a = 0

Passage – 3

31. (A)

$\sin x.\cos 2y = \left(a^2 - 1\right)^2 + 1$ …(1)

$\cos x.\sin 2y = a + 1$ … (2)

For any value of x & y

$-1 \le \sin x.\cos 2y \le 1$

For equation (1) $a^2 = 1$ is the only value

$\Rightarrow a = \pm 1$

Out of these 2 only a = –1 satisfy equation (2)

So only one value of a

32. (B)

If a = –1

Sin x . cos 2y = 1

$x = \pi/2, y = 0, \pi, 2\pi$

$x = 3\pi/2, y = \pi/2, 3\pi/2$

cos x . sin 2y = 0

(1) $x = \pi/2, 3x/2, y \in R$

(2) $y = 0, \pi/2, \pi, 3\pi/2, x \in R$

$\therefore$ total 2 solutions

33. (D)

From above y has 5 solutions for a = –1

$y \in \left\{0, \pi/2, \pi, 3\pi/2, 2\pi\right\}$

PASSAGE – 4

34. (A)

$x\left(\cos y + \sin y\right)^3 = 27$

$x\left(\cos y - \sin y\right)^3 = 1$

Taking power 2/3 on both the sides and adding

$x^{2/3}\left(2\right) = 9 + 1$

$x^{2/3} = 5$

$x = \pm 5\sqrt{5}$

35. (D)

Dividing (1)/(2) form above

$\frac{\cos y + \sin y}{\cos y - \sin y} = 3$

cos y + sin y = 3 cos y – 3 sin y

4 sin y = 2 cos y

$\tan y = 1/2$

Total 6 solutions in $\left(0, 6\pi\right]$

36. (B)

As $\tan y = 1/2$

$\cos = \frac{2}{\sqrt{5}}$

$\therefore \sin^2 y + 2\cos^2 y = 1 + \cos^2 y$

$= 1 + \frac{4}{5}$

$= 9/5$

PASSAGE – 5

37. (B)

ABCD is a quadrilateral

$\sin^2 A + \sin^2 B + \sin^2 C + \sin^2 D = (x + 1)^2 + 4$

for equality to hold true.

$A = B = C = D = 90^0$ & $(x + 1)^2 = 0 \Rightarrow x = -1$

Then ABCD must be a rectangle

38. (C)

$\tan\theta = x = -1$ (from above question)

$\theta = n\pi - z/4$

39. (D)

$\tan^4 x - 10\tan^2 x + 9 = 0$

$(\tan^2 x - 9)(\tan^2 x - 1) = 0$

$\tan x = \pm 3, \pm 1$

Total 8 solutions in $[0, 2\pi]$

40. (C)

$D > 0$

$(-10)^2 - 4 \times 1 \times a > 0$

a = 25

$a \in (-\infty, 25)$

Also roots should be positive

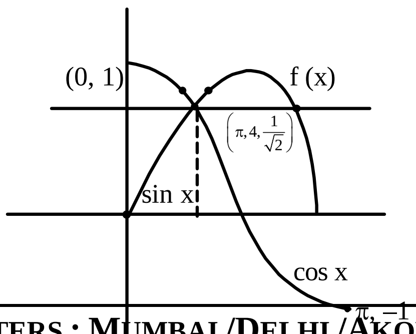
$\therefore$ product of roots > 0, $\frac{a}{1} > 0$

$\therefore a \in (0, 25)$

PASSAGE – 7

41. (C)

$f(x) = \max\{\sin x, \cos x\} = 4/5$



$\frac{4}{5} = 0.8$

$\frac{1}{\sqrt{2}} = 0.75$

$\because \frac{4}{5} > \frac{1}{\sqrt{2}}$, $y = \frac{4}{5}$ cuts f (x) at 3 points

$\therefore$ 3 solutions

42. (B)

Reasoning Type

43. (A)

Statement $(\sin x + \cos x)^{1+\sin^2 x}$

$= (\sin x + \cos x)^{(\sin x + \cos x)^2}$

$\because$ max. value of $\sin x + \cos x = \sqrt{2}$

Occurs at $x = \pi/4$

$\left(\sqrt{2}\right)^{\left(\sqrt{2}\right)^2} = 2$

44. (A)

45. (D)
Statement 1 is false
Statement 2 is true

46. (A)

$\sqrt{1 - \sin 2x} = \sin x$

$|\cos x - \sin x| = \sin x$

When $x \in \left[0, \pi/4\right]$ cos x > sin x

$\therefore$ cos x – sin x= sin x

$\Rightarrow \tan x = 1/2$

$\Rightarrow$ one solution

Statement 2 correct explanation

47. (D)

$\frac{\tan 4x - \tan 2x}{1 + \tan 4x \tan 2x} = 1$

$\Rightarrow$ tan (4x – 2x) = 1

tan 2x = 1

In this case tan 4x is always not designed

So no solution

Matrix Match

48. (A)

$\cos^2_{2x} + \cos^2 x = 1$

$\cos^2_{2x} = \sin^2 x$

$\cos^2_{2x} + \left(\cos\left(\pi/2 - x\right)\right)^2$

$2x = n\pi \pm \left(\pi/2 - x\right)$

$3x = n\pi + \pi/2$

$x = \frac{n\pi}{3} + \pi/6$ $\qquad$ $x = x\pi - \pi/2$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Hence $x = \left\{n\pi \pm \frac{\pi}{6}\right\} \cup \left\{2n\pi \pm \frac{\pi}{2}\right\}$ $\qquad$ (R)

(B) $\cos x + \sqrt{3}\sin x = \sqrt{3}$

$\Rightarrow \cos\left(x - \frac{\pi}{5}\right) = \cos \pi/6$

$\Rightarrow x - \frac{\pi}{3} = 2n\pi \pm \pi/6$

$\Rightarrow x = 2n\pi + \frac{\pi}{2}, 2n\pi + \pi/6; n \in I$ $\qquad$ (S)

(C) $1 + \sqrt{3}\tan^2 x = \left(1 + \sqrt{3}\right)\tan x$

$\Rightarrow \sqrt{3}\tan^2 x - \left(1 + \sqrt{3}\right)\tan x + 1 = 0$

$\Rightarrow \left(\sqrt{3}\tan x - 1\right)\left(\tan x - 1\right) = 0$

$\tan x = \frac{1}{\sqrt{3}}, \tan x = 1$

$x = \left\{n\pi + \frac{\pi}{4}\right\}, \left\{n\pi + \frac{\pi}{6}\right\}; n \in z$ $\qquad$ (P)

(D) tan 3x – tan2x – tan x = 0

$\Rightarrow$ tan 3x = tan2x + tan x

$\Rightarrow \frac{\tan 2x + \tan x}{1 - \tan 2x.\tan x} = \tan 2x$ of tan x

$\Rightarrow$ either tan 2x = – tan x

Or tan 2x . tan x= 0

$\Rightarrow x = n\pi$

Or $\qquad 2x = n\pi - x$

$\Rightarrow x = \frac{n\pi}{3}$

Hence $n \in \left(\frac{n\pi}{3}\right)$ $\qquad$ (Q)

49. (A) – (R)

$\cos^7 x + \sin^2 x = 1$

$\Rightarrow \cos^7 x = \cos^2 x$

$\Rightarrow \cos^2 x\,(1 - \cos^5 x) = 0$

$\Rightarrow$ cos x = 0, 1

Total 3 solution in $\left(-\pi, \pi\right)$

(B) – (Q)

$\sqrt{3}\cos ec20^0 - \sec 20^0$

$\Rightarrow \frac{\sqrt{3}}{\sin 20^0} - \frac{1}{\cos 20^0}$

$$\Rightarrow 4\times\frac{\frac{\sqrt{3}}{2}\cos 20^0-\frac{1}{2}\sin 20^0}{2\sin 20^0.\cos 20^0}$$

$$\Rightarrow 4\frac{\sin\left(60^0-20^0\right)}{\sin 40^0}$$

$\Rightarrow$ 4

(C) – (R)

$4\cos 36^0-4\cos 72^0+4\sin 18^0\cos 36^0$

= 4 cos 36^0 – 4 cos 72^0 + 2 [sin 54^0 – sin8^0]

= 6 cos 36^0 – 6 cos 72^0

$$=6\left(\frac{\sqrt{5}+1}{4}\right)-6\left(\frac{\sqrt{5}-1}{4}\right)$$

= 3

(D) – (S)

cosec x = 1 + cot x

cosec x + cot x = 1 {As cosec^{2}x – cot^{2}x = 1}

2cosec x = 2, cosec x = 1

$\therefore$ 2 solution in $[-2\pi, 2\pi]$

50. (A) – (P)

If $\cos\theta+\cos\phi=2$

$\Rightarrow \cos\theta=1$ & $\cos\phi=1$

$\Rightarrow \sin\theta=0$ & $\sin\phi=0$

So. No value of θ & ϕ will statisfy. $\sin\theta+\sin\phi=1/2$

So, no solution

(B) – (P)

$$\sin^2\alpha+\sin\left(\pi/3-\alpha\right)\sin\left(\pi/3+\alpha\right)=\sec\alpha$$

$$\cancel{\sin^2}\alpha\left(\sin^2\pi/3\right)-\cancel{\sin^2}\alpha=\sec\alpha$$

$$\frac{3}{4}=\sec\alpha$$

No solution

(C) – (Q)

$\tan\theta=3\tan\phi$

$\tan^2(\theta-\phi)$

$$=\left[\frac{\tan\theta-\tan\phi}{1+\tan\theta+\tan\phi}\right]^2$$

$$=\left[\frac{2\tan\phi}{1+0(\tan\phi)^2}\right]^2$$

Let $y=\frac{2x}{1+3x^2}$, take $x=\tan\phi$

y + 3x^{2}y = 2x

(3y)x^2 – 2x + y = 0

As x is real $D\ge 0$

$$(-2)^2 - 4\times 3y.y \ge 0$$

$$4 - 4.3y^2 \ge 0$$

$$3y^2 \le 1$$

$$y^2 \le 1/3$$

$$\therefore \left[\frac{2\tan\phi}{1+3(\tan\phi)^2}\right]^2 \le 1/3$$

TRIGO EQUATION
EXERCISE – 2 (B)

1. LHS $\le 2 \Delta$ RHS ≥ 2

$\therefore$ Equality appears when LHS = RHS = 2

$\therefore$ for RHS = 2 x = $\pm$ 1

But @ x = $\pm$ 1 LHS $\ne 2$

$\Rightarrow$ simultaneously LHS & RHS can't be $\Rightarrow$ No solution

2. $\frac{\sin^2 x + \cos^2 x}{\sin x \ \cos x} = \frac{2}{\sin x}$

$\Rightarrow \cos x = \frac{1}{2}$

$\therefore$ In $[-2\pi, 2\pi]$ No. of solution = 4

3. let $\sin x + \cos x = t$

$\Rightarrow \sin 2x = t^2 - 1$

$3t - 2(t)\left(1 - \frac{(t^2 - 1)}{2}\right) = 8$

$t - 2 + t^3 - t = 8$

$t^3 = 8$

$\Rightarrow \sin x + \cos x = 2$

No solution

4. $\sin^4 x + \cos^4 x = \sin x \cos x$

$\Rightarrow 1 - 2\sin^2 x \ \cos^2 x = \frac{\sin^2 x}{2}$

$\Rightarrow 1 - \frac{\sin^2 2x}{2} = \frac{\sin 2x}{2}$

$\sin^2 2x + \sin 2x - 2 = 0$

Sin 2x = - 2 or sin 2x = - 1

Discard in $[0, 2\pi]$ possible @ 2 values of x

5. $1 - \cos^2 \theta + 3\cos\theta = 3$

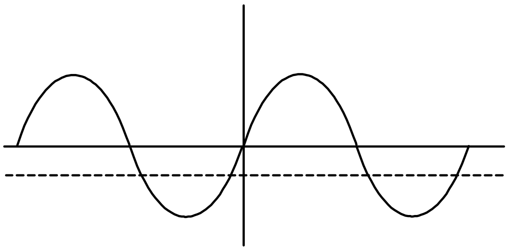
$\cos^2 \theta - 3\cos\theta + 2 = 0$

$\cos\theta = 2, \qquad \cos\theta = 1$

$\theta = 0$ 1 solution

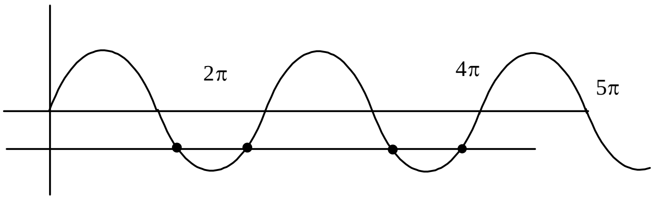
6. $\sin^2 x - \sin x - 1 = 0$

$\sin = \frac{1 \pm \sqrt{5}}{2}$ discard $\frac{1 + \sqrt{5}}{2}$



4 intersection point $\Rightarrow$ 4 soln.

7. $\sin\theta = 1+\sqrt{2}$ or $1-\sqrt{2}$



Discard

$\sin\theta = 1-\sqrt{2}$

Least n = 4

& Meα n = 5

Ans. 4

8. cos x + sin x = 2

$\Rightarrow \cos x = \sin x = 1$

$\Rightarrow \phi$

9. Me write equation

$2\sin\left(2e^{x}\right) = 2^{x} + 2^{-x}$

Now LHS ≤ 2 & RHS ≤ 2

$\therefore$ in solution to exist

LHS = RHS – 2

$\therefore$ for RHS = 2 x = 0

But @ x = 0 LHS $\neq$ 2

$\therefore$ no soln.

10. cos x sin y = 1

$\Rightarrow$ either cos x = -1 & sin y = - 1

$x = \pi, 3\pi$ & $y = \frac{3\pi}{2}$

$\left\langle \pi, \frac{3}{2} \right\rangle, \left\langle 3\pi, \frac{3\pi}{2} \right\rangle$

Or cos x = 1 sin y = 1

$x = 0, 2\pi,$ $y = \frac{\pi}{2}$ & $\frac{5\pi}{2}$

$\therefore$ Total ordered pair 6

11. $2\sin\theta = \left(r^{2}-1\right)^{2} + 2$

Now LHS ≤ 2, RHS ≥ 2

$\therefore$ for soln. LHS = RHS = 2

$\therefore$ $r = \pm 1$ & $\sin\theta = 1$

$\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

$\therefore$ ordered pair 6

12. $\sin x \cos x\left(\sin^2 x+\sin x\cos x+\cos x\right)=1$

$\Rightarrow \dfrac{\sin 2x}{2}\left(1+\dfrac{\sin 2x}{2}\right)=1$

Let sin 2x = y

$2y+y^2=4$

$y^2+2y-4=0$

$\Rightarrow \sin 2x = -1+\sqrt{5}, \quad -1-\sqrt{5}$

discard discard

13. $\sin^4 x-\sin x\left(1-\sin^2 x\right)+2\sin^2 x+\sin x=0$

$\sin^4 x+\sin^3 x+2\sin^2 x=0$

$\Rightarrow \sin^2 x=0$ or $\sin^2 x-\sin x12=0$

discard

$x=0,\pi,2\pi,3\pi$

14. $\left(1-\tan\theta\right)\left(1+\dfrac{2\tan\theta}{1+\tan^2\theta}\right)=1-1\tan\theta$

$\left(1-\tan\theta\right)\dfrac{\left(1+\tan\theta\right)^2}{1+\tan^2\theta}=1+\tan\theta$

$\Rightarrow \tan\theta=-1$ or $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}=1$

2 solution

In $\left[0,2\pi\right]$

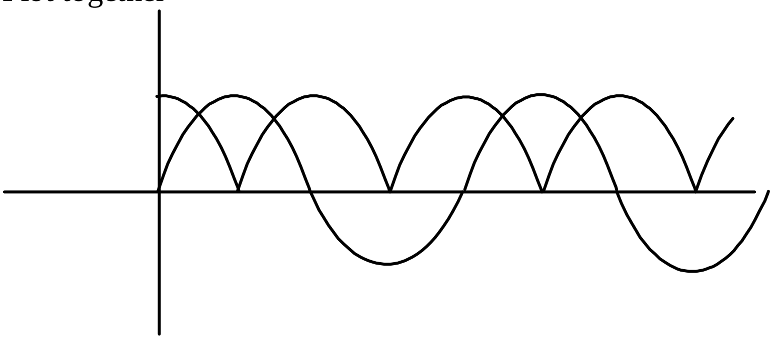
$1-\tan^2\theta=1+\tan^2\theta$

$\tan\theta=0$

3 soln.

Total 5 solution

15. Plot together



4 soln

16. $\cos x=\left|\cos x-\sin x\right|$

$\cos x\geq\sin x\Rightarrow\cos x=\cos x-\sin x$

Soln. here is $0,2\pi$

$\cos x<\sin x\Rightarrow\cos x=-\cos x+\sin x$

$nt\left(\dfrac{\pi}{4},\dfrac{5}{4}\right)$ tan x = 2

Only 1 soln. here also

A < 3

17. that is only possible when

$\log_{|\cos n|} |\sin x| = 1$

$\Rightarrow |\sin x| = |\cos x|$

tan x = y

ijn $(-2\pi, 2\pi)$

18. Case I : $\cot x \geq 0$

$\cot x = \cot x + \dfrac{1}{\sin x}$

$\Rightarrow \sin x = \infty$

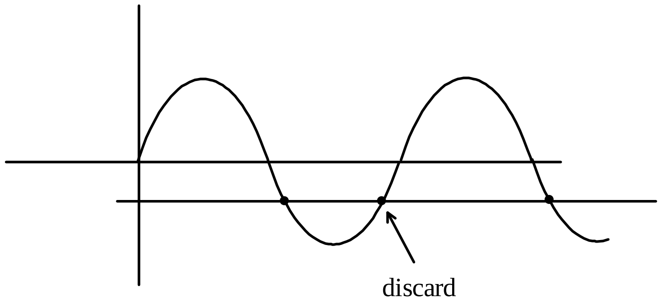
$\Rightarrow$ no soln

Case II: cot x < 0

$\Rightarrow -\cot x = \cot x + \dfrac{1}{\sin x}$

$\dfrac{-2\cos x}{\sin x} = \dfrac{1}{\sin x}$

$\Rightarrow \cos x = \dfrac{-1}{2}$



$\Rightarrow$ 2 soln.

19. $\sum \cos x = 5$ only possible when

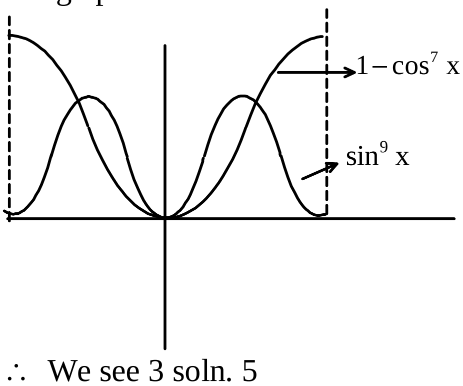
$\cos x = \cos 2x = \cos 3x = \cos^4 x = \cos^5 x = 1$

Simultaneously possible of x = 0

$\therefore$ 1 soln.

20. $\sin^4 x = 1 - \cos^7 x$

Rough plot $\sin^9 x$ & $1 - \cos^7 x$



$\therefore$ We see 3 soln. 5

21. $\text{LHS} \leq 1$ & $\text{RHS} = \left(x - \sqrt{3}\right)^2 + 1$

$\Rightarrow \text{RHS} \geq 1$

$\therefore$ for soln. to exist LHS= RHS = 1

$\Rightarrow x = \sqrt{3}$ only hence 1 soultion

22. $\sin x + \sin y = \sin(x+y)$

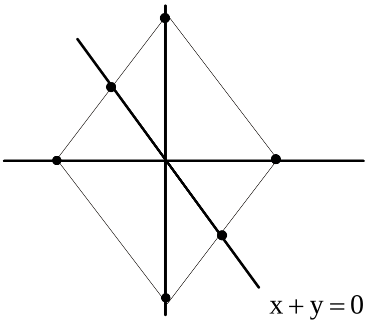
$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right)$

$\Rightarrow \sin\left(\frac{x+y}{2}\right)$ or $\cos\left(\frac{x-y}{2}\right) = \cos\left(\frac{x+y}{2}\right)$

Or $x = 2m\pi$ or $y = 2k\pi$

$x + y = 2n\pi$

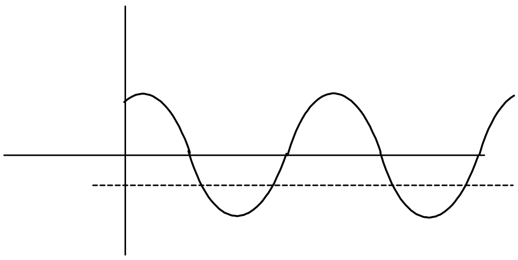
Here any x + y = 0, x = 0 y = 0 will intersect $|x| + |y| = 1$



6 soln.

23. $\sin x \cos = \frac{3}{4}$ & $\cos x \cos y = \frac{1}{4}$

$\Rightarrow \cos(x-y) = 1$ & $\cos(x+y) = \frac{-1}{2}$



(i) x = y $\Rightarrow \cos^2 x = \frac{-1}{2}$

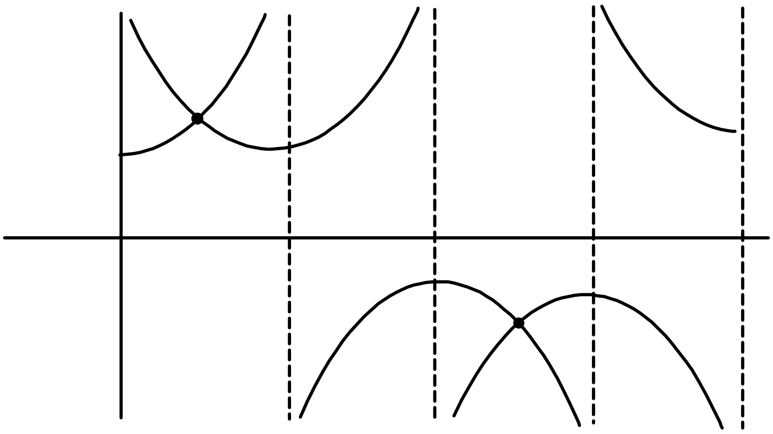
4 soln.

(ii) $x = y + 2\pi$ & $x = y - 2\pi$ not possible as $0 < x, y < 2\pi$

A 4 soln.

24. $\sin^5 x + \frac{1}{\sin x} = \frac{1}{\cos x} + \cos^5 x$

Now plot LHS & RHS simultaneously



There are 2 intersection points but here $\sin x = \cos x$

$\Rightarrow$ Overall No soln. A 0

25. $a_1 + a_2 \sin x + a_3 \cos x + a_4 \sin 2x + a_5 \cos 2x = 0$

Since this is an identify in x

Put x = 0, $x = \frac{\pi}{2}, x = \frac{-\pi}{2}, x = \pi, x = -\pi$

& soln. to get

$a_1 = a_2 = a_3 = a_5 = 0$

$\therefore$ one possibility $\langle 0, 0, 0, 0, 0 \rangle$

26. $(\sin\theta + 2)(\sin\theta + 3)(\sin\theta + 4) = 6$

$\sin\theta = -1$ then only LHS = 6

Otherwise LHS > 6

$= \sin\theta = -1 \qquad \therefore [0, 4\pi]$

Possible at $\frac{3\pi}{2}, \frac{7\pi}{2}$

Sum = $5\pi \Rightarrow k = 5$

27. $1 - \cos^2 x + a\cos x + a^2 > 1 + \cos x$

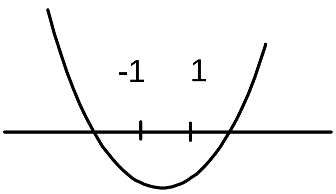
$\cos^2 x + (1-a)\cos x - a^2 < 0$

Put cos x = t

$f(t) = t^2 + (1-a)t - a^2 < 0 \, \forall t [-1, 1]$

$f(-1) < 0$

$f(1) < 0$



(i) $f(-1) \le 0 \Rightarrow a \in [-\infty, 0] \cup [1, \infty]$

(ii) $f(1) \le 0 \Rightarrow a \in (-\infty, -1) \cup [3, \infty]$

$a \in [-\infty, 1] \cup [3, \infty]$

28. simplify

cos y sin x – sin y cos x + sin y sin x + cos x cos y = 0

$\sin(x+y)+\cos(x-y)=0$

$\tan(x-y)=-1$ $\quad -2\pi < x-y < 2\pi$

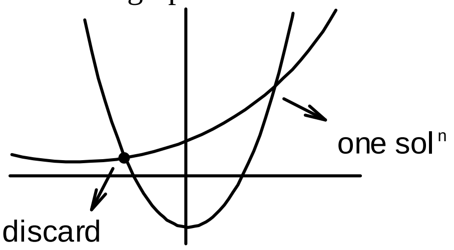
$x-y$ has 4 solutions in $(-2\pi, 2\pi)$

29. put $\tan^2\theta = t$

$(1-t^2)-2^t=0$

$2^t=-1+t^2$

Plot both graph when t > 0



We know t = 3 satisfies $\tan^2\theta = 3$

$\therefore$ 2 soln.

30. Use principal sol.

P sin x = $\frac{\pi}{2} - p\cos$

$P(\sin x + \cos x) = \frac{\pi}{2}$

Now least P = $\frac{\pi}{2\sqrt{2}}$ when is > 1

$\therefore$ least +in internal P = 2

31. sin x + cos x = 1

$\Rightarrow \sin x = 0 \;\&\; \cos x = 1$ $\quad$ cos x = 0 7 sin x = 4

$x = 2n\pi$ $\quad$ $x = (4n+1)\frac{\pi}{2}$

x = 0 $\quad$ $\frac{\pi}{2}$

2 soln.

32. Let $\sin x + \cos x = 1 \Rightarrow \sin 2x = t^2 - 1$

$\Rightarrow t = 2\left(\frac{t^2-1}{2}+1\right)$

$\Rightarrow t^2 = t^2 - 1 + 2 \Rightarrow 2 = 1$

No soln.

33. $\text{RHS} \geq 1 \;\&\; \text{LHS} \geq 1$

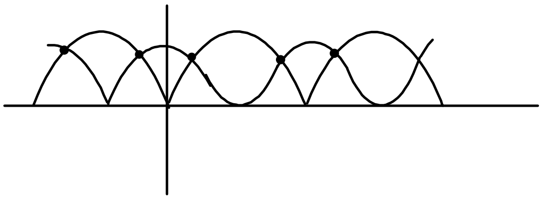
Only possible @ x = 0 & x = 1

Now pur x = 0 satisfies

Put x = 1 doesn't satisfy

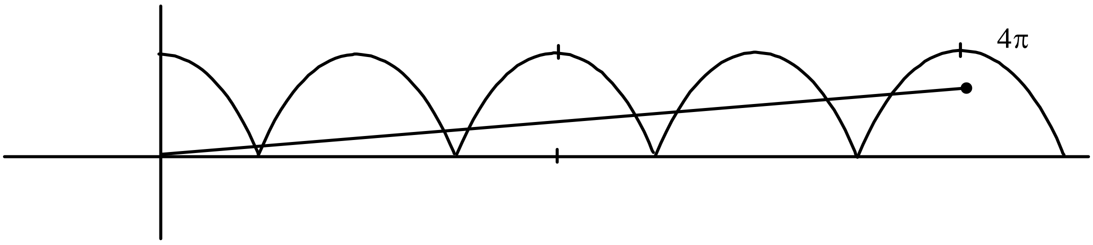
$\therefore$ one soln. x = 0

34.



6 soln.

35. $|\cos x| = \frac{x}{30} \rightarrow$ this passes through $\left(4\pi, \frac{4\pi}{30}\right)$



36. $3\left(2\cos^2 x - 1\right) - 10\cos x + 7 = 0$

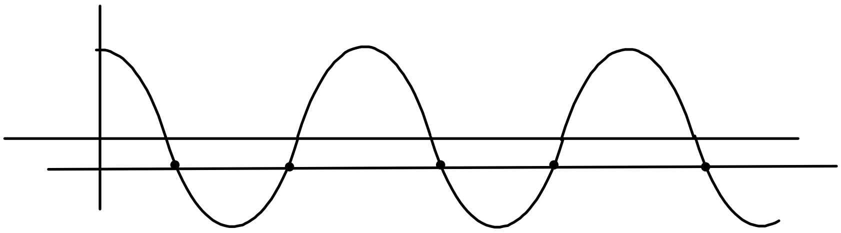
$6\cos^2 x - 10\cos x + 4 = 0$

$3\cos^2 x - 5\cos x + 2 = 0$

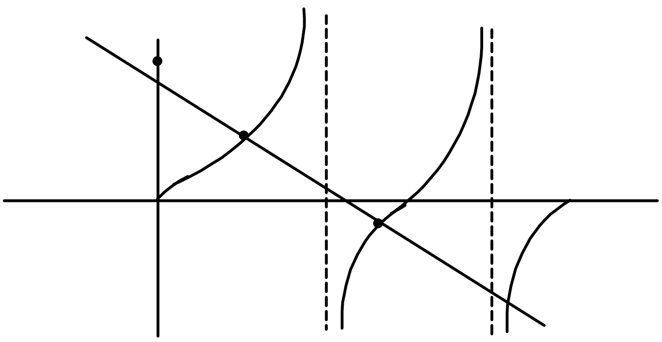
$3\cos^2 x - 6\cos x + \cos x - 2 = 0$

$(3\cos x + 1)(\cos x - 2) = 0$

$\cos x = \frac{-1}{3}$



37. $\tan x = \frac{5\pi}{4} - \frac{3}{2}x$



3 soln.

38. $|\cos x| + \cos^2 x = 0$

$0 \le x \le 4\pi$

$\cos x \ge 0 \Rightarrow \cos x + \cos^2 x = 0$

$\cos x = 0 \Rightarrow \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\cos x < 0$

$-\cos x+\cos^2 x=0$

$\cos x=0, \qquad \cos x=1$

4 soln.

39. $\sec^2(a+2)=1-a^2$

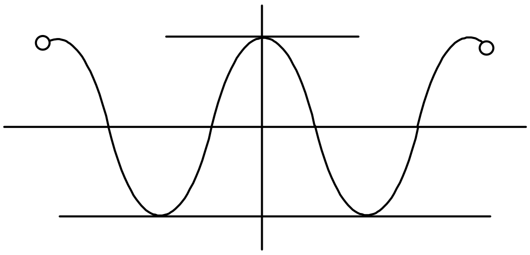
Only possible of a = 0

$\Rightarrow \qquad \sec^2 2x=1$

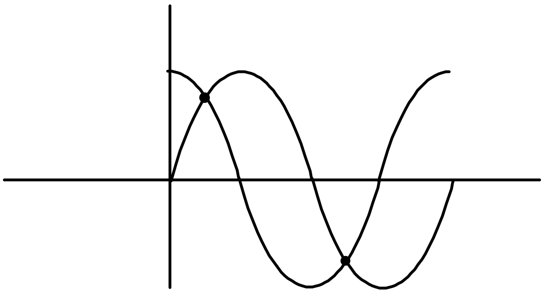
or $\qquad \sec^2 x=1 \qquad$ or $\qquad$ -1

$\Rightarrow \cos 2x=1$ or $\qquad$ -1

Total 3 soln.



40.



only after $x>\frac{5\pi}{4}$

i.e. x = 4