

# VECTOR SOLUTIONS

## LEVEL - 1

1. Conceptual

2. Conceptual

3. Magnitude = AB where A(4, -1, 0), B(-2, -2, 0)

$$AB = \sqrt{(-2-4)^2 + (-2+4)^2 + (0-0)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

4. A is unit vector so its magnitude is 1.

$$\Rightarrow \sqrt{(0.4)^2 + (0.3)^2 + c^2} = 1 \Rightarrow 0.16 + 0.09 + c^2 = 1$$

$$\Rightarrow c^2 = 0.75 \Rightarrow c = \sqrt{0.75}$$

5. Conceptual

6.  $PQ = (3-2)\hat{i} + (4-3)\hat{j} + (5-5)\hat{k} = \hat{i} + \hat{j}$

7.  $|A| = \sqrt{2^2 + 4^2 + 5^2} = \sqrt{45}$

$\Rightarrow$  direction cosines  $l, m, n$  are  $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, \frac{-5}{\sqrt{45}}$

8.  $\frac{2}{-4} = \frac{3}{-6} = \frac{-1}{-\lambda} \Rightarrow \lambda = -2$

9.  $\overline{AB} = (4-3)\hat{i} + (5-4)\hat{j} + (6-5)\hat{k} = \hat{i} + \hat{j} + \hat{k}$

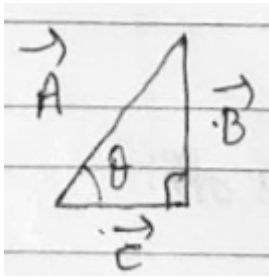
$$\overline{CD} = (4-7)\hat{i} + (6-9)\hat{j} + (0-3)\hat{k} = -3\hat{i} - 3\hat{j} - 3\hat{k} = -3\overline{AB}$$

So, AB and CD are anti parallel

10.  $m = \frac{|\vec{F}|}{|\vec{a}|} = \frac{\sqrt{6^2 + (-8)^2 + 10^2}}{1} = 10\sqrt{2}\text{kg}$

11.  $|\vec{A}| - |\vec{B}| \leq |\vec{A} + \vec{B}| \leq |\vec{A}| + |\vec{B}|$

12.



$$\cos \theta = \frac{C}{A} = \frac{3}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{5}\right)$$

13. Conceptual

$$14. \quad F\sqrt{10} = \sqrt{(2F)^2 + (\sqrt{2}F)^2 + 2(2F)(\sqrt{2}F)\cos \theta}$$

$$\Rightarrow F\sqrt{10} = F\sqrt{4 + 2 + 4\sqrt{2}\cos \theta} \Rightarrow 4\sqrt{2}\cos \theta = 4 \Rightarrow \theta = 45^\circ$$

15. resultant of  $\vec{a} + \vec{b}$

16. Conceptual

17. Conceptual

$$18. \quad \text{Let } \vec{A} + \vec{B} = \vec{C} = 3\hat{i} + 7\hat{j} + \hat{k} \Rightarrow \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{3\hat{i} + 7\hat{j} + \hat{k}}{\sqrt{59}}$$

$$19. \quad \vec{A} + (\hat{i} - 3\hat{j} + 2\hat{k}) + (3\hat{i} + 6\hat{j} - 7\hat{k}) = \vec{A} + 4\hat{i} + 3\hat{j} - 5\hat{k} = \hat{j} \Rightarrow \vec{A} = -4\hat{i} - 2\hat{j} + 5\hat{k}$$

$$20. \quad \text{displacement} = \sqrt{10^2 + 12^2 + 14^2} \approx 21$$

21. Conceptual

22. Conceptual

23. Conceptual

24. Conceptual

$$25. \quad \Delta \vec{P} = m\vec{v}_2 - m\vec{v}_1 = [(v \sin \theta \hat{i} + v \cos \theta \hat{j}) - (v \sin \theta \hat{i} - v \cos \theta \hat{j})]$$

$$\Rightarrow \Delta \vec{P} = 2mv \cos \theta \hat{j}$$

$$\Rightarrow |\Delta \vec{P}| = 2mv \cos \theta$$

26. Conceptual

27. Conceptual

$$28. \quad |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cos \theta} = |\vec{A}| = |\vec{B}|$$

$$\Rightarrow A^2 = A^2 + A^2 - 2A^2 \cos \theta \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

29. Conceptual

$$30. \quad \vec{A} \cdot \vec{C} = 0$$

31. Conceptual

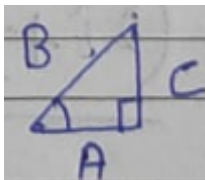
32. Let forces are F and 2F

$$\Rightarrow (2F)^2 = F^2 + (2F)^2 + 2F(2F) \cos \theta \Rightarrow \cos \theta = \frac{-1}{4}$$

33. Conceptual

34. Conceptual

$$35. \quad \vec{A} \perp \vec{B} \text{ and } \vec{A} \perp \vec{C} \text{ so } \vec{A} \perp (\vec{B} \times \vec{C})$$



37.

$$38. \quad w = \vec{F} \cdot \vec{S} = F \cos \theta = 50 \times 10 \times \cos 60 = 250J$$

$$39. \quad (\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0 \Rightarrow A^2 - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - B^2 = 0 \Rightarrow A^2 = B^2$$

$$40. \quad w = \vec{F} \cdot (10\hat{j}) = (-2\hat{i} + 15\hat{j} + 6\hat{k}) \cdot (10\hat{j}) = 150J$$

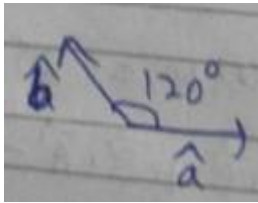
41. Conceptual

$$42. \quad (\hat{i} A \cos \theta + \hat{j} A \sin \theta) \cdot (\hat{i} B \sin \theta - \hat{j} B \cos \theta) = 0$$

43. Projection of  $\vec{A}$  on  $\vec{B} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = \frac{-2+9-4}{\sqrt{1^2+3^2+4^2}} = \frac{3}{\sqrt{26}}$
44.  $(4\hat{i} - 3\hat{j}) \cdot (0.6\hat{i} + 0.8\hat{j}) = 0$
45. Conceptual
46. Conceptual
47. Conceptual
48. Conceptual
49. Conceptual
50.  $\vec{C}$  is parallel to  $\vec{A} \times \vec{B}$

### LEVEL - 2

1. Conceptual
2. Conceptual
3. Conceptual
4. Conceptual
5. Conceptual
6. Conceptual
7. Conceptual
8. Resultant of  $\vec{OA}$  and  $\vec{OC}$  is  $R\sqrt{2}$  along  $\vec{OB}$   
 So final resultant =  $R\sqrt{2} + R = R(\sqrt{2} + 1)$
9.  $\vec{a} + \vec{b} = \vec{c} \Rightarrow a^2 + b^2 + 2ab\cos\theta = c^2 = a^2 + b^2$   
 $\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$
10. Conceptual
11.  $(A\cos\alpha)^2 + (A\cos\beta)^2 + (A\cos\gamma)^2 = A^2$   
 Now  $\alpha = \beta = \gamma \Rightarrow 3A^2\cos^2\alpha = A^2 \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$   
 And  $A\cos\alpha = \frac{A}{\sqrt{3}}$
12. Conceptual
13. component =  $\frac{(2\hat{i} + 3\hat{j}) \cdot (\hat{i} + \hat{j})}{|\hat{i} + \hat{j}|} = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$



14.

$$|\hat{a} - \hat{b}| = \sqrt{3}$$

15.  $\frac{P+Q}{P-Q} = \frac{3}{1} \Rightarrow 3P - 3Q = P + Q \Rightarrow 2P = 4Q \Rightarrow P = 2Q$

16.  $R^2 = 3^2 + 2^2 + 2(3)(2)\cos\theta = 13 + 12\cos\theta$  (i)

$$(2R)^2 = 6^2 + 2^2 + 2(6)(2)\cos\theta = 40 + 24\cos\theta$$

$$\Rightarrow R^2 = 10 + 6\cos\theta$$
 (ii)

From (i) and (ii)

$$13 + 12\cos\theta = 10 + 6\cos\theta$$

$$6\cos\theta = -3 \Rightarrow \cos\theta = \frac{-1}{2} \Rightarrow \theta = 120^\circ$$

17. Conceptual

18.  $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$$

$$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

19. Conceptual

20. Conceptual

21.  $\vec{F} = \frac{dP_x}{dt} \hat{i} + \frac{dP_y}{dt} \hat{j} = -2\sin t \hat{i} + 2\cos t \hat{j}$

$$\vec{F} \cdot \vec{P} = 0$$

22. Conceptual

23.  $\vec{V} = \vec{\omega} \times \vec{r}$

24. Conceptual

25. Conceptual

26. Conceptual

27. Power =  $\vec{F} \cdot \vec{V} = (20\hat{i} + 15\hat{j} - 5\hat{k}) \cdot (6\hat{i} - 4\hat{j} + 3\hat{k}) = 45$  unit

28. Conceptual
29. Conceptual
30. Conceptual
31. Conceptual
32.  $\text{Area} = |\vec{d}_1 \times \vec{d}_2|$
33. Conceptual
34. Conceptual
35. Conceptual
36. Conceptual
37.  $\text{Area} = |\vec{d}_1 \times \vec{d}_2|$