Exercise-1

(Unit & Dimenssions & Kine I & II)

① radian is a unit of angle.

② Impulse = $F.t = \frac{dP}{dt}.dt = dp$ = linear momentum

③ impulse and l.m. has same dimensions.

$E = \frac{hc}{d}$ and a.m. = mvr

$I = MR^2$ and $Y = \frac{F}{A} \frac{L}{\Delta l}$

$P = F/A$

④ $F = 6\pi r \eta v$

$\eta = \frac{F}{6\pi r v}$ $\therefore [\eta] = ML^{-1}T^{-1}$

⑤ Surface tension $= \frac{F}{L}$ $[T] = ML^{0}T^{-2}$

⑥ $F = \frac{G m_1 m_2}{r^2}$ SI unit of G is $= \frac{Nm^2}{kg^2}$

⑦ $H = \frac{KA\Delta\theta}{L}$ $\Rightarrow$ $K = \frac{HL}{A\Delta\theta} = MLT^{-3}K^{-1}$

$\frac{\text{energy}}{\text{time}} = \frac{KA\Delta\theta}{L}$

⑧ Energy $= \sigma T^4$

$[\sigma]$ = stefans const $= \frac{[ML^2T^{-2}]}{[K^4]} = J\,m^{-2}s^{-1}K^{-4}$

⑨ magnetic permeability $= \frac{H}{m} = Hm^{-1}$

(10) $1/R$ and RC $\tau = RC$ = time con. const.

(11) entropy $= S = \frac{Q}{T} = \frac{\text{Heat}}{\text{temp}} = ML^2T^{-2}K$

(12) ML^2T^{-2} = kinetic Energy

(13) $ML^{-1}T^{-2} = F/A$ = stress

(14) Angular momentum = $mvR = ML^2T^{-1}$

(15) $F = \frac{Gm_1m_2}{r^2}$

$[G] = M^{-1}L^3T^{-2}$

(16) $\left(P + \frac{a}{v^2}\right)(v-b) = CT$ unit of Pressure = unit of (a/v^2)

$\therefore [a] = [P][v^2] = ML^5T^{-2}$

(17) $PV = nRT$ $[R] = \frac{[PV]}{[nT]} = \frac{[P][V]}{[n][T]} = ML^2T^{-2}K^{-1}mol^{-1}$

(18) SI unit of the universal gas constant R is Joule $K^{-1}mol^{-1}$

(19) $E = h\nu$ $[h] = \frac{[E]}{[\nu]} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$

(20) SI unit unit of Planck's Constant

h = Joule second.

(21) dimensions of Planck's Constant are the same as those of angular momentum $= mvR = [ML^2T^{-1}]$

(22) $T = 2\pi\sqrt{l/g}$ $\Rightarrow T \propto l^{1/2} \cdot g^{-1/2} \cdot m^0$

$\therefore a = 0$ and $l = 1/2$ and $g = -1/2$

(23) $V \propto A^{\alpha} u^{\beta} t^{\gamma}$ where V - volume of water

$t \rightarrow$ seconds

$[V] = [M^0 L^3 T^0]$ $A \rightarrow$ Area

$[A] = [M^0 L^2 T^0]$ $u \rightarrow$ velocity of u

$[u] = [M^0 L^1 T^{-1}]$ $\therefore \alpha \mp \beta = \gamma$

$[t] = [M^0 L^0 T^{-1}]$

(24) $h = \dfrac{2T\cos\theta}{r\rho g}$ $\therefore [h] = \dfrac{[T]}{[r\rho g]}$

(25) $n = \dfrac{p}{2l}\sqrt{\dfrac{F}{m}}$ $[n] = [T^{-1}]$; $[l] = [L]$

$[F] = [ML^1T^{-2}]$;

$\therefore [m] = [ML^{-1}T^0]$

(26) $y = a\sin(bt - cx)$

all mathematical expression are dimensionless.

$\therefore [b] = \dfrac{1}{T}$ and $[c] = \dfrac{1}{x}$

(27) $[b/c] = [LT^{-1}]$ ∴ = velocity.

(28) $\left[P + \frac{a}{v^2}\right](v-b) = nRT$

dimension of $[a/v^2] = [P]$

$\therefore [a] = [Pv^2]$

(29) dimension of $[b] = [v]$

(30) $[nRT] = [Pv]$ = energy.

(31) $[ab] = [Pv^2][v] = [Pv^3] = ML^8T^{-2}$

(32) $Y = \frac{F}{A}\frac{L}{\Delta l}$ ∴ $[Y] = FA^2V^{-4}$

(33) $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

$[\varepsilon_0] = M^{-1}L^{-3}T^4A^2$

(34) Permeability (μ_0) $\frac{1}{\mu_0\varepsilon_0} = c^2 \Rightarrow [\mu_0] = \left[\frac{1}{\varepsilon_0 c^2}\right]$

$= MLT^{-2}A^{-2}$

(35) $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ = speed of light = velocity.

(36) Heat = $ms\Delta T$

$[s] = M^0L^2T^{-2}K^{-1}$

(37) Heat = mL = Energy

L = latent heat

$\therefore [L] = M^0 L^2 T^{-2}$

(38) Boltzman's constant $= \frac{\text{Energy}}{\text{temp}} = \frac{ML^2T^{-2}}{K} = ML^2T^{-2}K^{-1}$

(39) $V = IR \Rightarrow [V] = [I][R] = \frac{[E]}{d}$ Electric field

$[V] = ML^2T^{-3}A^{-1}$

(40) $[R] = \frac{[V]}{[I]} = \frac{ML^2T^{-3}A^{-1}}{A} = ML^2T^{-3}A^{-1}$

(41) electric field $= \frac{F}{q} = \frac{[F]}{[q]} = \frac{[F]}{[AT^{-1}]}$

$= MLT^{-3}A^{-1}$

(42) magnetic induction field

$= \frac{\phi}{A} = \frac{\text{magnetic flux}}{\text{Area}} = \frac{(\text{Potential})(\text{time})}{\text{Area}} = MT^{-2}A^{-1}$

(43) magnetic flux $= \frac{ML^2T^{-2}}{I} = ML^2T^{-2}I^{-1}$ $T\text{-}m^2$

(44) $V = L\frac{dI}{dt} \Rightarrow [L] = \frac{\text{Potential}}{dI/dt}$

Self Inductance

$= ML^2T^{-2}I^{-2}$

(45) $Q = CV$

$[Q] = [C][V] \Rightarrow [C] = \frac{[Q]}{[V]} = \frac{\varepsilon_0 A}{d}$

$[C] = M^{-1}L^{-2}T^{4}A^{2}$

(46) $\text{mass} = [v]^a [F]^b [E]^c$

$[M^1 L^0 T^0] = [M^0 L^1 T^{-1}]^a [M L^1 T^{-2}]^b [M L^2 T^{-2}]^c$

$[M^1 L^0 T^0] = [M^{b+c} L^{a+b+2c} T^{-a-2b-2c}]$

$\therefore \left.\begin{aligned} b+c &= 1 \\ a+b+2c &= 0 \\ -a-2b-2c &= 0 \end{aligned}\right\} \quad \begin{aligned} b &= 0 \\ c &= 1 \\ a &= -2 \end{aligned}$

(47) frequency$(n) \propto l^a \rho^b Y^c$

$[T^{-1}] = [L]^a [M L^{-3} T^0]^b [M L^{-1} T^{-2}]^c$

$b + c = 0$

$a - 3b - c = 0$

$-2c = -1 \Rightarrow c = 1/2$

$\therefore \ n \propto (Y)^{1/2}$

(48) $\frac{1}{2}\varepsilon_0 E^2$ = Energy density = $\frac{[E]}{[L^3]} = ML^{-1}T^{-2}$

(49) [Angular momentum] = $[ML^2T^{-1}]$

$[E/v] = ML^{-1}T^{-2}$

$[F/A] = ML^{-1}T^{-2}$

(64) magnitude of a Calorie

$$= \frac{4.2\,c^2}{a\,b^2}$$

(65) $v = \frac{4}{3}\pi r^3$

$$\therefore \frac{\Delta v}{v} = 3\frac{\Delta r}{r} \Rightarrow \frac{\Delta v}{v} = 3 \times (1) = 3\%.$$

(66) $$\frac{\Delta v}{v} = 6\%. = 3\frac{\Delta r}{r} \Rightarrow \frac{\Delta r}{r} = 2\%.$$

$$\therefore \frac{\Delta A}{A} = 2\frac{\Delta r}{r} = 4\%.$$

(67) $X = (m^x L^y T^{-z})$

$$\frac{\Delta X}{X} = x\frac{\Delta m}{m} + y\frac{\Delta L}{L} + z\frac{\Delta T}{T}$$

$$= (ax + by + cz)\%.$$

(68) $T = 2\pi\sqrt{l/g}$

$$T^2 = 4\pi^2\frac{l}{g} \Rightarrow g = \frac{4\pi^2 l}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T} = 2 + 6 = 8\%.$$

(69) $[M L^2 T^0] \rightarrow$ moment of inertia

$$n_1[M_1 L_1^2] = n_2[M_2 L_2^2]$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2 = \frac{6}{10 \times 10^{-3} \times (5 \times 10^{-2})^2} = \underline{2.4 \times 10^{5}}$$

(70) $X = \varepsilon_0 L \frac{\Delta V}{\Delta t}$

$[\varepsilon_0] = M^{-1}L^{-3}T^{4}A^{2}$

$[L] = [l]$

$\therefore X = [A] = \text{current}.$

$[\Delta V] = \frac{\text{Energy}}{\text{charge}} = ML^{2}T^{-3}A^{-1}$

$[\Delta t] = [T]$

(71) $\eta = \frac{\pi}{8} \frac{R^4}{l} \frac{P}{Q}$

$\frac{\Delta \eta}{\eta} = 4\frac{\Delta R}{R} + \frac{\Delta l}{l} + \frac{\Delta P}{P} + \frac{\Delta Q}{Q}$

$\therefore$ R must be measured most accurately.

(72) $[M^1 T^0 L^0] = [LT^{-1}]^a [ML^{-3}]^b [LT^{-2}]^c$

$b = 1$; $0 = a - 3b + c$, $0 = -a - 2c$ $\Big\}$ $a + c = 3$, $-a - 2c = 0$

$-c = 3$

$c = -3$ | $a = +6$

$\therefore m \propto v^6$

(73) Speed $= [M^0 L^1 T^{-1}] = [M^0 L^1 T^0]^a [ML^{-3}T^0]^b [ML^0T^{-2}]^c$
$\qquad d \qquad \rho \qquad T$

$0 = b + c$ — (1)

$1 = a - 3b$ — (2)

$-1 = -2c$ — (3)

$a = -1/2$

$\therefore v \propto \frac{1}{\sqrt{d}}$

(74) Time period $T = [r]^a [M]^b [G]^c$

$\therefore a = 3/2 \quad \therefore T \propto r^{3/2}$

(75) $[\text{Mass}] = [E]^a [P]^b [F]^c$

$[M^1 T^0 L^0] = [ML^2T^{-2}]^a [MLT^{-1}]^b [MLT^{-2}]^c$

$$\left.\begin{aligned} 1 &= a+b+c \\ 0 &= 2a+b+c \\ 0 &= -2a-b-2c \end{aligned}\right\} \quad \begin{aligned} a &= -1 \\ b &= 2 \\ c &= 0 \end{aligned}$$

(76) $[\text{time}] = [T] = [c]^a [G]^b [h]^c$

$[M^0 L^0 T^1] = [LT^{-1}]^a [M^{-1}L^3T^{-2}]^b [ML^2T^{-1}]^c$

$$\left.\begin{aligned} 0 &= -b+c \\ 0 &= a+3b+2c \\ 1 &= -a-2b-c \end{aligned}\right\} \quad \begin{aligned} a &= -5/2 \\ b &= 1/2 \\ c &= 1/2 \end{aligned}$$

(77) $A = A_0 e^{(-at/m)}$ → mass m and time t

$[at/m] = [M^0 L^0 T^0]$

$\therefore [a] = [M^{+1} T^{-1}]$

(78) $g = \frac{4\pi^2 L}{T^2}$ $\quad \therefore \quad \frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$

for Δg min ΔT should be min.

(79) $Y = \frac{F}{A}\frac{L}{l} = \frac{(mg)}{\pi r^2} \cdot \frac{(2\,m)}{0.8\,mm} = 2 \times 10^{11}\ N/m^2$ ← 1kg

$$\frac{\Delta Y}{Y} = \underbrace{\cancel{\frac{\Delta F}{F}}^{0} + \cancel{\frac{\Delta L}{L}}^{0}}_{\text{Const.}} + \frac{\Delta A}{A} + \frac{\Delta l}{l}$$

$$\therefore \frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \frac{\Delta l}{l} = \frac{2\Delta r}{r} + \frac{\Delta l}{l}$$

$$= \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

$$\therefore \Delta Y = \left(\frac{2 \times 0.05}{0.8} + \frac{0.01}{0.4} \right) \times 2 \times 10^{11} \simeq 0.2 \times 10^{11} \, N/m^2$$

(8) main scale division = x cm.

no. of division of vernier scale = n coincide with $(n-1)$ divisions of main scale.

$$L.C = x/n$$

———— a ————

Exercise - II

(1) The dimensional method can not be used to obtain dependence of

(a) the height to which a liquid rises in a capillary tube on the angle of contact.

(b) height to which a body, projected upwards with a certain velocity, will rise on time t.

(c) the decrease in energy of a damped oscillator on time t.

(2) In dimensional method, the dimensionless proportionally constant is to be determined

(a) experimentally

(b) by a detailed mathematical derivation.

(3) mass of bee = 0.000087 kg

mass of flower = 0.0123 kg

$\therefore$ total mass = 0.0124 kg

(4) $r = 0.021$ cm

$\pi = 3.142$

Area $= \pi r^2$

$= 3.142 \times (0.021)^2$

$= 0.0014$ cm^2

(5) $t = 10.3$ s

$d = 100.5$ m

$\therefore$ average speed $= \frac{100.2}{10.3} = 9.76$ m/s window to JEE Main

(1) Significant figure in

$23.023 = 5$

$0.0003 = 1$

$2.1 \times 10^{-3} = 2$

(2) Here x = half degree

$n = 30'$

$\therefore L.C. = \frac{x}{n} = \frac{1}{2 \times 30} = \frac{1}{60} = 1$ min

(3) Magnetic field $= B = \frac{E}{c}$ ← electric field

$[B] = [MT^{-1}C^{-1}]$

(4) $m = 3.513$ kg ; $v = 5$ m/s

$P = mv = 3.513 \times 5 \approx 17.6$ kg/m

(5) $[ML^2/Q^2]$ = henry (H)

(6) 'rad' is the correct unit used for biological effect of radiation.

(7) Moment of inertia $= I = ML^2$

moment of force $= \tau = FR$

(8) $F = 6\pi r \eta v \Rightarrow [\eta] = \frac{[F]}{[rv]} = ML^{-1}T^{-1}$

(9) $\frac{1}{\sqrt{\mu_0 \epsilon_0}} = c \Rightarrow \frac{1}{\mu_0 \epsilon_0} = c^2 \rightarrow [L^2T^{-2}]$

(10) momentum $= mv = ML^1T^{-1}$

$h = ML^2T^{-1}$

(11) τorque = Force x distance = work

$[ML^2T^{-2}]$

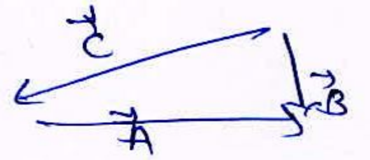
———^———

Vector

Ex-1

(1) unit vector $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{5\hat{i} - 12\hat{j}}{13}$

(2)



$\vec{A} + \vec{C} = \vec{B}$

(3) $F_1 = 3$ and $F_2 = 4$

$\therefore \quad |F_1 - F_2| \le R \le |F_1 + F_2|$

$1 \le R \le 7$

(4) $F_1 = 6N$ and $F_2 = 8\ N$

$|F_1 - F_2| \le R \le F_1 + F_2$

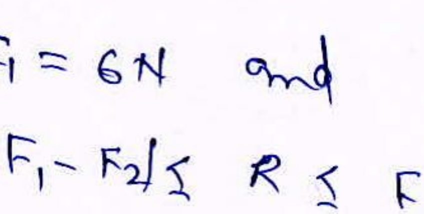
$2 \le R \le 16$

(5) $\vec{A} + \vec{B} = \vec{C}$ $\quad \therefore \quad \vec{A} = 1$

$\vec{B} = 1$

$\vec{C} = 2$

(6)



$\therefore$ displacement

$= \sqrt{8^2 + 24^2}$

$= 16\sqrt{2}$ m Ans

(7) θ angle b/w a and b is acute angle,

$(\vec{a}-\vec{b}) = \sqrt{a^2+b^2-2ab\cos\theta}$

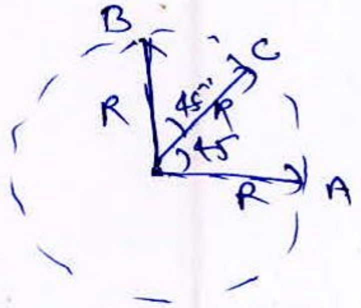
(8) $(\vec{p}+\vec{q})$ and $(\vec{p}-\vec{q})$

$$\Rightarrow \cos\theta = \frac{(\vec{p}+\vec{q})\cdot(\vec{p}-\vec{q})}{(p^2+q^2)^{1/2}\sqrt{(\vec{p}-\vec{q})}} = \frac{\vec{p}^2 + \cancel{\vec{q}\cdot\vec{p}} - \cancel{\vec{p}\cdot\vec{q}} - q^2}{(p^2+q^2)}$$

$$= \frac{p^2-q^2}{p^2+q^2} = \left(\frac{p^2-q^2}{p^2-q^2}\right)$$

$\therefore \theta = 0$ or $180'$ $\qquad = 1$ or -1

(9)



Resultant of A and B $= \sqrt{2}R$ along the $\vec{OC}$

$\therefore$ net sum $= \vec{OC} + \sqrt{2}R$

$= (\sqrt{2}+1)R$

(10) $\vec{p}$ and $\vec{q}$ are $\perp^r$ to each other then

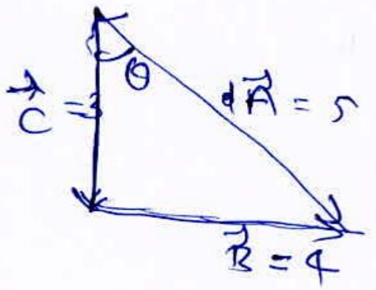
$\vec{p}\cdot\vec{q} = 0$

$\Rightarrow (2\hat{i}+b\hat{j}+2\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k}) = 0$

$\Rightarrow 2+b+2 = 0 \Rightarrow b = -4$

(11) A vector is not changed if it is a slide parallel to itself.

(12) $\vec{A} = \vec{B} + \vec{C}$ and $|\vec{A}| = 5$; $|\vec{B}| = 4$ and $|\vec{C}| = c$



$\cos\theta = 3/5$

$\theta = \cos^{-1}(3/5)$

(13) $(\vec{A} + \vec{B}) = (\vec{A} - \vec{B})$ multiply by $(\vec{A} - \vec{B})$

$\Rightarrow (\vec{A} - \vec{B})(\vec{A} + \vec{B}) = (\vec{A} - \vec{B})(\vec{A} - \vec{B})$

$\Rightarrow \cancel{A^2} + \vec{A}\cdot\vec{B} - \vec{B}\cdot\vec{A} - B^2 = \cancel{A^2} - \vec{A}\cdot\vec{B} - \vec{B}\cdot\vec{A} + \vec{B}^2$

$\Rightarrow \vec{B}^2 - 2\vec{A}\cdot\vec{B} = 0$

$|\vec{B}|^2 = 2\vec{A}\cdot\vec{B} \quad \therefore \vec{B} = 0$

(14) two Forces F have same magnitude Resultant $= F$

$F = \sqrt{F^2 + F^2 + 2F^2\cos\theta}$

$\Rightarrow \cos\theta = -1/2$ and $\theta = 120°$

(15) $\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$
$\vec{B} = \hat{i} - 3\hat{j} - 5\hat{k}$
$\vec{C} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$\vec{A} + \vec{B} = \vec{C}$ $(\vec{B}\cdot\vec{C} = 0)$

right angled triangle

(16) $\vec{A}, \vec{B}, \vec{C}$ vectors

$(\vec{A}\times\vec{B})\cdot\vec{C} \rightarrow$ scaler quantity

but $\vec{A}\times(\vec{B}\cdot\vec{C}) \longrightarrow$ not define.

(17) $|\vec{A}| = |\vec{B}|$ and $\theta = 90'$

$\therefore (\vec{A} + \vec{B})$ $\perp^{r}$ $\vec{A} \times \vec{B}$

$\perp^{r}$ $(\vec{A} - \vec{B})$

$\perp^{r}$ $3(\vec{A} - \vec{B})$

(18) $\vec{A} \times \vec{B}$ $\perp^{r}$ to $\vec{A}$ and $\vec{B}$.

$\therefore \vec{A} \cdot (\vec{B} \times \vec{A}) = 0$

(19) $|F_1| + |F_2| = 18N$

$|\vec{F_1} + \vec{F_2}| = 12$

$\therefore F_1 = 13\ N$

and $F_2 = 5N$

$\tan\alpha = \infty$ $\therefore \alpha = 90^{\circ} \Rightarrow \therefore$ $F_1 + F_2 \cos\theta = 0$

$\cos\theta = -F_1/F_2$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

(20) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} = \vec{B} \times \vec{A}$

$\Rightarrow \vec{B} \times \vec{A} = 0$ $\therefore \theta = 180^{\circ}$

(21) $\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$ and $|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}| \sin\theta$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = 45^{\circ}.$

(22) $|\vec{R}| = \sqrt{(\vec{A}+\vec{B})^2 + (A-B)^2 + 2(\vec{A}+\vec{B})(\vec{A}-\vec{B})\cos\theta}$

$\theta = 0'$ or $180'$ angle b/w $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$

$\therefore |\vec{R}| = 2|\vec{A}|$

(23)

$$\cos\theta = \frac{\hat{i} \cdot (\hat{i}+\hat{j}+\hat{k})}{\sqrt{1}\ \sqrt{3}} = 1/\sqrt{3}$$

$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(24) for unit vector

$\sqrt{0.2^2 + 0.6^2 + a^2} = 1$

$\Rightarrow a^2 = 1 - 0.04 - 0.36 = 0.6$

$\boxed{a = \sqrt{.6}}$

(25) $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$

Comp. of $\vec{A}$ along vector $\vec{B}$

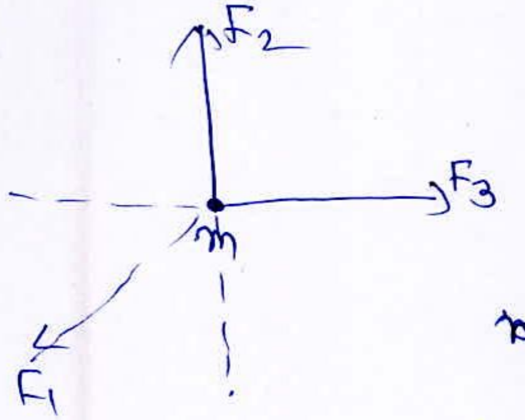
$= |\vec{A}|\ \hat{B}\cos\theta = (|\vec{A}|\cos\theta)\frac{\vec{B}}{|\vec{B}|}$

$= \left(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right)|\vec{A}|\ \left(\vec{B}/|\vec{B}|\right)$

$= \frac{5}{\sqrt{2}}(\hat{i}+\hat{j})$

window to JEE Main

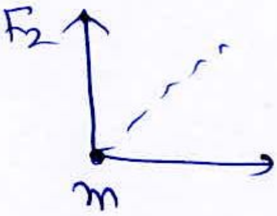
(1)



$F_1 = \sqrt{F_2^2 + F_3^2}$

for remains stationary.

after F_1 removed



$a = \dfrac{\sqrt{F_2^2 + F_3^2}}{m} = \dfrac{F_1}{m}$

(2) $|\vec{F_1}| + |\vec{F_2}| = 18$ and $|\vec{F_1} + \vec{F_2}| = 12$

$\alpha = 90^\circ$ with small force.

$\therefore \cos\theta = -F_1/F_2$

$\therefore F_1 = 5$ and $F_2 = 13$

(3) $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

$\Rightarrow -\vec{B} \times \vec{A} = \vec{B} \times \vec{A} \Rightarrow \vec{B} \times \vec{A} = 0$

$\therefore \sin\theta = 0$

$\therefore \boxed{\theta = \pi \text{ or } 180^\circ}$