Number Theory

Chapter Ex 1

(1) $P < q < t < r < S$

(2) $a = 2x+1$, ~~y=2~~
$b = 2y+1$, $c = 2z$, $d = 2l$
Use the option by putting the values

(3) a) last digit is 4, so is an even number
b) " " " 8, " " " "
c) " " " 6, " " " "
d) " " " 5, " " odd number

3, 7, 11, 13, 17, 19, 23, 29, 31, 37

(4) $(n-1)!$ is not divisible by n. The case will be only when n is a prime number. So, in $12 \le n \le 40$, there are 7 prime numbers.

(5) 10 has 2 & 5 as prime factors.

(6) 2, 3 & 7 will satisfy.

(7) A, B, A−B & A+B are all prime numbers. A prime number can be represent in form $6K \pm 1$, when number is greater than 5.

$A - B = (6a+1) - (6b-1) = 6(a-b)+2$ will be an even number. So, ~~prime~~ A & B must be less than 6. So, 2, 3 & 5 prime numbers are left. 2 & 3 won't satisfy, because $(3-2=1)$.

So, take 2 & 5.

$A = 5, \; B = 2$

$A + B + (A - B) + (A + B) = 3A + B = 3(5) + 2 = 17.$

So, Sum will be a prime number.

(8) $V = \lfloor 41$

$A = \lfloor 41 + 1$

~~All are factors~~

$A + 1 = \lfloor 41 + 2 = 2a$

$A + 2 = \lfloor 41 + 3 = 3b$

$A + 3 = \lfloor 41 + 4 = 4b$

$\vdots$

$A + 40 = \lfloor 41 + 41 = 41z$

So, all are composite numbers.

(9) 1, 4, 5, 6, 9 will satisfy the condition

(10) $X = 28m, \; Y = 17n$

So, option a satisfy

Chapter Ex - 2

(2)

$156 = 120 \times 1 + 36$

$120 = 36 \times 3 + 12$

$36 = 12 \times 3 + 0$

So, HCF = 12

(3) Apply Euclid's division lemma

$15 = \cancel{3 \times 4}\ 4 \times 3 + 3$

(4) $(9^n)^2 - (4^n)^2$

$\Rightarrow (9^n - 4^n)(9^n + 4^n)$

So, 5 & 13 satisfy

Ex-3

(1) $\frac{6}{7} = 0.\overline{857142}$

So, 100th digit will be 1

Because $100 = 16 \times 6 + 4$. and 4th digit is 1.

(2) Let $K = 0.\overline{xy}$

$100K = xy.\overline{xy}$

$-K = 0.\overline{xy}$

$99K = xy$

$K = \frac{xy}{99}$

(3) $x = 0.4545\ldots = \frac{\cancel{45}\,5}{\cancel{99}\,11}$

So, Sum of numerator & denominator is 16

(4) $abde = 1$ [using identity]

So, option b is correct

(5) let, $x = 3+\sqrt{2}$ is a rational number

~~$x-3 = \sqrt{2}$~~

S.B.S [Square of a rational number is always a rational number]

$\Rightarrow x^2 = 9+2+6\sqrt{2}$

$\Rightarrow x^2 = 11+6\sqrt{2}$

~~So~~ R.H.S is an irrational number but L.H.S is a rational number. So, our assumption is wrong & hence $(3+\sqrt{2})$ is an irrational number.

Ch- Ex-4

(1) Rationalise the denominator

$$\frac{4(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} - \frac{(2-\sqrt{3})^2}{4-3}$$

$$\Rightarrow \frac{\cancel{4}^{2}(\sqrt{3}-1)^2}{\cancel{2}} - \frac{(4+3-4\sqrt{3})}{1}$$

$$\Rightarrow 2(4-2\sqrt{3}) - (7-4\sqrt{3})$$

$$\Rightarrow 8-4\cancel{\sqrt{3}}-7+4\cancel{\sqrt{3}} = 1$$

(2) $\sqrt{x} = \sqrt{7+4\sqrt{3}}$

$\sqrt{a}+\sqrt{b} = \sqrt{7+4\sqrt{3}}$

S.B.S

$\Rightarrow a+b+2\sqrt{ab} = 7+4\sqrt{3}$

$a+b=7, \quad ab=12$

$a=4, \; b=3$

$\sqrt{7+4\sqrt{3}} = \sqrt{4}+\sqrt{3} = 2+\sqrt{3}$

$\sqrt{x} + \frac{1}{\sqrt{x}}$

$\Rightarrow (2+\sqrt{3}) + \frac{1}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$

$\Rightarrow (2+\cancel{\sqrt{3}}) + 2-\cancel{\sqrt{3}}$

$\Rightarrow 4$

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(3) Rationalise the Denominator

(4) $\sqrt{\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \cdots + \frac{1}{\sqrt{100}+\sqrt{99}}}$

$\Rightarrow \sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2})^2-1} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} + \cdots + \frac{\sqrt{100}-\sqrt{99}}{(\sqrt{100})^2-(\sqrt{99})^2}}$

$\Rightarrow \sqrt{\cancel{\sqrt{2}}-1 + \cancel{\sqrt{3}} - \cancel{\sqrt{2}} + \cancel{\sqrt{4}} - \cancel{\sqrt{3}} + \cdots + \sqrt{100} - \cancel{\sqrt{99}}}$

$\Rightarrow \sqrt{\sqrt{100}-1}$

$\Rightarrow \sqrt{9} = 3$

(5) $x = \frac{\sqrt{3}+1}{2}$

$2x - 1 = \sqrt{3}$

S.B.S

$\Rightarrow 4x^2 + 1 - 4x = 3$

$\Rightarrow 4x^2 - 4x = 2$

$\Rightarrow 4x^3 - 8x + 2x^2 + 7$

$\Rightarrow \cancel{4x^3 - 4x^2}$

$x(4x^2) + 2x^2 - 8x + 7$

$\Rightarrow x(2+4x) + 2x^2 - 8x + 7$

$\Rightarrow 2x + 4x^2 + 2x^2 - 8x + 7$

$\Rightarrow 6x^2 - 6x + 7$

$\Rightarrow 6(x^2 - x) + 7$

$\Rightarrow \cancel{6}^{3} \cdot \frac{1}{\cancel{2}} + 7$

$\Rightarrow 10$

(6) $\sqrt{34-24\sqrt{2}} \times (4+3\sqrt{2})$

$\Rightarrow \sqrt{34-24\sqrt{2}} = \sqrt{a} - \sqrt{b}$

$\Rightarrow 34-24\sqrt{2} = a+b-2\sqrt{ab}$

$a+b=34, \quad ab=288$

$a=18, \; b=16$

$\Rightarrow \sqrt{34-24\sqrt{2}} = 3\sqrt{2}-4$

$\Rightarrow (3\sqrt{2}-4) \times (4+3\sqrt{2})$

$\Rightarrow \cancel{12\sqrt{2}} - 16 + 18 - \cancel{12\sqrt{2}}$

$\Rightarrow 2$

(7) $\sqrt[4]{48}$

$\Rightarrow \sqrt[4]{16 \times 3}$

$\Rightarrow 2\sqrt[3]{3}$

$\Rightarrow \sqrt[3]{9}$ will be the rationalising factor

(8) $Y = \sqrt{2}+1$,

Rationalise the Denominator

(9) $N = \sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$

$N^3 = 2 + \cancel{\sqrt{5}} + 2 - \cancel{\sqrt{5}} + 3\sqrt[3]{4-5}\left(\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}\right)$

$N^3 = 4 - 3N$

$N^3 + 3N = 4$

N = 1 satisfy above eqn

$\boxed{\text{Ch} - \text{Ex} - 5}$

(1) $\dfrac{(5\times 125)^{0.25}}{(2^8)^{\frac{1}{10}} \times (2^8)^{-\frac{3}{20}}}$

$\Rightarrow \dfrac{(625)^{0.25}}{(2^8)^{\frac{1}{4}}} \Rightarrow \dfrac{(5^4)^{\frac{1}{4}}}{2^2} \Rightarrow \dfrac{5}{4}$

(2) $\dfrac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$

$\Rightarrow \dfrac{2^{r+s} \times 3^{r+s} \times (2^2)^{r-s} \times 3^{r-s}}{2^{3r} \times (3^2)^{r+2s}}$

$\Rightarrow 2^{\cancel{r}+s+\cancel{2r}-2s-\cancel{3r}} \times 3^{\cancel{r}+\cancel{s}+\cancel{r}-\cancel{s}-\cancel{2r}-4s}$

$\Rightarrow 2^{-s} \times 3^{-4s}$

$s \leq 0$

3) $P = 2^m, \quad Q = 3^n$

$(12)^{mn}$

$\Rightarrow (2^2 \times 3)^{mn}$

$\Rightarrow (2^2)^{mn} \times 3^{mn}$

$(2)^{2mn} \times Q^m$

$\Rightarrow P^{2n} Q^m$

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4) Cyphers means number of zeroes.

$20^{50} \times 50^{20} = 2^{50} \times 10^{50} \times 5^{20} \times 10^{20}$

$= 10^{20} \times 2^{30} \times 10^{70}$

$= 2^{30} \times 10^{90}$

So, number of cyphers is 90

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5) $2^a > 4^c, \quad 3^b > 9^a$

$2^a > 2^{2c}, \quad 3^b > 3^{2a}$

$a > 2c, \quad b > 2a$

$\Rightarrow c < a < b$

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6) $2^{2005}(2^3 - 2^2 - 2 + 1) = K \cdot 2^{2005}$

$2^{2005}(8 - 4 - 2 + 1) = K \cdot 2^{2005}$

$3 \times 2^{2005} = K \times 2^{2005}$

$\boxed{K = 3}$

(7) $\frac{23^8(23-1)}{22} = 23^x$

$23^8 = 23^x$

$\boxed{8 = x}$

(8) $3^a = 3^{4(b+2)}$, $5^{3b} = 5^{a-3}$

$a = 4b+8$, $3b = a-3$

$3b+3 = a$

$\Rightarrow 3b+3 = 4b+8$

$b = -5$

$a = -12$

$ab = 60$

(9) $2^x = 3^y = 6^{-2} = k$

$2^x = k$, $3^y = k$, $6^{-2} = k$

$2 = k^{\frac{1}{x}}$, $3 = k^{\frac{1}{y}}$, $6 = k^{-\frac{1}{2}}$

$\Rightarrow 2 \times 3 = k^{\frac{1}{x}+\frac{1}{y}}$

$\Rightarrow 6 = k^{\frac{1}{x}+\frac{1}{y}}$

$\Rightarrow k^{-\frac{1}{2}} = k^{\frac{1}{x}+\frac{1}{y}}$

$\Rightarrow -\frac{1}{2} = \frac{1}{x}+\frac{1}{y}$

$\Rightarrow 0 = \frac{1}{2}+\frac{1}{x}+\frac{1}{y}$

(10)

$$\frac{\frac{1}{x^{2007}} - \frac{1}{x^{2009}}}{\frac{1}{x^{2008}} - \frac{1}{x^{2010}}}$$

$$\Rightarrow \frac{\frac{\cancel{(x^2-1)}}{x^{2009}}}{\frac{\cancel{(x^2-1)}}{x^{2010}}}$$

$$\Rightarrow \frac{x^{2010}}{x^{2009}}$$

$$\Rightarrow x$$

Exercise - 1

a) $(2^6)^{\frac{1}{6}} = 2$

b) $(5^4)^{\frac{1}{4}} = 5$

c) $(3^3)^{\frac{2}{3}} = 9$

d) $(2^5)^{\frac{6}{5}} = 2^6 = 64$

e) $(343)^{\frac{2}{3}} = (7^3)^{\frac{2}{3}} = 7^2 = 49$

(2) a) $\frac{12^{3/2}}{3^{3/2}} = 4^{3/2} = (2^2)^{3/2} = 8$

b) $\left(\frac{1}{5^3}\right)^{4/3} = (5^{-3})^{4/3} = 5^{-4} = \frac{1}{625}$

c) $\frac{10^{-\frac{1}{6}}}{10^{-\frac{1}{3}}} = 10^{-\frac{1}{6}-\frac{1}{3}} = 10^{-\frac{1}{6}}$

d) $11^{\frac{1}{6}} \cdot 11^{\frac{1}{5}} = 11^{\frac{1}{6}+\frac{1}{5}} = 11^{\frac{11}{30}}$

(3) a) $2^n = 32$

$2^n = 2^5$

$\Rightarrow n = 5$

b) $3^n = 243$

$\Rightarrow 3^n = 3^5$

$\Rightarrow n = 5$

c) $5^n = 625$

$5^n = 5^4$

$n = 4$

d) $(0.2)^n = (0.2)^5$

$n = 5$

(4) $\frac{\sqrt[3]{.125} \times \sqrt[5]{(.00032)^{-2}}}{\sqrt[5]{(.00243)^{-3}} \times (27)^{2/3}}$

$\Rightarrow \frac{\left(\frac{1}{8}\right)^{\frac{1}{3}} \times \left((0.2)^{-10}\right)^{1/5}}{\left[(0.3)^{\cancel{5}\times -3}\right]^{1/\cancel{5}} \times (3)^{\cancel{3}\times\frac{2}{\cancel{3}}}}$

$\Rightarrow \frac{2^{-\cancel{3}\times\frac{1}{\cancel{3}}} \times (0.2)^{-2}}{(0.027)^{-1} \times (9)}$

$\Rightarrow \frac{\frac{1}{2} \times \cancel{25} \times \frac{\cancel{27}^{\,3}}{\cancel{1000}_{\,40}}}{\cancel{9}} = \frac{3}{80}$

5) a) $\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\frac{1}{\sqrt{5}+\sqrt{4}}$

$\Rightarrow \frac{\sqrt{2}-1}{2-1}+\frac{\sqrt{3}-\sqrt{2}}{3-2}+\frac{\sqrt{4}-\sqrt{3}}{4-3}+\frac{\sqrt{5}-\sqrt{4}}{1}$

$\Rightarrow \cancel{\sqrt{2}}-1+\cancel{\sqrt{3}}-\cancel{\sqrt{2}}+\cancel{\sqrt{4}}-\cancel{\sqrt{3}}+\sqrt{5}-\cancel{\sqrt{4}}$

$\Rightarrow -1+\sqrt{5}$

b) $\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$

Rationalise the Denominator

$\Rightarrow \frac{\sqrt{6}-\sqrt{5}}{6-5}+\frac{\sqrt{7}-\sqrt{6}}{7-6}+\frac{\sqrt{8}-\sqrt{7}}{8-7}+\frac{\sqrt{9}-\sqrt{8}}{9-8}$

$\Rightarrow \cancel{\sqrt{6}}-\sqrt{5}+\cancel{\sqrt{7}}-\cancel{\sqrt{6}}+\cancel{\sqrt{8}}-\cancel{\sqrt{7}}+\sqrt{9}-\cancel{\sqrt{8}}$

$\Rightarrow -\sqrt{5}+3$

c) $\frac{\sqrt{5}+\sqrt{3}}{4\sqrt{5}+4\sqrt{3}-3\sqrt{5}-3\sqrt{3}}$

$\Rightarrow \frac{\cancel{\sqrt{5}+\sqrt{3}}}{\cancel{\sqrt{5}+\sqrt{3}}} = 1$

~~del 6)~~

$\cancel{x = \sqrt{5}-2}$

$\cancel{\frac{1}{x} = \frac{1}{\sqrt{5}-2}}$

$\cancel{\frac{1}{x} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2}$

(7) $x = \dfrac{\sqrt{5}-2}{\sqrt{5}+2}$, $y = \dfrac{\sqrt{5}+2}{\sqrt{5}-2}$

$x = \dfrac{(\sqrt{5}-2)^2}{5-4}$, $y = \dfrac{(\sqrt{5}+2)^2}{5-4}$

$x = 9-4\sqrt{5}$, $y = 9+4\sqrt{5}$

$x^2 - y^2 = (9-4\sqrt{5})^2 - (9+4\sqrt{5})^2$

$= -144\sqrt{5}$

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(8) $x = \dfrac{1}{2-\sqrt{3}}$

Rationalise the Denominator

$x = \dfrac{2+\sqrt{3}}{2^2-(\sqrt{3})^2}$

$x = 2+\sqrt{3}$

$x-2 = \sqrt{3}$

S.B.S

$x^2-4x+4 = 3$

$x^2-4x = -1$

$x(x^2-4x)+9x+10$

$\Rightarrow -x+9x+10$

$\Rightarrow 8x+10$

$\Rightarrow 8(2+\sqrt{3})+10$

$\Rightarrow 26+8\sqrt{3}$

$- 2\sqrt{2}(3-2\sqrt{3})$

(9) $a = \dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$, $b = \dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

Rationalise the Denominator

$a = \dfrac{7+2\sqrt{10}}{3}$, $b = \dfrac{7-2\sqrt{10}}{3}$

$ab = \dfrac{49-40}{3}$, $a^2 = \dfrac{89+28\sqrt{10}}{9}$, $b^2 = \dfrac{89-28\sqrt{10}}{9}$

$ab = 3$

$3(a^2-b^2) + 4ab$

$\Rightarrow 3\left[\dfrac{89+28\sqrt{10}}{9} - \dfrac{(89-28\sqrt{10})}{9}\right] + 12$

$\Rightarrow 3\left[\dfrac{56\sqrt{10}}{9}\right] + 12$

$\Rightarrow \dfrac{56\sqrt{10}}{3} + 12$

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(10) $x = \dfrac{1}{\sqrt{5}-2}$

$x = \dfrac{\sqrt{5}+2}{1}$

$x - 2 = \sqrt{5}$

S.B.S

$\Rightarrow x^2 + 4 - 4x = 5$

$\Rightarrow x^2 - 4x = 1$

$\Rightarrow x^3 - 4x^2 + 2x^2 - 9x + 18$

$\Rightarrow x(x^2-4x) + 2(x^2-4x) - x + 18$

$\Rightarrow \cancel{x} + 2 - \cancel{x} + 18$

$\Rightarrow 20$

(11)

$$x = \frac{4\sqrt{3}(2+\sqrt{2})}{2} - \frac{30(4\sqrt{3}+3\sqrt{2})}{48-18} - \frac{3\sqrt{2}(3-2\sqrt{3})}{-3}$$

~~x => 2~~

$$x = 2\sqrt{3}(2+\sqrt{2}) - (4\sqrt{3}+3\sqrt{2}) + \sqrt{2}(3-2\sqrt{3})$$

$$x = \cancel{4\sqrt{3}} + \cancel{2\sqrt{6}} - \cancel{4\sqrt{3}} - \cancel{3\sqrt{2}} + \cancel{3\sqrt{2}} - \cancel{2\sqrt{6}}$$

$$x = 0$$

(12)

$$\frac{70}{\sqrt{10}+2\sqrt{5}+2\sqrt{10}-4\sqrt{5}}$$

$$\Rightarrow \frac{70}{3\sqrt{10}-2\sqrt{5}}$$

$$\Rightarrow \frac{70(3\sqrt{10}+2\sqrt{5})}{90-20}$$

$$\Rightarrow 3\sqrt{10}+2\sqrt{5}$$

$$\cancel{\Rightarrow 3(2.236)+}$$

15

$$2^x \cdot 3^{2y} = 2^4 \cdot 3^2$$

$$\Rightarrow x = 4, \quad y = 1$$

$$x + y = 5$$

16

$$\frac{6}{2\sqrt{3}+\sqrt{6}} - \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{4}{\sqrt{6}-\sqrt{2}}$$

$$\Rightarrow \frac{6(2\sqrt{3}-\sqrt{6})}{12-6} - (\sqrt{3}+\sqrt{2}) + \frac{\cancel{4}(\sqrt{6}+\sqrt{2})}{\cancel{4}}$$

$\Rightarrow 2\sqrt{3} - \cancel{\sqrt{6}} - \sqrt{3} - \cancel{\sqrt{2}} + \cancel{\sqrt{6}} + \cancel{\sqrt{2}}$

$\Rightarrow \sqrt{3}$

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(17) $11^{1/3},\ 7^{1/4},\ 5^{1/2}$

$11^{4/12},\ 7^{3/12},\ 5^{6/12}$

$(14641)^{1/12},\ 343^{1/12},\ 15625^{1/12}$

$\Rightarrow 7^{1/4} < 11^{1/3} < 5^{1/2}$

(18) $\sqrt{\sqrt{45}-\sqrt{40}} = \sqrt{a}-\sqrt{b}$

$\sqrt{45}-\sqrt{40} = a+b-2\sqrt{ab}$

$3\sqrt{5}-2\sqrt{10} = a+b-2\sqrt{ab}$

$\sqrt{5}(3-2\sqrt{2}) = a+b-2\sqrt{ab}$

$a+b=3,\ ab=2$

$a=2,\ b=1$

$\Rightarrow \sqrt{\sqrt{45}-\sqrt{40}} = 5^{1/2}(\sqrt{2}-1)$

$$\sqrt{30-10\sqrt5} = \sqrt a - \sqrt b$$
$$30-10\sqrt5 = a+b-2\sqrt{ab}$$
$$a+b=30, \quad ab=125$$
$$a=25, b=5$$
$$\sqrt{30-10\sqrt5} = 5-\sqrt5$$

$$\Rightarrow \frac{10\sqrt2}{\sqrt{18}-\sqrt{3+\sqrt5}} + \sqrt{30-10\sqrt5}$$

$$\Rightarrow \frac{10\sqrt2}{3\sqrt2 - \frac{\sqrt5+1}{\sqrt2}} + 5-\sqrt5$$

$$\Rightarrow \frac{20}{6-\sqrt5-1} + 5-\sqrt5$$

$$\Rightarrow \frac{\cancel{20}(5+\sqrt5)}{\cancel{20}} + 5-\sqrt5$$

$$\Rightarrow 5+\cancel{\sqrt5}+5-\cancel{\sqrt5} = 10$$

3 48

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23 Rationalise the Denominator

24 $x = 928y + 244$

928 is multiple of 58. So, x will be give the same remainder for 58 as when ~~244~~ is divided by 58.

25 Rationalise the denominator

$$x+\frac1x = 4$$

$$x^2+\frac{1}{x^2} = 14, \quad x^2-\frac{1}{x^2} = \sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4}$$
$$= \sqrt{196-4} = \sqrt{192}$$

(19) Rationalise the Denominator

(20)
$$x = 6 + \sqrt{7}$$
$$x - 6 = \sqrt{7}$$
$$x^2 + 36 - 12x = 7$$
$$x^2 - 12x = -29$$
$$\Rightarrow x^3 - 18x^2 + 101x - 132$$
$$\Rightarrow x^3 - 12x^2 - 6x^2 + 72x + 29x - 132$$
$$\Rightarrow x(x^2 - 12x) - 6(x^2 - 12x) + 29x - 132$$
$$\Rightarrow -\cancel{29}x + 174 + \cancel{29}x - 132$$
$$\Rightarrow 42$$

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(21)
$$x = \frac{1}{2}\left(\frac{5+1}{\sqrt{5}}\right)$$
$$x = \frac{3}{\sqrt{5}}$$
Put value of x in $\frac{\sqrt{x^2-1}}{x - \sqrt{x^2-1}}$

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(22)
$$\frac{10\sqrt{2}}{3\sqrt{2} - \sqrt{3+\sqrt{5}}} + \sqrt{30 - 10\sqrt{5}}$$
$$\sqrt{3+\sqrt{5}} = \sqrt{a} + \sqrt{b}$$
$$3 + \sqrt{5} = a + b + 2\sqrt{ab}$$
$$a + b = 3, \quad 4ab = 5$$
$$\cancel{a = \frac{1}{4}, b = 5} \quad a = \frac{5}{2}, \quad b = \frac{1}{2}$$

NTSE

(1) $\dfrac{1}{\sqrt{11-2\sqrt{30}}} - \dfrac{3}{\sqrt{7-2\sqrt{10}}} - \dfrac{4}{2\sqrt{2+\sqrt{3}}}$

$\sqrt{11-2\sqrt{30}} = \sqrt{a} - \sqrt{b}$

$11 = a+b, \quad 30 = ab$

$a = 6, \; b = 5$

$\sqrt{11-2\sqrt{30}} = \sqrt{6} - \sqrt{5}$

Similarly,

$\sqrt{7-2\sqrt{10}} = \sqrt{5} - \sqrt{2}$

$\sqrt{2+\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{2}} + \sqrt{\dfrac{1}{2}}$

$\Rightarrow \dfrac{1}{\sqrt{6}-\sqrt{5}} - \dfrac{3}{\sqrt{5}-\sqrt{2}} - \dfrac{\cancel{4}\,2}{\cancel{2}\left(\dfrac{\sqrt{3}+1}{\sqrt{2}}\right)}$

$\Rightarrow \sqrt{6}+\sqrt{5} - \dfrac{\cancel{3}(\sqrt{5}+\sqrt{2})}{\cancel{3}} - \dfrac{\cancel{2}\sqrt{2}(\sqrt{3}-1)}{\cancel{2}}$

$\Rightarrow \cancel{\sqrt{6}} + \cancel{\sqrt{5}} - \cancel{\sqrt{5}} - \cancel{\sqrt{2}} - \cancel{\sqrt{6}} + \cancel{\sqrt{2}}$

$\Rightarrow 0$

(3)

(2) $2^{1/2}, 3^{1/3}, 8^{1/8}, 9^{1/9}$

$2^{1/2}, 3^{1/3}, 2^{3/8}, 3^{2/9}$

$3^{\frac{1}{3}} = 3^{3/9} = 27^{1/9}$

$27^{1/9} > 3^{2/9}$

$2^{4/8} > 2^{3/8}$

$2^{1/2} > 2^{3/8}$

$2^{1/2}, 3^{1/3}$

$2^{3/6}, 3^{2/6}$

$8^{1/6} < 9^{1/6}$

$3^{1/3}$ is the greatest

(3) $\cancel{\left[\left(\frac{2}{3}\right)^{2/3} + \left(\frac{2}{3}\right)^{1/3} + 1\right]}\left[1 - \sqrt[3]{\frac{2}{3}}\right]$

$\Rightarrow \cancel{\left(\frac{2}{3}\right)} \Rightarrow \cancel{\left[1 - \sqrt[3]{\frac{2}{3}}\right]} \Rightarrow \cancel{1 - \left(\sqrt[3]{\frac{2}{3}}\right)^3} = 1 - \frac{2}{3} =$

~~$\left(\frac{2}{\ }\right)$ is the rationalising factor~~

(4)

$$\sqrt[3]{\frac{4}{9}} + \sqrt[3]{\frac{2}{3}} + 1$$

$$\frac{\left[\left(\frac{2}{3}\right)^{2/3} + \left(\frac{2}{3}\right)^{1/3} + 1\right]\left(1 - \sqrt[3]{\frac{2}{3}}\right)}{\left(1 - \sqrt[3]{\frac{2}{3}}\right)}$$

$$\Rightarrow \frac{\left[1 - \left(\sqrt[3]{\frac{2}{3}}\right)^3\right]\sqrt[3]{3}}{\sqrt[3]{3} - \sqrt[3]{2}}$$

$$\Rightarrow \frac{\left(1 - \frac{2}{3}\right)\sqrt[3]{3}}{\sqrt[3]{3} - \sqrt[3]{2}}$$

$$\Rightarrow \frac{\sqrt[3]{3}}{3\left(\sqrt[3]{3} - \sqrt[3]{2}\right)}$$

$$N = \frac{\sqrt{\sqrt{5}+2} + \sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+2}} - \sqrt{3-2\sqrt{2}}$$

$$= 1 + \sqrt{\frac{\sqrt{5}-2}{\sqrt{5}+2}} - \sqrt{3-2\sqrt{2}}$$

$$= 1 + \sqrt{\frac{(\sqrt{5}-2)^2}{5-4}} - (\sqrt{2}-1)$$

$$= \cancel{1} + \sqrt{5} - \cancel{2} - \sqrt{2} + \cancel{1}$$

$$= \sqrt{5} - \sqrt{2}$$

(7)

3^{210}, 7^{140}, 17^{105}, 31^{84}

$L.C.M = 420$

$(3^{1/2})^{420}$, $(7^{1/3})^{420}$, $(17^{1/4})^{420}$, $(31^{1/5})^{420}$

$[3^{1/2} < 2,\ 7^{1/3} < 2,\ 17^{1/4} > 2,\ 31^{1/5} < 2]$

$\Rightarrow$ 17^{105} is the greatest

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(8)

$$\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}} + \frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$$

$$\Rightarrow \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2} + \frac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{(\sqrt{2}-\sqrt{3})^2-(\sqrt{5})^2}$$

$$\Rightarrow \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{\cancel{5}+2\sqrt{6}-\cancel{5}} + \frac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{\cancel{5}-2\sqrt{6}-\cancel{5}}$$

$$\Rightarrow \frac{\cancel{\sqrt{2}}+\sqrt{3}+\cancel{\sqrt{5}}-\cancel{\sqrt{2}}+\sqrt{3}-\cancel{\sqrt{5}}}{2\sqrt{6}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}$$

(9)

$$\left(\frac{1}{a}\right)^{\frac{1}{b}} = \frac{1}{3}$$

$$\Rightarrow \quad a^{-1/b} = 3^{-1}$$

$$\Rightarrow \quad a = 3^{b}$$

$$ab = b \cdot 3^{b}$$

So, product of ab should be in multiple of 3.

---

(10)

$$x = 5k + 2$$

$$y = 5l + 4$$

$$x + y = 5(k + l) + 6$$

$$x + y = 5(k + l + 1) + 1$$

$$z = 1$$

$$\frac{2z - 5}{3} = \frac{-\cancel{3}}{\cancel{3}} = -1$$

$$\frac{1}{x^4} = \ldots$$

$$= \cancel{12\sqrt{4\times 35}}$$

$$= 24\sqrt{35}$$

$$x^4 - \frac{1}{x^4} = \left(x^2+\frac{1}{x^2}\right)\left(x^2-\frac{1}{x^2}\right)$$

$$= 14\,(192)^{1/2}$$

$$= 14\sqrt{4\times 48}$$

$$= 14\sqrt{64\times 3}$$

$$= \cancel{56\sqrt{3}}\ 112\sqrt{3}$$

Exercise – 2

(2) $$\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$$

$$\Rightarrow \frac{1}{1+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^b}{x^a}}$$

$$\Rightarrow \frac{x^b}{x^a+x^b} + \frac{x^a}{x^a+x^b} = 1$$

---

(3) $$2^x = 3^y = 12^z = k$$

$$2 = k^{\frac{1}{x}},\quad 3 = k^{\frac{1}{y}},\quad 12 = k^{\frac{1}{z}}$$

$$2^2\cdot 3 = k^{\frac{2}{x}+\frac{1}{y}}$$

$$12 = k^{\frac{2}{x}+\frac{1}{y}}$$

$$k^{\frac{1}{z}} = k^{\frac{2}{x}+\frac{1}{y}}$$

$$\boxed{\frac{1}{z} = \frac{2}{x}+\frac{1}{y}}$$

4) $5^{x-3} \cdot 3^{2x-8} = 5^2 \cdot 3^2$

$x-3=2$

$\boxed{x=5}$

5) $$\frac{3^{2(n+1)} \; 3^{-\frac{n}{2} \; x-2} \cdots - 3^{3n}}{3^{3m} \times 8}$$

$$\Rightarrow \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 8} = 9^{-3}$$

$$\Rightarrow \frac{3^{3n}(\cancel{9-1})}{3^{3m} \times \cancel{8}} = 9^{-3}$$

$$\Rightarrow 3^{3(n-m)} = 3^{-6}$$

$$3(n-m) = -6$$

$$\Rightarrow m-n = 2$$

6) $abc = 1$

$$\Rightarrow \left(1+a+\frac{1}{b}\right)^{-1} + \left(1+b+\frac{1}{c}\right)^{-1} + \left(1+c+\frac{1}{a}\right)^{-1}$$

$$\Rightarrow \left(1+\frac{1}{bc}+\frac{1}{b}\right)^{-1} + \left(1+b+\frac{1}{c}\right)^{-1} + \left(1+c+\frac{abc}{a}\right)^{-1}$$

$$\Rightarrow \left(\frac{bc+1+c}{bc}\right)^{-1} + \left(\frac{c+bc+1}{c}\right)^{-1} + \left(1+c+bc\right)^{-1}$$

$$\Rightarrow \frac{bc}{bc+1+c} + \frac{c}{c+bc+1} + \frac{1}{1+c+bc}$$

$$\Rightarrow 1$$

(8) $2^x = 7^y = 14^z = k$

$2 = k^{\frac{1}{x}}, \quad 7 = k^{\frac{1}{y}}, \quad 14 = k^{\frac{1}{z}}$

$2 \times 7 = k^{\frac{1}{x}+\frac{1}{y}}$

$14 = k^{\frac{1}{x}+\frac{1}{y}}$

$k^{\frac{1}{z}} = k^{\frac{1}{x}+\frac{1}{y}}$

$\frac{1}{z} = \frac{1}{x}+\frac{1}{y}$

$z = \frac{xy}{x+y}$

---

(9) $\sqrt{11\sqrt{11\sqrt{11 \cdots \infty}}} = 14641^x$

$K = \sqrt{11\sqrt{11\sqrt{11 \cdots \infty}}} = (11^4)^x$

$\cancel{11 = 11^{4x}}$

~~L.H.S~~

Let, $K = \sqrt{11\sqrt{11 \cdots \infty}}$

$K^2 = 11K$

$K = 11$

$\Rightarrow 11 = 11^{4x}$

$\Rightarrow x = \frac{1}{4}$