WAVE OPTICS SOLUTIONS

EXERCISE - I (single choice)

1. Radius of secondary wavelets in denser medium is less than that in rarer medium hence (A)

2. frequency depends on source and hence remains unchanged. (C)

3. All wavelengths interfere constructively at centre (D) for white light whereas for monochromatic light all bright fringes look alike.

4. Position of maxima on screen $y = \frac{nD\lambda}{d}$

   hence $n_1\lambda_R = n_2\lambda_B$, $\frac{\lambda_R}{\lambda_B} = \frac{2}{3}$ hence (B)

5. lower orders of shorter wavelength interfere constructively next to central bright fringe (C)

6. fringe width is $\frac{D\lambda}{d}$, de broglie wavelength of electrons decreases with increasing voltage (B)

7. Resultant intensity

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

   for coherent sources with $I_1 = I_2 = I_0$; $I_{max} = 4I_0$

   for incoherent sources $\langle \cos\phi \rangle = 0$, $I = 2I_0$

   hence (B)

8. If I_0 is the intensity of each wave then for interference of two such waves $I_{max} = 4I_0$ (C)

9. for first minima

$$d\sin\theta = \frac{\lambda}{2}$$

$$d\left(\frac{\pi\theta^\circ}{180}\right) = \frac{\lambda}{2} \quad \text{hence } d = \frac{180\lambda}{2\pi\theta^\circ} \quad \text{(A)}$$

10. Resultant intensity

$$I = I_0 \cos^2 \phi/2$$

$$\frac{3I_0}{4} = I_0 \cos^2\phi/2 \; ; \; \cos\phi/2 = \pm\frac{\sqrt{3}}{2}$$

$$\frac{\phi}{2} = n\pi \pm \frac{\pi}{6} \quad (n = 0, 1, 2 \ldots)$$

$$\boxed{\phi = 2n\pi \pm \frac{\pi}{3}}$$

$$\phi = \frac{2\pi}{\lambda}(d\sin\theta) \quad \text{where } d\sin\theta = \frac{yd}{D}$$

$$y = \frac{D\lambda}{d}\left(\frac{\phi}{2\pi}\right)$$ gives positions on screen where waves interfere with phase difference ϕ.

for $\phi = 2n\pi \pm \frac{\pi}{3}$

$$y = \frac{D\lambda}{d}\left(\frac{2n \pm \frac{1}{3}}{2}\right)$$ gives positions where intensity is $\frac{3}{4}I_0$. for minimum separation choose $n = 0$ or 1 or 2 ....

$$\Delta y = \frac{D\lambda}{3d} \quad \text{(D)}$$

11. Resultant intensity $I = I_0 \cos^2 \phi/2$

for $I = \frac{I_0}{4}$ i.e intensity due to single slit

$\cos \phi/2 = \pm 1/2$

$\frac{\phi}{2} = \left(n\pi \pm \frac{\pi}{3}\right)$ or $\phi = \left(2n\pi \pm \frac{2\pi}{3}\right)$

Positions on screen where phase difference of interfering waves is ϕ are

$$\boxed{y = \frac{D\lambda}{d}\left(\frac{\phi}{2\pi}\right)}$$

In the above case $y = \frac{D\lambda}{d} \frac{(2n \pm 2/3)}{2}$

Nearest to centre $\Rightarrow n = 0$ i.e $y = \pm \frac{D\lambda}{3d}$ (c)

12. If n_1^{th} bright order of λ_1 coincides with n_2^{th} bright order of λ_2 then

$n_1 \lambda_1 = n_2 \lambda_2$ $\quad \frac{n_1}{n_2} = \frac{4}{5}$

Position of n^{th} order maxima on screen $y = \frac{n_1 D\lambda_1}{d}$ or $\frac{n_2 D\lambda_2}{d}$

for least distance from centre $n_1 = 4$ or $n_2 = 5$

(13) Fringe width

$$\beta = \frac{D\lambda}{d}$$

$$\Delta\beta = \frac{(\Delta D)\lambda}{d} \qquad \boxed{\lambda = \frac{(\Delta\beta)\, d}{\Delta D}}$$

(14) Phase difference on Screen $\phi = \frac{2\pi}{\lambda} d\sin\theta$

$$\phi = \frac{2\pi}{\lambda}\left(\frac{dy}{D}\right)$$

for $y = \frac{1}{4}\frac{D\lambda}{d}$ $\quad \phi = \pi/2$

~~hence~~ $I = I_0 \cos^2 \phi/2$ hence $\frac{I_0}{I} = 2$ (A)

(15) Phase difference of interfering waves at P

$\phi = \frac{2\pi}{\lambda}\left(\frac{dy}{D}\right)$ hence as D increases ϕ decreases.

Initially at P $\phi = 2\pi$ and now decreases towards zero. hence intensity decreases as ϕ varies from 2π to π and then intensity increases as ϕ varies from π towards zero..

(C)

(16) $I = I_0 \cos^2 \phi/2$ and $\phi = \frac{2\pi}{\lambda}\left(\frac{d\,x}{D}\right)$ where $x = 4\times10^{-5}$ m.

(17) Similar to solutions 10 and 11

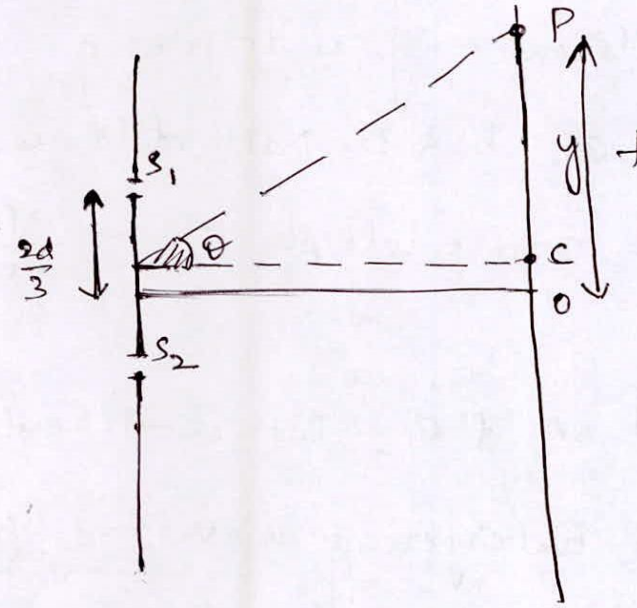
(18) for maxima $\quad d\sin\theta = n\lambda$; $\sin\theta = \frac{n\lambda}{d}$

In the range -30^0 to 30^0; $-\frac{1}{2} < \sin\theta < \frac{1}{2}$ or

$-d/2\lambda < n < d/2\lambda$ or $-300 < n < 300$

(C)

19)



If C is center of screen then $OC = d/6$, let $OP = y$

Path difference at P

$S_2P - S_1P = d \sin\theta$

$$\boxed{P.d = \frac{d\left(y - \frac{d}{6}\right)}{D}}$$

for white spot $P.d = 0$

or $\boxed{y = d/6}$ (D)

20) At O, path difference $S_1P - S_2P = \frac{d^2}{6D}$ $(\because y = 0)$

for maxima at O; Path difference $= n\lambda$

i.e $\frac{d^2}{6D} = n\lambda$ or $\lambda = \frac{d^2}{6Dn}$ $[n = 1, 2 \ldots]$

hence $\frac{d^2}{3D}$ is not possible (A)

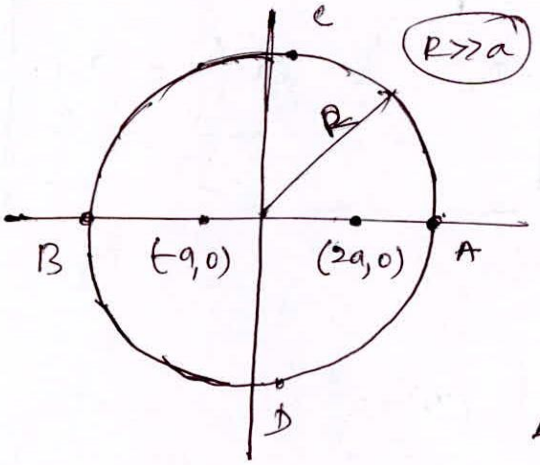
21) Path difference is zero below the centre of screen hence central maxima shifts downward and so do the other orders without change in fringe width (D)

22) Phase difference of the interfering waves at the centre of the screen $\boxed{\phi = \frac{2\pi}{\lambda}(\mu - 1)t}$

$I = I_0 \cos^2\phi$; At $\mu = 1$ $\phi = 0$ $I = I_0$ hence (C)

23. At A and C path difference of interfering waves is zero whereas at B & D path difference is 5 μm which is an odd multiple of λ. (D)

24.



At A & B, Path difference of interfering waves is $3a$

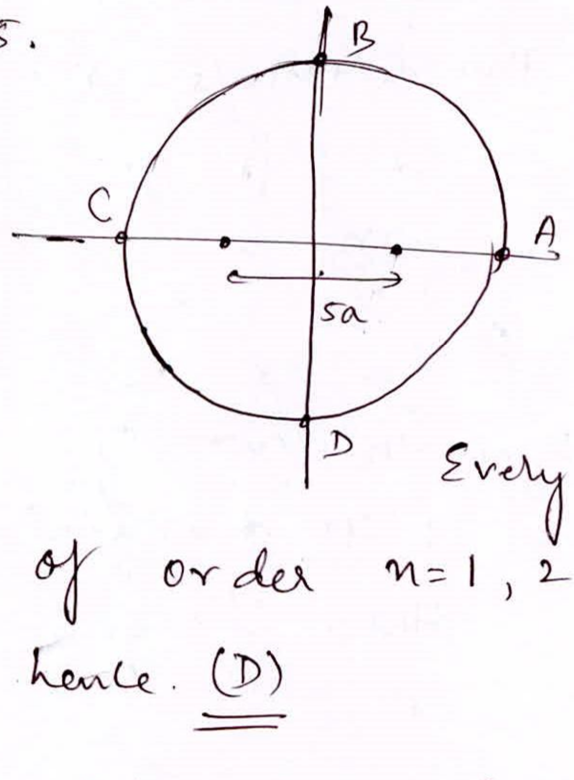
$3a = n\lambda$ for maxima

$n = 15$ Since $\lambda = \frac{a}{5}$

A & B have 15^{th} order maxima

Positions C & D have zeroeth order maxima. (A)

25.



At A and C path difference is $5a$.

hence $n = 3.75$ at A & C

whereas $n = 0$ at B & D

Every Quadrant has maxima of order $n = 1, 2$ & 3 with zeroth order at B & D

hence. (D)

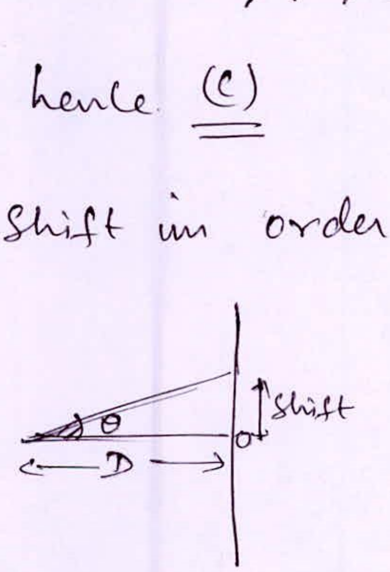
26. Fringe Pattern is narrow if d is large hence to observe the pattern d should be decreased.

(B)

27. Optical path difference at a distance "x" apart in a medium of refractive index μ is μx.
hence phase difference $\phi = \frac{2\pi}{\lambda}(\mu x)$ where λ is wavelength in air. $\underline{\underline{(A)}}$

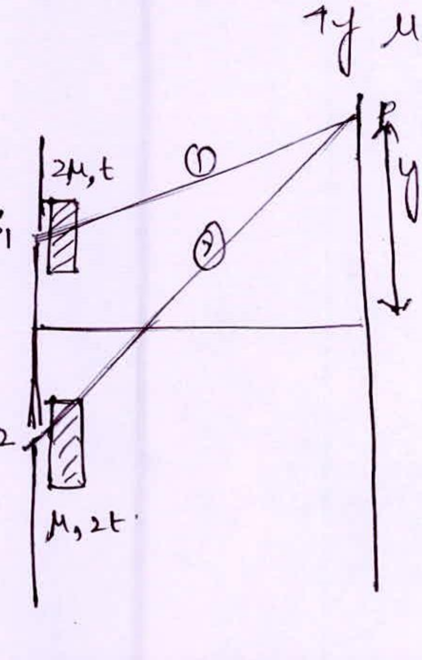
28. Path difference at Centre of Screen O
is $(\mu_1 - 1)t - (\mu_2 - 1)t = (\mu_1 - \mu_2)t$
for maxima at O
$$(\mu_1 - \mu_2)t = n\lambda \quad \text{or} \quad \lambda = \frac{1248}{n} \text{ (nm)}$$
hence $\underline{\underline{(C)}}$

29. Shift in order due to glass plates $= (\mu_1 - \mu)t \times \frac{D}{d}$



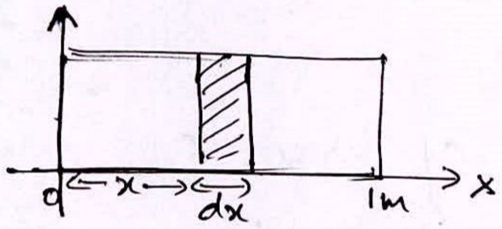
Shift $= D\,\theta^c$
$$(\mu_1 - \mu)t\frac{\cancel{D}}{d} = \cancel{D}\frac{\pi}{180}(6)$$
If $\mu_1 = 1.8$ then $\underline{\underline{\mu = 1.6}}$ (A)

30.



Path difference at P is
optical path of ② − optical path of ①
$$\text{P.d} = [(S_2P - 2t) + \mu 2t] - [(S_1P - t) + 2\mu t]$$
$$= (S_2P - S_1P) - t$$
for Central maxima. P.d = 0
$$(S_2P - S_1P) - t = 0$$
$$\frac{dy}{D} = t \quad \boxed{y = \frac{Dt}{d}} \text{ (B)}$$

31. If second minima is above then phase difference should be greater than 3π and less then 6π since third maxima is below hence (A)

32. 

optical path of element dx is $\mu\, dx$

hence total optical path is $\int_{x=0}^{x=1} \mu\, dx = \frac{4}{3}m$ (C)

WAVE OPTICS SOLUTIONS

EXERCISE II

1. frequency remains unchanged at $\frac{c}{\lambda_a}$ but wavelength varies as $\frac{\lambda_a}{\mu}$. (A) & (C)

2. At a point directly opposite to one of the slits $y = d/2$ hence path difference of the interfering waves is $\frac{dy}{D}$ or $\frac{d^2}{2D}$

for destructive interference $\frac{d^2}{2D} = (2n-1)\frac{\lambda}{2}$

$$\text{or } \lambda = \frac{d^2}{D(2n-1)} \quad (n = 1, 2, \ldots) \quad \text{(A) \& (C)}$$

3. All wavelengths interfere constructively at centre and shorter wavelengths slightly above (lower orders) completely dark fringe is never formed because destructive interference for one wavelength may be constructive for the other wavelengths. (B) (C) & (D)

4. for maxima $d\sin\theta = n\lambda$ & $|\sin\theta| \le 1$

hence $-1 \le \frac{n\lambda}{d} \le 1$ or $\frac{-d}{\lambda} \le n \le \frac{d}{\lambda}$

reducing wavelength increases the orders

(B) & (C)

5. Fringe width $\beta = \frac{D\lambda}{d}$ hence $\underline{(B)}$

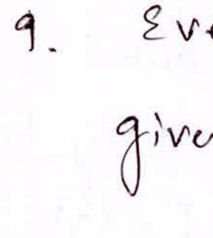
6. If $I_1 \neq I_2$ then $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ is non zero.
hence $\underline{(B)}$

7. Introducing a glass plate increases optical path and orders shift to compensate accordingly
(A) & (D). Intensity at bright & dark fringe increase.

8. At O, path difference is 5.5λ, an odd multiple of λ hence dark fringe appears at O
Also path difference decreases with increasing "y" hence 5 maxima and 6 minima are observed on screen. (A) & (D).

9. Every order shifts by same length given by $\frac{(\mu-1)tD}{d}$ hence (A) & (C)

10. Path difference at any point at height y from centre of screen is



$$P.d = (\mu_2 - 1)t_2 - (\mu_1 - 1)t_1 + d\sin\theta.$$

for Central maxima $P.d = 0$

$$y = \frac{D}{d}\left[(\mu_2-1)t_2 + (\mu_1-1)t_1\right]$$

$$y = \frac{D}{d}\left[(t_2 - t_1)\right]$$ hence $\underline{B \,\&\, C}$

11. $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \neq 0$ hence (A)

12. $SS_2 - SS_1 = (\sqrt{2}-1)d$ hence mica sheet of refractive index $\mu = 1.5$ & thickness $2(\sqrt{2}-1)d$ is placed in front of S_1 (A)

Path difference due to glass plate of thickness t and refractive index μ is $(\mu-1)t$.