X- PHYSICS - 4. CIRCULAR MOTION SOLUTIONS

LEVEL 1:

1. (a) Angular displacement is rotation of position vector through a small angle.
2. (c) f = 3000 r.p.m = $\frac{3000}{60}$ = 50 Hz
   $\omega = 2\pi f$ = 100π rad/s
   $\theta = \omega t$ = 100π rad
3. (a) Circumference of the wheel x rotations = distance covered
   Thus, πd x 2000 = 9500
   d = 1.5 m
4. (a) a = rα; v = rω; a = $\frac{v\alpha}{\omega}$
5. (c) $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30}$ rad/s
6. (b) a = rα; a = rω^2; $\alpha = \omega^2$ i.e. constant
7. (b) C = 2πr; r = $\frac{C}{2\pi}$; $\omega = 2\pi f$
   v = rω = $\frac{C}{2\pi} 2\pi f$ = Cf
8. (a) f = 240 r.p.m = $\frac{240}{60}$ = 4 Hz
   $\omega = 2\pi f$ = 8π rad/s
   v = rω = (0.6)(8π) = 4.8π m/s
9. (c) v = 72 km/hr = 20 m/s; v = rω; $\omega_0 = \frac{20}{0.25}$ = 80 rad/s
   Using motion equation, $\omega^2 = {\omega_0}^2 + 2\alpha\theta$
   Then, $0^2 = 80^2 + 2\alpha(40\pi)$
   $\alpha = \frac{-80^2}{80\pi} = \frac{-80}{\pi}$ = -25.48 rad/s^2
10. (c) f = n r.p.m = $\frac{n}{60}$ Hz
   $\omega = 2\pi f = 2\pi\frac{n}{60}$ rad/s
   v = rω = $(\pi)(2\pi\frac{n}{60}) = 2\pi^2\frac{n}{60}$ m/s
11. (c) $\omega = \omega_0 + \alpha t$; 36 = 0 + α(6); α = 6 rad/s^2
   Then, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ = 108 rad
12. (c) $\omega = 2\pi f = 2\pi\frac{1200}{60}$ = 40π rad/s
   $\omega_0 = 2\pi f_0 = 2\pi\frac{600}{60}$ = 20π rad/s
   But, $\omega = \omega_0 + \alpha t$; $\alpha = \frac{40\pi - 20\pi}{5}$ = 4π rad/s^2
   $\omega^2 = {\omega_0}^2 + 2\alpha\theta$; $\theta = \frac{(40\pi)^2 - (20\pi)^2}{2(4\pi)}$ = 150π rad
   No of revolutions = $\frac{150\,\pi}{2\pi}$ = 75
13. (d) f = 60 r.p.m = $\frac{60}{60}$ = 1 Hz
   $\omega = 2\pi f$ = 2π rad/s
   v = rω = (0.2)(2π) = 0.4π m/s
14. (d) Average speed = $\frac{2\pi r}{t} = \frac{(2)(3.14)(100)}{62.8}$ = 10 m/s
   Average velocity = 0 m/s
15. (c)

LEVEL 2:

1. (c) Tangential acceleration = a_t = a
   Radial acceleration = $a_r = \frac{v^2}{r}$
   Resultant acceleration = a = $\sqrt{a_t^2 + a_r^2} = \sqrt{a^2 + \frac{v^4}{r^2}}$

2. (d)

3. (d) Tangential acceleration = a_t = 2 m/s^2

Radial acceleration = $a_r = \frac{v^2}{r} = \frac{30^2}{500} = \frac{9}{5}$ m/s^2

Resultant acceleration = a = $\sqrt{2^2 + (\frac{9}{5})^2}$ = 2.69 m/s^2

4. (b) $\omega = \omega_0 + \alpha t$; $\alpha = \frac{15\pi - 10\pi}{2} = 2.5\pi$ rad/s^2

5. (c) P = mv; CPF = $\frac{mv^2}{r}$; Required ratio = $\frac{v}{r}$

6. (d)

7. (a) Tension in the string = CPF = $mr\omega^2$

32 = (2)(1)$(2\pi f)^2$

f = $\frac{2}{\pi}$ rps = $\frac{120}{\pi}$ rpm = 38 rpm

8. (b) Frictional force = μmg = (0.5)(800)(10) = 4000 N

4000 = $\frac{mv^2}{r}$; v^2 = (4000)(40)/(800) = 200

v = 14 m/s

9. (d) $\tan\theta = \frac{v^2}{rg} = \frac{(10)^2}{(80)(10)} = \frac{1}{8}$

10. (b) $\mu = \tan\theta$ = 0.24

SUBJECTIVE QUESTIONS:

1. (a) Work done by the centripetal force will be zero.

2. S = rθ = 6.38 x 10^6 x $\frac{\pi}{180}$ = 1.11 x 10^5 m

3. Vertically downwards

4. $\omega = \frac{2\pi}{T}$; $\frac{\omega_u}{\omega_s} = \frac{T_s}{T_u}$; $\omega_u = \frac{xk}{y}$

5. f = x r.p.m = $\frac{x}{60y}$ Hz

T = $\frac{60y}{x}$ s

$\omega = 2\pi f = 2\pi \frac{x}{60y} = \frac{\pi x}{30y}$ rad/s

6. $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30}$ rad/s

7. a) $\omega = \frac{2\pi}{T}$; T = $\frac{2\pi}{\omega} = \frac{2\pi}{40} = \frac{\pi}{20}$ s

b) v = rω; r = $\frac{v}{\omega} = \frac{20}{40}$ = 0.5m

8. f = 400 r.p.m = $\frac{400}{60} = \frac{20}{3}$ Hz

$\omega = 2\pi f = \frac{40}{3}\pi$ rad/s

v = rω = (2)$(\frac{40}{3}\pi) = \frac{80}{3}\pi$m/s

9. C = 2πr; r = $\frac{C}{2\pi}$; $\omega = \frac{2\pi}{T}$

v = rω = $(\frac{C}{2\pi})(\frac{2\pi}{T}) = \frac{C}{T}$ m/s

10. C = 2πr; r = $\frac{C}{2\pi}$; $\omega = \frac{2\pi}{T}$

v = rω = $(\frac{C}{2\pi})(\frac{2\pi}{T}) = \frac{C}{T}$

T = $\frac{C}{v}$ s

11. v = rω; $r_c\omega_c = r_s\omega_s$

Required ratio is $\frac{\omega_c}{\omega_s}$ = 1 : 2

12. $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30}$ rad/s

v = rω = (0.1)$(\frac{\pi}{30}) = \frac{\pi}{300}$m/s

13. v = rω; $\omega = \frac{v}{r} = \frac{10^8}{1.5 \times 10^{-10}}$ = 0.67 x 10^{18} rad/s

14. v = rω; As T is constant, $\omega$is constant

    Then, $\frac{v_2}{v_1} = \frac{r_2}{r_1} = \frac{2R}{R}$; v_2 = 2v m/s

15. v = rω = (2.2 x 10^{20})(1.2 x 10^{-15}) = 2.64 x 10^5 m/s

    T = $\frac{2\pi}{\omega} = \frac{2x3.14}{1.2x10^{-15}}$ = 5.23 x 10^{15} s = 1.66 x 10^8 years

16. $a_r = \frac{v^2}{r} = \frac{(40)^2}{160}$ = 10 m/s^2

17. $a_r = \frac{v^2}{r}$; Acceleration increases 8 times

18. $a_r = r\omega^2$; As T is constant, ω is constant

    Then, $\frac{a_2}{a_1} = \frac{r_2}{r_1}$

19. $\omega = \frac{2\pi}{T} = \frac{2\pi}{(24)(3600)} = \frac{\pi}{43200}$ rad/s

    But, $a_r = r\omega^2$ and $a_r = r\alpha$

    Then, $\alpha = \omega^2$ = 5.43 x 10^{-8} rad/s^2

20. v = rω and $a_r = r\alpha$

    Required ratio is 1:1

21. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ = 0 + (15)(100) = 1500 rad

22. f = 200 r.p.m = $\frac{200}{60} = \frac{10}{3}$ Hz

    $\omega_0 = 2\pi f = \frac{20}{3}\pi$ rad/s

    $\omega = \omega_0 + \alpha t$; t = $\frac{0-\frac{20}{3}\pi}{-2} = \frac{10}{3}\pi$ s

23. (a)

24. (b) Electrostatic force provides the necessary CPF for circular motion of electrons around the nucleus.

25. CPF = $mr\omega^2$; As CPF is constant, $r\omega^2$ = constant i.e. $\omega \propto \frac{1}{\sqrt{r}}$

    Angular velocity will become $\frac{1}{\sqrt{2}}$ times the initial value.

26. (c) Linear velocity is directed along the tangent in a circular motion.

27. (c) It remains at the neck as the water inside the bottle experiences centrifugal force.

28. Tangential direction as linear velocity is directed along the tangent in a circular motion.

29. (c) It is due to higher centrifugal force along the equator than poles.

30. (c)

31. more (four times)

32. $\omega = \frac{2\pi}{T} = \frac{2\pi}{\sqrt{22}}$ = 1.34 rad/s

    C = 2πr; r = $\frac{34.3}{2\pi}$= 5.46 m

    v = rω = (5.46)(1.34) = 7.32 m/s

    tan $\theta = \frac{v^2}{rg} = \frac{(7.32)^2}{(5.46)(9.8)}$ = 1

    Thus, θ = 45^0

33. tan $\theta = \frac{v^2}{rg}$ = tan $\theta = \frac{(\sqrt{rg})^2}{rg}$ = 1

    Thus, θ = 45^0

34. Cycle will also overturn as tan $\theta = \frac{v^2}{rg}$

35. tan $\theta = \frac{v^2}{rg} = \frac{(10)^2}{(10)(10)}$ = 1

    Thus, θ = 45^0

36. tan $\theta = \frac{v^2}{rg}$; As tan θ is constant, r $\propto$ v^2

    Thus, if speed is doubled, radius should be changed to four times.

37. (b) tan $\theta = \frac{v^2}{rg}$ i.e. tan $\theta \propto$ v^2