TRIGO – I (SOLUTION)

EXERCISE – 1(A)

1. (B)

π radian = 180 degree $\Rightarrow 1$ radian = $\frac{180}{\pi}$ degree $\simeq 57.3$ degree, hence $\sin 1^c > 1^0$.

2. (B)

Length of arc of circle of radius r subtending θ at the center = $r\theta$.

Hence $15 = r\times\frac{3}{4}$ or r = 20 cm

3. (D)

In second quadrant sin A > 0, cos A < 0 & tan A < 0.

Given $\tan A = -\frac{4}{3}$, hence $\sin A = \frac{4}{5}$ & $\cos A = -\frac{3}{5}$

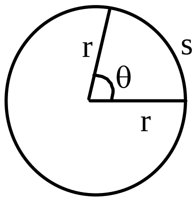
Now $2\cot A - 5\cos A + \sin A = -2\times\frac{3}{4}\times\left(-\frac{3}{5}\right)+\frac{4}{5} = \frac{23}{10}$

4. (D)

S + 2r = mr

$\Rightarrow$ S = (m – 2) r

$\therefore \theta = \frac{S}{r} = (m-2)^c$



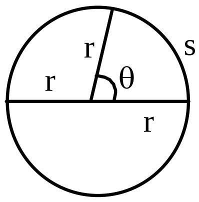
5. (C)

$\sin 15^0 \cos 15^0 = \frac{1}{2}\sin 30^0 = \frac{1}{4}$

6. (C)

$\pi r = S + 2r$

$\Rightarrow S = (\pi - 2)r$



7. (B)

$$\frac{\cot\theta + \text{cosec}\,\theta - 1}{\cot\theta - \text{cosec}\,\theta + 1}$$

$$= \frac{\frac{1}{\text{cosec}\,\theta - \cot\theta} - 1}{\cot\theta - \text{cosec}\,\theta + 1} \quad \left(\because (\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 1\right)$$

$$= \frac{1 - \text{cosec}\,\theta + \cot\theta}{(\text{cosec}\,\theta - \cot\theta)(\cot\theta - \text{cosec}\,\theta + 1)}$$

$$= \frac{1}{\text{cosec}\,\theta - \cot\theta} = \text{cosec}\,\theta + \cot\theta$$

$$= \frac{1+\cos\theta}{\sin\theta} = \frac{\sin\theta}{1-\cos\theta}$$

8. (A)

$\tan(90^0 - \theta) = \cot\theta \Rightarrow \tan 89^0 = \cot 1^0$, $\tan 88^0 = \cot 2^0$, $\tan 87^0 = \cot 3^0$,…etc

Also $\tan\theta\times\cot\theta = 1$, hence $\tan 1^0 \tan 2^0 \tan 3^0 \ldots \tan 89^0 = 1$

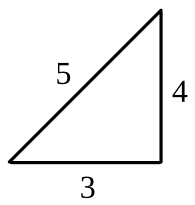
9. (C)

$\tan A = -\frac{4}{3}$

$\cot A = -\frac{3}{4}$

$\cos A = -\frac{3}{5}$

$\sin A = \frac{4}{5}$



($\because$ In 2^{nd} quadrant sin A > 0, cos A < 0, tan A < 0, cot A < 0)

$\therefore 2\cot A - 5\cos A + \sin A = -\frac{6}{4} + 3 + \frac{4}{5}$

10. (A)

$\sin\theta - \cos\theta = 1 \Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 1$

$\Rightarrow \sin\theta\cos\theta = 0$

11. (C)

$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5\tan\theta - 3}{5\tan\theta + 2}$

$5\tan\theta = 4 \Rightarrow \frac{5\tan\theta - 3}{5\tan\theta + 2} = \frac{4-3}{4+2} = \frac{1}{6}$

12. (A)

Sin x + cosec x = 2 $\Rightarrow$ $\sin^2 x$ – 2sin x + 1 = 0

$\Rightarrow$ sin x = 1 $\Rightarrow$ $\sin^n x + \text{cosec}^n x = 2$

13. (C)

$$x\sin 45^0 \cos^2 60^0 = \frac{\tan^2 60^0 \text{cosec} 30^0}{\sec 45^0 \cot^2 30^0} \Rightarrow x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)^2 = \frac{\left(\sqrt{3}\right)^2 (2)}{\left(\sqrt{2}\right)\left(\sqrt{3}\right)^2} \Rightarrow x = 8$$

14. (A)

$90^0 < 130^0 < 135^0$ hence sin A > 0 & cos A < 0 & $|\sin A| > |\cos A|$

$\Rightarrow \sin A + \cos A > 0$

15. (A)

$\frac{\cos 17^0 + \sin 17^0}{\cos 17^0 - \sin 17^0} = \frac{1 + \tan 17^0}{1 - \tan 17^0}$

$= \tan\left(45^0 + 17^0\right) = \tan 62^0$

16. (A)

$\tan 75^0 - \cot 75^0 = -2\cot 150^0$

$= 2\tan 60^0 = 2\sqrt{3}$

17. (A)

$\cos^2\alpha + \cos^2\left(\alpha + 120^0\right) + \cos^2\left(\alpha - 120^0\right)$

$$= \frac{1+\cos\alpha}{2} + \frac{1+\cos 2\left(\alpha + 120^0\right)}{2} + \frac{1+\cos 2\left(\alpha - 120^0\right)}{2}$$

$$= \frac{3+\cos 2\alpha + 2\cos 2\alpha \cos 120^0}{2} = \frac{3+\cos 2\alpha - \cos 2\alpha}{2} = \frac{3}{2}$$

18. (A)

$\cos 24^0 + \cos 5^0 + \cos 175^0 + \cos 204^0 + \cos 300^0$

$= \cos 24^0 + \cos 5^0 + \cos\left(180^0 - 5^0\right) + \cos\left(180^0 + 24^0\right) + \cos\left(360^0 - 60^0\right)$

$= \cos 24^0 + \cos 5^0 - \cos 5^0 - \cos 24^0 + \cos 60^0$

$= \cos 60^0 = \frac{1}{2}$

19. (B)

$\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

$\Rightarrow \cos^2 48^0 - \sin^2 12^0 = \cos 60^0 \cos 36^0 = \frac{\sqrt{5}+1}{8}$

20. (B)

$\sin\alpha + \sin\beta = a \Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = a$ …(i)

& $\cos\alpha - \cos\beta = b \Rightarrow 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} = -b$ … (ii)

From (i) & (ii) $\tan\frac{\alpha-\beta}{2} = -\frac{b}{a}$

21. (D)

$\frac{1}{2}\sin^2\left(\frac{\pi}{4}+\theta\right) = \cos 2\left(\frac{\pi}{4}+\theta\right)$

$= \cos\left(\frac{\pi}{2}+2\theta\right) = -\sin 2\theta$

22. (D)

$\cos\alpha + \cos\beta = 0 \Rightarrow \cos^2\alpha + \cos^2\beta = -2\cos\alpha\cos\beta$

$\& \sin\alpha + \sin\beta = 0 \Rightarrow \sin^2\alpha + \sin^2\beta = -2\sin\alpha\sin\beta$

Now $\cos 2\alpha + \cos 2\beta = 2\left(\cos^2\alpha + \cos^2\beta - 1\right) = 2\left(1 - \sin^2\alpha - \sin^2\beta\right)$

$\Rightarrow \cos\alpha + \cos\beta = 2\left(-2\cos\alpha\cos\beta - 1\right)$ ...(i)

& $\cos 2\alpha + \cos 2\beta = 2\left(1 + 2\sin\alpha\sin\beta\right)$ …(ii)

(i) + (ii) $\Rightarrow \cos\alpha + \cos 2\beta = 2\cos(\alpha+\beta)$

23. (B)

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \Rightarrow \tan(A+B) = \frac{-\frac{1}{2}-\frac{1}{3}}{1-\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)}$

$\Rightarrow$ tan (A + B) = –1

$\Rightarrow A + B = \frac{3\pi}{4}$

24. (A)

$\frac{\sin 3\theta - \cos 3\theta}{\sin\theta + \cos\theta} + 1 = \frac{2\sin\theta - 4\sin^2\theta - 4\cos^3\theta + 3\cos\theta}{\sin\theta + \cos\theta} + 1$

$= 4\frac{\sin\theta + \cos\theta - \sin^3\theta - \cos^3\theta}{\sin\theta + \cos\theta}$

$= 4\frac{\sin\theta + \cos\theta - \left(\sin^3\theta + \cos^3\theta\right)\left(1 - \sin\theta\cos\theta\right)}{\sin\theta + \cos\theta}$

$= 4\sin\theta\cos\theta = 2\sin\theta$

25. (A)

$\cos A = m\cos B \Rightarrow \frac{\cos A}{\cos B} = m$

Apply component to get $\frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$

$\Rightarrow -\frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\sin\frac{A+B}{2}\sin\frac{A-B}{2}} = \frac{m+1}{m-1}$

$\Rightarrow \cot\frac{A+B}{2} = \frac{m+1}{m-1}\tan\frac{B-A}{2}$

26. (D)

$\tan 45^0 = \tan\left(180^0 + 45^0\right) \Rightarrow \tan 225^0 = \tan\left(100^0 + 125^0\right)$

$\Rightarrow \frac{\tan 100^0 + \tan 125^0}{1 - \tan 100^0 \tan 125^0} = 1$

$\Rightarrow \tan 100^0 + \tan 125^0 + \tan 100^0 \tan 125^0 = 1$

27. (D)

As A & B are the acute angles hence,

$\sin A = \frac{1}{\sqrt{10}} \Rightarrow \cos A = \frac{3}{\sqrt{10}} \;\&\; \sin B = \frac{1}{\sqrt{5}} \Rightarrow \cos B = \frac{2}{\sqrt{5}}$

Now cos (A + B) = cos A cos B – sin A sin B

$\Rightarrow \cos\left(A+B\right) = \frac{3}{\sqrt{10}} \times \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{10}} \times \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{2}}$

$\Rightarrow A + B = \frac{\pi}{4}$

28. (B)

$\sin 50^0 - \sin 70^0 + \sin 10^0 = -2\cos 60^0 \sin 10^0 + \sin 10^0 = 0$

29. (C)

$\sin 12^0 \sin 48^0 \sin 54^0 = \sin\left(60^0 - 12\right)\sin 12^0 \sin\left(60^0 + 12^0\right)\frac{\sin 54^0}{\sin 72^0}$

$= \frac{\sin 36^0 \sin 54^0}{4\sin 72^0} = \frac{\sin 36^0 \sin 54^0}{8\sin 36^0 \cos 36^0}$

$= \frac{\sin 54^0}{8\cos 36^0}$, but $\sin 54^0 = \sin\left(90^0 - 36^0\right) = \cos 36^0$

$\therefore \sin 12^0 \sin 48^0 \sin 54^0 = \frac{1}{8}$

30. (C)

$$\frac{\sin 3\theta+\sin 5\theta+\sin 7\theta+\sin 9\theta}{\cos 3\theta+\cos 5\theta+\cos 7\theta+\cos 9\theta} = \frac{(\sin 3\theta+\sin 9\theta)+(\sin 5\theta+\sin 7\theta)}{(\cos 3\theta+\cos 9\theta)+(\cos 5\theta+\cos 7\theta)}$$

$$= \frac{2\sin 6\theta\cos 3\theta+2\sin 6\theta\cos\theta}{2\cos 6\theta\cos 3\theta+2\cos 6\theta\cos\theta} = \tan 6\theta$$

31. (B)

$$\frac{\sin(B+A)+\cos(B-A)}{\sin(B-A)+\cos(B+A)} = \frac{\sin(B+A)+\sin\left(\frac{\pi}{2}-(B-A)\right)}{\cos\left(\frac{\pi}{2}-(B-A)\right)+\cos(B+A)}$$

$$= \frac{2\sin\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-B\right)}{2\cos\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-B\right)} = \tan\left(\frac{\pi}{4}+A\right)$$

$$= \frac{1+\tan A}{1-\tan A} = \frac{\cos A+\sin A}{\cos A-\sin A}$$

32. (A)

$$\frac{\sin 2x}{\sin 2y} = n \Rightarrow \frac{\sin 2x+\sin 2y}{\sin 2x-\sin 2y} = \frac{n+1}{n-1} \ \text{\{By componendo \& dividendo\}}$$

$$\Rightarrow \frac{2\sin(x+y)\cos(x-y)}{2\cos(x+y)\sin(x-y)} = \frac{n+1}{n-1}$$

$$\Rightarrow \frac{\tan(x+y)}{\tan(x-y)} = \frac{n+1}{n-1}$$

33. (D)

$$2\cos^2\theta-2\sin^2\theta = 1 \Rightarrow \cos 2\theta = \frac{1}{2} \therefore \theta = 60^0$$

34. (C)

$\cos^2 A(3-4\cos^2 A)^2 + \sin^2 A\ (3-4\sin^2 A)^2 = (3\cos A - 4\cos^3 A)^2 + (3\sin A - 4\sin^3 A)^2$
$= \cos^2 3A + \sin^2 3A = 1$

35. (C)

$$2\sin^2\beta+4\cos(\alpha+\beta)\sin\alpha\sin\beta+\cos 2(\alpha+\beta)$$

$$= 2\sin^2\beta+2\cos(\alpha+\beta)\big(\cos(\alpha-\beta)-\cos(\alpha-\beta)\big)+\cos 2(\alpha+\beta)$$

$$= 2\sin^2\beta+2\cos(\alpha+\beta)\cos(\alpha-\beta)-2\cos^2(\alpha+\beta)+\cos 2(\alpha+\beta)$$

$$2\sin^2\beta+2\cos^2\alpha-2\sin^2\beta-2\cos^2(\alpha+\beta)+2\cos^2(\alpha+\beta)-1$$

$$= 2\cos^2\alpha-1 = \cos 2\alpha$$

36. (C)

$$\frac{3\cos\theta+\cos 3\theta}{3\sin\theta-\sin 3\theta}=\frac{3\cos\theta+\left(4\cos^3\theta-3\cos\theta\right)}{3-\sin\theta-\left(3\sin\theta-4\sin^3\theta\right)}=\cot^3\theta$$

37. (B)

$$2\sin A\cos^3 A-2\sin^3 A\cos A=2\sin A\cos A\left(\cos^2 A-\sin^2 A\right)$$
$$=\sin 2A\cos 2A=\frac{\sin 4A}{2}$$

38. (C)

$$32\sin\left(\frac{A}{2}\right)\sin\left(\frac{5A}{2}\right)=16\left(\cos 2A-\cos 3A\right)$$
$$=16\left(2\cos^2 A-1-4\cos^3 A+3\cos A\right)$$
$$=16\left(2\times\frac{9}{16}-1-4\times\frac{27}{64}+3\times\frac{3}{4}\right)=11$$

39. (D)

$$-\sqrt{3^2+4^2}+8\le 3\cos x+4\sin x+8\le\sqrt{3^2+4^2}+8$$
$$\Rightarrow 3\le 3\cos x+4\sin x+8\le 13$$

40. (B)

$$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3},\cos\frac{4\pi}{3}=\cos\left(\pi+\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}$$
$$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3},\cos\frac{4\pi}{3}=\cos\left(\pi+\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}$$
$$\Rightarrow x=y\cos\frac{2\pi}{3}=z\cos\frac{4\pi}{3}\Rightarrow x=-\frac{y}{1}=-\frac{z}{2}\Rightarrow y=z=-2x$$

Now xy + yz + zx = x (–2x) + (–2x) (–2x) + (–2x) x = 0

41. (C)

$\sin\theta<0\ \&\ \tan\theta>0\Rightarrow\theta$ lies in 3^{rd} quadrant.

42. (A)

$$8x^2-26x+15=0\Rightarrow\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}=\frac{13}{4}\ \&\ \tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{15}{8}$$

Now $\tan\frac{\alpha+\beta}{2}=\dfrac{\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}\Rightarrow\tan\frac{\alpha+\beta}{2}=\dfrac{\frac{13}{4}}{1-\frac{15}{8}}=-\frac{26}{7}$

Further $\cos(\alpha+\beta)=\dfrac{1-\tan^2\frac{\alpha+\beta}{2}}{1+\tan^2\frac{\alpha+\beta}{2}}\Rightarrow\cos(\alpha+\beta)=-\frac{627}{725}$

43. (C)

$$\tan 9^0-\tan 27^0-\tan 63^0+\tan 81^0=\tan 9^0-\tan 27^0-\tan\left(90^0-27^0\right)+\tan\left(90^0-9^0\right)$$
$$=\tan 9^0+\cot 9^0-\tan 27^0-\cot 27^0=2\left(\operatorname{cosec}18^0-\operatorname{cosec}54^0\right)$$

$$= 2\frac{\sin 54^0 - \sin 18^0}{\sin 54^0 \sin 18^0} = \frac{4\sin 18^0 \cos 36^0}{\sin 54^0 \sin 18^0}$$
$$= \frac{4\cos 36^0}{\sin\left(90^0 - 36^0\right)} = 4$$

44. (A)

$$\left(1+\cos\alpha\right)\left(1+\cos 3\alpha\right)\left(1+\cos 5\alpha\right)\left(1+\cos 7\alpha\right) = 16\cos^2\frac{\alpha}{2}\cos^2\frac{3\alpha}{2}\cos^2\frac{5\alpha}{2}\cos^2\frac{7\alpha}{2}$$
$$= \left(2\cos\frac{\pi}{16}\cos\frac{7\pi}{16}\right)^2\left(2\cos\frac{3\pi}{16}\cos\frac{5\pi}{16}\right)^2$$
$$= \left(\cos\frac{\pi}{2} + \cos\frac{3\pi}{8}\right)^2\left(2\cos\frac{\pi}{2} + \cos\frac{\pi}{8}\right)^2$$
$$= \left(\cos\frac{\pi}{8}\cos\frac{3\pi}{8}\right)^2 = \left(\cos\frac{\pi}{8}\sin\frac{\pi}{8}\right)^2$$
$$= \frac{1}{4}\sin^2\frac{\pi}{4} = \frac{1}{8}$$

45. (C)

$$\sqrt{3}\cos ec20^0 - \sec 20^0 = \frac{\frac{\sqrt{3}}{2}\cos 20^0 - \frac{1}{2}\sin 20^0}{\frac{1}{2}\cos 20^0 \sin 20^0}$$
$$= \frac{4\sin\left(60^0 - 20^0\right)}{2\cos 20^0 \sin 20^0} = \frac{4\sin 40^0}{\sin 40^0} = 4$$

46. (C)

sin 20^0 sin 40^0 sin 60^0 sin 80^0 = (sin (60^0 – 80^0) sin 20^0 sin (60^0 + 20^0)) sin 60^0

$$= \frac{1}{4}\sin\left(3\times 20^0\right)\sin 60^0 = \frac{1}{4}\sin^2 60^0 = \frac{3}{16}$$

47. (D)

$$\cos 20^0 \cos 40^0 \cos 80^0 = \frac{2\sin 20^0 \cos 20^0 \cos 40^0 \cos 80^0}{2\sin 20^0}$$
$$= \frac{2\sin 40^0 \cos 40^0 \cos 80^0}{4\sin 20^0} = \frac{2\sin 80^0 \cos 80^0}{8\sin 20^0}$$

$= \frac{\sin 160^0}{8\sin 20^0}$, but sin 160^0 = sin (180^0 – 20^0) = sin 20^0, hence

$$\cos 20^0 \cos 40^0 \cos 80^0 = \frac{1}{8}$$

Alternately

cos 20^0 cos 40^0 cos 80^0 = cos (60^0 – 20^0) cos 20^0 cos (60^0 + 20^0)

$$\frac{1}{4}\cos\left(3\times 20^0\right) = \frac{1}{8}$$

48. (A)

$\sin\left(\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{\pi}{6}\right)$ acquires maximum at $\alpha + \frac{\pi}{6} = \frac{\pi}{4}$ i.e. $\theta = \frac{\pi}{12}$

49. (D)

$5\sin^2\theta + 4\cos^2\theta = 4 + \sin^2\theta \geq 4$

50. (B)

$x + \frac{1}{x} = 2\cos\theta \; \& \; x^3 + \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 - 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

$\Rightarrow x^3 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta = 2\cos 3\theta$

51. (B)

$0 < 2 < \pi, \pi < 3 < 2\pi \; \& \; \pi < 5\pi$

$\Rightarrow$ sin 2 > 0, sin 3 < 0, sin 5 < 0

$\Rightarrow$ sin 2 sin 3 sin 5 > 0

52. (A)

(sec A – tan A) (sec B – tan B) (sec C – tan C) = X &

(sec A – tan A) (sec B – tan B) (sec C – tan C) = X

$\Rightarrow X^2 = (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C) = 1$

$\Rightarrow X = \pm 1$

TRIGO – I (SOLUTION)
EXERCISE – 1(B)

1. (A)

$$\frac{-\tan\left(\frac{\pi}{2}-n\right)x-\cos\left(\frac{\pi}{2}+n\right)-\sin^3\left(\frac{\pi}{2}+n\right)}{\cos\left(\frac{\pi}{2}-n\right)+\left(\frac{\pi}{2}+n\right)}$$

$$=\frac{-\cot x\sin n-\cos^3 x}{-\sin n\times\cot x}$$

$$=\frac{\cos x+\cos^3 x}{\cos x}=1+\cos^2 x$$

2. (A)

$$\frac{\sin\theta}{\cos\theta}\left(\sec^2\theta\right)^3+\frac{\cos\theta}{\sin\theta}\left(1+\cot^2\theta\right)^3$$

$$=\tan\theta\left(1+\tan^2\theta\right)^3+\frac{1}{\tan\theta}\left(\frac{1+\tan^2\theta}{\tan^2\theta}\right)^3$$

$$=\sqrt{\frac{a}{b}}\left(1+\frac{a}{b}\right)^3+\sqrt{\frac{b}{a}}\left(1+\frac{b}{a}\right)^3$$

$$=\left(a+b\right)^3\left(\frac{a^{\frac{1}{2}}}{b^{\frac{7}{2}}}+\frac{b^{\frac{1}{2}}}{a^{\frac{7}{2}}}\right)$$

$$=\frac{\left(a+b\right)^3\left(a^4+b^4\right)}{\left(ab\right)^{\frac{7}{2}}}$$

3. (C)

$$f\left(x\right)=3\left[\sin^4\left(\frac{\pi}{2}-n\right)+\sin^4 x\right]-2\left[\cos^6 n+\sin^6 n\right]$$

$$=3\left[\left(\cos^2 n\right)^2+\left(\sin^2 n\right)^2\right]-2\left[\left(\sin^2 n\right)^3+\left(\cos^2 n\right)^3\right]$$

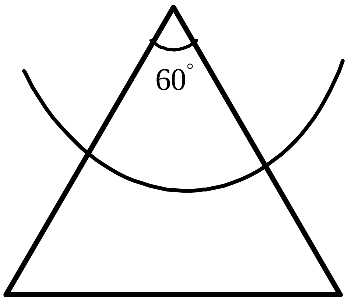
$$=3\left[1-2\sin^2 n\cos^2 n\right]-2\left[1-3\sin^2 n\cos^2 n\right]$$

$$=1$$

4. (D)

$$\frac{\sin^2 20\dfrac{\left(1-\cos^2 20\right)}{\cos^2 20}}{\dfrac{\sin^2 20.\sin^2 20}{\cos^2 20}}=1$$

5. (A)



Area of $\Delta = \frac{\sqrt{3}}{a}.4 = 4\sqrt{3}$

$2z \_\_\_\_\_\_ zr^2$

$1 \_\_\_\_\_\_ \frac{zr^2}{2\pi}$

$\frac{\pi}{3} \_\_\_\_\_ \frac{zr^2}{2r}.\frac{z}{3}$

Given $\frac{zr^2}{0} = \frac{4\sqrt{3}}{z}$

$zr^2 = 12\sqrt{3}$

$r = \sqrt{\frac{12\sqrt{3}}{z}}$

6. (A)

$2\cos 10 + \sin(90+10) + \sin(3\times 300 - 80) + \sin(27\times 360 + 280)$

$= 2\cos 10 + \cos 10 - 2\sin 80$

$= \cos 10 + 2(\cos 10 - \sin(90-10))$

$= \cos 10 + 2(\cos 10 - \cos 10) = \cos 10$

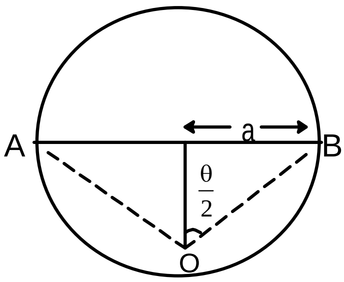
7. (B)

$\sin^2 n + \cos ec^2 x + 2 + \cos^2 n + \sec^2 n + 2 - t^2 n - \cot^2 n - 2$

$= 3 + (\cos ec^2 n - \cot^2 n) + (\sec^2 n - t^2 n)$

$= 5$

8. (C)



$\sin\frac{\theta}{2} = \frac{a}{0B}$

$OB = \frac{a}{\sin\frac{\theta}{2}}$

Required area $= zr^2 = \pi a^2 \cos ec^2 \frac{\theta}{2}$

$$= za^2\left(1+\cot^2\frac{\theta}{2}\right)$$

9. (C)

$$\cos^2(90-17)+\cos^2 47-\sin^2(90-47)+\sin^2(90+17)$$
$$=\sin^2 17+\cos^2 47-\cos^2 47+\cos^2 17$$
$$=1$$

10. (D)

(i) $\sin(2\times 360+4s)=\sin 45=\frac{1}{\sqrt{2}}$

(ii) $\frac{1}{-\sin(3\times 360+330)}=\frac{1}{-\sin(360-30)}=\frac{1}{\sin 30}=2$

(iii) $\tan\left(4z+\frac{\pi}{3}\right)=\tan\left(\frac{z}{3}\right)=\sqrt{3}$

(iv) $\frac{1}{-t\left(4z-\frac{3}{4}\right)}=\frac{1}{t_{\frac{z}{a}}}=1$

11. (C)

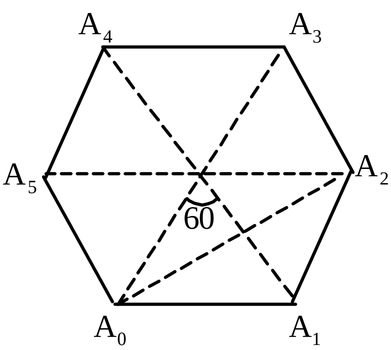
$$\ell^2=\left[\sin\theta-\sin\left(\frac{\pi}{2}-\theta\right)\right]^2+\left[\cos\theta+\cos\left(\frac{\pi}{2}-\theta\right)\right]$$
$$=(\sin\theta-\cos\theta)^2+(\cos\theta+\sin\theta)^2$$
$$=1+1=2$$
$$\ell=\sqrt{2}$$

12. (B)

Given $y=\frac{2\sin\alpha}{(1+\cos\alpha+\sin\alpha)}$

$$\frac{(1+\sin\alpha-\cos\alpha)(1+\sin\alpha+\cos\alpha)}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$
$$=\frac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}=\frac{(1+2\sin\alpha+\sin^2\alpha+\sin^2\alpha)}{(1+\sin\alpha)(1+\sin x+\cos)}$$
$$=\frac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}=y$$

13. (C)



$A_0A_1 = 1$

$\cos 120^\circ = \dfrac{1^2 + 1^2 - A_0A_2{}^2}{2.1.1.}$

$-\dfrac{2}{2} = 2 - A_0A_2{}^2$

$A_0A_2{}^2 = 3$

$A_0A_2 = \sqrt{3}$

$A_0A_4 = \sqrt{3}$

$A_0A_1 \times A_0A_2 + A_0A_4 = 1.\sqrt{3}.\sqrt{3} = 3$

14. (A)

$$\tan^2 30 + 4\sin^2 45 + \frac{1}{3}\cos^2 30 = \frac{1}{3} + \frac{4}{2} + \frac{1}{3} \times \frac{3}{4}$$
$$= \frac{1}{3} + \frac{1}{4} + 2$$
$$= 2.\frac{7}{12}$$

15. (D)

16. (D)

$$\frac{\tan^2 60 - 2\tan^2 45 + \sec^2 30}{3\sin^2 45 \sin 90 + \cos^2 60.\cos^3 0} = \frac{3 - 2 + \frac{4}{3}}{3.\frac{1}{2} + \frac{1}{4}}$$

$$= \frac{1 + \frac{4}{3}}{\frac{1}{2}\left(3 + \frac{1}{2}\right)} = \frac{\frac{7}{3}}{\frac{1}{2}.\frac{7}{2}} = \frac{4}{3}$$

17. (D)

$$\frac{\tan\theta}{\left(\tan^3\theta + t\theta\right)\cos^2\theta} = \frac{1}{\left(1 + t^2\theta\right)\cos^2\theta}$$

$$= \frac{1}{\frac{\left(\sin^2\theta + \cos^2\theta\right)}{\cos^2\theta}.\cos^2\theta}$$
$$= 1$$

18. (D)

$$\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = \cos ec^2\theta \sec^2\theta$$

19. (b)

$\sin(180 + 20) + \cos(180 + 20)$

$= -\sin 20 - \cos 20 < 0$

20. (A)

$$\frac{1}{2}\left[2\cos\alpha\sin(\beta-r)+2\cos\beta\sin(r-\alpha)+2\cos r\sin(\alpha-\beta)\right]$$

$$=\frac{1}{2}\left[\sin(\alpha+\beta r)-\sin(\alpha+r-\beta)+\sin(\beta+r-\alpha)-\sin(\alpha+\beta-r)+\sin(\alpha+r-\beta)-\sin(r+\beta-\alpha)\right]$$

$$=0$$

21. (D)

$$\frac{\sin\alpha}{\sin\beta}=\frac{5}{3}$$

$$\frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\frac{8}{2}=4\ (C\,\&\,D)$$

$$\frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}=4$$

$$\frac{t\left(\frac{\alpha+\beta}{2}\right)}{t\left(\frac{\alpha-\beta}{2}\right)}=4$$

22. (B)

$$(\sin 2\theta+\sin 2\phi)^2+(\cos 2\theta+\cos 2\phi)^2=\frac{1}{\alpha}+\frac{9}{4}$$

$$1+1+2\cos(2\theta-2\phi)=\frac{10}{4}=\frac{5}{2}$$

$$2\cos(2\theta-2\phi)=\frac{5}{2}-2$$

$$\cos 2(\theta-\phi)=\frac{1}{4}$$

$$2\cos^2(\theta-\phi)=\frac{1}{4}+1$$

$$\cos^2(\theta-\phi)=\frac{5}{8}$$

23. (A)

$$(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=1$$

$$(\sec\theta+\tan\theta)-(\sec\theta-\tan\theta)=2a-\frac{1}{2a}$$

Let $\sec\theta-\tan\theta=t$

$$\frac{1}{t}-t=\left(2a-\frac{1}{2a}\right)$$

$$1-t^2=\left(2a-\frac{1}{2a}\right)t$$

$$t^2+\left(2a-\frac{1}{2a}\right)t-1=0$$

$$t=\frac{\left(\frac{1}{2a}-2a\right)\pm\sqrt{\left(2a-\frac{1}{2a}\right)^2+4}}{2}$$

$$t=\frac{\left(\frac{1}{2a}-2a\right)\pm\left(2a+\frac{1}{2a}\right)}{2}$$

$$t=\frac{1}{2a},-2a$$

24. (B)

$\sec^2\theta\ge 1$

$\frac{4xy}{(x+y)^2}\ge 1$

$(x+y)^2-4xy\le 0$

$(x-y)^2\le 0$

$x=y \quad \text{only} \begin{pmatrix} x\ne 0 \\ y\ne 0\end{pmatrix}$

25. (C)

$f(\theta)=\sin\theta\left(\sin\theta+3\sin\theta-4\sin^3\theta\right)$

$f(\theta)=4\sin^2-4\sin^4\theta$

$f(\theta)=4\sin^2\theta\left(1-\sin^2\theta\right)\ge 0 \quad \forall\theta$

26. (C)

$A+B+C=\pi$

$\tan A+\tan B+\tan C=\tan A+B+C$

27. (B)

$A+B+C=\frac{3\pi}{2}$

$\cos 2A+\cos 2B+\cos 2C+4\sin A\sin B\sin C$

$=2\cos(A+B)\cos(A-B)+2\cos^2C-1+a\sin A\sin B\sin C$

$=2\cos\left(3\frac{\pi}{2}-c\right)\cos(A-B)+2\cos^2C-1+4\sin A.\sin B.\sin C$

$=-2\sin C\cos(A-B)+1-2\sin^2C+4\sin A\sin B\sin C$

$=-2\sin C\left[\cos(A-B)-\cos(A+B)\right]+4\sin A\sin B\sin C$

$=-4\sin A\sin B\sin C+4\sin A\sin B\sin C+1$

28. (A)

$$\frac{A+B+C}{2}=\frac{\pi}{2}$$

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{c}{2}\right)$$

$$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\frac{1}{\tan\frac{c}{2}}$$

$$\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{A}{2}\tan\frac{C}{2}=1$$

$$\tan\frac{C}{2}=1-\frac{2}{9}=\frac{7}{9}$$

29. (D)

30. (D)

$$\frac{1}{4}\left(4\cos\theta\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\right)$$

$$=\frac{\cos 3\theta}{4}$$

31. (B)

$$\frac{\cos 20+4(\cos 40-\cos 60)\sin 70}{\cos^2 10}$$

$$=\frac{\cos 20+4\cos 40\cos 20-2\cos 20}{\cos^2 10}$$

$$=\frac{2(\cos 20+2\cos 60+2\cos 20-2\cos 20)}{2\cos^2 10}$$

$$=\frac{2(\cos 20+1)}{(1+\cos 20)}=2$$

32. (B)

$$\cot\theta-2\cot 2\theta$$

$$\frac{1}{\tan\theta}-\frac{2\times(1-\tan^2\theta)}{2\tan\theta}$$

$$=\frac{\tan^2\theta}{\tan\theta}=\tan\theta$$

33. (D)

$$f(\theta)=5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+3$$

$$=5\cos\theta+3\cos\theta.\cos\frac{\pi}{3}-3\sin\theta\sin\frac{\pi}{3}+3$$

$$=\frac{13}{2}\cos\theta-\frac{3\sqrt{3}}{2}\sin\theta+3$$

$$-\sqrt{\frac{13^2}{4}+\frac{27}{4}}+3\le f(\theta)\le\sqrt{\frac{13^2}{4}+\frac{27}{4}}+3$$

$$-4\le f(\theta)\le 10$$

34. (D)

$$A+B+C=\pi$$

$$\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{\pi}{2}$$

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

$$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\frac{1}{\tan\frac{C}{2}}$$

$$\Rightarrow\sum\tan\frac{A}{2}\tan\frac{B}{2}=1$$

35. (B)

$$a=\sin 170^{\circ}+\cos 170^{\circ}$$

$$=\sin(180-10)+\cos(180-10)$$

$$=\sin 10-\cos 10$$

$$\Rightarrow a=-ve$$

$$\{\because \cos 10^{\circ}>\sin 10^{\circ}\}$$

36. (B)

$$\sin^2 A+\sin^2(A-B)+2\sin A\cos B\sin(B-A)$$

$$=\sin^2 A+\sin^2(A-B)+[\sin(A+B)+\sin(A-B)]\sin(B-A)$$

$$=\sin^2 A+\sin^2(A-B)+\sin(A+B)\sin(B-A)-\sin^2(A-B)$$

$$=\sin^2 A+\sin^2 B-\sin^2 A$$

$$=\sin^2 B$$

37. (C)

$$\sin\theta+\cos\beta$$

Maximum Value $=2$

38. (A)

$$\sin\alpha-\cos\alpha=\frac{1}{5}$$

$$\sin^2\alpha+\cos^2\alpha-2\sin\alpha\cos\alpha=\frac{1}{25}$$

$$\sin 2\alpha = \frac{24}{25}$$

$$\frac{2\tan\alpha}{1+\tan^2\alpha} = \frac{24}{25}$$

$$\Rightarrow \tan\theta = \frac{4}{3} \text{ or } \frac{3}{4}$$

Now, $$\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{4}{3} \text{ or } \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{3}{4}$$

$$\Rightarrow \tan\alpha = \frac{1}{2} \text{ or } \tan\alpha = -3$$

39. (C)

$$y = \frac{12}{9+5\left(\frac{3}{5}\cos n + \frac{4}{5}\sin x\right)}$$

$$= \frac{12}{9+5\sin(\alpha+n)} \text{ let}\left(\sin\alpha = \frac{3}{5}\right)$$

$$y_{\text{maximum}} = \frac{12}{4} = 3$$

40. (A)

Let $\lambda = 2\cos\beta - 5\sin\beta$

Let $S = 3\sin\beta + 5\cos\beta$

$\lambda^2 + S^2 = 9 + 25$

$\therefore \lambda^2 = 9$

41. (C)

$$m^2 - n^2 = \frac{\sin^2\alpha}{\sin^2\beta} - \frac{\cos^2\alpha}{\cos^2\beta}$$

$$= \frac{(\sin\alpha\cos\beta)^2 - (\sin\beta\cos\alpha)^2}{\sin^2\beta\cos^2\beta}$$

$$\therefore (m^2 - n^2)\sin^2\beta = \frac{\sin(\alpha+\beta)\sin(\alpha-\beta)}{\cos^2\beta}$$

$$(m^2 - n^2)(\sin^2 B) = \sin^2\alpha - \sin^2\beta$$

$$= \frac{\cos^2\beta - \cos^2\alpha}{\cos^2\beta}$$

$$= 1 - \left(\frac{\cos\alpha}{\cos\beta}\right)^2$$

$$= 1 - n^2$$

42. (C)

$\frac{b}{y} = \cot\theta$ & $\frac{a}{x} = \operatorname{cosec}\theta$

$\operatorname{cosec}^2\theta - \cot^2\theta = \frac{a^2}{n^2} - \frac{b^2}{y^2}$

$\frac{a^2}{n^2} - \frac{b^2}{y^2} = 1$

43. (C)

$\sin\beta = \frac{12}{13}, \cos\beta = \frac{5}{13}$

$\therefore \frac{13\sin\beta + \frac{5}{\cos\beta}}{5\tan\beta + \frac{6}{\sin\beta}} = \frac{13\times\frac{12}{13} + \frac{5}{5}\times 13}{5.\frac{12}{5} + \frac{6}{12}\times 13}$

$= \frac{25\times 2}{37} = \frac{50}{37}$

44. (C)

If $A + B + C = n\pi$ then

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

45. (D)

$y_{\text{minimum}} = \frac{1}{9}$

46. (D)

$4\sin\frac{4}{a}\cos\frac{4}{a}\cos\frac{4}{2}$

$= 2\sin\frac{4}{2}\cos\frac{4}{2} = \sin 4$

47. (B)

$\left(\frac{\cos B}{\cos B} + \frac{\cos C}{\sin C}\right)\left(\frac{\cos C}{\sin C} + \frac{\cos A}{\sin A}\right)\left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}\right)$

$= \frac{\sin(B+C).\sin(A+C).\sin(A+B)}{\sin^2 A.\sin^2 B\sin^2 C}$

$= \operatorname{cosec}A.\operatorname{cosec}B.\operatorname{cosec}C$

48. (A)

$\tan^2\alpha + \cot^2\alpha + 2 = m^2$

$(\tan^2\alpha + \cot^2\alpha)^2 = (m^2 - 2)^2$

$\tan^4\alpha + \cot^4\alpha + 2 = m^4 + 4 - 4m^2$

$\tan^4\alpha + \cot^4\alpha = m^4 - 4m^2 + 2$

49. (A)

$$\frac{\frac{\sin\left(\frac{3\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}\sin\left(\theta+\frac{2\theta}{2}\right)}{\frac{\sin\left(\frac{3\theta}{2}\right)}{\sin\frac{\theta}{2}}\cos\left(\theta+\frac{2\theta}{2}\right)}=\frac{\sin\alpha}{\cos\alpha}$$

$$\tan 2\theta = \tan\alpha$$

$$\theta=\frac{\alpha}{2}$$

50. (D)

$$\frac{\cos 10 - \sqrt{3}\sin 10}{\sin 10.\cos 10}$$

$$=2.2\frac{\left(\frac{1}{2}\cos 10-\frac{\sqrt{3}}{2}\sin 10\right)}{2\sin 10\cos 10}$$

$$=\frac{4\sin(30-10)}{\sin 20}=4$$

51. (B)

$$B+C=\pi-A$$

$$\cos B\cos C-\sin B\sin C=-\cos A$$

$$\cos B\cos C-\sin B\sin C=-\cos B\cos C$$

$$2\cos B\cos C=\sin B\sin C$$

$$\tan B\tan C=2$$

52. (D)

$$\cos 3\theta=4\cos^3\theta-3\cos\theta$$

$$\cos^3\theta=\left(\frac{\cos 3\theta+3\cos\theta}{4}\right)$$

$$\frac{1}{4}\sum_{r=0}^{10}\left[\cos(\pi r)+3\cos\left(\frac{\pi r}{3}\right)\right]$$

$$=\frac{1}{4}\left[\left(\cos\theta+\cos\pi+\cos\lambda-\cos 10\pi\right)+3\left(\cos\theta+\cos\frac{\pi}{3}+----+\cos 10\frac{\pi}{2}\right)\right]$$

$$=\frac{1}{4}\left(\frac{\sin\left(11.\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)}\left(10.\frac{\pi}{2}\right)+3\frac{\sin\left(11.\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)}\cos\left(10.\frac{\pi}{36}\right)\right)$$

$$=\frac{1}{4}\left(\tan 1+3\frac{\sin\left(2\pi-\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)}\cos\left(2\pi-\frac{\pi}{3}\right)\right)$$

$$=\frac{1}{4}\left(\tan+1-3\frac{\sin\frac{\pi}{6}}{\sin\frac{\pi}{6}}\times\cos\frac{\pi}{3}\right)$$

$$=+\frac{1}{4}\left(+1-\frac{3.1}{2}\right)=\frac{-1}{8}$$

53. (B)

$$4_1 4_n - 4_{n-1}$$

$$= 2\cos\theta 2\cos n\theta - 2\cos(n-1)\theta$$

$$= 2[2\cos\theta.\cos n\theta - \cos n\theta\cos\theta - \sin n\theta\sin\theta]$$

$$= 2\cos(n+1)\theta = 4_{n+1}$$

54. (C)

$$\cos(5\theta) = \cos(2\theta+3\theta)$$

$$= \cos 2\theta.\cos 3\theta - \sin 2\theta\sin 3\theta$$

$$= (2\cos^2\theta-1)(4\cos^3\theta-3\cos\theta) - 2\sin\theta\cos\theta(3\sin\theta-4\sin^3\theta)$$

$$= 8\cos^5\theta - 6\cos^3\theta - 4\cos^3\theta + 3\cos\theta - 2\sin^2\theta.\cos\theta(3-4\sin^2\theta)$$

$$= 8\cos^5\theta - 6\cos^3\theta - 4\cos^3\theta + 3\cos\theta - 2(1-\cos^2\theta).\cos\theta(4\cos^2\theta-1)$$

$$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

55. (D)

$$y = \frac{\sin\theta(3-4\sin^2\theta)}{\sin\theta}$$

$$\sin^2\theta = \left(\frac{3-y}{4}\right)$$

$$0 < \sin^2\theta \le 1$$

$$0 < \frac{3-y}{4} \le 1$$

$$0 < 3-y \le 4$$

$$-3 < -y \le 1$$

$$-1 \le y < 3$$

TRIGO SOLUTION

EXERCISE - 1(C)

No solution: Q No 39

1. (B)

$\tan A = \sqrt{2}$ $\qquad\qquad\qquad\qquad \tan^2 A = 2$

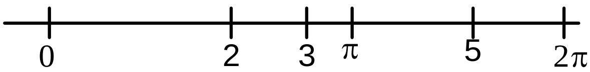
$$\frac{\sin^4 A - 3\sin^2 A\cos^2 A + 7\cos^4 A}{1+\sin^2 A\cos^2 A + 5\cos^4 A}$$

$$= \frac{\sin^4 A - 3\sin^2 A\cos^2 A + 7\cos^4 A}{\left(\sin^2 A + \cos^2 A\right)^2 + \sin^2 A\cos^2 A + 5\cos^4 A}$$

$$= \frac{\tan^4 A - 3\tan^2 A + 7}{\left(\tan^2 A + 1\right)^2 + \tan^2 A + 5}$$

$$= \frac{4-6+7}{9+2+5} = \frac{5}{16}$$

2. (A)



$\sin 2 > 0$

$\sin 3 > 0$

$\sin 5 < 0$

$\sin 2 \sin 3 \sin 5 < 0$

3. (A)

$f(x) = \sec x - \tan x$

$g(x) = \sec x + \tan x$

$f(x).g(x) = 1$

$f(A).f(B).f(C) = g(A).g(B).g(C)$

$f^2(A).f^2(B).f^2(C) = f(A)g(A)f(B).g(B).f(C)g(C)$

$f(A)f(B)f(C) = \pm 1$

4. $\cos(A) = \cos B\cos C$

$\cos(\pi - (B+C)) = \cos B\cos C$

$-[\cos B\cos C - \sin B\sin\ ] = \cos B\cos C$

$\sin B\sin C = 2\cos B\cos C$

$\tan B\tan C = 2$

5. (D)

$$\sum_{r=0}^{10}\cos^3\left(\frac{\pi r}{3}\right)$$

$$1+\left(\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^3+(-1)^3+\left(-\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)+(1)^3+\left(\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^3+(-1)^3+\left(-\frac{1}{2}\right)^3+(-1)^3+\left(-\frac{1}{2}\right)^3=-\frac{1}{8}$$

6. (B)

$U_n = 2\cos n\theta$

$U_1 U_n - U_{n-1}$

$= 2(2\cos n\theta.\cos\theta) - 2\cos(n-1)\theta$

$= 2(\cos(n+1)\theta + \cos(n-1)\theta) - 2\cos(n-1)\theta$

$= 2\cos(n+1)\theta$

$= U_{n+1}$

7. (C)

$\cos 5\theta = a\cos^5\theta + b\cos^3\theta + c\cos\theta$

$\frac{\cos 5\theta}{\cos\theta} = a\cos^4\theta + b\cos^2\theta + c$

$\lim_{\theta\to\frac{\pi}{2}} \frac{\cos 5\theta}{\cos\theta} = c$

$c = \lim_{\theta\to\frac{\pi}{2}} + \frac{5(\sin 5\theta)}{\sin\theta}$

$c = +5$

8. (D)

$f(x) = \frac{\sin 3x}{\sin x}$ $\qquad \sin x \neq 0$

$3 - 4\sin^2 x$

Range in $[-1, 3)$

9. (C)

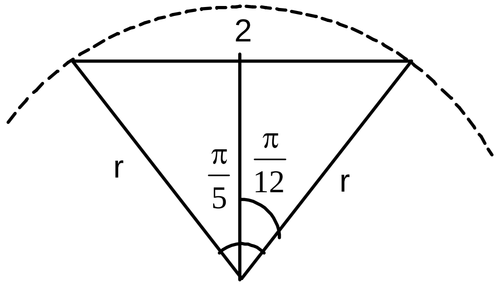
$\cos(\theta+\phi) = m(\cos(\theta-\phi))$

$\frac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\cos\theta\cos\phi + \sin\theta\sin\phi} = \frac{m}{1}$

$\frac{\sin\theta\sin\phi}{\cos\theta\cos\phi} = \frac{1-m}{1+m}$

$\tan\theta = \left(\frac{1-m}{1+m}\right)\cot\phi$

10. (B)



$\sin\frac{\pi}{12} = \frac{1}{r}$

$r = \frac{1}{\sin\frac{\pi}{12}} = \sqrt{6} + \sqrt{2}$

11. (B)

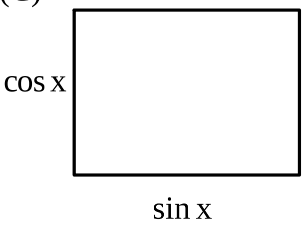
$$\frac{1-2\sin^2\frac{\alpha}{2}}{1+\sin\alpha}=\frac{\cos\alpha}{1+\sin\alpha}$$

$$=\frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)^2}$$

$$=\frac{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{3}}{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}$$

$$=\frac{1-m}{1+m}$$

12. (C)



$$A=\sin x\cos x$$

$$A=\frac{1}{2}\sin 2x$$

$$A\le\frac{1}{2}$$

13. (D)

$$\sin^8 75^\circ-\cos^8 75^\circ$$

$$=\left(\sin^4 75^\circ+\cos^4 75^\circ\right)\left(\sin^2 75^\circ-\cos^2 75^\circ\right)(1)$$

$$=\left(1-\frac{1}{2}\sin^2 150\right)\left(-\cos 150^\circ\right)$$

$$=\left(1-\frac{1}{8}\right)\left(+\frac{53}{2}\right)$$

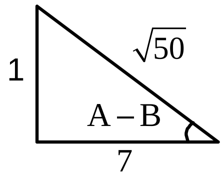
$$=\frac{753}{16}$$

14. (D)

$$\tan A=3$$

$$\tan B=2$$

$$\tan(A-B)=\frac{3-2}{1+6}=\frac{1}{7}$$



$$\sin\left(A-B\right)=\frac{1}{\sqrt{50}}$$

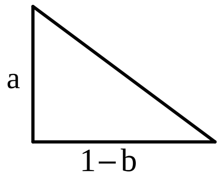
$$\sin 2\left(A-B\right)=2\times 1\frac{1}{\sqrt{50}}\times\frac{7}{550}$$

$$=\frac{7}{25}$$

15. $\tan A+\tan B=a$

$\tan A.\tan B=b$

$$\tan\left(A+B\right)=\frac{a}{1-b}$$



$$\sin\left(A+B\right)=\frac{a}{\sqrt{a^2+\left(1-b\right)^2}}$$

$$\sin^2\left(A+B\right)=\frac{a^2}{a^2+\left(1-b\right)^2}$$

16. (A)

$$\tan B=\frac{n\sin A\cos A}{1-n\cos^2 A}$$

$$\tan\left(A+B\right)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$=\frac{\frac{\sin A}{\cos A}+\frac{n\sin A\cos A}{1-n\cos^2 A}}{1-\frac{\sin A}{\cos A}\frac{n\sin A\cos A}{1-n\cos^2 A}}$$

$$=\frac{\frac{\sin A-n\cos^2 A\sin A+n\cos^2 A\sin A}{\cos A\left(1-n\cos^2 A\right)}}{\frac{1-n\cos^2 A-n\sin^2 A}{1-n\cos^2 A}}=\frac{\sin A}{\cos A\left(1-n\right)}$$

17. (B)

$$\tan\beta=\frac{1}{3}$$

$$\tan 2\beta=\frac{\frac{2}{3}}{1-\frac{1}{9}}=\frac{\frac{2}{5}}{\frac{8}{9}}=\frac{3}{4}$$

$$\tan\alpha = \frac{1}{7}$$

$$\tan(\alpha+2\beta) = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{3}{28}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$$

$$\alpha + 2\beta = \frac{\pi}{24}$$

18. (C)

$$\sin A = \sin B$$
$$\cos A = \cos B$$
$$\Rightarrow A = B + 2n\pi$$
$$\sin\left(\frac{A-B}{2}\right) = 0$$

19. (C)

$$1 - \frac{1}{\sin\theta+\cos\theta}\left(\sin^3\theta+\cos^3\theta\right)$$
$$= 1 - \frac{1}{\sin\theta+\cos\theta}(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)$$
$$= \sin\theta\cos\theta$$

20. (B)

$$\frac{\cos x\cos y+\sin x\sin y}{\cos x\cos y-\sin x\sin y} = \frac{a}{1}$$
$$\cot x\cot y = \frac{1+a}{a-1}$$

21. $2\sin A = \sqrt{3}\sin B$ ________(1)

$2\cos A = \sqrt{5}\cos B$ _________-(2)

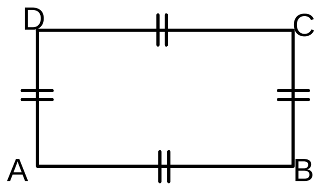
Square and add (1) & (2)

$$3\sin^2 B + 5\cos^2 B = 4$$
$$3\tan^2 B + 5 = 4\sec^2 B \text{ or } \tan B = 1$$
$$\tan A = \frac{\sqrt{3}}{\sqrt{5}}$$
$$\tan A + \tan\beta = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}}$$

22. $P_n - P_{n+2} = \cos^n\theta + \sin^n\theta - \cos^{n-2}\theta - \sin^{n-2}\theta$

$$= \cos^{n-2}\theta\left(-\sin^2\theta\right) + \sin^{n-2}\theta\left(-\cos^2\theta\right)$$
$$= -\sin^2\theta\cos^2\theta\left(\cos^{n-4}\theta+\sin^{n-4}\theta\right)$$
$$= k\,P_{n-4}$$
$$K = -\sin^2\theta\cos^2\theta$$

23. Given



$AB^2 = AC \times BD$

$AC^2 + BD^2 = 4AB \,\&\, \tan\theta = \dfrac{BD}{AC}$

$\Rightarrow AC^2 + BD^2 = 4AC\,BD$

$\Rightarrow \left(\dfrac{BD}{AC}\right)^2 + 1 = 4\left(\dfrac{BD}{AC}\right)$

$\tan^2\theta + 1 = 4\tan\theta$

$\sin 2\theta = \dfrac{1}{2} \Rightarrow \theta = 15^\circ$

24. (A)

$\sin A\left(\dfrac{A}{2} + \left(\dfrac{A}{2} + \dfrac{C}{2}\right)\right) = K\sin\left(\dfrac{\pi - (A+B)}{2}\right)$

$\sin\left(\dfrac{A}{2} + \dfrac{\pi - B}{2}\right) = K\cos\left(\dfrac{A+B}{2}\right)$

$\cos\left(\dfrac{A-B}{2}\right) = K\cos\left(\dfrac{A+B}{2}\right)$

$$\frac{\cos\frac{A}{2}\cos\frac{B}{2} + \sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2} - \sin\frac{A}{2}\sin\frac{B}{2}} = \frac{K}{1}$$

$\dfrac{K-1}{K+1} = \tan\dfrac{A}{2}\tan\dfrac{B}{2}$

25. $\dfrac{3 + \cot 16^\circ \cot 16^\circ}{\cot 76^\circ + \cot 16^\circ}$

$\cot 17^\circ + \cot 16^\circ = \dfrac{\sin 92^\circ}{\sin 76^\circ \sin 16} = \dfrac{\sin 88}{\sin 16 \sin 16^\circ}$

$1 + \cot 76 \cot 16 = \dfrac{1}{2\sin 76 \sin 16}$

$$LHS = \frac{3 + \cot 17\cot 16}{\cot 76 + \cot 16} = \frac{2 + \dfrac{1}{2\sin 76\sin 16}}{\dfrac{\sin 88}{\sin 76 \sin 16}} = \frac{2(\cos 60 - \cos 92) + 1}{\sin 88^\circ}$$

$= \dfrac{2\sin^2 46}{2\sin 44^\circ \cos 44} = \cot 44^\circ = \tan 46^\circ$

26. $\dfrac{2\left(\sin 1^\circ + \sin 2^\circ --------\sin 89^\circ\right)}{2\left(\cos 1^\circ + \cos 2^\circ -------\cos 44h^\circ\right) + 1}$

$$N^r = 2\left(\sin 1^\circ + \sin 2^\circ ------ \sin 89^\circ\right) = \frac{2\sin\left(\frac{89}{2}\right)\sin 45}{\sin\left(\frac{1}{2}\right)}$$

$$D^r = 2\left(\cos 1^\circ + \cos 2^\circ ------- \cos 89^\circ\right) = \frac{2\sin 22^\circ \cos\frac{45}{2}}{\sin\frac{1^\circ}{2}} + 1$$

$$= \frac{-\sin\frac{1^\circ}{2} + \sin\frac{89^\circ}{2}}{\sin\frac{1^\circ}{2}} + 1$$

$$\frac{N^r}{D^r} = \frac{\frac{2\sin\frac{89}{2}\sin 45}{\sin\frac{1}{2}}}{\frac{\sin\frac{89}{2}}{\sin\frac{1}{2}}} = \sqrt{2}$$

$$= \frac{\sin\frac{89}{2}}{\sin\frac{1}{2}}$$

27. (A)

$x \in \left(\frac{5\pi}{4}, \frac{9\pi}{4}\right)$

$\sin x < -\frac{1}{2}$

$\cos x > -\frac{1}{2}$

$\sin x - \cos x < 0$

$\sin x - \cos x = t$

$1 - \sin 2x = t^2$

$t^2 = 1 - \frac{2024}{2025}$

$t^2 = \frac{1}{2025}$

$t = \frac{-1}{45}$

28. (D)

$(\tan 4\theta + \tan 2\theta)(1 - \tan^2 3\theta \tan^2 \theta)$

$= \frac{\sin 6\theta}{\cos 4\theta \cos 2\theta}(1 - \tan 2\theta \tan\theta)(1 + \tan 3\theta \tan\theta)$

$= \frac{\sin 6\theta}{\cos 4\theta \cos 2\theta}\cos^2 3\theta \cos^2\theta$

$= 2\tan 3\theta \sec^2\theta$

29. (D)

$$\frac{1}{\tan x}+\frac{1-\sqrt{3}\tan x}{\sqrt{3}+\tan x}+\frac{1+\sqrt{3}\tan x}{\tan x-\sqrt{3}}$$
$$=\frac{3}{\tan 3x}=\frac{3\left(1-3\tan^2 x\right)}{3\tan x-\tan^3 x}$$

30. $$\frac{a\cos\alpha-b\sin\alpha}{\sin\alpha\cos\alpha}$$
$$=\sqrt[2]{a^2+b^2}\,\frac{\left(\sin\theta\cos\alpha-\cos\alpha\sin\alpha\right)}{\sin 2\alpha}$$
$$=\sqrt[2]{a^2+b^2}\qquad\frac{\sin\left(\theta-\alpha\right)}{\sin 2\alpha}\qquad\left(\theta=3\alpha\right)$$
$$=\sqrt[2]{a^2+b^2}$$

31. (D)
$$2\sin\frac{A}{2}=\sin\frac{A}{2}+\cos\frac{A}{2}-\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)$$
$$=-\sqrt{1+\sin A}+\sqrt{1-\sin A}$$

32. (A)
$$p^2=1-\sin 40^\circ$$
$$\sin 40^\circ=1-p^2\qquad\left(p<0\right)$$
$$\cos 40^\circ=\sqrt{1-\left(1-p^2\right)^2}$$
$$=-p\sqrt{2-p^2}$$

33. (C)
$$\tan\frac{\alpha}{2}+\cot\frac{\alpha}{2}$$
$$=\frac{2}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$$
$$=\frac{2}{p}$$
$$x^2-\frac{2}{p}+1=0$$
$$px^2-2x+p=0$$

34. (B)
$$\frac{1}{\cos 290^\circ}+\frac{1}{\sqrt{3}\sin 250^\circ}$$
$$\frac{1}{\sin 20^\circ}-\frac{1}{\sqrt{3}\cos 20^\circ}$$
$$\frac{2\left(\sqrt{3}\cos 20^\circ-\sin 20^\circ\right)}{\sqrt{3}\,2\sin 20^\circ\cos 20^\circ}$$

$$=\frac{4}{13}\frac{\sin\left(60^\circ-20^\circ\right)}{\sin 40^\circ}=\frac{4}{\sqrt{3}}$$

35. (B)

$\sin x+\sin y=a$

$\cos x+\cos y=b$

$=2$

$$a=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$b=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\frac{a}{b}+\frac{b}{a}=\frac{1}{\sin\left(\frac{x+y}{2}\right)\cos d\left(\frac{x+y}{2}\right)}$$

$$\frac{a^2+b^2}{ab}=\frac{2}{\sin\left(x+y\right)}$$

$$\sin\left(x+y\right)=\frac{2ab}{a^2+b^2}$$

36. (C)

$$\sqrt{2}\sin\frac{3\pi}{20}+\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\frac{\pi}{10}-\frac{1}{\sqrt{2}}\cos\frac{\pi}{10}\right)$$

$$=\sqrt{2}\sin\frac{3}{20}-\sqrt{2}\left(\sin\left(\frac{\pi}{4}-\frac{\pi}{10}\right)\right)=0$$

37. (A)

If $A+B=\frac{\pi}{4}$

$\tan\left(A+B\right)=1$

$\tan A+\tan B+\tan A\tan B+1=2$

$\left(1+\tan A\right)\left(1+\tan\right)=2$

$A=11^\circ$ $\qquad$ $B=34^\circ$

$\left(1+\tan A\right)\left(1+\tan 34^\circ\right)=2$

$A=17^\circ$ $\qquad$ $B=28^\circ$

$\left(1+\tan 17^\circ\right)$ $\qquad$ $\left(1+\tan 28^\circ\right)=2$

Hence $\dfrac{\left(1+\tan 11^\circ\right)\left(1+\tan 34^\circ\right)}{\left(1+\tan 17^\circ\right)\left(1+\tan 28^\circ\right)}=1$

38. $$\frac{4\sin 9^\circ\sin 21^\circ\sin 39^\circ\sin 51^\circ\sin 69^\circ\sin 81^\circ}{\sin 54^\circ}$$

$$=\frac{4\left(\sin 9^\circ\sin 31^\circ\sin 69^\circ\right)\left(\sin 21^\circ\sin 39^\circ\sin 81^\circ\right)}{\sin 54^\circ}$$

$$=\frac{\frac{4\left(\sin 27\right)}{4}\left(\frac{\sin 63}{4}\right)}{\sin 54}$$

$$=\frac{1}{4}\frac{\sin 27^\circ \cos 27^\circ}{\sin 54}=\frac{1}{8}$$

39. NO SOLUTION

40. (A)

$$\sin^2\theta=\frac{1}{2}\left(\frac{x}{y}+\frac{y}{x}\right)$$

Possible only when $x=y\left(\frac{x}{y}+\frac{y}{x}\in(-\infty,-2]\cup[2,\infty)\right)$

41.

$$\cos\frac{6\pi}{7}=-\cos\frac{\pi}{7}$$
$$\cos\frac{5\pi}{7}=-\cos\frac{2\pi}{7}$$
$$\cos\frac{3\pi}{7}=-\cos\frac{4\pi}{7}$$
$$\left(\cos\frac{\pi}{7}+\cos\frac{6\pi}{7}\right)+\left(\cos\frac{2\pi}{7}+\cos\frac{5\pi}{7}\right)+\left(\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}\right)+\cos\pi=-1$$

42. (D)

$$\cot(\alpha+\beta)=0$$
$$\alpha+\beta=\frac{\pi}{2}$$
$$\sin(\alpha+2\beta)=\sin\left(\frac{\pi}{2}+\beta\right)=\cos\beta$$

43. (C)

$$\sin x=1-\sin^2 x$$
$$\sin x=\cos^2 x$$
$$=\sin^6+3\sin^5 x+3\sin^4 x+\sin^3 x-2$$
$$=\left(\sin x+\sin^2 x\right)^3-2$$
$$1-2=-1$$

44. (D)

$$\frac{1}{4}\left[\sqrt{3}\cos 30^\circ-\sin 23^\circ\right]$$
$$\frac{1}{2}\left[\sin 60^\circ\cos 23-\cos 60\sin 23\right]$$
$$\frac{1}{2}\sin 37^\circ$$

45. (B)

$$\tan 60^\circ=\sqrt{3}$$
$$\tan\left(20^\circ+40^\circ\right)=\sqrt{3}$$
$$\tan 20^\circ+\tan 40^\circ=\sqrt{3}-\sqrt{3}\tan 20^\circ\tan 40^\circ$$

$$\tan 20^\circ + \tan 40^\circ + \sqrt{3}\tan 20^\circ \tan 40^\circ = \sqrt{3}$$

46. (A)

$$\tan(\alpha+\beta)=1$$
$$\tan\alpha+\tan\beta=1-\tan\alpha\tan\beta$$
$$(1+\tan\alpha)(1+\tan\beta)=2$$
$$f(\theta)=\frac{1}{1+\tan\theta}$$
$$f(\alpha)f(\beta)=\frac{1}{(1+\tan\alpha)(1+\tan\beta)}=\frac{1}{2}$$

47. (A)

If $A+B+C=\pi$ $\quad \tan A+\tan B+\tan C=\tan A+\tan B+\tan C$

$\tan A\tan B\tan C=6$ $\quad \tan A+\tan B=3$

$\tan A\tan B=2$ $\quad \tan A=1$

$\Rightarrow \tan C=3$ $\quad \tan B=2$

48. (D)

$$\frac{(\sin\alpha+\cos\alpha)^2}{\cos 2\alpha\left(\frac{\tan\alpha+1}{\tan\alpha-1}\right)}-\frac{1}{4}\sin 2\alpha\left[\cot\frac{\alpha}{2}-\tan\frac{\alpha}{2}\right]$$

$$\frac{(\sin\alpha+\cos\alpha)^2(\sin\alpha-\cos\alpha)}{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)(\sin\alpha+\cos\alpha)}-\frac{1}{42}\sin\alpha\cos\alpha\frac{\cos\alpha}{\left(\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)}$$

$$1-\cos^2\alpha=\sin^2\alpha$$

49. $x=\sqrt{2+\sqrt{2-\sqrt{2+x}}}$

$$x=2\cos\theta$$
$$2\cos\theta=\sqrt{2+\sqrt{2-2\cos\frac{\theta}{2}}}$$
$$=\sqrt{2+\sqrt{2(1-\cos)\left(\frac{\theta}{2}\right)}}$$
$$=\sqrt{2+2\sin\frac{\theta}{4}}$$
$$=\sqrt{2\left(1+\cos\left(\frac{\pi}{2}-\frac{\theta}{4}\right)\right)}$$
$$2\cos\theta=2\cos\left(\frac{\pi}{4}-\frac{\theta}{8}\right)$$
$$\theta=\frac{\pi}{4}-\frac{\theta}{8}$$
$$\frac{9\theta}{8}=45^\circ$$
$$\theta=40^\circ$$

Hence $x=2\cos 40^\circ$

50. (B)

$$\frac{1}{\sin 1^\circ}\left(\frac{\sin\left(46^\circ-45^\circ\right)}{\sin 46\sin 45}+\frac{\sin\left(48-47\right)}{\sin 48\sin 47}-------\right)$$

$$\frac{1}{\sin 1^\circ}\left(\cot 45-\cot 46+\cot 47-\cot 48------\right)$$

$$=\frac{1}{\sin 1^\circ}=\operatorname{cosec}1^\circ$$

51. $2\sin x+4\cos x+12\sin y+5\cos y+5\cos y=18$

$$5\cos\left(x-\tan^{-1}\frac{3}{4}\right)+13\cos\left(y-\tan^{-1}\frac{12}{5}\right)=8$$

Possible $x=\tan^{-1}\frac{3}{4}$

$y=\tan^{-1}\frac{12}{5}$

$$\tan\left(x+y\right)=\frac{\frac{3}{4}+\frac{12}{5}}{1-\frac{3}{4}\times\frac{12}{5}}=-\frac{63}{16}$$

52. $\sum\tan B\tan C=x$

$$\left(1-\tan B\tan C\right)=\frac{\cos\left(B+C\right)}{\cos B\cos C}=\frac{-\cos A}{\cos B\cos C}$$

$$\tan B\tan C=1+\frac{\cos A}{\cos B\cos C}$$

$$\sum\tan B\tan C=3+\sum\frac{\cos A}{\cos B\cos C}$$

$$=3+\sum\frac{\cos^2 A}{\cos A\cos B\cos C}$$

$$\sum\cos^2 A=\frac{3+\cos 2A+\cos 2B+\cos 2C}{2}$$

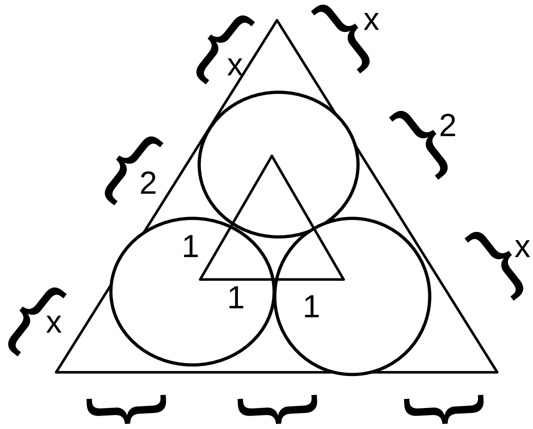
$$\frac{2-4\pi\cos A}{2}=1-2\pi\left(\cos A\right)$$

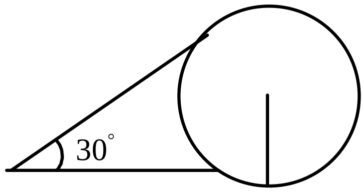
$$\sum\tan B\tan C=3+\frac{1-2\pi\cos A}{\pi\cos A}$$

$$=1+\frac{1}{\pi\cos A}$$

$$=\frac{9+1}{9}$$

53. $\tan 30^\circ=\frac{1}{x}=\frac{1}{\sqrt{3}}$





$x = \sqrt{3}$

Side of equilateral triangle

$= 2 + 2\sqrt{3}$

Area $= \dfrac{\sqrt{3}}{4}\left(2 + 2\sqrt{3}\right)^2$

$= 4\sqrt{3} + 6$

54. $\displaystyle\sum_{m=1}^{6} \cos ec\left(\theta + (m-1)\frac{\pi}{4}\right) = 4\sqrt{2}$

$$\frac{1}{\sin\frac{\pi}{4}}\sum_{m=1}^{6}\frac{\sin\frac{\pi}{4}}{\sin\left(\theta + (m-1)\frac{\pi}{4}\right)\sin\left(\theta + \frac{M\pi}{4}\right)} = 4\sqrt{2}$$

$$= \sqrt{2}\sum_{M=1}^{6}\frac{\sin\left(\theta + m\frac{\pi}{4} - \left(\theta + (m-1)\frac{\pi}{4}\right)\right)}{\sin\left(\theta + (m-1)\frac{\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)}$$

$$\frac{\sin(A-B)}{\sin A \sin B} = \cot B - \cot A$$

$$\sqrt{2}\sum_{m=1}^{6}\left(\cot\left(\theta + (m-1)\frac{\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)\right)$$

$$= \sqrt{2}\left(\cot\theta - \cot\left(\theta + \frac{3\pi}{2}\right)\right) = 4$$

$$\cot\theta + \tan\theta = 4$$

$$\sin 2\theta = \frac{1}{2}$$

$$2\theta = \frac{\pi}{6} \qquad \text{or } \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{12} \qquad \text{or } \frac{5\pi}{12}$$

55. $3\sin^2 A + 2\sin^2 B = 1$

$\Rightarrow 3\sin^2 A = \cos 2B$ ________(1)

Also $3\sin 2A = 2\sin 2B$ __________(2)

From (1) & (2)

$$\frac{3\sin^2 A}{3(2\sin A\cos A)} = \frac{\cos 2B}{2\sin 2B}$$

$\Rightarrow \tan A = \cot 2B$

$$\tan A = \tan\left(\frac{\pi}{2} - 2B\right)$$

$$A + 2B = \frac{\pi}{2}$$

56. $\tan 20^\circ \tan 80^\circ \cot 50^\circ$

$= \tan 20 \tan(60+20)\tan(60-20)$

$= \tan(3\times 20^\circ)$

$= \sqrt{3}$

57. $a\cos x + b\sin x = c$

Put $\cos x = \frac{1-t^2}{1+t^2}$ and $\sin x = \frac{2t}{1+t^2}$

Where $t = \tan\frac{x}{2}$

$$a\left(\frac{1-t^2}{1+t^2}\right) + b\left(\frac{2t}{1+t^2}\right) = c$$

$(a+c)t^2 - 2bt + c - a = 0$

Roots of this equation are $\tan\frac{x_1}{2}$ and $\tan\frac{x_2}{2}$

$$\tan\frac{x_1}{2} + \tan\frac{x_2}{2} = \frac{2b}{a+c}$$

$$\tan\frac{x_1}{2}, \tan\frac{x_2}{2} = \frac{c-a}{a+c}$$

$$\tan\left(\frac{x_1+x_2}{2}\right) = \frac{\frac{2b}{a+c}}{1-\frac{c-a}{c+1}} = \frac{2b}{2a} = \frac{b}{a}$$

58. $\sin x + \cos x = \frac{\sqrt{7}}{2}$

$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = \frac{\sqrt{7}}{2}$ Where $t = \tan\frac{x}{2}$

$$\left(\sqrt{7}+2\right)t^2-4t+\left(\sqrt{7}-2\right)=0$$

$$t=\frac{4\pm\sqrt{16-4(57+2)(57-2)}}{2\left(\sqrt{7}+2\right)}=\frac{1}{3}\left(\sqrt{7}-2\right)\text{ or }\left(\sqrt{7}-2\right)$$

$$\tan\frac{x}{2}<\tan\frac{\pi}{8}$$

$$x\in\left[0,\frac{\pi}{4}\right]\quad\frac{x}{2}\in\left[0,\frac{\pi}{8}\right]$$

So $\tan\frac{x}{2}$ will have lower value

$$\tan\frac{x}{2}=\frac{\sqrt{7}-2}{3}$$

59. $\cos x+\cos y-\cos(x+y)=\frac{3}{2}$

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)-2\cos^2\left(\frac{x+y}{2}\right)+1=\frac{3}{2}$$

$$4\cos^2\left(\frac{x+y}{2}\right)-4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)+1=0$$

$D\geq 0$

$$16\cos^2\left(\frac{x-y}{2}\right)-16\geq 0$$

$$-16\sin^2\left(\frac{x-y}{2}\right)\geq 0$$

Possible only at $x=y$

60. $\tan x=n\tan y$ $\quad(\tan y=t)$

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$$

$$=\frac{nt-t}{1+nt^2}$$

$$\tan(x-y)=\frac{t(n-1)}{1+nt^2}=\alpha$$

$\sec^2(x-y)$ will be max if $\tan(x-y)$ is max

$$\alpha+n\alpha t^2=(n-1)t$$

$D\geq 0$

$$(n-1)^2-4n\alpha^2\geq 0$$

$$\alpha^2\leq\frac{(n-1)^2}{4n}$$

$$\tan^2(x-y)\leq\frac{(n-1)^2}{4n}$$

$$\sec^2(x-y)\leq\frac{(n-1)^2}{4n}+1$$

$$\leq \frac{(n+1)^2}{4n}$$

TRIGO – 1

EX – 2A

1. **(ABC)**

$$a = \cos x + \cos\left(x + \frac{2\pi}{3}\right) + \cos\left(x \frac{4\pi}{3}\right)$$
$$= \cos x + 2\cos x \cdot \cos(2\pi/3)$$
$$= \cos x - \cos x = 0$$
$$b = \sin x + \sin\left(x + \frac{2\pi}{3}\right) + \sin\left(x + \frac{4\pi}{3}\right)$$
$$= \sin x + 2\sin x \cdot \cos\left(\frac{2\pi}{3}\right)$$
$$= \sin x - \sin x = 0$$

2. **(BD)**

$$\left.\begin{array}{l} \sin 1 \approx \sin 57^o \\ \sin 2 \approx \sin 114^o \approx \sin 66^o \\ \sin 3 \approx \sin 171^o \approx \sin 9^o \end{array}\right\} \sin 2 > \sin 1 > \sin 3$$

$$\left.\begin{array}{l} \cos 1 \approx \cos 57^o \\ \cos 2 \approx \cos 114^o \approx -\cos 66^o \\ \cos 3 \approx \cos 171^o \approx -\cos 9^o \end{array}\right\} \cos 1 > \cos 2 > \cos 3$$

3. **(AB)**

$$x = r\sin A \cos B$$
$$y = r\sin A \sin B$$
$$z = r\cos A$$
$$x^2 + y^2 + z^2 = r^2\sin^2 A + r^2\cos^2 A = r^2$$

4. **(BCD)**

Quadrilateral is cyclic hence $\left.\begin{array}{l} P + R = \pi \\ Q + S = \pi \end{array}\right\}$

5. **(ABD)**

$$\tan\frac{x}{2} \in \theta$$

(i) $\sin x = \dfrac{2\tan x/2}{1 + \tan^2 x/2} \in \theta$

(ii) $\cos x = \dfrac{1 - \tan^2 x/2}{1 + \tan^2 x/2} \in \theta$

(iii) $\sec\dfrac{x}{2} = \sqrt{1 + \tan^2\dfrac{x}{2}}$ (not necessarily)

(iv) $\tan x = \dfrac{2\tan x/2}{1 - \tan^2 x/2} \in \theta$

6. **(ABCD)**

$$2\sin A/2 = \pm\sqrt{1+\sin A} \pm \sqrt{1-\sin A}$$

7. **(AD)**

$$\left(4\cos^2 9^o - 3\right)\left(4\cos^2 27^0 - 3\right)$$

$$= \frac{\left(4\cos^3 9^o - 3\cos 9^o\right)\left(4\cos^3 27^o - 3\cos 27^0\right)}{\cos 9^o \cdot \cos 27^o}$$

$$= \frac{\cancel{\cos 27^o} \cdot \cos 80^o}{\cos 9^o \cdot \cancel{\cos 27^o}} = \frac{\sin 9^o}{\cos 9^o} = \tan 9^o = \cot 81^o$$

8. **(AD)**

$$\cos\beta = \sqrt{\sin\alpha \cdot \cos\alpha}$$

$$\Rightarrow \cos^2\beta = \sin\alpha \cdot \cos\beta$$

$$\Rightarrow \frac{1+\cos 2\beta}{2} = \frac{\sin 2\alpha}{2} \Rightarrow \cos 2\beta = -\left(1-\sin 2\alpha\right)$$

$$= -\left(1-\cos\left(\pi/2 - 2\alpha\right)\right)$$

$$= -2\sin^2\left(\pi/4 - \alpha\right)$$

$$= -2\cos^2\left(\pi/4 + \alpha\right)$$

9. **(BC)**

$$\frac{\sqrt{1+\cos\alpha} + \sqrt{1-\cos\alpha}}{\sqrt{1+\cos\alpha} - \sqrt{1-\cos\alpha}} \quad ; \quad \alpha \in \left(\pi, 2\pi\right)$$

$$= \frac{\cancel{\sqrt{2}}\left(\left|\cos\frac{\alpha}{2}\right| + \left|\sin\frac{\alpha}{2}\right|\right)}{\cancel{\sqrt{2}}\left(\left|\cos\frac{\alpha}{2}\right| - \left|\sin\frac{\alpha}{2}\right|\right)}$$

$$= \frac{-\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}}{-\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}} = \frac{1-\tan\frac{\alpha}{2}}{1+\tan\frac{\alpha}{2}} = \tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$$

$$= \cot\left(\pi/4 + \alpha/2\right)$$

10. **(CD)**

$$\cos ec\theta - \cot\theta = q$$

$$\cos ec\theta + \cot\theta = \frac{1}{q}$$

So, $\cos ec\theta = \dfrac{\left(a + \dfrac{1}{q}\right)}{2}$

$$\cot\theta = \frac{\left(\frac{1}{q} - q\right)}{2}$$

11. **(AD)**

$$\cos A \cdot \cos B + \sin A \sin B = \frac{3}{5} \qquad \ldots(1)$$

$$\sin A \sin B = 2\cos A \cos B \qquad \ldots(2)$$

So, $\cos A \cos = \frac{1}{5}$

$$\sin A \sin B = \frac{2}{5}$$

12. **(BCD)**

13. **(ABD)**

$$\frac{\sin 22^o \cos 8^o + \cos 158^o \cos 98^o}{\sin 23^o \cos 7^o + \cos 157 \cos 97^o}$$

$$= \frac{\frac{1}{2}\left(\sin 30^o + \sin 14^o + \cos 256^o + \cos 60^o\right)}{\frac{1}{2}\left(\sin 30^o + \sin 16^o + \cos 254^o + \cos 60^o\right)}$$

$$\sin 30^o = \cos 60^o = \frac{1}{2}$$

$$\cos 254^o = \cos\left(270^o - 16^o\right) = -\sin 16^o$$

$$\cos 256^o = \cos\left(270^o - 14^o\right) = -\sin 14^o$$

$$\therefore = \frac{\frac{1}{2}\left(\frac{1}{2} + \sin 14^o - \sin 14^o + \frac{1}{2}\right)}{\frac{1}{2}\left(\frac{1}{2} + \sin 16^o - \sin 16^o + \frac{1}{2}\right)}$$

$$\sec\left(-100\pi\right) = 1$$

$$\operatorname{cosec}\left(\frac{-3\pi}{2}\right) = \operatorname{cosec}\left(\frac{\pi}{2}\right) = 1 \qquad \ldots(B)$$

$$\sin\left(\frac{7\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1 \neq 1$$

$$\cot\left(\frac{5\pi}{4}\right) = 1 \qquad \ldots(D)$$

14. **(AD)**

$$\sqrt{\frac{1-\sin A}{1+\sin A}} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$$

$$\sqrt{\frac{1-\sin A}{1+\sin A}}$$

$$\Rightarrow \sqrt{\left(\frac{1-\sin A}{1+\sin A}\right)\left(\frac{1+\sin A}{1+\sin A}\right)}$$

$\Rightarrow \sqrt{\frac{1-\sin^2 A}{(1+\sin A)^2}} = \sqrt{\frac{\cos^2 A}{(1+\sin A)^2}}$

$\Rightarrow \left|\frac{\cos A}{1+\sin A}\right|$ now 1 + sin A is always positive

$\Rightarrow \left|\frac{\cos A}{(1+\sin A)}\frac{(1-\sin A)}{(1-\sin A)}\right|$

$\Rightarrow \left|\frac{\cos A(1-\sin A)}{(1-\sin^2 A)}\right| \Rightarrow \left|\frac{1-\sin A}{\cos A}\right|$

L.H.S

$\Rightarrow \frac{1-\sin A}{|\cos A|} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$

$\frac{1-\sin A}{|\cos A|} = \frac{1-\sin A}{\cos A}$

$\Rightarrow \sin A = 1$ or $|\cos A| = \cos A$

But $\sin A \neq 1$

$\because$ then $\cos A = 0$

$|\cos A| = \cos A$ means cos A is positive i.e. 1^{st} and 4^{th} Quadrant

15. **(ABCD)**

$\sec A = \frac{17}{8}$ $\qquad$ $\operatorname{cosec} B = \frac{5}{4}$

$\cos A = \frac{8}{17}$ $\qquad$ $\sin B = \frac{4}{5}$

$\sin A = \pm\frac{15}{17}$ $\qquad$ $\cos B = \pm\frac{3}{5}$

$\therefore \sec(A+B) = \frac{1}{\cos(A+B)} = \frac{1}{\cos A \cos B - \sin A \sin B}$

$= \frac{1}{\left(\frac{8}{17}\right)\left(\pm\frac{3}{5}\right) - \left(\pm\frac{15}{17}\right)\left(\frac{4}{5}\right)}$

$= \frac{85}{\pm 24 \pm 60}$

Possible answer are

$\frac{85}{84}, \frac{85}{-84}, \frac{85}{36}, \frac{85}{-36}$

16. **(ABCD)**

(A) $\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}-\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$

$= \frac{\cos 2\alpha}{2\tan\left(\frac{\pi}{4}-\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$

$$= \frac{\cos 2\alpha}{2\sin\left(\frac{\pi}{4}-\alpha\right)\cos\left(\frac{\pi}{4}-\alpha\right)}$$

$$= \frac{\cos 2\alpha}{\sin\left(2\left(\frac{\pi}{4}-\alpha\right)\right)}$$

$$= \frac{\cos 2\alpha}{\sin\left(\frac{\pi}{2}-2\alpha\right)} = \frac{\cos 2\alpha}{\cos 2\alpha} = 1$$

(B) $$\frac{\sin(\pi-\alpha)}{\sin\alpha - \cos\alpha\tan\frac{\alpha}{2}} + \cos(\pi-\alpha)$$

$$= \frac{\sin\alpha}{\frac{\left(\sin\alpha\cos\frac{\alpha}{2} - \cos\alpha\sin\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}}} - \cos\alpha$$

$$= \frac{\sin\alpha\cos\frac{\alpha}{2}}{\sin\left(\alpha - \frac{\alpha}{2}\right)} - \cos\alpha$$

$$= \frac{\left(2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\right)\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} - \cos\alpha$$

$$= 2\cos^2\frac{\alpha}{2} - \cos\alpha$$

$$= 1 + \cos\alpha - \cos\alpha = 1$$

(C) $$\frac{\left(1-\tan^2\alpha\right)}{4\tan^2\alpha}$$

$$= \left(\frac{1\tan^2\alpha\pi}{2\tan\alpha}\right)^2 = \cot^2 2\alpha$$

$$\frac{1}{4\sin^2\alpha\cos^2\alpha} = \cos ec^2 2\alpha$$

$$\therefore \cos ec^2 2\alpha - \cot^2 2\alpha = 1$$

(D) $$1 + \sin 2\alpha = (\sin\alpha + \cos\alpha)^2$$

$$\therefore \frac{1+\sin 2\alpha}{(\cos\alpha + \sin\alpha)^2} = 1$$

17. **(AD)**

$$\frac{\cos A}{\sin B\sin C} + \frac{\cos B}{\sin C\sin A} + \frac{\cos C}{\sin A\sin B}$$

$$= \frac{\sin A\cos A + \sin B\cos B + \sin C\cos C}{\sin A\sin B\sin C}$$

$$A + B + C = \pi$$

$$\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$$

$\therefore$ Numerator = 2sin A sin B sin C
L.H.S = 2.

18. **(ABC)**

$$\frac{\sqrt{3}\sin(\alpha+\beta)-\dfrac{1}{\cos\left(\dfrac{\pi}{6}\right)}\cos(\alpha+\beta)}{\sin\alpha}$$

$$=\frac{\sqrt{3}\sin(\alpha+\beta)-\dfrac{4}{\sqrt{3}}\cos(\alpha+\beta)}{\sin\alpha}$$

$\sin\beta=\frac{4}{5}$

If $\beta\in\left(0,\frac{\pi}{2}\right)$ and $\tan\beta>0$

Then $\cos\beta=\frac{3}{5}$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$=\frac{3}{5}\sin\alpha+\frac{4}{5}\cos\alpha$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=\frac{3}{5}\cos\alpha-\frac{4}{5}\sin\alpha$

$=\frac{1}{\sqrt{3}\sin\alpha}\left(3\sin(\alpha+\beta)-4\cos(\alpha+\beta)\right)$

$=\frac{1}{\sqrt{3}\sin\alpha}\left(\frac{9}{5}\sin\alpha+\frac{12}{5}\cos\alpha-\frac{12}{5}\cos\alpha+\frac{16}{5}\sin\alpha\right)$

$=\frac{1}{\sqrt{3}\sin\alpha}\left(\frac{25}{5}\sin\alpha\right)=\frac{5}{\sqrt{3}}$

(A) & (B) true

For $\tan\beta<0 \quad \cos\beta=\frac{-3}{5}$

$\sin(\alpha+\beta)=\frac{-3}{5}\sin\alpha+\frac{4}{5}\cos\alpha$

$\cos(\alpha+\beta)=\frac{-3}{5}\cos\alpha-\frac{4}{5}\sin\alpha$

L.H.S. $=\frac{1}{\sqrt{3}\sin\alpha}\left(3\sin(\alpha+\beta)-4\cos(\alpha+\beta)\right)$

$=\frac{1}{\sqrt{3}\sin\alpha}\left(\frac{-9}{5}\sin\alpha+\frac{12}{5}\cos\alpha+\frac{12}{5}\cos\alpha+\frac{16}{5}\sin\alpha\right)$

$=\frac{1}{\sqrt{3}\sin\alpha}\left(\frac{24}{5}\cos\alpha+\frac{7}{5}\sin\alpha\right)$

$= \left(\frac{24\cos\alpha + 7\sin\alpha}{5\sqrt{3}\sin\alpha} \right)$

$= \frac{\sqrt{3}}{15}\left(24\cot\alpha + 7\right)$

$\therefore$ (A) (B) (C)

19. **(ACD)**

$\cos 20^o + \cos 40^o + \cos 60^o + ... + \cos 140^o$

$= \left(\cos 40^o + \cos 140^o\right) + \left(\cos 60^o + \cos 120^o\right) + ... + \left(\cos 80^o + \cos 100^o\right) + \cos 20^o$

As $\cos\left(\pi - \theta\right) = -\cos\theta$

$\therefore \cos\left(\pi - \alpha\right) + \cos\theta = 0$

L.H.S

$= 0 + \cos 20^o$ (A)

$= \cos 20^o$

$= \sin 70^o$ (D)

$= \cos 20^o$

$= \cos\left(30^o - 10^o\right)$

$= \frac{\sqrt{3}}{2}\cos 10^o + \frac{1}{2}\sin 10^o$ (C)

20. **(ABC)**

$x\cos\alpha + 4\sin\alpha = K$ where

$\alpha = A, B$ are roots of this equation

$x\cos\theta = k - y\sin\theta$

Square

$x^2\cos^2\theta = \left(k - y\sin\theta\right)^2$

$x^2\left(1 - \sin^2\theta\right) = k^2 + y^2\sin^2\theta - 2ky\sin\theta$

$x^2 - x^2\sin^2\theta = k^2 + y^2\sin^2\theta - 2ky\sin\theta$

$0 = \left(x^2 + y^2\right)\sin\theta - 2ky\sin\theta + \left(k^2 - x^2\right)$

Quadratic in $\sin\theta$ roots are sin t, sin B

$\sin A\sin B = \frac{k^2 - x^2}{x^2 + y^2}$

$\sin A + \sin B = \frac{2ky}{x^2 + y^2}$

$y\sin\theta = k - x\cos\theta$

Square

$y^2\sin^2\theta = \left(k - x\cos\theta\right)^2$

$y^2\left(1 - \cos^2\theta\right) = k^2 + x^2\cos^2\theta - 2kx\cos\theta$

$\left(x^2 + y^2\right)\cos^2\theta - 2kx\cos\theta + k^2 - y^2 = 0$

Quadratic in cos B having roots cos A, cos B

$\cos A + \cos B = \frac{2kx}{x^2 + y^2}$

$\cos A \cos B = \frac{k^2 - y^2}{x^2 + y^2}$

21. **(AC)**

$\frac{2\sin 2^o + 4\sin 4^o + ...+180\sin 180^o}{90} = A$

$A = \frac{\left(2\sin 2^o + 4\sin 4^o + ...+176\sin 176^o + 178\sin 178^o\right)}{90}$

Supplementary angles have the same sign value

$\sin 2^o = \sin 178^o$

$\sin 4^o = \sin 176^o$ etc

$90A = \left(2\sin 2^o + 178\sin 178^o\right) + \left(4\sin 4^o + 176\sin 176^o\right) + ... + \left(88\sin 88^o + 92\sin 92^o\right) + 90\sin 90^o$

$= \left(2\sin 2^o + 178\sin 2^o\right) + \left(4\sin 4^o + 176\sin 176^o\right) + ... + \left(88\sin 88^o + 92\sin 88^o\right) + 90$

$= \left(180\sin 2^o + 180\sin 4^o + ... + 180\sin 88^o\right) + 90$

$90A = 180\left(\sin 2^o + \sin 4^o + ... + \sin 88^o\right) + 90$

$\sin 2^o + \sin 4^o + ... + \sin 88^o$

$= \frac{\sin\left(44^o\right)\sin\left(45^o\right)}{\sin 1^o}$

$90A = 180\frac{\sin 44^o \sin 45^o + 90^o}{\sin 1^o}$

$= 90\left(\frac{2\sin 44^o \sin 45^o}{\sin 1^o} + 1\right)$

$= 90\left(\frac{\cot 1^o - \cos 89^o}{\sin 1^o} + 1\right)$

$= 90\left(\cot 1^o - \frac{\cos 89^o}{\sin 1^o} + 1\right)$

$\cos 89^o = \sin 1^o$

$\therefore 90A = 90\left(\cot 1^o - 1 + 1\right)$

$= 90\cot 1^o$

$A = \cot 1^o$ (A)

(B) is $\tan 1^o$

(C) $= \frac{\cos\left(31^o\right)\cos 1^o}{\sin 51^o \sin 1^o}$

(D) is $\tan 1^o$

22. **(ABCD)**

$\sin\theta + \cos\alpha = -\frac{1}{5}$

$\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} + \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{-1}{5}$ Let $\tan\frac{\theta}{2} = x$

$\frac{2x + 1 - x^2}{1 + x^2} = \frac{-1}{5}$

$5\left(2x+1-x^2\right)+1\left(1+x^2\right)=0$

$10x+5-5x^2+1+x^2=0$

$6+10x-4x^2=0$

$2x^2-5x-3=0$ (C)

$x=\dfrac{5\pm\sqrt{25+4(2)(3)}}{2}$

$=\dfrac{5\pm 7}{2}=6-1$

Now question is value of $\tan\dfrac{\theta}{2}$ is a root of which equation. If we see the other equations putting $x=-1$, or 6 will satisfy them. So All (A) (B) (C) (D) Have roots either -1 or 6

23. **(ABC)**

$\cos(A-B)=\dfrac{3}{5}$

$\cos A\cos B+\sin A\sin B=\dfrac{3}{5}$

$\tan A\tan B=2$

$\sin A\sin B=2\cos A\cos B$

$\therefore 3\cos A\cos B=\dfrac{3}{5}$

$\cos A\cos B=\dfrac{1}{5}$ (A)

$\sin A\sin B=\dfrac{2}{5}$ (B)

$\cos(A+B)=\cos A\cos B-\sin A\sin B$

$=\dfrac{1}{5}-\dfrac{2}{5}=\dfrac{-1}{5}$ (C)

$\cos(A+B)=\dfrac{-1}{5}$

$\sec^2(A+B)=25$

$\tan^2(A+B)=24$ (D) is wrong

24. **(ABCD)**

$\sin a+\sin b=\dfrac{1}{\sqrt{2}}$ …(1)

$\cos a+\cos b=\dfrac{\sqrt{3}}{\sqrt{2}}$ …(2)

Square and add the equations we get

$2+2\cos a\cos b+2\sin a\sin b=\dfrac{1}{2}+\dfrac{3}{2}$

$\therefore 2\cos(a-b)=0$

$\cos(a-b)=0$

$2\cos^2\left(\dfrac{a-b}{2}\right)-1=0$

$\cos^2\left(\frac{a-b}{2}\right)=\frac{1}{2}$

$\sec^2\left(\frac{a-b}{2}\right)=2$

$\tan^2\left(\frac{a-b}{2}\right)=1$

$\therefore \cot^2\left(\frac{a-b}{2}\right)=1$ (C)

$\sin a+\sin b=\frac{1}{\sqrt{2}}$

$2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)=\frac{1}{\sqrt{2}}$

$\cos a+\cos b=\frac{\sqrt{3}}{\sqrt{2}}$

$2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)=\frac{\sqrt{3}}{\sqrt{2}}$

Hence $\tan\left(\frac{a+b}{2}\right)=\frac{1}{\sqrt{3}}$ (D)

$$\sin(a+b)=\frac{2\tan\left(\frac{a+b}{2}\right)}{1+\tan^2\left(\frac{a+b}{2}\right)}=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\sqrt{3}}{2}$$

$\sin(a+b)=\frac{\sqrt{3}}{2}$ (A)

25. **(AC)**

$\frac{\sin A}{\sin B}=p,\frac{\cos A}{\cos B}=q$

Dividing we get $\frac{\tan A}{\tan B}=\frac{p}{q}$

Now $\sin A=p\sin B$ …(1)

$\cos A=q\cos B$ …(2)

$\sin^2 A+\cos^2 A=1$

$p^2\sin^2 B+q^2\cos^2 B=1$

$p^2\sin^2 B+q^2\left(1-\sin^2 B\right)=1$

$\left(p^2-q^2\right)\sin^2 B=1-q^2$

$\sin^2 B=\left(\frac{1-q^2}{p^2-q^2}\right)$

$\cos^2 B=1-\sin^2 B$

$=\left(\frac{p^2-1}{p^2-q^2}\right)$

$\tan^2 B=\left(\frac{1-q^2}{p^2-1}\right)=\frac{q^2-1}{1-p^2}$ (C)

From (1) & (2)

$\sin^2 A = p^2 \sin^2 B$

$= p^2 \left(\frac{1-q^2}{p^2 - q^2} \right)$

$\cos^2 A = q^2 \left(\frac{p^2 - 1}{p^2 - q^2} \right)$

$\tan^2 A = \frac{p^2 \left(1 - q^2\right)}{q^2 \left(p^2 - 1\right)}$ (A)

26. **(ABCD)**

$0 \le \theta \le \pi$

$81^{\sin^2\theta} + 81^{\cos^2\theta} = 30$

$81^{\sin^2\theta} + 81^{-\sin^2\theta} = 30$

Let $81^{\sin^2\theta} = x$

$x + \frac{81}{x} = 30$

$x^2 + 81 = 30x$

$x^2 - 30x + 81 = 0$

$x = 27$ or 3

$\therefore\ 81^{\sin^2\theta} = 27, 3$

$3^{4\sin^2\theta} = 3^3, 3^1$

$\therefore\ 4\sin^2\theta = 3, 1$

$\sin^2\theta = \frac{3}{4}, \frac{1}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}, \pm\frac{1}{2}$

For $0 \le \theta \le \pi$ $\theta = 30^o, 60^o, 120^o, 150^o$

27. **(BCD)**

For a cyclic Quadrilateral B + D = π or B = π – D

sin B = sin D $\Rightarrow$ cosec B = cosec D

cot B = – cot D or tan B = – tan D

similarly A = π – C

cot A = – cot C

cot A + cot C = 0

sec B = –sec D

28. **(ABCD)**

$\sin\left(\frac{11\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right)$

$= \frac{1}{2}\left(\cos\left(\frac{6\pi}{12}\right) - \cos\left(\frac{16\pi}{12}\right)\right)$

$= \frac{1}{2}\left(0 - \cos\left(\frac{4\pi}{3}\right)\right)$

$= \frac{1}{2}\left(+\frac{1}{2}\right) = \frac{1}{4}$ … (A)

$\operatorname{cosec}\left(\frac{9\pi}{10}\right)\sec\left(\frac{4\pi}{5}\right)$

$= \frac{1}{\sin\frac{\pi}{10}\cos\left(\frac{\pi}{5}\right)} = \frac{1}{(-1)\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)}$

= (–1) (4) … (B)

$\sec^4\theta + \cos^4\theta$

$= 1 - 2\sin^2\theta\cos^2\theta$

$= 1 - \frac{\sin^2 2\theta}{2} \quad \theta = \frac{\pi}{8}$

$= 1 - \frac{\sin^2\frac{\pi}{4}}{2}$

$= 1 - \frac{\frac{1}{2}}{2}$

$= \frac{3}{4}$ …(C)

$\left(1+\cos\frac{2\pi}{9}\right)\left(1+\cos\frac{4\pi}{9}\right)\left(1+\cos\frac{8\pi}{9}\right)$

$= 2\cos^2\frac{\pi}{9} \times 2\cos^2\frac{2\pi}{9} \times 2\cos^2\frac{4\pi}{9}$

$= 8\left(\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\right)$

$= 8\left(\frac{1}{8}\frac{\sin\left(\frac{8\pi}{9}\right)}{\sin\frac{\pi}{9}}\right)^2$

$= \frac{8}{64} = \frac{1}{8}$ …(D)

29.

(A) $\tan\alpha\tan 2\alpha\tan 3\alpha$

$\tan 3\alpha - \tan 2\alpha - \tan\alpha$

Is true for all angles

Hint: $2\alpha + \alpha - 3\alpha$

$\therefore$ take tan on both sides to get the answer

(B) $\operatorname{cosec}2\alpha + \operatorname{cosec}4\alpha - \operatorname{cosec}\alpha$

$= \frac{1}{\sin\frac{2\pi}{7}} + \frac{1}{\sin\frac{4\pi}{7}} - \frac{1}{\sin\frac{\pi}{7}}$

$$=\frac{2\sin\left(\frac{3\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)}{\left[2\sin\left(\frac{\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)\right]\sin\frac{4\pi}{7}}-\frac{1}{\sin\frac{\pi}{7}}$$

$$=\frac{1}{\sin\frac{\pi}{7}}=\frac{1}{\sin\frac{\pi}{7}}=0$$

(D) $8\cos\alpha\cos 2\alpha\cos 4\alpha$

$$=\frac{\sin 8\alpha}{\sin\alpha}=\frac{\sin\frac{8\pi}{7}}{\sin\frac{\pi}{7}}=-1$$

(C) $\cos\alpha+\cos 3\alpha-\cos 2\alpha$

$= 2\cos 2\alpha\cos\alpha-\cos 2\alpha$

$=\cos 2\alpha\left(2\cos\alpha-1\right)$

$$=\cos\left(\frac{2\pi}{7}\right)\left(2\cos\frac{\pi}{7}-1\right)$$

$$=\cos\left(\frac{2\pi}{7}\right)\left(2\cos\frac{\pi}{7}-1\right)\frac{\sin\frac{\pi}{7}}{\sin\frac{\pi}{7}}$$

$$=\cos\left(\frac{2\pi}{7}\right)\left(\frac{2\sin\frac{\pi}{7}\cos\frac{\pi}{7}-\sin\frac{\pi}{7}}{\sin\frac{\pi}{7}}\right)$$

$$=\frac{\cos\left(\frac{2\pi}{7}\right)}{\sin\frac{\pi}{7}}\left(\sin\frac{2\pi}{7}-\sin\frac{\pi}{7}\right)$$

$$=\frac{\cos\left(\frac{2\pi}{7}\right)\left(2\sin\frac{\pi}{14}\right)\left(\cos\frac{3\pi}{14}\right)}{2\sin\frac{\pi}{14}\cos\frac{\pi}{14}}$$

$$=\frac{\cos\frac{3\pi}{14}\cos\frac{2\pi}{7}}{\left(\cos\frac{\pi}{14}\right)}$$

$$=\frac{1}{2}\frac{\left(2\cos\left(\frac{3\pi}{14}\right)\sin\frac{3\pi}{14}\right)}{\cos\frac{\pi}{14}}=\frac{1}{2}\frac{\sin\left(\frac{6\pi}{14}\right)}{\cos\frac{\pi}{14}}$$

$\frac{6\pi}{14},\frac{\pi}{14}$ are complementary

$\therefore \text{L.H.S.} = \frac{1}{2} \times 1 = \frac{1}{2}$

30. **(A)**

$\cot\alpha - \tan\alpha = 2\cot 2\alpha$ …(A)

(A) is true

$\tan\alpha + \cot\alpha = 2\operatorname{cosec}2\alpha$ …(D)

(D) is wrong

$\tan\left(45^0 + \alpha\right) - \tan\left(45^0 + 1 - \alpha\right)$

$= \cot\left(45^0 - \alpha\right) - \tan\left(45^0 - \alpha\right)$

$= 2\cot\left(2\left(45^0 - \alpha\right)\right)$

$2\cot\left(90^0 - 2\alpha\right)$

$= \tan 2\alpha$

(B) (C) wrong

TRIGO SOLUTION

EXERCISE - 2B

1. (A)

$$45^\circ = \frac{45}{180}\times 200 = 50$$

2. (D)

$$\frac{23\pi^C}{4} = \frac{23\times 180^\circ}{4} = 1035^\circ$$

3. $200^g = \frac{200}{300}\times 180^\circ = 180^\circ$

$$= \frac{200}{200}\times \pi^C = \pi^C$$

4. (B)

Sum of angles of a hexagon $= 180(n-2)$

$$= 180\times 4 = 720^\circ = 800^g = 4\pi^C$$

5. $\sin\alpha + \sin\beta = \frac{1}{3}$ $\qquad$ $\cos\alpha + \cos\beta = \frac{1}{4}$

6. $\Rightarrow 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3}$ ______(1)

7. (A,B,D)

$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4}$ ________(2)

$$\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{4}{3}$$

Hence, $\sin(\alpha+\beta) = \frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)} = \frac{2\times\frac{4}{3}}{1+\frac{16}{9}} = \frac{8\times 3}{25} = \frac{24}{25}$

$$\cos(\alpha+\beta) = \frac{1-\tan^2\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)} = \frac{1-\frac{16}{9}}{1+\frac{169}{}} = \frac{-7}{25}$$

$$\tan(\alpha+\beta) = \frac{24}{25}\times\frac{25}{7} = -\frac{24}{7}$$

8. (B)

$$P_n - P_{n-2}$$

$$= \sin^n\theta + \cos^n\theta - \sin^n\theta - \cos^{n-2}\theta$$

$$= \sin^{n-2}\theta\left(\sin^2\theta - 1\right) + \cos^{n-2}\left(\cos^2\theta - 1\right)$$

$$= -\sin^2\theta\cos^2\theta\left(\sin^{n-4}\theta + \cos^{n-4}\theta\right)$$

$$= -\sin^2\theta.\cos^2\theta.P_{n-4}$$

$$\Rightarrow k = -\sin^2\theta.\cos^2\theta$$

9. (B)

$\sin\theta + \cos\theta = m \Rightarrow \sin\theta.\cos\theta = \left(m^2 - 1\right)/2$

$4\left(1 - P_6\right) = 4\left(1 - \left(\sin^6\theta + \cos^6\theta\right)\right)$

$= 4\left(1 - \left(1 - 3\sin^2\theta.\cos^3\theta\right)\right)$

$= 12\sin^2\theta.\cos^2\theta = 12\left(\frac{m^2 - 1}{2}\right)^2$

$= 3\left(m^2 - 1\right)^2$

10. (D)

$P_{n-2} - P_n = \sin^2\theta.\cos^2\theta P_\lambda$

$\lambda = n - 4$

11. (C)

$2P_6 - 3P_4 + 10 = 2\left(\sin^6\theta + \cos^6\theta\right) - 3\left(\sin^4\theta + \cos^4\theta\right) + 10$

$= 2\left(1 - 3\sin^2\theta\cos^3\theta\right) - 3\left(1 - 25\sin^2\theta\cos^2\theta\right) + 10$

$= 2 - 3 + 10 = 9$

12. (B)

$\sin\frac{\pi}{7} + \sin\frac{2\pi}{7} + \sin\frac{3\pi}{7}$

$= \frac{2\sin\frac{2\pi}{7}.\sin\frac{3\pi}{14}}{2.\sin\frac{\pi}{14}} = \frac{\cos\left(\frac{\pi}{14}\right) - \cos\left(\frac{\pi}{2}\right)}{2\cos\frac{\pi}{14}}$

$\frac{1}{2}\cot\left(\frac{\pi}{14}\right)$

13. (B)

$\sin\frac{\pi}{4} + \sin\frac{3\pi}{4} + \sin\frac{5\pi}{4} + - - - - - - n \text{ terms}$

$= \frac{\sin\left(\frac{\pi}{4} + \frac{(n-1)}{2}\frac{2\pi}{n}\right).\sin\left(\frac{2\pi.n}{2n}\right)}{\sin\left(\frac{2\pi}{2n}\right)}$

$= \frac{\sin\pi.\sin\pi}{\sin\frac{\pi}{4}} = 0$

14. (C)

$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$

$$=\frac{\cos\left(\frac{\pi}{11}+\frac{(5-1)}{2}\frac{2\pi}{11}\right).\sin\left(\frac{5}{2}\frac{2\pi}{11}\right)}{\sin\left(\frac{2\pi}{11}/2\right)}$$

$$=\frac{2\cos\frac{5\pi}{11}.\sin\frac{5\pi}{11}}{2\sin\frac{\pi}{11}}=\frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}$$

$$=\frac{\sin\frac{\pi}{11}}{2\sin\frac{\pi}{11}}=\frac{1}{2}$$

15. (C)

$$\sum_{r=0}^{n}\sin^2\frac{r\pi}{n}=\frac{1}{2}\sum_{r=0}^{n}\left(1-\cos\frac{2r\pi}{n}\right)$$

$$=\frac{n+1}{2}-\frac{1}{2}\left(\cos 2+\cos\frac{2\pi}{n}+\cos\frac{9\pi}{n}+........\cos 2\pi\right)$$

$$=\frac{n+1}{2}-\frac{\cos\left(0+\frac{2\pi}{2}\right).\sin\left(\frac{2\pi}{n}.\frac{(n+1)}{2}\right)}{2\sin\left(\frac{2\pi}{n.2}\right)}$$

$$=\frac{n+1}{2}-\frac{1}{2}\frac{\sin\left(\pi+\frac{\pi}{n}\right)(-1)}{\sin\frac{\pi}{n}}=\frac{n+1}{2}-\frac{1}{2}$$

$$=n/2$$

16. (B)

$\cos 1\approx\cos 57^\circ$

$\cos 57\approx\cos(6.28+0.72)$

$\approx\cos(0.72)$

So, $\cos 7>\cos 1$

(but not a correct explanation)

17. (A)

$\left(27^{\sin 2x}.81^{\sin 2x}\right)_{max}$

$=\left(2^{3\cos 2x+4\sin 2x}\right)_{min}$

$=3^{-5}=\frac{1}{3^5}$

18. (B)

$\sin B\simeq\sin 9^\circ$

$\sin 1\simeq\sin 57$

$\sin 2\simeq\sin 66^\circ$

Sin x is possible but and in clearly as decreasing

(Not a correct explanation)

19. (A)

$\sin^2 A+\sin^2 B+\sin^2 C=2$

$\Rightarrow \sin^2 A+2\sin^2 B+2\sin^2 C=4$

$\Rightarrow 1-\cos 2A+1-\cos 2B+1-\cos 2C=4$

$\Rightarrow \cos 2A+\cos 2B+\cos 2C=-1$

$\Rightarrow -1-4\cos A\cos B\cos C=-1$

$\Rightarrow \cos A\cos B\cos C=0$

Hence are of them is 90°

$\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$

20. (B)

$xy+yz+zx=1$

Let so y $x=\tan\frac{A}{2}, y=\tan\frac{B}{2}, z=\tan\frac{C}{2}$

$xy+yz+zx=\tan\frac{A}{2}.\tan\frac{B}{2}+\tan\frac{B}{2}.\tan\frac{C}{2}+\tan\frac{C}{2}.\tan\frac{A}{2}=1$

Hence, $\left(\frac{A}{2}+\frac{B}{2}+\frac{C}{2}\right)=n\pi+\frac{\pi}{2}\Rightarrow A+B+C=2n\pi+\pi$

We know $\frac{2x}{1+x^2}=\sin A, \frac{2y}{1+y^2}=\sin B, \frac{2z}{1+z^2}=\sin C$

$\sin A+\sin B+\sin C=4\cos\frac{A}{2}.\cos\frac{B}{2}\cos\frac{C}{2}$

$\Rightarrow \frac{2x}{1+x^2}+\frac{2y}{1+y^2}+\frac{2z}{1+z^2}=4\frac{1}{\sqrt{1+x^2}}.\frac{1}{\sqrt{1+y^2}}.\frac{1}{\sqrt{1+z^2}}$

The identify $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$ is true but not used here

21. (A) $A=\sin^2\theta+\cos^4\theta$

$=\sin^2\theta+\left(1-\sin^2\theta\right)^2$

$=\sin^4\theta-\sin^2\theta+1$

$A\in\left[\frac{3}{4},3\right]$ (minimum at $\sin^2\theta=\frac{1}{2}$,maximum of $\sin^2\theta=1$)

Ans: (Q)

(B) $A=3\cos^2\theta+\sin^4\theta$

$=\sin^4\theta-3\sin^2\theta+3$

$A\in[1,3]$

(min of $\sin^2\theta=0$, max of $\sin^2\theta=1$)

Ans: (S)

(C) $A=\sin^2\theta-\cos^4\theta$

$=\left(1-\cos^2\theta\right)-\cos^4\theta$

$=-\left(\cos^4\theta+\cos^2\theta\right)+1$

$A\in[-1,1]$

Ans: (P)

(D) $A=\tan^2\theta+2\cot^2\theta$

By A.M $\geq$ G.M., $A\geq 2\sqrt{2}$

$A \in \left[2\sqrt{2}, \infty\right)$

22. $(A-R);(B-P);(C-Q),(D-S)$

$\cos\alpha + \cos\beta = \frac{1}{2}$; $\sin\alpha + \sin\beta = \frac{1}{3}$

$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{2}$ ……..(i)

$2\text{sdin}\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3}$ …….(ii)

(C) $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{1/3}{1/2} = \frac{2}{3}$

(A) $\cos\left(\frac{\alpha+\beta}{2}\right) = \pm\frac{3}{\sqrt{13}}$

$(i)^2 + (ii)^2$

$$4\cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4} + \frac{1}{9} = \frac{13}{36}$$

$$\cos\left(\frac{\alpha-\beta}{2}\right) = \pm\frac{\sqrt{13}}{12}$$

$$\tan\left(\frac{\alpha-\beta}{2}\right) = \pm\frac{\sqrt{131}}{\sqrt{13}}$$

23. (A) $\cos 20^\circ + \cos 80^\circ - \sqrt{3}\cos 50^\circ$

$= 2\cos 50^\circ - \cos 30^\circ - \sqrt{3}\cos 50^\circ$

$= 0$

(B) $1 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + ... + \cos\frac{6\pi}{7}$

$$= 1 + \frac{\cos\left(\frac{\pi}{7} + \frac{5}{2}.\frac{\pi}{7}\right).\sin\left(\frac{6}{2}.\frac{\pi}{7}\right)}{\sin\left(\frac{\pi}{14}\right)}$$

$$= 1 + \frac{\cos\left(\frac{\pi}{2}\right).\sin\left(\frac{3\pi}{7}\right)}{\sin\frac{\pi}{14}} = 1 + 0 = 1$$

(C) $\cos 20^\circ + \cos 40^\circ + \cos 60^\circ - 4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$= 2\cos 30^\circ.\cos 10^\circ + 2\cos^2 30^\circ - 1 - 4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$= 2\cos 30^\circ\left(\cos 10^\circ + \cos 30^\circ\right) - 1 - 4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$= 4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ - 1 - 4\cos 10^\circ.\cos 20^\circ.\cos 30^\circ$

$= -1$

(D) $\frac{1}{2}\left[2\cos 20^\circ.\cos 100^\circ + 2\cos 100^\circ.\cos 140^\circ - 2\cos 140^\circ.\cos 200^\circ\right]$

$= \frac{1}{2}\left[\cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \cos 340^\circ - \cos 60^\circ\right]$

$$=\frac{1}{2}\left[-\frac{1}{2}-\frac{1}{2}-\frac{1}{2}+\cos 80^\circ+\cos 40^\circ-\cos 20^\circ\right]$$

$$=-\frac{3}{4}+\frac{1}{2}\left(2\cos 60^\circ.\cos 20^\circ-\cos 20^\circ\right)$$

$$=-\frac{3}{4}$$

Ans: $(A)-(S);(B)-(R),(C)-(P),(D)-(Q)$

24. (A) $\tan\theta\left(\cot\theta.\cos\theta+\sin\theta\right)$

$$=\tan\theta.\cot\theta.\cos+\frac{\sin\theta}{\cos\theta}-\sin\theta$$

$$=\frac{\cos^2\theta+\sin^2\theta}{\cos\theta}=\sec\theta$$

Ans: (R,T)

(B) $\frac{\tan\theta.\cos ec^2\theta}{1+\tan^2\theta}$

$$=\frac{\sin\theta}{\cos\theta}.\frac{1}{\sin^2\theta}.\cos^2\theta=\cot\theta$$

Ans: (P)

(C) $\frac{\cos ec^2\theta-1}{\cos ec\theta.\cos^2\theta}=\frac{\cot^2\theta}{\frac{1}{\sin\theta}.\cos^2\theta}$

$$=\frac{\cos^2\theta}{\sin^2\theta.\cos^2\theta}.\sin\theta$$

$$=\cos ec\theta$$

Ans: (S)

(D) $\frac{\cot\theta.\sec^2\theta}{1+\cot^2\theta}=\frac{\frac{\cos\theta}{\sin\theta}.\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}}$

$$=\frac{\sin\theta}{\cos\theta}=\tan\theta$$

Ans: (Q)

25. (A)

$3\sin 2\theta+4\cos 2\theta+3$

$\in[-5+3,5+3]$ $\qquad \lambda+k=6$ $\qquad$ (R)

$\in[-2,8]$ $\qquad \lambda-k=10$ $\qquad$ (S)

(B) $5\cos\theta+3\cos\theta.\cos\frac{\pi}{3}-2\sin\theta.\sin\frac{\pi}{3}+3$

$$=\left(\frac{13}{2}\cos\theta-\frac{3\sqrt{3}}{2}\sin\theta\right)+3$$

$$\in\left[\sqrt{\frac{169}{4}+\frac{27}{4}}+3,\sqrt{\frac{169}{4}+\frac{27}{4}}+3\right]$$

$\in[-4,10]$ $\qquad \lambda+k=6$ $\qquad$ (R)

$\qquad \lambda-k=14$ $\qquad$ (T)

(C) $1+\sin\theta.\frac{1}{\sqrt{2}}+\cos.\frac{1}{\sqrt{2}}+2.\frac{\cos\theta}{\sqrt{2}}+\frac{2\sin\theta}{\sqrt{2}}$

$=1+\frac{3}{\sqrt{2}}\cos\theta+\frac{3}{\sqrt{2}}\sin\theta$

$=\in\left[1-\frac{3\sqrt{2}}{\sqrt{2}},1+\frac{3\sqrt{2}}{\sqrt{2}}\right]$

$\in[-2,4]$ $\lambda+k=2$ (P)

$\lambda-k=6$ (Q)

26. $\cos A=\frac{1}{3}$

$A\in\left(1350^{\circ},1440^{\circ}\right)$

$\Rightarrow\in\left(270^{\circ},360^{\circ}\right)$ IV quad

$\sin A=-25\frac{2}{3}$

$\frac{A}{2}\in\left(675^{\circ},720^{\circ}\right)$ IV quad

$\sin\frac{A}{2}=\sqrt{\frac{1-\cos A}{2}}=\sqrt{\frac{1-\frac{1}{3}}{2}}=-\frac{1}{\sqrt{3}}$

$\cos\frac{A}{2}=\sqrt{\frac{1+\cos A}{2}}=\sqrt{\frac{2}{3}}$ $\tan\frac{A}{2}=-\frac{1}{\sqrt{2}}$

$(A)-(R);(B)-(S);(C)-(Q);(D)-(P)$

27. (A) $\sqrt{3}\sin x-\cos x\quad\in[-2,2]$ (R)

(B) $4\cos^2 x-4\cos x+3$

$=(2\cos\alpha-1)^2+2$

$\in[0,9]+2$

$\in[2,11]$ (P)

(C) $\frac{2\tan\alpha}{\tan^2\alpha+1}=\sin 2x\in[-1,1]$ (S)

(D) $\sin^4+\cos^4 x=1-2\sin^2 x.\cos^2 x$

$=1-\frac{1}{2}\sin^2 2x$

$\in[0,1]$

So, $\in\left[\frac{1}{2},1\right]$ (Q)

28. Standard solutions

$\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

$\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$

$\cos 2A+\cos 2B+\text{cps}2C=-1-4\cos 2A\cos B\cos C$

$\cos^2 A+\cos^2 B+\cos^2 c=1-2\cos A\cos B\cos C$

$(A)-(Q);(B)-(P),(C)-(R);(D)-(S)$

29. $\sin 2\theta = k$

(A) $\cos ec\, 20 + \cot 2\theta - \cos 2\theta$

$$= \frac{1}{\sin 2\theta} + \frac{1-\sin\theta}{\sin 2\theta} = \frac{1}{k} + \frac{1-k^2}{k} = \frac{2-k^2}{k}$$ (s)

(B) $\left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2 = \frac{1+\sin 2\theta}{1-\sin 2\theta}$

$$= \frac{1+k}{1-k}$$ (p)

(C) $\sin 2\theta - \frac{1}{2}\left(2\cos^2 2\theta\right)$

$$= k - \frac{b}{2}\left(1-k^2\right)$$

$$= k^2 + k - 1$$ (Q)

(D) $\sin 60 = 3\sin\theta - 4\sin^3 2\theta$

$$= 3k - 4k^3$$ (R)

30. (A) $\sin B = \frac{4}{5}$, $\tan\left(\frac{A}{2}\right) = 1 \Rightarrow A = 90^\circ$

$$B + C = 90^\circ$$

$$\cos C = \frac{4}{5}$$

(A) $\frac{a}{b} = \frac{\sin A}{\sin B} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$ ________(R)

(B) $\frac{b}{c} = \frac{\sin B}{\sin C} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$ ______(S)

(C) $\frac{c+b}{a} = \frac{\sin C + \sin B}{\sin A} = \frac{3}{5} + \frac{45}{} = \frac{7}{5}$ ________ (P)

(D) $\sqrt{\frac{a^2+c^2-b^2}{20c}} = \sqrt{\frac{\sin^2 A + \sin^2 C - \sin^2 B}{2\sin A \sin C}} = \sqrt{\frac{3}{5}}$ ________ (Q)

TRIGO SOLUTION
EXCRCISE 2-(C)

1. $\frac{\sin 2\alpha + \sin 4\alpha - \sin 3\alpha}{\cos 2\alpha + \cos 4\alpha - \cos 3\alpha}$

$= \frac{2\sin 3\pi \cos x - \sin 3x}{2\cos 3x \cos x - \cos 3x} = \frac{\sin 3x}{\cos 3x} = \tan 3x$

$K = 3$

2. $\tan 45^{\circ} = 1 \Rightarrow \tan\left(27^{\circ} + 18^{\circ}\right) = 1$

$\Rightarrow \frac{\tan 27^{\circ} + \tan 18^{\circ}}{1 - \tan 27^{\circ} + \tan 18^{\circ}} = 1$

$\Rightarrow \tan 27^{\circ} + \tan 18^{\circ} = 1 - \tan 27^{\circ} \tan 18^{\circ}$

$\Rightarrow \tan 27^{\circ} + \tan 18^{\circ} + \tan 27^{\circ} \tan 18^{\circ} = 1$

3. LHS $\frac{\cos 96^{\circ} + \sin 96^{\circ}}{\cos 96^{\circ} - \sin 96^{\circ}} = \frac{1 + \tan 96^{\circ}}{1 - \tan 96^{\circ}}$

$\Rightarrow \frac{\tan 45^{\circ} + \tan 96^{\circ}}{1 - \tan 45^{\circ} \tan 96^{\circ} \Rightarrow} = \tan(141)$

Or $\tan(180 + 141) = \tan(321)$

$\therefore n = 3$

4. In ABC $\sum \tan A = \pi \tan A$

$6 = 2 \times \tan C$

$\Rightarrow \tan C = 3$

5. $a \sin\theta - \frac{a}{\sin\theta} = \frac{b}{\cos\theta} - \cos\theta$

$a\frac{\left(\sin^2\theta - 1\right)}{\sin\theta} = b\frac{\left(1 - \cos^2\theta\right)}{\cos\theta}$

$\Rightarrow -a\cos^3\theta = b\sin^3\theta \Rightarrow a\cos^3\theta + b\sin^3\theta = 0$

Square both side

$a^2\cos^6\theta + b^2\sin^6\theta + 2ab\cos^3\theta\sin^3\theta = 0$

6. $\left(\frac{c}{a} + \frac{c}{b}\right)^2$

Diagram

$= \left(\sec\theta + \cos ec\theta\right)^2$

$= 4\frac{\left(\sin\theta + \cos\theta\right)^2}{\left(\sin 2\theta\right)^2} \Rightarrow 4\left(\frac{1}{\sin^2 2\theta} + \frac{1}{\sin 2\theta}\right)$

$\therefore$ **For** $\min\left(\sin 2\theta\right)$ $\max = 1$

Hence $\min = 8$

7. $\sqrt{2\cot\alpha + \cos ec^2\alpha} + \cot\alpha + 2$

$\sqrt{(1+\cot\alpha)^2}+\cot\alpha+2$

$|1+\cot\alpha|+\cot\alpha+2$

Since $\frac{3\pi}{4}<\alpha<\pi \Rightarrow 1+\cot\alpha<0$

$\Rightarrow 2-1-\cot\alpha+\cot\theta$

Ans: 1

8. Let $\alpha\,\&\,\beta$ be the roots

$\alpha\,\&\,\beta \;=-\cos\theta+1, \alpha\beta=\frac{1}{2}\cos^2\theta$

$\therefore \alpha^2\beta^2=(1-\cos\theta)^2-2\times\frac{1}{2}\cos^2\theta$

$=1-2\cos\theta$

Maximum as 3

9. $\sin^4\theta+\cos^4\theta=1-2\sin^2\theta\cos^2\theta$

$=1-\frac{\sin^2 2\theta}{2}$

For maximum $\sin 2\theta=0$

i.e1

10. If $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$

That is only possible when all

$\sin\theta=1 \qquad \Rightarrow \cos\theta_1=0$

$\therefore \cos\theta_1+\cos\theta_2+\cos\theta_3=0$

11. $\tan\frac{\pi}{4n}\tan\frac{2\pi}{4n}----\tan\frac{n\pi}{4\pi}-----\tan\left(\frac{\pi}{2}-\frac{2\pi}{4n}\right)\tan\left(\frac{\pi}{2}-\frac{\pi}{4n}\right)$

(use $\left[\text{Use}\tan\left(\frac{\pi}{2}-\alpha\right)=\cot\alpha\right]$

OR

$\tan\frac{4\pi}{4n}.\tan\frac{2\pi}{4n}-------1------\cot\frac{2\pi}{4n}.\cot\frac{\pi}{4n}$

$(\text{use}\tan\theta\cot\theta=1)$

Ans: 1

12. $\sin x=1-\sin^2 x$ or $\sin x=\cos^2 x$ of $\cos^2 x+\cos^4 x=1$

Expression

$\cos^6 x\left[\cos^6 x+3\text{co}^4 x+3\cos^4 x+1\right]+\cos^4 x+\cos^2 x+3$

$=\cos^6 n\left[\cos^2 n+1\right]^3+4$

$=\left(\cos^4 n+\cos^2 n\right)^3+4$

$=1+4=5$

13. Same as Q 2

14. On simplification the expression reduces to

$\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}$

When is minimum when all gytes equal $=60$

$\therefore$ Min value is 1

15. $\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta-\frac{2\pi}{3}\right)}=\frac{z}{\cos\left(\theta+\frac{2\pi}{3}\right)}=k$

$\Rightarrow x+y+z------k\left(\cos\theta+\cos\left(\theta-\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{2\pi}{3}\right)\right)$

$=k\left(\cos\theta+\alpha\cos\theta\cos\frac{2\pi}{3}\right)$

$=k\left(\cos\theta-2\cos\theta\times\frac{1}{2}\right)$

$=0$

16. $\frac{\tan 6^\circ\tan 54^\circ\tan 66^\circ\tan 42^\circ\tan 78^\circ}{\tan 54^\circ}$

$\frac{\tan 18^\circ\tan 42^\circ\tan 78^\circ}{\tan 54^\circ}$

Use $\tan\theta\tan(60-\theta)\tan(160^\circ+\theta)=\tan 3\theta$

17. Angle moved in 60 min $=360^\circ$

Angle moved in 20 min $=120^\circ$

$\therefore d=10\times120^\circ\times\frac{\pi}{180}$

$d=\frac{20\pi}{3}$

Ans : 2

18. During every motion particle rotates by 1 radian angle (Approx 57.16°)

$\therefore$ To rotate 360° it must be in its 7^{th} round

19. $\sec^4 A-\tan^4 A-2\tan^2 A$

$=(\sec^2 A-\tan^2 A)(\sec^2 A+\tan^2 A)-2\tan^2 A$

$=\sec^2 A+\tan^2 A-2\tan^2 A$

$=\sec^2 A-\tan^2 A$

$=1$

20. We know $\sin^3\theta\geq\sin^8\theta$ ______(1)

$\cos^2\theta\geq\cos^{14}\theta$ _______(2)

(1) & (2)

$\Rightarrow\sin^8\theta+\cos^{14}\theta\leq1$

$0<A\leq1$

$a=0, b=1$

21. We know $x+\frac{1}{x}$ lies in interval

$(-\infty,-2]\text{or}[2,\infty)$

$\therefore$ For is to be equal to $2\cos\theta$

$\cos\theta$ must be ± 1

$\therefore$ in $[0,2\pi]$ only 3 possible angle $0,\pi,2\pi$

22. Use $\sin^2 A-\sin^2 p=\sin(A+B)\sin(A-B)$

$\therefore \sin^2 24-\sin^2 6=\sin 30\sin 18$

$$=\frac{1}{2}\times\frac{\sqrt{5}-1}{4}$$

$$=\frac{\sqrt{5}-1}{8}$$

$\therefore a=5, b=1, c=8$

Ans: 2

23. $A+B=228$ (similar to question 2)

$\Rightarrow \tan A\tan B+\tan A\tan B=1$

Now $\frac{(1+\cot A)(1+\cot B)}{\cot A\cot B}$

Simplifying to $1+\text{tam}\,A+\tan B+\tan A\tan B=2$

24. $\cos 3\theta=4\cos^3\theta-3\cos\theta$

Or $\cos^3\theta=\frac{\cos 3\theta+3\cos\theta}{4}$

$\therefore$ Expression is

$$\frac{\cos 3\theta+3\cos\theta}{4}+\frac{\cos(360+3\theta)+3\cos(120+\theta)}{4}+\frac{\cos(720+3\theta)+\cos(240+\theta)}{4}$$

$$=\frac{3}{4}\cos 3\theta+\left(\frac{\cos\theta+\cos(120+\theta)+\cos(240+\theta)}{4}\right)$$

$$=\frac{3}{4}\cos 3\theta+\theta$$

$a+b=7$

25. $\frac{\sin(A+B)\cos A}{\cos(A+B)\sin A}=3$

Apply components & dividuals

$\frac{\sin(2A+B)}{\sin(B)}=\frac{3}{3}=2$

26. $\cos A=\cos(\pi-(B+C))$

$=-\cos(B+C)$

$\therefore \frac{\cos A}{\sin B\sin C}=-\frac{\cos B\cos C+\sin B\sin C}{\sin B\sin C}$

$\frac{\cos A}{\sin B\sin C}=(1-\cos B\cot C)$

$\therefore$ Expression becomes

$$\left(\cot A\cot B+\mathrm{coB}\cos C+\cot C\cot A\right)$$

In $\Delta ABC\ \sum\cot A\cot B=1$

$=2$

27. $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\left(-\frac{\cos\pi}{15}\right)$

$=-\left[\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]$

$=-\left[\frac{\sin\left(\frac{16\pi}{15}\right)^{-1}}{16\sin\frac{\pi}{15}}\right]$

Use trick formula

$\prod_{r=0}^{n-1}\cos 2r\theta=\frac{\sin 2^n\theta}{2^n\sin\theta}$

$=\frac{1}{16}$

$\frac{1}{\sqrt{x}}=4$

28. $\sin A\cos dB=\frac{1}{4}\ 3\sin A\cos B=\cos A\sin B$

$\therefore\ \cos A\sin B=\frac{3}{4}$

$=\sin\left(A+B\right)=1,\sin\left(A-B\right)=-\frac{1}{2}$

$=A+B=90$ & $A-B=-30^\circ$

$A=30^\circ,B=60^\circ$

$=\cot^2A=3$

29. $a^2+b^2=2-2\sin A\sin 2B+2\cos A\cos B$

$=2-2\cos\left(ACB\right)$

$=4\cos^2\left(\frac{ACB}{2}\right)$

Maximum $=4$

30. Put values directly (Alied angle)

31. For minimum in ΔABC $A=B=C=60^\circ$

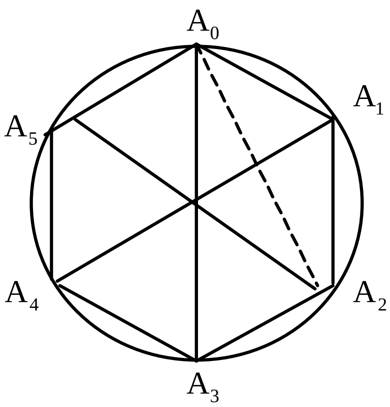
$\therefore\ \left|\sec A\sec B\sec C\right|=8$

32. Same as question 32

33. Use cosine Rule

$A_0A_4=A_0A_2=\sqrt{1^2+1^2-2\cos 120^\circ}$

$=\sqrt{3}$



$\therefore A_0A_1 = 1$
$A_0A_2 = \sqrt{3}$
$A_0A_4 = \sqrt{3}$
$\therefore$ Ans 3

34. Expression is
$3 + 2\sin^2\theta - 3\sin^2\theta$
Or $3 + 1 - \cos 2\theta - 3\sin 2\theta$
$4 - [\cos 2\theta + 3\sin 2\theta]$
Minimum $4 \pm \sqrt{10}$ [use also $\theta + \sin\theta$]
Ans:-8

35. $\left|9\tan^2 A + 4\cot^2 B\right|$
For min put both $\tan^2 A = 0$ & $\cot^2 B = 0$

36. Use $AM \geq GM$
$\therefore \left|\tan A + \cot A\right| \geq 2$

37. Same As Q 23

38. $\cos 20^\circ + 1 - \cos 110^\circ - \sqrt{2}\sin 65^\circ$
$1 + \cos 20^\circ - \cos 110^\circ - \sqrt{2}\sin 65^\circ$
$1 + 2\sin 65 \sin 45^\circ - \sqrt{2}\sin 65^\circ$
$1 + \sqrt{2}\sin 65^\circ - \sqrt{2}\sin 65^\circ$
Ans: 1

39. $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

OR $\dfrac{3\tan\frac{\pi}{9} - \tan^3\frac{\pi}{9}}{1 - 3\tan^2\frac{\pi}{9}} = \sqrt{3}$

Square
$9\tan^2\frac{\pi}{9} + \tan^6\frac{\pi}{9} - 6\tan^4\frac{\pi}{9} = 3 + 27\tan^4\frac{\pi}{9} - 18\tan^2\frac{\pi}{9}$
Solving to get 3

40. $\sum \cos A = 0$

$\sum \sin A = 0$

$\Rightarrow B = 120 + A, C = 240 + A$

Expression become $\frac{3\cos 3}{3\cos 3A} = 3$

Trigonometry – 1

Exercise – 3(A)

1.

$$\sec A - \tan A = p \Rightarrow \frac{1}{p} = \frac{1}{\sec A - \tan A} = \sec A + \tan A$$

$$\therefore \frac{1}{p} - p = 2\tan A \;\;\&\;\; \frac{1}{p} + p = 2\sec A$$

$$\therefore \sin A = \frac{\frac{1}{p} - p}{\frac{1}{p} + p} \text{ or } \sin A = \frac{1-p^2}{1+p^2}$$

2.

$$\frac{\sin(270+\theta)\cos^3(720^\circ - \theta) - \sin(270-\theta)\sin^3(540+\theta)}{\sin(90+\theta)\sin(-\theta) - \cos^2(180-\theta)} = \frac{(-\cos\theta)\cos^3\theta - (-\cos\theta)(-\sin^3\theta)}{\cos\theta(-\sin\theta) - \cos^2\theta}$$

$$= \frac{-\cos^4\theta - \cos\theta\sin^3\theta}{-\sin\theta\cos\theta - \cos^2\theta}$$

$$= \frac{-\cos\theta}{-\cos\theta}\,\frac{(\cos^3\theta + \sin^3\theta)}{(\sin\theta + \cos\theta)}$$

$$= \frac{(\cos\theta + \sin\theta)\,(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\sin\theta + \cos\theta)}$$

$$= 1 - \sin\theta\cos\theta$$

$$\text{and } \frac{\cot(270^\circ)}{\operatorname{cosec}^2(450^\circ + \theta)} = \frac{\tan\theta}{\sec^2\theta} = \sin\theta\cos\theta$$

$$\therefore \text{L.H.S.} = 1 - \sin\theta\cos\theta + \sin\theta\cos\theta$$

$$\tan\theta = -\frac{4}{3}$$

case I : $\sin\theta = \frac{4}{5}, \cos\theta = \frac{-3}{5}$

$$\frac{\sin\theta + \cos\theta - \tan\theta}{\sec\theta + \operatorname{cosec}\theta - \cot\theta} = \frac{\frac{4}{5} - \frac{3}{5} - \left(-\frac{4}{3}\right)}{-\frac{5}{3} + \frac{5}{4} - \left(-\frac{3}{4}\right)}$$

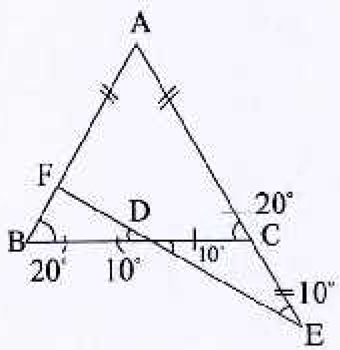
$$= \frac{\frac{1}{5} + \frac{4}{3}}{2 - \frac{5}{3}} = \frac{\frac{23}{15}}{\frac{1}{3}} = \frac{23}{5}$$ Ans (i)

case II : $\sin\theta = \frac{-4}{5}, \cos\theta = \frac{3}{5}$

$$\frac{\sin\theta + \cos\theta - \tan\theta}{\sec\theta + \operatorname{cosec}\theta - \cot\theta} = \frac{-\frac{4}{5} + \frac{3}{5} - \left(-\frac{4}{3}\right)}{\frac{5}{3} - \frac{5}{4} - \left(-\frac{3}{4}\right)}$$

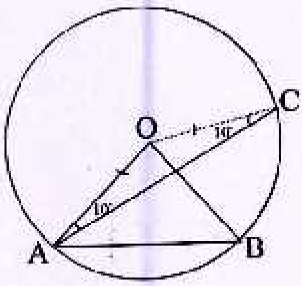
$$= \frac{\frac{4}{3} - \frac{1}{5}}{\frac{5}{3} - \frac{1}{2}} = \frac{\frac{17}{15}}{\frac{7}{6}} = \frac{17 \times 2}{7 \times 5} = \frac{34}{35}$$ Ans (II)

4.



$\angle BFD = 150° \Rightarrow \cos\angle BFD = \frac{-\sqrt{3}}{2}$

5.



$OA = OC = \text{radius}$

$\angle AOC = 180^\circ - 10^\circ - 10^\circ = 160^\circ$

$OA = AB = 2$ units

ΔOAB is Equilateral so $\angle AOB = 60^\circ$

$\therefore \angle BOC = 160^\circ - 60^\circ = 100^\circ = \dfrac{5\pi}{9}$

$\therefore \text{Arc } BC = r\theta = (2)\left(\dfrac{5\pi}{9}\right)$

$\text{Arc } BC = \dfrac{10\pi}{9}$

6.

$\cos\left(x - \frac{3\pi}{2}\right) + \sin\left(\frac{3\pi}{2} + x\right) + \sin(32\pi + x) - 78\cos(19\pi - x) + \cos(56\pi + x) - 9\sin(x + 17\pi)$

$= -\sin x + (-\cos x) + \sin x + 18\cos x + \cos x + 9\sin x$

$= 9\sin x + 18\cos x$

$a = 9, b = 18$

$a + b = 27$

7.

$\cos\left(\frac{3\pi}{2} + 4\alpha\right) + \sin(3\pi - 8\alpha) - \sin(4\pi - 12\alpha) = \sin 4\alpha + \sin 8\alpha + \sin 12\alpha$

$=\sin 4\alpha+\sin 12\alpha+\sin 8\alpha$

$=2\sin 8\alpha\cos 4\alpha+\sin 8\alpha$

$=\sin 8\alpha\left(2\cos 4\alpha+1\right)$

$=\sin 8\alpha\dfrac{\left(2\cos 4\alpha+1\right)}{\sin 4\alpha}\sin 4\alpha$

$=\dfrac{\left(2\sin 4\alpha\cos 4\alpha\right)\left(2\sin 4\alpha\cos 4\alpha+\sin 4\alpha\right)}{\sin 4\alpha}$

$=\left(2\cos 4\alpha\right)\left(\sin 8\alpha+\sin 4\alpha\right)$

$=\left(2\cos 4\alpha\right)\left(2\sin 6\alpha\cos 2\alpha\right)$

$=4\cos 2\alpha\cos 4\alpha\sin 6\alpha$

(8) **8.**

$$\frac{\cos 5x+\cos 4x}{2\cos 3x-1}=\frac{\left(\cos 5x+\cos 4x\right)}{\left(2\sin 3x\cos 3x-\sin 3x\right)}\sin x$$

$$=\frac{\left(2\cos\frac{9x}{2}\cos\frac{x}{2}\right)}{\left(\sin 6x-\sin 3x\right)}\sin 3x$$

$$=\left(\frac{2\cos\frac{9x}{2}\cos\frac{x}{2}}{2\sin\frac{3x}{2}\cos\frac{9x}{2}}\right)\left(2\sin\frac{3x}{2}\cos\frac{3x}{2}\right)$$

$$=2\cos\frac{x}{2}\cos\frac{3x}{2}$$

$$=\cos x+\cos 2x$$

(9) **9.**

$$\sin 2\alpha\left(1+\tan 2\alpha\tan\alpha\right)+\frac{1+\sin\alpha}{1-\sin\alpha}=\tan 2\alpha+\tan^2\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$$

$$\sin 2\alpha(1+\tan 2\alpha \tan\alpha) = \sin 2\alpha\left(\frac{\cos(2\alpha-\alpha)}{\cos 2\alpha \cos\alpha}\right) \qquad \left[1+\tan A \tan B = \frac{\cos(A-B)}{\cos A \cos B}\right]$$

$$= \frac{\sin 2\alpha \cos\alpha}{\cos 2\alpha \cos\alpha} = \tan 2\alpha$$

and $\dfrac{1+\sin\alpha}{1-\sin\alpha} = \dfrac{(\sin\alpha + \cos\alpha)^2}{(\sin\alpha - \cos\alpha)^2}$

$$= \left(\frac{1+\tan\alpha}{1-\tan\alpha}\right)^2 = \tan^2\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$$

$\therefore$ L.H.S. $= \tan 2\alpha + \tan^2\left(\dfrac{\pi}{4}+\dfrac{\alpha}{2}\right) =$ R.H.S.

10.

$\sin 2\alpha = 4\sin 2\beta$

$$\frac{\sin 2\alpha}{\sin 2\beta} = 4$$

By componendo & dividendo

$$\frac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha - \sin 2\beta} = \frac{4+1}{4-1}$$

$$\frac{2\sin(\alpha+\beta)\cos(\alpha-\beta)}{2\sin(\alpha-\beta)\cos(\alpha+\beta)} = \frac{5}{3}$$

$$\therefore \frac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)} = \frac{5}{3}$$

$$3\tan(\alpha+\beta) = 5\tan(\alpha-\beta)$$

11.

Given $0 < x < \dfrac{\pi}{4}$, $\sin x + \cos x = \dfrac{5}{4}$

squaring both the sides gives

$$1+\sin x\cos x=\frac{25}{16}\text{ or }2\sin x\cos x=\frac{9}{16}$$

$\therefore$ if $(\cos x-\sin x)=y$

$$y^2=1-2\sin x\cos x=1-\frac{9}{16}$$

Hence $\cos x-\sin x=\frac{\sqrt{7}}{4}$

12.

$$\frac{\sin a+\sin b+\sin c}{\cos a+\cos b+\cos c}=\frac{\sin a+\sin c+\sin b}{\cos a+\cos c+\cos b}$$

$$=\frac{2\sin\left(\frac{a+c}{2}\right)\cos\left(\frac{a-c}{2}\right)+\sin b}{2\cos\left(\frac{a+c}{2}\right)\cos\left(\frac{a-c}{2}\right)+\cos b}$$

$$\frac{a+c}{2}=b$$

$\therefore$ L.H.S.

$$=\frac{2\sin b\cos\left(\frac{a-c}{2}\right)+\sin b}{2\cos b\cos\left(\frac{a-c}{2}\right)+\cos b}$$

$$=\frac{\sin b}{\cos b}\left(\frac{2\cos\left(\frac{a-c}{2}\right)+1}{2\cos\left(\frac{a-c}{2}\right)+1}\right)=\tan b$$

13.

$$\left(\frac{\sin 3\theta}{\sin\theta}\right)^2-\left(\frac{\cos 3\theta}{\cos\theta}\right)^2=\left(3-4\sin^2\theta\right)^2-\left(4\cos^2\theta-3\right)^2$$

$$\frac{\sin 3\theta}{\sin\theta}=3-4\text{son}^2\theta\ \&\ \frac{\cos 3\theta}{\cos\theta}=4\cos^2\theta-3$$

$$\Rightarrow \left(\frac{\sin 3\theta}{\sin\theta}\right)^2-\left(\frac{\cos 3\theta}{\cos\theta}\right)^2=\left(4\cos^2\theta-4\sin^2\theta\right)\left(3-4\sin^2\theta+3-4\cos^2\theta\right)=4\cos^2\theta-3$$

$$=(4\cos 2\theta)(2)=8\cos 2\theta$$

14.

$$\cos^2 36^\circ+\sin^2 18^\circ=\frac{1+\cos 72^\circ}{2}+\frac{1-\cos 36^\circ}{2}$$

$$=1+\frac{\sin 18^\circ-\cos 36^\circ}{2}$$

$$=1+\frac{\left(\frac{\sqrt{5}-1}{4}\right)-\left(\frac{\sqrt{5}+1}{4}\right)}{2}$$

$$=1-\frac{1}{4}=\frac{3}{4}$$

15.

$$\tan A+\tan B=\frac{-b}{a},\ \tan A\ \tan B=\frac{c}{a}$$

$$a\sin^2(A+B)+b\sin(A+B)\cos(A+B)+c\cos^2(A+B)=?$$

$$\tan(A+B)=\frac{-\frac{b}{a}}{1-\frac{c}{a}}=\frac{-b}{a-c}=\frac{b}{c-a}=t(\text{say})$$

$$a\sin^2(A+B)+b\sin(A+B)\cos(A+B)+c\cos^2(A+B)$$

$$=a\left(\frac{t^2}{1+t^2}\right)+b\frac{\sin(2A+2B)}{2}+c\left(\frac{1}{1+t^2}\right)$$

$$=a\left(\frac{t^2}{1+t^2}\right)+\frac{b}{2}\left(\frac{2t}{1+t^2}\right)+c\left(\frac{1}{1+t^2}\right)$$

$$=\frac{at^2+bt+c}{\left(1+t^2\right)}$$

$$= \frac{a\left(\frac{b}{c-a}\right)^2 + b\left(\frac{b}{c-a}\right) + c}{\left[1+\left(\frac{b}{c-a}\right)^2\right]}$$

$$= \frac{ab^2 + b^2(c-a) + c(c-a)^2}{\left[(c-a)^2 + b^2\right]}$$

$$= \frac{ab^2 + b^2c - b^2a + c\left(c^2 + a^2 - 2ac\right)}{\left((c-a)^2 + b^2\right)}$$

$$= \frac{b^2c + c(c-a)^2}{(c-a)^2 + b^2}$$

$$= \frac{c(b^2 + (c-a)^2)}{\left(b^2 + (c-a)^2\right)} = c$$

16.

$$\sin\theta = -\frac{323}{325}, \quad \theta \in \left(\pi, \frac{3\pi}{2}\right)$$

$$\cos\theta = -\sqrt{1-\left(\frac{323}{325}\right)^2} = -\sqrt{\frac{648 \times 2}{(325)^2}} \qquad \frac{\theta}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$$

$$\Rightarrow \cos\theta = \frac{-36}{325} = 2\cos^2\left(\frac{\theta}{2}\right) - 1$$

$$\Rightarrow 2\cos^2\left(\frac{\theta}{2}\right) = \frac{289}{325} \ \& \ \sin^2\frac{\theta}{2} = \frac{361}{650}$$

$$\Rightarrow \cos\left(\frac{\theta}{2}\right) = \frac{-17}{\sqrt{650}} \ \& \ \sin\frac{\theta}{2} = \frac{18}{\sqrt{650}}$$

17.

$$\tan\left(\frac{\pi}{6} + \frac{\theta}{2}\right)\tan\left(\frac{\pi}{6} - \frac{\theta}{2}\right) = \left(\frac{\tan 30^\circ + \tan\frac{\theta}{2}}{1 - \tan 30^\circ \ \tan\frac{\theta}{2}}\right)\left(\frac{\tan 30^\circ - \tan\frac{\theta}{2}}{1 + \tan 30^\circ \tan\frac{\theta}{2}}\right)$$

$$=\frac{\tan^2 30^\circ-\tan^2\frac{\theta}{2}}{1-\tan^2 30^\circ\tan^2\frac{\theta}{2}}=\frac{\frac{1}{3}-\tan^2\frac{\theta}{2}}{1-\frac{\tan^2\frac{\theta}{2}}{3}}$$

$$=\frac{\frac{1}{3}-\left(\frac{1-\cos\theta}{1+\cos\theta}\right)}{1-\frac{1}{3}\left(\frac{1-\cos\theta}{1+\cos\theta}\right)}=\frac{(1+\cos\theta)-3(1-\cos\theta)}{3(1+\cos\theta)-(1-\cos\theta)}$$

$$=\frac{4\cos\theta-2}{4\cos\theta+2}=\frac{2\cos\theta-1}{2\cos\theta+1}$$

18 **18.**

$$y=\frac{\sin x+\sin 2x+\sin 4x+\sin 5x}{\cos x+\cos 2x+\cos 4x+\cos 5x}$$

$$y=\frac{(\sin x+\sin 5x)+(\sin 2x+\sin 4x)}{(\cos x+\cos 5x)+(\cos 2x+\cos 4x)}$$

$$y=\frac{2\sin 3x\cos 2x+2\sin 3x\cos x}{2\cos 3x\cos 2x+2\cos 3x\cos x}$$

$$y=\frac{2\sin 3x}{2\cos 3x}\left(\frac{\cos 2x+\cos x}{\cos 2x+\cos x}\right)$$

$$y=\tan 3x=\tan\left(\frac{3\pi}{36}\right)$$

$$y=2-\sqrt{3}$$

19 **19.**

$$\cos(\alpha+\beta)+\sin(\alpha-\beta)=0$$

$$\cos\alpha\cos\beta-\sin\alpha\sin\beta+\sin\alpha\cos\beta-\cos\alpha\sin\beta=0$$

$$\cos\alpha\cos\beta+\sin\alpha\cos\beta=\sin\alpha\sin\beta+\cos\alpha\sin\beta$$

$$\cos\beta(\cos\alpha+\sin\alpha)=\sin\beta(\sin\alpha+\cos\alpha)$$

$\because \tan\beta \neq 1$

$\Rightarrow \cos\alpha + \sin\alpha = 0$

$\tan\alpha = -1$

20.

(20)

$\sin a + \sin b = \frac{1}{\sqrt{2}}, \cos a + \cos b = \frac{\sqrt{6}}{2}$

squaring and adding gives

$\sin^2 a + 2\sin a \sin b + \sin^2 b + \cos^2 a + 2\cos a \cos b + \cos^2 b = \frac{1}{2} + \frac{6}{4} = 2$

$2 + 2(\sin a \sin b + \cos a \cos b) = 2$

$\Rightarrow 2 + 2\cos(a-b) = 2$

(i) $\boxed{\cos(a-b) = 0}$

(ii) $\sin a + \sin b = \frac{1}{\sqrt{2}}$

$2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) = \frac{1}{\sqrt{2}}$ ...(i)

$\cos a + \cos b = \frac{\sqrt{6}}{2}$

$2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) = \frac{\sqrt{6}}{2}$ ....(ii)

From (i) & (ii), $\tan\left(\frac{a+b}{2}\right) = \frac{2}{\sqrt{12}} = \frac{1}{\sqrt{3}}$

$$\therefore \sin(a+b) = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\boxed{\sin(a+b)=\frac{\sqrt{3}}{2}}$$

21.

(21)

$$2^n\cos\theta\cos 2\theta\ldots\ldots\ldots\cos 2^{n-1}\theta \quad =2^n\left(\frac{1}{2^n}\frac{\sin 2^n\theta}{\sin\theta}\right)$$

$$=\frac{(\sin 2^n\theta)}{\sin\theta}$$

$$=\frac{\sin\left(\frac{2^n\pi}{2^n+1}\right)}{\sin\left(\frac{\pi}{2^n+1}\right)}$$

The angles $\frac{2^n\pi}{2^n+1}, \frac{\pi}{2^n+1}$ are supplementary

$$=\frac{\sin(\pi-\theta)}{\sin\theta}=1$$

If $\theta-\frac{\pi}{2^n-1}$

$$\text{L.H.S.}=\frac{\sin 2^n\theta}{\sin 0}$$

$$=\frac{\sin\left(\frac{2^n\pi}{2^n-1}\right)}{\sin\theta}$$

$$=\frac{\sin\left(\pi+\frac{\pi}{2^n-1}\right)}{\sin\theta}$$

$$=\frac{\sin(\pi+\theta)}{\sin\theta}$$

$$=-1$$

22.

$$\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}-\sin\frac{6\pi}{7}=4\sin\frac{\pi}{7}\sin\frac{3\pi}{7}\sin\frac{5\pi}{7}$$

$$\text{R.H.S.} = 4\sin\frac{\pi}{7}\sin\frac{3\pi}{7}\sin\frac{5\pi}{7}$$

$$=2\sin\frac{\pi}{7}\left(2\sin\frac{3\pi}{7}\sin\frac{5\pi}{7}\right)$$

$$=2\sin\frac{\pi}{7}\left(\cos\left(\frac{2\pi}{7}\right)-\cos\frac{8\pi}{7}\right)$$

$$=2\sin\frac{\pi}{7}\left(\cos\left(\frac{2\pi}{7}\right)+\cos\frac{\pi}{7}\right)$$

$$=2\sin\frac{\pi}{7}\cos\left(\frac{2\pi}{7}\right)+2\sin\frac{\pi}{7}+\cos\frac{\pi}{7}$$

$$=\sin\frac{3\pi}{7}+\sin\left(\frac{-\pi}{7}\right)+\sin\left(\frac{2\pi}{7}\right)$$

$$=\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}-\sin\frac{\pi}{7}$$

$$=\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}-\sin\frac{6\pi}{7}$$

23.

$$\sin 7x+\sin 5x+\sin 3x+\sin x=(\sin 7x+\sin x)+(\sin 5x+\sin 3x)$$

$$=2\sin 4x\cos 3x+2\sin 4x\cos x$$

$$=2\sin 4x(\cos 3x+\cos x)$$

$$\cos 7x+\cos 5x+\cos 3x+\cos x=(\cos 7x+\cos x)+(\cos 5x+\cos 3x)$$

$$=2\cos 4x\cos 3x+2\cos 4x\cos x$$

$$=2\cos 4x(\cos 3x+\cos x)$$

$$\text{L.H.S.}=\frac{2\sin 4x}{2\cos 4x}\ \frac{(\cos 3x+\cos x)}{(\cos 3x+\cos x)}$$

$= \tan 4x = \tan \frac{\pi}{12} = 2 - \sqrt{3}$.

24.

(24)

$$X = \sin\left(\theta + \frac{7\pi}{12}\right) + \sin\left(\theta - \frac{\pi}{12}\right) + \sin\left(\theta + \frac{3\pi}{12}\right)$$

$$X = 2\sin\left(\theta + \frac{3\pi}{12}\right)\cos\left(\frac{\pi}{3}\right) + \sin\left(\theta + \frac{3\pi}{12}\right)$$

$$= \sin\left(\theta + \frac{3\pi}{12}\right) + \sin\left(\theta + \frac{3\pi}{12}\right)$$

$$= 2\sin\left(\theta + \frac{\pi}{4}\right)$$

$$Y = \cos\left(\theta + \frac{7\pi}{12}\right) + \cos\left(\theta - \frac{\pi}{12}\right) + \cos\left(\theta + \frac{3\pi}{12}\right)$$

$$= 2\cos\left(\theta + \frac{3\pi}{12}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\theta + \frac{3\pi}{12}\right)$$

$$= \cos\left(\theta + \frac{3\pi}{12}\right) + \cos\left(\theta + \frac{3\pi}{12}\right)$$

$$= 2\cos\left(\theta + \frac{\pi}{4}\right)$$

$$\frac{X}{Y} = \tan\left(\theta + \frac{\pi}{4}\right)$$

$$\frac{X}{Y} - \frac{Y}{X} = \tan\left(\theta + \frac{\pi}{4}\right) - \cot\left(\theta + \frac{\pi}{4}\right)$$

$$= -2\left(\cot\left(2\left(\theta + \frac{\pi}{4}\right)\right)\right)$$

$$= \frac{-2}{\tan\left(2\theta + \frac{\pi}{2}\right)}$$

$$= \frac{-2}{-\cot 2\theta}$$

$$= 2\tan 2\theta$$

25.

$$\cot 9^\circ + \cot 27^\circ + \cot 63^\circ + \cot 81^\circ = \cot 9^\circ + \cot 81^\circ + \cot 27^\circ + \cot 63^\circ$$

$$= \frac{\sin(90^\circ)}{\sin 9^\circ \sin 81^\circ} + \frac{\sin(90^\circ)}{\sin 27^\circ \sin 63^\circ}$$

$$= \frac{1}{\sin 9^\circ \cos 9^\circ} + \frac{1}{\sin 27^\circ \cos 27^\circ}$$

$$= \frac{2}{\sin 18^\circ} + \frac{2}{\sin 54^\circ} = 2\left(\frac{1}{\sin 18^\circ} + \frac{1}{\cos 36^\circ}\right)$$

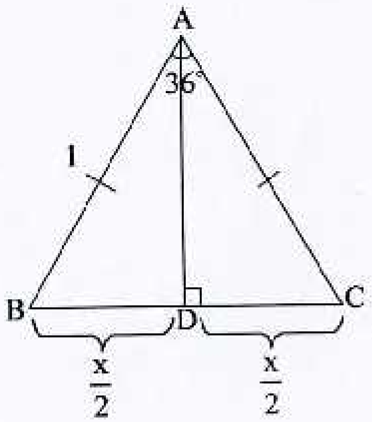
$$= 2\left(\frac{4}{\sqrt{5}-1} + \frac{4}{\sqrt{5}+1}\right)$$

$$= 8\left(\frac{\sqrt{5}+1+\sqrt{5}-1}{5-1}\right)$$

$$= \frac{8}{4}\left(2\sqrt{5}\right)$$

$$= 4\sqrt{5}$$

26.



$\angle BAD = 18^\circ \Rightarrow \sin 18^\circ = \dfrac{\frac{x}{2}}{1}$

$\dfrac{x}{2} = \sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$

$x = \dfrac{\sqrt{5}-1}{2}$

$p = -1, q = 5$

$(p, q) \equiv (-1, 5)$

27.

$\cos^4 t = \dfrac{3}{8} + \dfrac{1}{2}\cot 2t + \dfrac{1}{8}\cos 4t$

L.H.S. $= \cos^4 t$

$= \left(\dfrac{1+\cos 2t}{2}\right)^2$

$= \dfrac{1}{4}\left(1 + \cos^2 2t + 2\cos 2t\right)$

$= \dfrac{1}{4}\left(1 + \left(\dfrac{1+\cos 4t}{2}\right) + 2\cos 2t\right)$

$= \dfrac{1}{4}\left(1 + \dfrac{1}{2} + \dfrac{\cos 4t}{2} + 2\cos 2t\right)$

$= \dfrac{3}{8} + \dfrac{\cos 4t}{8} + \dfrac{\cos 2t}{2} =$ R.H.S.

28.

$\dfrac{\cos x - \cos 3x}{\sin 3x - \sin x} = \dfrac{2\sin 2x \sin x}{2\sin x \cos 2x} = \tan 2x$

$= \tan 15^\circ$

$=2-\sqrt{3}$

29.

$$\frac{\sin x-\cos x+1}{\sin x+\cos x-1}=\frac{(\sin x-\cos x+1)\,(\sin x+\cos x+1)}{(\sin x+\cos x-1)\,(\sin x+\cos x+1)}$$

$$=\frac{(1+\sin x)^2-\cos^2 x}{(\sin x-\cos x)^2-1}=\frac{1+\sin^2 x+2\sin x-\cos^2 x}{1+2\sin x\cos x-1}$$

$$=\frac{2\sin^2 x+2\sin x}{2\sin x\cos x}$$

$$=\frac{2\sin x\,(1+\sin x)}{2\sin x\cos x}$$

$$=\frac{1+\sin x}{\cos x} \qquad \text{.......(I)}$$

$$\frac{1+\sin x}{\cos x}=\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}$$

$$=\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}=\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$$

$$=\tan\left(\frac{\pi}{4}+\frac{x}{2}\right).$$

30.

$$\cos 24^\circ-\cos 12^\circ+\cos 48^\circ-\cos 84^\circ=\cos 24^\circ+\cos 48^\circ-\cos 12^\circ-\cos 84^\circ$$

$$=\left(2\cos 36^\circ\cos 12^\circ\right)-\left(2\cos 36^\circ\cos 48^\circ\right)$$

$$=2\cos 36^\circ\left(\cos 12^\circ-\cos 48^\circ\right)$$

$$=2\cos 36^\circ\left(2\sin 30^\circ\sin 18^\circ\right)$$

$$=2\times\left(\frac{\sqrt{5}+1}{4}\right)\times 2\times\frac{1}{2}\times\left(\frac{\sqrt{5}-1}{4}\right)$$

$$=2\times\frac{(4)}{16}=\frac{1}{2}$$

31.

$$\tan 20^\circ\left(\cot 10^\circ-\tan 10^\circ\right)=\tan 20^\circ\left(\frac{1}{\tan 10^\circ}-\tan 10^\circ\right)$$

$$=\tan 20^\circ\left(\frac{1-\tan^2 10^\circ}{\tan 10^\circ}\right)$$

$$=\tan 20^\circ\,(2)\left(\frac{1-\tan^2 10^\circ}{2\tan 10^\circ}\right)$$

$$=2\tan 20^\circ\times\frac{1}{\tan 20^\circ}=2.$$

32.

$$\sin^2\left(\frac{15\pi}{8}-4x\right)-\sin^2\left(\frac{17\pi}{8}-4x\right)$$

$$=\sin(4\pi-8x)\,\sin\left(-\frac{2\pi}{8}\right)$$

$$=-\sin(8x)\times\left(\frac{-1}{\sqrt{2}}\right)$$

$$=\frac{1}{\sqrt{2}}\sin 8x$$

$$-1\le\sin 8x\le 1$$

$$-\frac{1}{\sqrt{2}}\le\frac{\sin 8x}{\sqrt{2}}\le\frac{1}{\sqrt{2}}$$

Max value is $\frac{1}{\sqrt{2}}$

33. In a Cyclic Quadrilateral

$A+C=\pi \qquad A=\pi-C$

$B+D=\pi \qquad B=\pi-D$

$\therefore \sin A=+\sin C, \quad \cos A=-\cos C$

$\sin B=+\sin D, \quad \cos B=-\cos D$

$\therefore \cos A+\cos B+\cos C+\cos D=0$

$\sin A+\sin B+\sin C+\sin D$

$=2\sin A+2\sin B \neq 0$

34.

$$\sqrt{\tan x+\sin x}+\sqrt{\tan x-\sin x}=2\sqrt{\tan x}\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

L.H.S.

$$=\sqrt{\tan x+\tan x\cos x}+\sqrt{\tan x-\tan x\cos x}$$

$$=\sqrt{\tan x}\left(\sqrt{1+\cos x}+\sqrt{1-\cos x}\right)$$

$$=\sqrt{\tan x}\left(\sqrt{2\cos^2\frac{x}{2}}+\sqrt{2\sin^2\frac{x}{2}}\right) \qquad \left[\sqrt{x^2}=|x|\right]$$

$$=\sqrt{\tan x}\left(\sqrt{2}\right)\left(\left|\cos\frac{x}{2}\right|+\left|\sin\frac{x}{2}\right|\right)$$

$$=\sqrt{2}\sqrt{\tan x}\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)$$

$$=\sqrt{2}\sqrt{\tan x}\left(\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\cos\frac{x}{2}+\frac{1}{\sqrt{2}}\sin\frac{x}{2}\right)$$

$$=2\sqrt{\tan x}\left(\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)=\text{R.H.S.}$$

35.

$$\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}-2\cos\frac{3\pi}{7}\cos\frac{\pi}{7}+\cos\frac{6\pi}{7}$$

$$=2\cos\frac{3\pi}{7}\cos\frac{\pi}{7}-\cos\frac{\pi}{7}$$

$$=\cos\frac{\pi}{7}\left(2\cos\frac{3\pi}{7}-1\right)$$

$$=\cos\frac{\pi}{7}\frac{\left(2\cos\frac{3\pi}{7}-1\right)\sin\frac{3\pi}{7}}{\sin\frac{3\pi}{7}}$$

$$=\frac{\cos\frac{\pi}{7}\left(2\sin\frac{3\pi}{7}\cos\frac{3\pi}{7}-\sin\frac{3\pi}{7}\right)}{\sin\frac{3\pi}{7}}$$

$$=\frac{\cos\frac{\pi}{7}\left(\sin\frac{6\pi}{7}-\sin\frac{3\pi}{7}\right)}{\sin\frac{3\pi}{7}}$$

$$=\frac{\cos\frac{\pi}{7}\left(2\sin\frac{3\pi}{14}\cos\frac{9\pi}{14}\right)}{2\sin\frac{3\pi}{14}\cos\frac{3\pi}{14}}$$

$$=\frac{\cos\frac{\pi}{7}\cos\frac{9\pi}{14}}{\cos\frac{3\pi}{14}}$$

$$=\frac{1}{2}\frac{\left(\cos\frac{11\pi}{14}+\cos\frac{7\pi}{14}\right)}{\cos\frac{3\pi}{14}}$$

$$=\frac{1}{2}\left(\frac{-\cos\frac{3\pi}{14}+0}{\cos\frac{3\pi}{14}}\right)=\frac{-1}{2}$$

36.

$$\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \cos 60^\circ \left(\cos 20^\circ \cos 40^\circ \cos 80^\circ\right)$$

$$= \cos 60^\circ\left(\frac{\cos 60^\circ}{4}\right)$$

$$=\frac{1}{16}$$

37.

$$\sqrt{3}\operatorname{cosec} 20^\circ - \sec 20^\circ = \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}$$

$$=\frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{\frac{(\sin 40^\circ)}{2}}$$

$$=\frac{2\left(\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ\right)}{\left(\frac{\sin 40^\circ}{2}\right)}$$

$$=\frac{4\sin\left(60^\circ - 20^\circ\right)}{\sin 40^\circ}$$

$$=4.$$

38.

$$A+B+C=\pi$$

$$\cot A = \cot B \cot C$$

$$\cos A - \cos B \cos C$$

$$\therefore \frac{-\cos(B+C)}{\cos B \cos C} = 1$$

$$-(1 - \tan A \tan B) = 1$$

$$\therefore \boxed{\tan A \tan B = 2}$$

39.

$$\cos(A + B)\sin(C + D) = \cos(A - B)\sin(C - D)$$

$$\Rightarrow \frac{\cos(A-B)}{\cos(A+B)} = \frac{\sin(C+D)}{\sin(C-D)}$$

$$\Rightarrow \frac{\cos(A+B) + \cos(A-B)}{\cos(A-B) - \cos(A+B)} = \frac{\sin(C+D) + \sin(C-D)}{\sin(C+D) - \sin(C-D)}$$

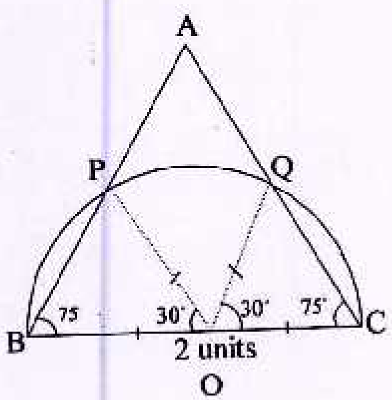
$$\Rightarrow \frac{2\cos A \cos B}{2\sin A \sin B} = \frac{2\sin(\cos 1)}{2\cos(\sin 1)}$$

$$\cot A \cot B = \frac{\cot D}{\cot C}$$

$$\therefore \cot A \cot B \cot C = C + D$$

40.

$B = C = 75°$

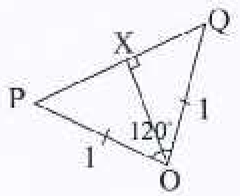


Let O be center of the semicircle

In ΔBOP, $\angle BOP = 180 - 75^\circ - 75^\circ = 30^\circ$

In ΔCOQ, $\angle COQ = 180^\circ - 75^\circ - 75^\circ = 30^\circ$

$\therefore \angle POQ = 180^\circ - 30^\circ - 30^\circ = 120^\circ$



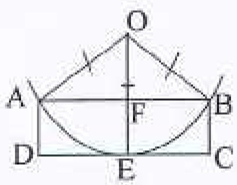
PO = QO = unit (radius of semi circle)

$PX = XQ = \frac{PQ}{2} = OQ \sin 60^\circ$

$\therefore \boxed{PQ = \sqrt{3}}$

41.

41



$AD = 1 \quad AB = 2\sqrt{3}$

$\therefore AF = \sqrt{3}$

OF = (radius) R – FE

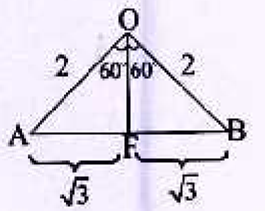
$= R - AD$

$= R - 1$

$OF = R - 1$

$OA = R$

In ΔOAF, $OF^2 + AF^2 = OA^2$



$(R-1)^2+\left(\sqrt{3}\right)^2=R^2$

$R^2+1-2R+3=R^2$

$\therefore R=2$ units

$\therefore OF=1$units

Area of Δ AOB $=\frac{1}{2}\left(2\sqrt{3}\right)\times 1=\sqrt{3}$ sq units

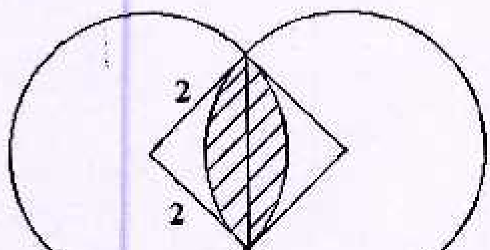
Area of sector OAEB $=\frac{1}{2}r^2\theta$, where $\theta=\frac{2\pi}{3}$

$=\frac{1}{2}(2)^2\left(\frac{2\pi}{3}\right)$

$=\frac{4\pi}{3}$ sq. units

$\therefore$ Area of shaded part $=\left(\frac{4\pi}{3}-\sqrt{3}\right)$sq . units

42.



(Fig : 1)

Area of shaded region $=2\times$ area of shaded region in figure 2

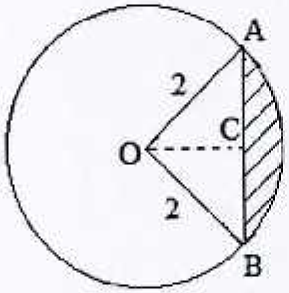


Fig. 2

where OC = $\sqrt{2}$

In ΔOCB $\angle BOC = 45^\circ$ ....$\left(\because \dfrac{OC}{OB} = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}\right)$

$\therefore \cos\theta = \dfrac{1}{\sqrt{2}}$

$\theta = 45^\circ$

Similarly $\angle COA = 45^\circ$

So angle $\angle BOA = 90^\circ$

Δ BOA area $= \dfrac{1}{2} \times AP \times BO$

$= \dfrac{1}{2}(2)\,(2) = 2$ sq. units

Sector Area $= \dfrac{1}{2} r^2\theta$

$= \dfrac{1}{2}(2)^2\left(\dfrac{\pi}{2}\right)$

$= 2\pi$

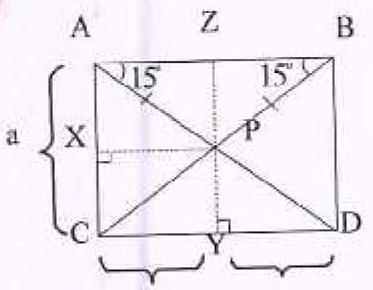
Area of shaded region (Fig. 2)

$= 2\pi - 2 = 2(\pi - 2)$

$\therefore$ Area of shaded position in Fig (1) $= 2 \times (2(\pi - 2))$

$= 4(\pi - 2)$

43.



$AZ = \frac{a}{2}$

$\frac{PZ}{AZ} = \tan 15^\circ \quad (\Delta PAZ)$

$PZ = \frac{a}{2}(2 - \sqrt{3})$

$PZ = a - \frac{\sqrt{3}}{2}a$

$\therefore AX = a - \frac{\sqrt{3}}{2}a \qquad (AX + XC = a)$

$XC = PY = a - AX$

$= a - \left(a - \frac{\sqrt{3}}{2}a\right)$

$= \frac{\sqrt{3}}{2}a$

So $PY = \frac{\sqrt{3}}{2}a$

$CY = \frac{a}{2} \qquad \ldots\ldots \Delta PCY$

$\therefore CP = \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}}$ ....(Pythagoras)

CP = a

By squaring PD = PC = a

So all sides of ΔPCD are equal to a,

$\therefore$ its an Equilateral Δ

44.

$0 < \alpha, \beta < \frac{\pi}{2}$

$\sqrt{(1-\cos\alpha\cos\beta)^2 - \sin^2\alpha\sin^2\beta}$

$= \sqrt{(1-\cos\alpha\cos\beta - \sin\alpha\sin\beta)\,(1-\cos\alpha\cos\beta+\sin\alpha\sin\beta)}$

$= \sqrt{(1-\cos(\alpha-\beta))\,(1-\cos(\alpha+\beta))}$

$\sqrt{2\sin^2\left(\frac{\alpha-\beta}{2}\right)2\sin^2\left(\frac{\alpha+\beta}{2}\right)}$

$= \sqrt{4}\left|\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)\right|$

$0 < 2, \beta < \frac{\pi}{2}$

$0 < \alpha+\beta < \pi$

$0 < \alpha+\beta < \frac{\pi}{2}$

$\beta > \alpha$ so $\sin\left(\frac{\alpha-\beta}{2}\right) < 0$

L.H.S. $= \sqrt{4}\sin\left(\frac{\beta-\alpha}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$

$$= 2\sin\left(\frac{\beta-\alpha}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$$

$$= (\cos\alpha - \cos\beta)$$

45 **45.**

$$P = (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma) = (1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)$$

$$P^2 = (1-\cos\alpha)(1-\cos\beta)(1+\cos\gamma) \times (1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)$$

$$P^2 = (1-\cos^2\alpha)(1-\cos^2\beta)(1-\cos^2\gamma)$$

$$P^2 = \sin^2\alpha\,\sin^2\beta\,\sin^2\gamma$$

$$P = \pm\sin\alpha\sin\beta\sin\gamma$$

Trigonometry – 1

Exercise – 3(C)

1.

$$\text{R.H.S.}=(1+\sec 2\theta)\,(1+\sec 4\theta)\,(1+\sec 8\theta)$$

$$=\frac{(1+\cos 2\theta)(1+\cos 4\theta)(1+\cos 8\theta)}{\cos 2\theta\,\cos 4\theta\,\cos 8\theta}$$

$$=\frac{(2\cos^2\theta)(2\cos^2 2\theta)(2\cos^2 4\theta)}{\cos 2\theta\,\cos 4\theta\,\cos 8\theta}$$

$$=\frac{8\left(\frac{1}{8}\frac{\sin 8\theta}{\sin\theta}\right)}{\left(\frac{1}{8}\frac{\sin 16\theta}{\sin 2\theta}\right)}$$

$$=8\times\frac{1}{64}\frac{\sin^2 8\theta}{\sin^2\theta}\times\frac{\sin 2\theta}{\sin 16\theta}\times 8$$

$$=\frac{\sin^2 8\theta}{2\sin 8\theta\,\cos\theta}\times\frac{2\sin\theta\,\cos\theta}{\sin^2\theta}$$

$$=\frac{\tan 8\theta}{\tan\theta}=\text{L.H.S.}$$

2.

$\cos\theta+\cos\phi=a,\ \sin\theta+\sin\phi=b$

squaring and adding we get

$$2+2\cos(\theta-\phi)=a^2+b^2$$

$$\cos(\theta-\phi)=\frac{a^2+b^2-2}{2}$$

$$\sin\theta+\sin\phi=b$$

$$2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)=b$$

$\cos\theta + \cos\phi = a$

$$2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = a$$

dividing we get

$$\tan\left(\frac{\theta+\phi}{2}\right) = \frac{b}{a}$$

$$\therefore \cos(\theta+\phi) = \frac{1-\left(\frac{b}{a}\right)^2}{1+\left(\frac{b}{a}\right)^2} = \frac{a^2-b^2}{a^2+b^2}$$

$\cos 2\theta + \cos 2\phi$

$= 2\cos(\theta+\phi)\cos(\theta-\phi)$

$$= 2\left(\frac{a^2+b^2-2}{2}\right)\left(\frac{a^2-b^2}{a^2+b^2}\right)$$

$$= \frac{(a^2-b^2)(a^2+b^2-2)}{(a^2+b^2)}$$

= L.H.S.

3.

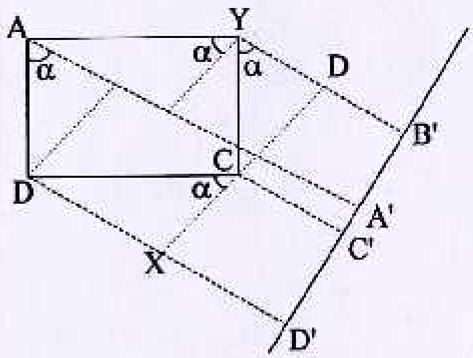
$$\tan^2\frac{\pi}{16} + \tan^2\frac{3\pi}{16} + \tan^2\frac{5\pi}{16} + \tan^2\frac{7\pi}{16}$$

$$\tan\frac{7\pi}{16} = \cot\left(\frac{\pi}{16}\right) \;\; \& \;\; \tan\frac{5\pi}{16} = \cot\left(\frac{3\pi}{16}\right)$$

$$\text{L.H.S.} = \tan^2\frac{\pi}{16} + \tan^2\frac{3\pi}{16} + \cot^2\frac{3\pi}{16} + \cot^2\frac{\pi}{16}$$

$$= \tan^2\frac{\pi}{16} + \cot^2\frac{\pi}{16} + \tan^2\frac{3\pi}{16} + \cot^2\frac{3\pi}{16}$$

14.



Let side length be a assume angle α

$CX = C'D' = a\cos\alpha$ ..... Projection of CD is C'D'.

and

$CY = A'B' = a\sin\alpha$ ..... Projection of AB is A'B' and so on

Similarly we will get

$A'D'$ as $a\sin\alpha$ and

$A'B'$ as $a\cos\alpha$

So $A'B'^2 + A'D'^2 + B'C'^2 + C'D'^2$

$= a^2\cos^2\alpha + a^2\cos^2\alpha + a^2\sin^2\alpha + a^2\sin^2\alpha$

$= 2a^2$, which is independent of α.

15.

$$\frac{\pi}{2} < \alpha < \pi$$

$$\left[\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} + \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}\right]$$

$$\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$$

$$\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} = \sqrt{\frac{\left(\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\right)^2}{\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)^2}}$$

$$= \sqrt{\left(\frac{\tan\frac{\alpha}{2}-1}{\tan\frac{\alpha}{2}+1}\right)^2}$$

$$= \left|\frac{\tan\frac{\alpha}{2}-1}{\tan\frac{\alpha}{2}+1}\right| = \frac{\tan\frac{\alpha}{2}-1}{\tan\frac{\alpha}{2}+1}$$

$$\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \sqrt{\left(\frac{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}\right)^2}$$

$$= \left|\frac{1+\tan\frac{\alpha}{2}}{1-\tan\frac{\alpha}{2}}\right|$$

$$= \frac{1+\tan\frac{\alpha}{2}}{\tan\frac{\alpha}{2}-1}$$

$$\therefore \text{L.H.S.} = \frac{\tan\frac{\alpha}{2}-1}{\tan\frac{\alpha}{2}+1} + \frac{1+\tan\frac{\alpha}{2}}{\tan\frac{\alpha}{2}-1}$$

$$= \frac{\left(\tan\frac{\alpha}{2}-1\right)^2 + \left(1+\tan\frac{\alpha}{2}\right)^2}{\left(\tan^2\frac{\alpha}{2}-1\right)}$$

$$= \frac{2\left(1+\tan^2\frac{\alpha}{2}\right)}{\left(\tan^2\frac{\alpha}{2}-1\right)} = -2\left(\frac{1+\tan^2\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}\right)$$

$$= -\frac{2}{\cos\alpha}$$

$$= -2\sec\alpha$$

61 16 $\cos(A+B+C) = (1-s_2)\cos A \cos B \cos C$

where $s_2 = \Sigma \tan B \tan C$

$$\cos\left(\frac{\pi}{3}\right) = (1-\Sigma\tan B\tan C)\cos A \cos B \cos C$$

$$\frac{1}{2} = (1-\Sigma\tan B\tan C)\cos A \cos B \cos C$$

$$\sec A \sec B \sec C = 2(1-\Sigma\tan B \tan C)$$

$$\therefore \boxed{\sec A \sec B \sec C + 2\Sigma\tan B \tan C = 2}$$

17.

$$\left(\frac{1-\tan B \tan C}{\cos^2 A}\right) + \left(\frac{1-\tan C \tan A}{\cos^2 B}\right) = 2\left(\frac{1-\tan A \tan B}{\cos^2 C}\right)$$

Let $\tan A = x$, $\tan B = y$ & $\tan C = z$

$\frac{1}{\cos^2 x} = \sec^2 A = 1+\tan^2 A = 1+x^2$ and so on

$$\therefore (1+x^2)(1-y^2) + (1+y^2)(1-xz) = 2(1+z^2)(1-xy)$$

$$1+x^2+1+y^2-yz-xz-x^2yz-xy^2z-2-2z^2+2xy+2xyz^2=0$$

$$\therefore x^2+y^2-2z^2+2xy-yz-xz+2xyz^2-x^2yz-xy^2z=0$$

$$x^2+y^2+2xy-4z^2+2z^2-yz-xz+xyz(2z-x-y)=0$$

[we have to take $2z-x-y$ as the common factor because that is what we ***missing***]

$(x+y)^2-4z^2+2(2z-x-y)+xyz\,(2z-x-y)$

$(x+y+2z)\,(x+y-2z)+z\,(2z-x-y)+xyz\,(2z-x-y)$

$\therefore\ (x+y-2z)\,[x+y+2z-z-xyz]=0$

$(x+y-2z)\,(x+y+z-xyz)=0$

$\therefore\ x+y=2z$

$\therefore$ tan A + tan B = 2 tan C proved result

OR $\quad x+y+z=xyz$

i.e. $\quad$ tan A +tan B + tan C = tan A tan B tan C

$\therefore$ A + B + C is an integral multiple of π.

18.

$$\frac{\tan(\alpha+\beta-\gamma)}{\tan(\alpha-\beta+\gamma)}=\frac{\tan\beta}{\tan\beta}$$

Taking componendo and dividendo we get

$$\frac{\tan(\alpha+\beta-\gamma)+\tan(\alpha-\beta+\gamma)}{\tan(\alpha+\beta-\gamma)-\tan(\alpha-\beta+\gamma)}=\frac{\tan\gamma+\tan\beta}{\tan\gamma-\tan\beta}$$

$$\Rightarrow\frac{\sin(2\alpha)}{\sin(2\beta-2\gamma)}=\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$$

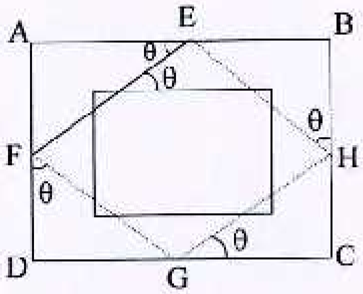
$$\left[\frac{\tan A+\tan B}{\tan A-\tan B}=\frac{\dfrac{\sin(A+B)}{\cos A\cos B}}{\dfrac{\sin(A+B)}{\cos A\cos B}}=\frac{\sin(A+B)}{\sin(A-B)}\right]$$

$$\text{So, }\sin 2\alpha=\frac{\sin(2\beta-2\gamma)\sin(\beta+\gamma)}{\sin(\gamma-\beta)}=\frac{2\sin(\beta-\gamma)\cos(\beta-\gamma)\sin(\beta+\gamma)}{\sin(\gamma-\beta)}$$

$$=-(\sin 2\beta+\sin 2\gamma)$$

so $\boxed{\sin 2\alpha+\sin 2\beta+\sin 2\gamma=0}$

19.



θ is the angle of rotation

$EF = 3 \Rightarrow AE = 3\cos\theta$

$EH = 3 \Rightarrow EB = 3\sin\theta$

$AE + EB = 3\sin\theta + 3\cos\theta$ or $AB = 4$

$3(\sin\theta + \cos\theta) = 4$

$\sin\theta + \cos\theta = \frac{4}{3}$

squaring we get $1 + 2\sin\theta\cos\theta = \frac{16}{9}$

$\sin 2\theta = \frac{7}{9} = \frac{2t}{1+t^2}$, $t = \tan\theta$

or $7t^2 - 18t + 7 = 0$

$\Rightarrow t = \frac{18 \pm 8\sqrt{2}}{14} = \frac{9 \pm 4\sqrt{2}}{7}$

$0 < \theta < \frac{\pi}{4} \quad \tan\theta < 1$

so $\boxed{\tan\theta = \frac{9 - 4\sqrt{2}}{7}}$

20.

$$\prod_{k=1}^{7}\cos k\theta = \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$$

$$=\left(\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}\right)\left(\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right)\cos\frac{5\pi}{15}$$

$$-\left(\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right)\left(\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right)\left(\cos\frac{\pi}{3}\right)\times(-1)$$

$$=\frac{1}{16}\frac{\sin\left(\frac{16\pi}{15}\right)}{\sin\left(\frac{\pi}{15}\right)}\times\frac{1}{4}\frac{\sin\left(\frac{12\pi}{15}\right)}{\sin\left(\frac{3\pi}{15}\right)}\times\frac{1}{2}\times(-1)$$

$$=\frac{1}{128}\left(\frac{\sin\left(\pi+\frac{\pi}{15}\right)}{\sin\left(\frac{\pi}{15}\right)}\times\frac{\sin\left(\pi-\frac{3\pi}{15}\right)}{\sin\left(\frac{3\pi}{15}\right)}\times(-1)\right)$$

$$=\frac{1}{128}\left(\frac{-\sin\left(\frac{\pi}{15}\right)}{\sin\left(\frac{\pi}{15}\right)}\times\frac{\sin\left(\frac{3\pi}{15}\right)}{\sin\left(\frac{3\pi}{15}\right)}\times-1\right)=\frac{1}{128}$$

21.

$$\sin 36^\circ\ \sin 72^\circ\ \sin 108^\circ\ \sin 144^\circ = \left(\sin^2 36^\circ\ \sin^2 72^\circ\right)$$

$$=\left(\cos^2 18^\circ\ \sin^2 36^\circ\right)$$

$$=\left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2\left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)^2=\frac{5}{16}.$$

22.

$\sum_{cyclic}\cos(B-C)=-\frac{3}{2}$ then show that $\Sigma\cos A-\Sigma\sin A=0$

$$\cos(A-B)+\cos(B-C)+\cos(C-A)=-\frac{3}{2}$$

$\therefore$ Expanding we get

$$2\cos A\cos B+2\cos B\cos C+2\cos C\cos A+2\sin A\sin B+2\sin B\sin C+2\sin C\sin A+3=0$$

Now we can take $3=\cos^2 A+\sin^2 A+\cos^2 B+\sin^2 B+\cos^2 C+\sin^2 C$

$$\therefore \sum\cos^2 A+2\sum\cos A\cos B+\sum\sin^2 A+2\sum\sin A\sin B=0$$

$$(\cos A+\cos B+\cos C)^2+(\sin A+\sin B+\sin C)^2=0$$

Sum of square of 2 numbers is zero means the numbers must be zero individually

$\therefore \cos A+\cos B+\cos C=0$ and $\sin A+\sin B+\sin C=0$

23.

$$\sum_{\text{cyclic}}\sin x\ \sin y\ \sin(x-y)+\prod\sin(x-y)=0$$

Now $\sum_{\text{cyclic}}\sin x\ \sin y\ \sin(x-y)=\sum_{\text{cyclic}}\frac{1}{2}(\cos(x-y)-\cos(x+y))\sin(x-y)$

$$=\sum_{\text{cyclic}}\frac{1}{2}(\sin(x-y)\cos(x-y)-\sin(x-y)\cos(x+y))$$

$$=\sum_{\text{cyclic}}\frac{1}{4}(\sin(2x-2y)-\sin 2x+\sin 2y)$$

$$=\frac{1}{4}\left[\sum_{\text{cyclic}}\sin(2x-2y)+\sum_{\text{cyclic}}(\sin 2y-\sin 2x)\right]$$

$$=\frac{1}{4}\left[\sum_{\text{cyclic}}\sin(2x-2y)+(\sin 2y-\sin 2x+\sin 2x-\sin 2z+\sin 2z-\sin 2y)\right]$$

$$=\frac{1}{4}\left[\sum_{\text{cyclic}}\sin(2x-2y)\right]$$

$$=\frac{1}{4}(\sin(2x-2y)+\sin(2y-2z)+\sin(2z-2x))$$

$$=\frac{1}{4}\left(2\sin(x-z)\cos(x+z-2y)+2\sin(z-x)\cos(z-x)\right)$$

$$=\frac{2}{4}\left(\sin(x-z)\right)\left(\cos(x+z-2y)-\cos(z-x)\right)$$

$$=\frac{1}{2}\sin(x-z)\left(2\sin(z-y)\sin(y-x)\right)$$

So, $\sum_{cyclic}\sin x\sin y\sin(x-y)=-\prod_{cyclic}\sin(x-y)$

$$\sum_{cyclic}\sin x\sin y\sin(x-y)+\prod_{cyclic}\sin(x-y)=0$$

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24.

$$m\sin(\alpha+\beta)=\cos(\alpha-\beta)\Rightarrow m=\frac{\cos(\alpha-\beta)}{\sin(\alpha+\beta)}$$

$$\frac{1}{1-m\sin 2\alpha}+\frac{1}{1-m\sin 2\beta}=\frac{1}{1-\frac{\cos(\alpha-\beta)\sin 2\alpha}{\sin(\alpha+\beta)}}+\frac{1}{\frac{1-\cos(\alpha-\beta)\sin 2\beta}{\sin(\alpha+\beta)}}$$

$$=\frac{\sin(\alpha+\beta)}{\sin(\alpha+\beta)-\cos(\alpha-\beta)\sin 2\alpha}+\frac{\sin(\alpha+\beta)}{\sin(\alpha+\beta)-\cos(\alpha-\beta)\sin 2\beta}$$

$$=\frac{\sin(\alpha+\beta)}{\sin(\alpha+\beta)-\frac{1}{2}\left(\sin(\alpha+\beta)+\sin(3\alpha-\beta)\right)}+\frac{\sin(\alpha+\beta)}{\sin(\alpha+\beta)-\frac{1}{2}\left(\sin(\alpha+\beta)+\sin(3\beta-\alpha)\right)}$$

$$=\frac{\sin(\alpha+\beta)}{\frac{1}{2}\left(\sin(\alpha+\beta)-\sin(3\alpha-\beta)\right)}+\frac{\sin(\alpha+\beta)}{\frac{1}{2}\left(\sin(\alpha+\beta)-\sin(3\beta-\alpha)\right)}$$

$$=\frac{\sin(\alpha+\beta)}{\sin(\beta-\alpha)\cos 2\alpha}+\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)\cos 2\beta}$$

$$=\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}\left(\frac{1}{\cos 2\beta}-\frac{1}{\cos 2\alpha}\right)$$

$$=\frac{\sin(\alpha+\beta)}{\sin\alpha-\beta}\left(\frac{\cos 2\alpha-\cos 2\beta}{\cos 2\alpha\,\cos 2\beta}\right)$$

$$=\frac{\sin(\alpha+\beta)\left(2\sin(\alpha+\beta)\sin(\beta-\alpha)\right)}{\sin(\alpha-\beta)\left(\cos^2(\alpha+\beta)-\sin^2(\alpha-\beta)\right)}$$

$$\left[\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B=\cos^2 B-\sin^2 A\right]$$

$$=\frac{-2\sin^2(\alpha+\beta)}{\left(\cos^2(\alpha-\beta)-\sin^2(\alpha+\beta)\right)}$$

$$=\frac{2}{\left(1-\frac{\cos^2(\alpha-\beta)}{\sin^2(\alpha+\beta)}\right)}=\frac{2}{1-m^2}$$

$$\left[\begin{array}{l}\cos 2\alpha\,\cos 2\beta=\cos^2(\alpha+\beta)-\sin^2(\alpha-\beta)\\ \qquad\qquad\qquad=\cos^2(\alpha-\beta)-\sin^2(\alpha+\beta)\\ \text{you can also prove this using }\cos\theta\text{ formula}\end{array}\right]$$

25.

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Given $\tan 60=\frac{p}{q}$

Now $\frac{1}{2}(p\,\text{cosec}\,2\theta = q\sec 2\theta)=\frac{1}{2}\left(\frac{p}{\sin 2\theta}-\frac{q}{\cos 2\theta}\right)$

$$=\frac{1}{2}\left(\frac{p\cos 2\theta-q\sin 2\theta}{\sin 2\theta\,\cos 2\theta}\right)$$

$$=\frac{1}{2}\sqrt{p^2+q^2}\,\frac{\left(\frac{p}{\sqrt{p^2+q^2}}\cos 2\theta-\frac{q}{\sqrt{p^2+q^2}}\sin 2\theta\right)}{\frac{(\sin 4\theta)}{2}}$$

$$=\frac{\sqrt{p^2+q^2}}{(\sin 4\theta)}(\sin 6\theta\cos 2\theta-\cos 6\theta\sin 2\theta)$$

$$= \frac{\sqrt{p^2+q^2}}{\sin 4\theta}\left(\sin\left(6\theta - 2\theta\right)\right)$$

$$= \sqrt{p^2+q^2}$$

26.

$$\frac{\cos\alpha}{\cos\theta} + \frac{\sin\alpha}{\sin\theta} = \frac{\cos\beta}{\cos\theta} + \frac{\sin\beta}{\sin\theta} = 1$$

From $\frac{\cos\phi}{\cos\theta} + \frac{\sin\phi}{\sin\theta} = 1$ we have $\frac{\cos\phi}{\cos\theta} = 1 - \frac{\sin\phi}{\sin\theta}$

squaring both the sides gives

$$\frac{\cos^2\phi}{\cos^2\theta} = 1 + \frac{\sin^2\phi}{\sin^2\theta} - \frac{2\sin\phi}{\sin\theta}$$

$$\theta = \frac{\sin^2\phi}{\sin^2\theta} + \frac{\sin^2\phi}{\cos^2\theta} - \frac{2\sin\phi}{\sin\theta} + 1 - \frac{1}{\cos^2\theta}$$

$$\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right)\sin^2\phi - \left(\frac{2}{\sin\theta}\right)\sin\phi - \tan^2\theta = 0$$

This is a quadratic in $\sin\phi$ with $\sin\alpha$, $\sin\beta$ as the roots so.

$$\sin\alpha\sin\beta = \frac{-\tan^2\theta}{\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right)} = -\sin^4\theta$$

Also $\frac{\sin\phi}{\sin\theta} = 1 - \frac{\cos\phi}{\cos\theta}$

squaring both the sides gives

$$\frac{\sin^2\phi}{\sin^2\theta} = 1 + \frac{\cos^2\phi}{\cos^2\theta} - \frac{2\cos\phi}{\cos\theta}$$

converting $\sin^2\phi = 1 - \cos^2\phi$

$$\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right)\cos^2\phi - \left(\frac{2}{\cos\theta}\right)\cos\phi - \cot^2\theta = 0$$

This is a quadratic in $\cos\phi$ with $\cos\alpha$, $\cos\beta$ as the roots so

$$\cos\alpha\cos\beta = \frac{-\cot^2\theta}{\left(\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}\right)}$$

$$= -\cos^4\theta$$

$$\therefore \frac{\cos\alpha\cos\beta}{\cos^2\theta}+\frac{\sin\alpha\ \sin\beta}{\sin^2\theta}+1$$

$$= \frac{-\cos^4\theta}{\cos^2\theta}-\frac{\sin^4\theta}{\sin^2\theta}+1$$

$$= -\cos^2\theta-\sin^2\theta+1=0$$

27.

$\sin x+\sin y=a$ ....(I)

$\cos x+\cos y=b$ ....(II)

$\tan x+\tan y=c$ ....(III)

$\tan x+\tan y=c$

So, $\dfrac{\sin(x+y)}{\cos x\ \cos y}=c$

$\dfrac{2\sin(x+y)}{\cos(x+y)+\cos(x-y)}=c$ ....(IV)

Now squaring and adding (I) and (II) we get

$$2+2\cos(x-y)=a^2+b^2$$

$$\cos(x-y)=\frac{a^2+b^2-2}{2}$$

From (I) $2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)=a$

(II) $2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = b$

Taking ratio we get $\tan\left(\frac{x+y}{2}\right) = \frac{a}{b}$

$$\cos(x+y) = \frac{1-\frac{a^2}{b^2}}{1+\frac{a^2}{b^2}} = \frac{b^2-a^2}{b^2+a^2}$$

$$\sin(x+y) = \frac{2\left(\frac{a}{b}\right)}{1+\frac{a^2}{b^2}} = \frac{2ab}{a^2+b^2}$$

$\therefore$ substituting in equation (IV) we get

$$\frac{2\left(\frac{2ab}{a^2+b^2}\right)}{\left(\frac{b^2-a^2}{b^2+a^2}\right)+\left(\frac{a^2+b^2-2}{2}\right)} = c$$

$$\frac{8ab}{2\left(b^2-a^2\right)+\left(a^2+b^2-2\right)\left(b^2+a^2\right)} = c$$

$$8ab = c\left(2b^2-2a^2+\left(a^2+b^2\right)-2b^2-2a^2\right)$$

$$\boxed{8ab = c\left(\left(a^2+b^2\right)^2-4a^2\right)}$$

28.

$$\tan\alpha\,\tan\beta = \sqrt{\frac{x-y}{x+y}}$$

$$\frac{x-y}{x+y} = \tan^2\alpha\,\tan^2\beta = \left(\frac{1-\cos 2\alpha}{1+\cos 2\alpha}\right)\left(\frac{1-\cos 2\beta}{1+\cos 2\beta}\right)$$

Taking componendo & dividendo

$$\frac{x}{-y}=\frac{(1-\cos 2\alpha-\cos 2\beta+\cos 2\alpha\cos 2\beta)+(1+\cos 2\alpha+\cos 2\beta+\cos 2\alpha\cos 2\beta)}{(1+\cos 2\alpha\ \cos 2\beta-\cos 2\alpha-\cos 2\beta)-(1+\cos 2\alpha+\cos 2\beta+\cos 2\alpha\cos 2\beta)}$$

$$=\frac{2(1+\cos 2\alpha\ \cos 2\beta)}{-2(\cos 2\alpha+\cos 2\beta)}$$

$$\frac{x}{y}=\frac{1+\cos 2\alpha\ \cos 2\beta}{\cos 2\alpha+\cos 2\beta}$$

$$\frac{x-y\cos 2\alpha}{y}=\frac{1+\cos 2\alpha\ \cos 2\beta}{\cos 2\alpha+\cos 2\beta}-\cos 2\alpha$$

$$=\frac{1-\cos^2 2\alpha}{\cos 2\alpha+\cos 2\beta}=\frac{\sin^2 2\alpha}{\cos 2\alpha+\cos 2\beta}$$

$$\frac{x-y\cos 2\beta}{y}=\frac{1+\cos 2\alpha\ \cos 2\beta}{\cos 2\alpha+\cos 2\beta}-\cos 2\beta$$

$$=\frac{1-\cos^2 2\beta}{\cos 2\alpha+\cos 2\beta}=\frac{\sin^2 2\beta}{\cos 2\alpha+\cos 2\beta}$$

$$\frac{(x-y\ \cos\alpha)\ (x-y\cos 2\beta)}{y^2}=\frac{\sin^2 2\alpha\ \sin^2 2\beta}{(\cos 2\alpha\ \cos 2\beta)} \qquad \text{......(A)}$$

$$\frac{x}{y}=\frac{1+\cos 2\alpha\cos 2\beta}{\cos 2\alpha+\cos 2\beta}$$

$$\frac{x^2}{y^2}=\left(\frac{1+\cos 2\alpha\ \cos 2\beta}{\cos 2\alpha+\cos 2\beta}\right)^2$$

$$\frac{x^2-y^2}{y^2}=\left(\frac{1+\cos 2\alpha\ \cos 2\beta}{\cos 2\alpha+\cos 2\beta}\right)^2-1$$

$$=\frac{(1+\cos 2\alpha\ \cos 2\beta-\cos 2\alpha-\cos 2\beta)\ (1+\cos 2\alpha\ \cos 2\beta+\cos 2\alpha+\cos 2\beta)}{(\cos 2\alpha+\cos 2\beta)^2}$$

$$=\frac{(1-\cos 2\alpha)\ (1-\cos 2\beta)\ (1+\cos 2\alpha)\ (1+\cos 2\beta)}{(\cos\alpha+\cos 2\beta)^2}$$

$$= \frac{\left(1-\cos^2 2\alpha\right)\left(1-\cos^2 2\beta\right)}{\left(\cos 2\alpha + \cos 2\beta\right)^2}$$

$$= \frac{\sin^2 2\alpha \sin^2 2\beta}{\left(\cos 2\alpha + \cos 2\beta\right)^2} \qquad \ldots\ldots(B)$$

$$\therefore \frac{\left(x-y\cos 2\alpha\right)\left(x-y\cos 2\beta\right)}{y^2} = \frac{x^2-y^2}{y^2}$$

$$\therefore \left(x-y\cos 2\alpha\right)\left(x-y\cos 2\beta\right) = x^2-y^2$$

29.

$$a^2+b^2+2ab\cos\theta = 1 \Rightarrow a^2+2ab\cos\theta = 1-b^2$$

$$\Rightarrow a^2+2ab\cos\theta + b^2\cos^2\theta = 1-b^2+b^2\cos^2\theta$$

$$\Rightarrow \left(a+b\cos\theta\right)^2 = 1-b\sin^2\theta$$

Similarly $c^2+d^2+2cd\cos\theta = 1 \Rightarrow \left(c+d\cos\theta\right)^2 = 1-d\sin^2\theta$

$$\Rightarrow \left(a+b\cos\theta\right)\left(c+d\cos\theta\right) = ac + bd\cos^2\theta + \left(ad+bc\right)\cos\theta$$

$$\Rightarrow \left(ad+bc\right)\cos\theta = -\left(ac+bd\right)$$

$$\therefore \left(a+b\cos\theta\right)\left(c+d\cos\theta\right) = ac + bd\cos^2\theta - \left(ac+bd\right) = -bd\sin^2\theta$$

$$\therefore \left(a+b\cos\theta\right)^2\left(c+d\cos\theta\right)^2 = b^2d^2\sin^4\theta$$

or $\left(1-b^2\sin^2\theta\right)\left(1-d^2\sin^2\theta\right) = b^2d^2\sin^4\theta$

$$\Rightarrow 1-\left(b^2+d^2\right)\sin^2\theta + b^2d^2\sin^4\theta = b^2d^2\sin^4\theta$$

$$\therefore \sin^2\theta = \frac{1}{b^2+d^2}$$

$$\operatorname{cosec}^2\theta = b^2+d^2$$

Now we can similarly do this

$$(a\cos\theta + b)^2 = (1 - a^2\sin^2\theta)$$

$$(d\cos\theta + c)^2 = (1 - c^2\sin^2\theta)$$

and

$$(a\cos\theta + b)(d\cos\theta + c) = -ac\sin^2\theta$$

Hence we will get

$$\operatorname{cosec}^2\theta = a^2 + c^2$$

30.

$$\frac{\sin^4\theta}{a} + \frac{\cos^4\theta}{b} = \frac{1}{a+b}$$

$$b(a+b)\sin^4\theta + a(a+b)(1-\sin^2\theta)^2 = ab$$

$$b(a+b)\sin^4\theta + a(a+b)(\sin^4\theta + 1 - 2\sin^2\theta) = ab$$

$$[b(a+b) + a(a+b)]\sin^4\theta + a(a+b) - 2a(a+b)\sin^2\theta = ab$$

$$(a+b)^2\sin^4\theta - 2a(a+b)\sin^2\theta + a^2 = 0$$

$$((a+b)\sin^2\theta - a)^2 = 0$$

$$\sin^2\theta = \frac{a}{a+b} \quad \therefore \cos^2\theta = \frac{b}{a+b}$$

$$\frac{\sin^8\theta}{a^3} + \frac{\cos^8\theta}{b^3} = \frac{\frac{a^4}{(a+b)^4}}{a^3} + \frac{\frac{b^4}{(a+b)^4}}{b^3}$$

$$= \frac{a+b}{(a+b)^4} = \frac{1}{(a+b)^3}$$

31.

$A+B+C+D = (2n+1)\pi$ gives $A+B = (2n+1)\pi - (C+D)$