EXERCISE 1

LEVEL 1

1. $(\sin A+\cos A)(1-\sin A\cos A)=\sin^3 A+\cos^3 A$

$\Rightarrow$ *R.H.S*

$\Rightarrow sin^3A + cos^3A$

$\Rightarrow$ (sinA+cosA)($sin^2A + cos^2A - sinAcosA$)

$\Rightarrow$ (sinA+cosA)(1-sinA cosA) [$1 = sin^2A + cos^2A$]

2. $\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}=2\cos ecA$

L.h.s

$\Rightarrow \frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}=2\cos ecA$

$\Rightarrow \frac{sinA(1-cosA)}{(1+cosA)(1-cosA)} + \frac{(1+cosA)}{sinA}$

$\Rightarrow \frac{sinA(1-cosA)}{1-COS^2A} + \frac{1+cosA}{sinA}$

$\Rightarrow \frac{sinA(1-cosA)}{sin^2A} + \frac{1+cosA}{sinA}$

$\Rightarrow \frac{(1-cosA)}{sinA} + \frac{1+cosA}{sinA}$

$\Rightarrow$ 2cosecA

3. $\frac{\cos ecA}{\cos ecA-1}+\frac{\cos ecA}{\cos ecA+1}=2\sec^2 A$

$\Rightarrow$ *L.H.S*

$\Rightarrow \frac{1/sinA}{1/sinA-1} + \frac{1/sinA}{1/sinA+1}$

$\Rightarrow \frac{1}{1-sinA} + \frac{1}{1+sinA}$

$\Rightarrow \frac{2}{1-sin^2A} \Rightarrow 2sec^2A$

4. $\frac{\cos ecA}{\cot A+\tan A}=\cos A$

L.H.S

$\Rightarrow \frac{cosecA}{cotA+tanA}$

$\Rightarrow \frac{1/sinA}{(cosA/sinA)+(sinA/cosA)}$

$\Rightarrow \frac{cosA}{cos^2A+sin^2A}$

$\Rightarrow$ cosA

5. $\frac{1}{\cot A+\tan A}=\sin A\cos A$

L.H.S

$\Rightarrow \frac{1}{(cosA/sinA)+(cosA/sinA)}$

$\Rightarrow \frac{sinAcosA}{cos^2A+sin^2A}$

$\Rightarrow$ sinAcosA

6. $\frac{1}{\sec A - \tan A} = \sec A + \tan A$

L.H.S

$\Rightarrow \frac{sec^2A - tan^2A}{secA - tanA}$ $[sec^2A - tan^2A = 1]$

$\Rightarrow$ SecA+TanA

7. $\frac{1-\tan A}{1+\tan A} = \frac{\cot A - 1}{\cot A + 1}$

L.H.S

$\Rightarrow$ put TanA=1/cotA

$\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

8. l.h.s

$\Rightarrow \frac{cos^2A}{cosA - sinA} + \frac{Sin^2A}{SinA - CosA}$

$\Rightarrow \frac{cos^2A}{CosA - sinA} - \frac{Sin^2A}{cosA - sinA}$

$\Rightarrow cosA + sinA$

9. (i) 2 sinA = 1 $\Rightarrow sinA = 1/2 \Rightarrow A = 30\circ$

(ii)2 cos 2A = 1 $\Rightarrow$ cos2A=1/2 $\Rightarrow$ 2A=60∘ $\Rightarrow$ A=30∘

(iii)sec 2A = 2 $\Rightarrow$ sec2A=2 $\Rightarrow$ 2A=60∘ $\Rightarrow$ A=30∘

(iv) tan 3A = 1 $\Rightarrow$ tan3A=tan45 $\Rightarrow$ 3A=45 $\Rightarrow$ A=15∘

(v) 2 sin 3A = 1 $\Rightarrow$ Sin3A=½, $\Rightarrow$ Sin3A=sin30 $\Rightarrow$ 3A=30, A=10∘

10. (i) $\sin (x + 10)^{o} = \frac{1}{2}$

$\Rightarrow$ sin(x+10)=sin30, $\Rightarrow$ x=20∘

(ii) $\cos (2x - 30^{o}) = 0$ $\Rightarrow$ cos(2x-30)=cos90

$\Rightarrow$ 2x-30=90,x=60∘

(iii) $2 \cos (2x - 15^{o}) = 1$

$\Rightarrow cos(2x - 15) = 1/2$

$\Rightarrow \cos(2x-15)=\cos 60 \Rightarrow 2x-15=60,\ x=75/2^\circ$

(iv) $\cos\left(\frac{x}{2}+10\right)=\frac{\sqrt{3}}{2}$

$\Rightarrow \cos(x/2+10)=\cos 30,\ \Rightarrow x/2+10=30, x=40^\circ$

(v) $\cos^2 30^\circ + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - 3/4$

$\Rightarrow \cos^2 x = 1/4$

$\Rightarrow \cos x = \pm 1/2$

$x=60^\circ$ or $x=120^\circ$ (**need to change**)

(vi) $\cos^2 30^\circ + \sin^2 2x=1$

$\Rightarrow 3/4 + \sin^2 2x = 1$

$\Rightarrow \sin^2 2x = 1/4$

$\Rightarrow \sin 2x = \pm 1/2$ (**need to change)**

(vii) $\sin^2 60^\circ + \cos^2(3x-9)^\circ = 1 \Rightarrow$

$\Rightarrow$ (**need to change)**

11. (i) $\sin\pi + 2\cos\pi + 3.\sin\left(\frac{3\pi}{2}\right) + 4\cos\left(\frac{3\pi}{2}\right) - 5\sec\pi - 6\operatorname{cosec}\left(\frac{3\pi}{2}\right)$

$\Rightarrow 0-2+3(-1)+0-5(-1)-6(-1)$

$\Rightarrow -2-3+5+6=6$

(ii) $\sin 0 + 2\cos 0 + 3.\sin\left(\frac{\pi}{2}\right) + 4\cos\left(\frac{\pi}{2}\right) + 5\sec 0 + 6\operatorname{cosec}\left(\frac{\pi}{2}\right)$

$\Rightarrow 0+2+3+0+5+6=16$

(iii) $4.\cot 45^\circ - \sec^2 60^\circ + \sin^2 30^\circ$

$\Rightarrow 4-4+1/4=1/4$

12. (i) $\sqrt{2}.\sin\left(\frac{\pi}{4}+\theta\right)=\cos\theta+\sin\theta$

L.H.S

$\Rightarrow \sqrt{2}[\sin\frac{\pi}{4}.\cos\theta + \cos\frac{\pi}{4}\sin\theta]$

$\Rightarrow \sqrt{2}[\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta]$

$\Rightarrow \cos\theta + \sin\theta$

(ii) $\sqrt{2}.\sin\left(\frac{\pi}{4}-\theta\right)=\cos\theta-\sin\theta$

L.H.S

$\Rightarrow \sqrt{2}[sin\frac{\pi}{4}.cos\theta - cos\frac{\pi}{4}sin\theta]$
$\Rightarrow \sqrt{2}[\frac{1}{\sqrt{2}}cos\theta - sin\theta]$
$\Rightarrow \cos\theta - sin\theta$

(iii) $\sqrt{2}.\cos\left(\frac{\pi}{4}+A\right)=\cos A-\sin A$

L.H.S

$\Rightarrow \sqrt{2}[cos\frac{\pi}{4}cosA - sinAsin\frac{\pi}{4}]$
$\Rightarrow \sqrt{2}\ [\frac{1}{\sqrt{2}}cosA - \frac{1}{\sqrt{2}}sinA]$
$\Rightarrow$ cosA-sinA

(iv) $\sqrt{2}.\cos\left(\frac{\pi}{4}-A\right)=\cos A+\sin A$

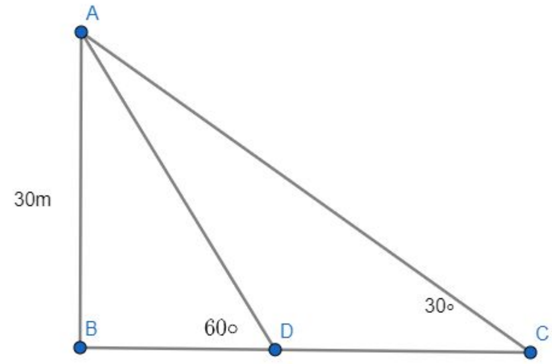
L.H.S

$\Rightarrow \sqrt{2}[cos\frac{\pi}{4}cosA + sinAsin\frac{\pi}{4}]$
$\Rightarrow \sqrt{2}\ [\frac{1}{\sqrt{2}}cosA + \frac{1}{\sqrt{2}}sinA]$
$\Rightarrow$ cosA+sinA

(v) $\tan\left(\frac{\pi}{4}+\theta\right)=\frac{1+\tan\theta}{1-\tan\theta}$

L.H.S
$\Rightarrow \frac{tan\frac{\pi}{4}+tan\theta}{1-tan\frac{\pi}{4}tan\theta}$
$\Rightarrow \frac{1+tan\theta}{1-tan\theta}$

13. (30 m in question)



AB be the building

The boy moved from point C to D ,thus angle changed from 30∘ to 60∘

In triangle ABD

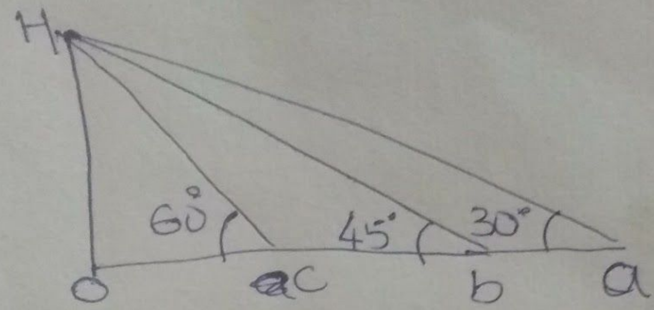
$\Rightarrow Tan\ 60^\circ = \frac{30}{BD} \Rightarrow BD = 10\sqrt{3}$

In triangle ABC

$\Rightarrow Tan\ 30 = \frac{30}{BC} \Rightarrow BC = 30\sqrt{3}$

$CD = BC - BD = 30\sqrt{3} - 10\sqrt{3} = 20\sqrt{3}$

14.



Let $OH = h$;

From triangle HOC

$\tan 60^\circ = \frac{h}{OC} \Rightarrow OC = \frac{h}{\sqrt{3}}$

Δle HOB; $\tan 45^\circ = \frac{h}{OB} \Rightarrow OB = h$

Δle HOa $\tan 30^\circ = \frac{h}{Oa} \Rightarrow Oa = h\sqrt{3}$

$ab = Oa - Ob = h(\sqrt{3}-1)$

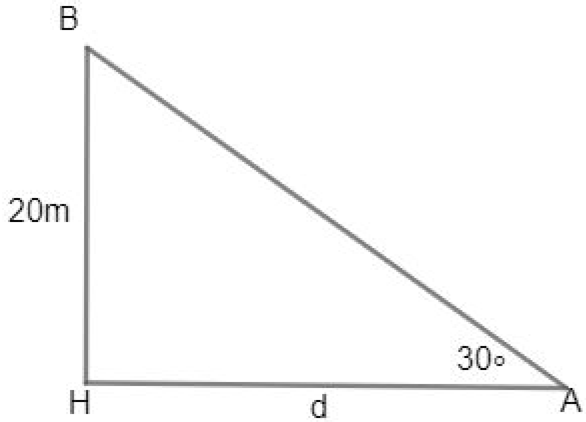
$bc = Ob - Oc = h\left(1 - \frac{1}{\sqrt{3}}\right)$

$= \frac{h}{\sqrt{3}}(\sqrt{3}-1)$

$ab : bc = h(\sqrt{3}-1) : \frac{h}{\sqrt{3}}(\sqrt{3}-1)$

$ab : bc = \sqrt{3} : 1$

15.



tan30= $\frac{p}{b} = \frac{20}{d}$

$\frac{1}{\sqrt{3}} = 20/d, d = 20\sqrt{3}$

Level 2

1. $\cos(45^\circ - A)\cos(45^\circ - B) - \sin(45^\circ - A)\sin(45^\circ - B) = \sin(A+B)$

L.H.S

⇒ cos[(45-A)+(45-B)]

⇒ cos[90-(A+B)]

⇒ Sin(A+B)

2. $\sin(45^\circ + A)\cos(45^\circ - B) + \cos(45^\circ + A)\sin(45^\circ - B) = \cos(A-B)$

L.H.S

⇒ Sin(45+A+45-B)

⇒ Sin(90+A-B)

⇒ Cos(A-B)

3. $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} = 0$

L.H.S

⇒ $\frac{sinA.cosB - sinB.cosA}{cosAcosB} + \frac{sinB.cosC - sinCcosB}{cosBcosC} + \frac{sinC.cosA - sinAcosC}{CosAcosC}$

⇒ tanA-tanB+tanB-tanC+tanC-tanA

⇒ 0

4. $\tan\left(\frac{\pi}{4} + \theta\right) \times \tan\left(\frac{3\pi}{4} + \theta\right) = -1$

L.H.S

⇒ tan($\frac{\pi}{4} + \theta$) $X\ tan(\pi - (\frac{\pi}{4} - \theta)$

⇒ tan($\frac{\pi}{4} + \theta$) X(-1) tan($\frac{\pi}{4} - \theta$)

$\Rightarrow$ -tan($\frac{\pi}{2} - \frac{\pi}{4} + \theta$) x tan($\frac{\pi}{4} - \theta$)
$\Rightarrow$ -tan($\frac{\pi}{2} - (\frac{\pi}{4} - \theta)$) x tan($\frac{\pi}{4} - \theta$)
$\Rightarrow$ -cot($\frac{\pi}{4} - \theta$) x tan($\frac{\pi}{4} - \theta$)
$\Rightarrow$ -1

5. $1 + \tan A \tan \frac{A}{2} = \tan A \cot \frac{A}{2} - 1 = \sec A$

L.H.S

$\Rightarrow$ 1 + [tan A. tan (A/2)]

= 1 + [sin A. sin (A/2) / cos A. cos (A/2)]

= [cos A. cos (A/2) + sin A. sin (A/2)] / [cos A. cos (A/2)]

= [cos (A - (A/2))] / [cos A. cos (A/2)]

= [cos (A/2)] / [cos A. cos (A/2)]

= 1 / (cos A)

6. $\cos(36^\circ - A)\cos(36^\circ + A) + \cos(54^\circ + A)\cos(54^\circ - A) = \cos 2A$

L.H.S
= 1/2 * [cos 72 + Cos 2A] + 1/2 [Cos 108 + Cos 2A]
= Cos 2A + 1/2 [cos 72 + cos 108]
= cos 2 A + 1/2 [cos 72 + Cos (180-72)]
= cos 2 A + 1/2 [cos 72 - cos 72]
= cos 2A

7. $\tan x + \cot x = 3,$

S.B.S
$\Rightarrow tan^2x + cot^2x + 2tanxcotx = 9$
$\Rightarrow tan^2x + cot^2x = 7$
S.B.S
$\Rightarrow tan^4x + cot^4x + 2tan^2xcot^2x = 49$
$\Rightarrow tan^4x + cot^4x = 47$

8. secx+tanx=k
secx-tanx=1/k [$sec^2x - tan^2x = 1$]
$\frac{k^2-1}{k^2+1} = \frac{k(k-1/k)}{k(k+1/k)}$

$\Rightarrow \frac{k-\frac{1}{k}}{k+\frac{1}{k}} = \frac{secx+tanx-secx+tanx}{secx+tanx+secx-tanx}$

$\Rightarrow \frac{2tanx}{2secx} = sinx$

9. (i) $\frac{\tan 5A - \tan 3A}{\tan 5A + \tan 3A} = \frac{\sin 2A}{\sin 8A}$

L.H.S

$\Rightarrow = \frac{\frac{sin5A}{cos5A} - \frac{sin3A}{cos3A}}{\frac{sin5A}{cos5A} + \frac{sin3A}{cos3A}}$

$= \frac{sin5A.cos3A - cos5A.sin3A}{sin5A.cos3A + cos5A.sin3A}$

$= \frac{sin(5A-3A)}{sin(5A+3A)}$

$= \frac{sin2A}{sin8A}$

(ii) $\frac{\cot A.\cot 4A + 1}{\cot A.\cot 4A - 1} = \frac{\cos 3A}{\cos 5A}$

L.H.S

$= \frac{\frac{cosA\ cos4A}{sinA\ sin4A} + 1}{\frac{cosA\ cos4A}{sinA\ sin4A} - 1}$

$= \frac{\frac{cosA\ cos4A + sinA\ sin4A}{sinA\ sin4A}}{\frac{cosA\ cos4A - sinA\ sin4A}{sinA\ sin4A}}$

$= \frac{cosA\ cos4A + sinA\ sin4A}{cosA\ cos4A - sinA\ sin4A}$

$= \frac{cos(A-4A)}{cos(A+4A)}$

$= \frac{cos(-3A)}{cos(5A)}$

$= \frac{cos\ 3A}{cos\ 5A}$

(iii) $\frac{\cot 6A - \tan 2A}{\cot 6A + \tan 2A} = \frac{\cos 8A}{\cos 4A}$

⇒ cos 6A /sin 6A -sin 2A/cos 2A

⇒ cos 6A/sin 6A+ sin 2A/cos2A

⇒ cos6A cos2A-sin6A sin2A

⇒ cos 6A cos2A+ sin 6A sin 2A

⇒ cos(6A+2A)

⇒ cos(6A-2A)

⇒ cos8A/cos2A

10. $2\cos^2 x + 7\sin x = 5,$

⇒ $2(1 - sin^2x) + 7sinx = 5$

⇒ $2sin^2x - 7sinx + 3 = 0$

⇒ $2sin^2x - 6sinx - sinx + 3 = 0$

⇒ 2sinx(sinx-3)-(sinx-3)=0

⇒ (2sinx-1)(sinx-3)=0

⇒ sinx=½,sinx=3(not possible)

11.

$\tan(30) = \frac{p}{b}$

$\frac{1}{\sqrt{3}} = \frac{h}{60}$

$h = \frac{60}{\sqrt{3}}$

$h = 20\sqrt{3}$

12.

$\tan m\angle ACB = \frac{perpendicular}{base}$

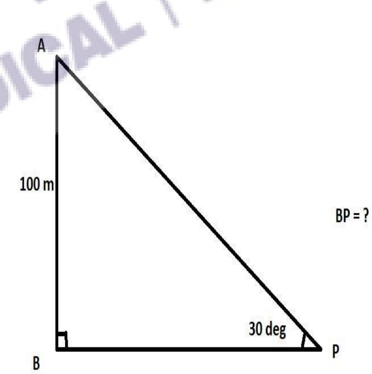
$\Rightarrow \tan 30° = \frac{AB}{BP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BP}$

$\Rightarrow BP = 100\sqrt{3}$

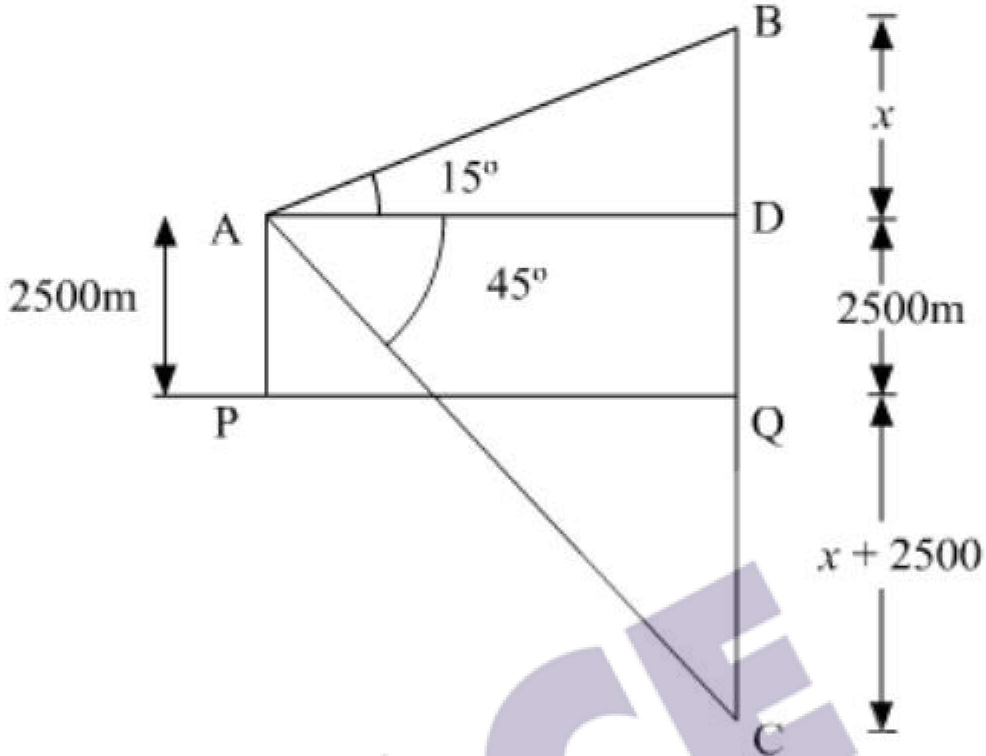
$\Rightarrow BP = 173.20$

$\Rightarrow BP = 173$ approx.



13. Incorrect q

14.



tan45= $\frac{x+5000}{AD}$,AD=X+5000…..(I)

Tan15= $\frac{x}{AD}, AD = xcot15$ …..(II)

From I & II

x= $\frac{5000}{cot15-1}$

Exercise 2

Level 2

1. $\cos^2\left(\frac{3\pi}{5}\right)+\cos^2\left(\frac{4\pi}{5}\right)$

⇒ $cos^2 108 + cos^2 144$

⇒ $sin^2 18 + cos^2 36$

⇒ $\frac{6-2\sqrt{5}}{16} + \frac{6+2\sqrt{5}}{16}$

⇒ 3/4

2. $\frac{\cos 10^o + \sin 10^o}{\cos 10^o - \sin 10^o}$

⇒ $\frac{1+tan10}{1-tan10}$

⇒ $\frac{tan45+tan10}{1-tan45tan10}$

$\Rightarrow$ tan55

3. $\tan x = \frac{b}{a}$

$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$

$\Rightarrow \sqrt{\frac{1+b/a}{1-b/a}} + \sqrt{\frac{1-b/a}{1+b/a}}$

$\Rightarrow \sqrt{\frac{1+tanx}{1-tanx}} ++ \sqrt{\frac{1-tanx}{1+tanx}}$

$\Rightarrow \frac{2}{\sqrt{1-tan^2x}}$

$\Rightarrow \frac{cosx}{\sqrt{cos2x}}$

4. Ithe roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^o$ and $\tan 15^o$,

Tan 30+tan15 =-p, tan30tan15=q

tan45=tan(30+15)= $\frac{tan30+tan15}{1-tan30tan15} = \frac{-p}{1-q}$

$\Rightarrow$ 1= $\frac{-p}{1-q}$ $\Rightarrow$ q-p=1

5. Tan 25 = x

=> tan 155 - tan 115

= tan (180-25) - tan (90+25)

= -tan 25 - (-cot 25)

= -tan 25 - (-1/tan 25)

= -x + 1/x

= $-x^2$+1/x

= $(1-x^2)$/x

=> 1+ tan 155 . tan 115

= 1 + tan (180-25) . tan (90+25)

= 1 + (-tan 25) . (-cot 25)

= 1 +tan 25 . cot 25

$= 1 + \tan 25 \,.\, 1/\tan 25$

$= 1 + x.1/x$

$= 1 + 1$

$= 2$

$$\frac{\tan 155^o - \tan 115^o}{1 + \tan 155^o \tan 115^o}$$

$= (1 - x^2)/2x$

6.

$\sin\left(120^o - \alpha\right) = \sin\left(120^o - \beta\right)$, $0 < \alpha, \beta < \pi,$

$\Rightarrow sin(180 - 60 - \alpha) = \sin(120 - \beta)$, $\Rightarrow \sin(120 - \alpha) = sin(120 - \beta)$

$\Rightarrow 60 + \alpha = 120 - \beta, \alpha + \beta = 60$ $\Rightarrow \alpha = \beta$

7. $15\sin^4 x + 10\cos^4 x = 6,$

$\Rightarrow 15tan^4x + 10 = 6sec^4x$

$\Rightarrow 15tan^4x + 10 = 6(1 + tan^2x)^2$

$\Rightarrow 9tan^4x - 12tan^2x - 6 = 0$

$\Rightarrow 9a^2 - 12a + 4 = 0$

a=tan^2x=2/3

8. $2\sin^2\theta + \sin^2 2\theta = 2; 0 \le \theta \le \pi/2$

$\Rightarrow sin^2 2\theta = 2cos^2\theta$

$\Rightarrow 4sin^2\theta cos^2\theta = 2cos^2\theta$

$\Rightarrow sin^2\theta = 1/2$

$\Rightarrow sin\theta = \pm \frac{1}{\sqrt{2}}, \theta = \pi/4$

also, $\theta = \pi/2$ *satisfy the condition*

sum=3 $\pi/4$

9. $\tan 203^o + \tan 22^o + \tan 203^o \tan 22^o =$

$\Rightarrow tan(225) = tan(203 + 22)$

$\Rightarrow 1 = \frac{tan203 + tan22}{1 - tan203tan22}$

$\Rightarrow 1 = tan203 + tan22 + tan203tan22$

10. $\sin 30^o = k$,k=1/2

$\Rightarrow cosx = 1 - 2k^2 = 1/2$

x=60 or 300

$\alpha = 60, \beta = 300$

11. Need to change

12. cos132 + cos 12 + cos 84 + cos 156

=2cos72cos60+2cos120cos36

=2{(root5-1)/4}1/2 + 2(-1/2)(root5+1)/4

=-1/2

13. $\frac{\sin 3\theta - \cos 3\theta}{\sin\theta + \cos\theta} + 1 =$

$\Rightarrow \frac{3sin\theta - 4sin^3\theta - 4cos^3\theta + 3cos\theta}{sin\theta + cos\theta} + 1$

$\Rightarrow 3 - 4(1 - sin\theta\cos\theta) + 1$

$\Rightarrow 4\sin\theta cos\theta = 2\sin 2\theta$

14. $\cos A = m\cos B$,

using componendo dividendo ie if a/b = c/d then (a+b)/(a-b)= (c+d)/(c-d)

so (CosA + CosB)/(CosA - CosB)=(m +1)/(m -1)

using formulas

[2Cos {(A + B)/2}Cos{(B - A)/2}]/[2Sin{(B- A)/2}Sin{(A+ B)/2}]=(m +1)/(m -1)

Cos{(A + B)/2}/Sin{(A + B)/2}={(m +1)/(m -1)} × Sin{(B - A)/2}/Cos {(B - A)/2}

we know that cosθ/sinθ = cotθ and sinθ/cosθ= tanθ

so we get

Cot{(A + B)/2}={(m +1)/(m -1)} Tan{(B - A)/2}

15. $\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta} =$

$\Rightarrow \sin 3\theta + sin5\theta + sin7\theta + sin9\theta$ $\Rightarrow cos3\theta + cos5\theta + cos7\theta + cos9\theta$

$\Rightarrow$ (sin3 θ +sin9 θ)+(sin5 θ +sin7 θ) $\Rightarrow$ (cos3 θ +cos9 θ)+(cos5 θ +cos7 θ)

$\Rightarrow 2\sin 6\theta \cos 3\theta + 2\sin 6\theta \cos\theta$

$\Rightarrow 2sin6\theta(cos3\theta + cos\theta)$

$\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta} =$

$\Rightarrow tan6\theta$

$\Rightarrow 2cos6\theta cos3\theta + 2cos6\theta cos\theta$

$\Rightarrow 2\cos 6\theta (\cos 3\theta + \cos\theta)$

16. $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$

$\Rightarrow cot(\frac{\pi}{2} - \frac{\pi}{4} + \theta)cot(\frac{\pi}{4} - \theta)$

$\Rightarrow \tan(\frac{\pi}{4} - \theta)\cot(\frac{\pi}{4} - \theta)$

$=1$

17. $\sin\theta + \cos\theta = 1$

S.B.S

$\Rightarrow sin^2\theta + cos^2\theta + 2sin\theta cos\theta = 1$

$\Rightarrow \sin 2\theta = 0$

18. $\sin 1^\circ < sin1$ ($1^c = 180/\pi$)

19. $\cos\ \theta = x + \frac{1}{x}$

$x + 1/x \geq 2$

$\cos\theta \neq 2,$ *no values of* θ

Level 2

1. $\sin 50^o - \sin 70^o + \sin 10^o =$

$\Rightarrow - 2cos60sin10 + sin10$

$\Rightarrow - sin10 + sin10 = 0$

2. $A.M \geq G.M$

3. $\sqrt{3}\operatorname{cos ec}20^o - \sec 20^o$

$2/\sin20\cos20(\sqrt{3}/2cos20 - 1/2\ sin20)$

4cosec40(sin40)=4

4. $\sin^2 x = \frac{a^2+b^2}{2ab}$,

$0 \le sin^2x \le 1$

$0 \le \frac{a^2+b^2}{2ab} \le 1$

$\Rightarrow a^2+b^2=0, \quad \Rightarrow a^2+b^2-2ab=0$

a=b=0 $\quad \Rightarrow a=b$

5. $\tan^2\theta = (1-e^2)$,

$Tan^2\theta=1-a^2$

LHS

$=\sec\theta+\tan^3\theta\text{cosec}\theta$

$=\sqrt{(1+\tan^2\theta)}+\tan^2\theta\times\tan\theta\times\sqrt{(1+\cot^2\theta)}$

[∵, $\sec^2\theta-\tan^2\theta=1$ and $\text{cosec}^2\theta-\cot^2\theta=1$]

$=\sqrt{1+(1-a^2)}+(1-a^2)\times\sqrt{(1-a^2)}\times\sqrt{\{1+(1/\tan^2\theta)\}}$

$=\sqrt{(2-a^2)}+(1-a^2)\times\sqrt{(1-a^2)}\times\sqrt{\{1+1/(1-a^2)\}}$

$=\sqrt{(2-a^2)}+(1-a^2)\times\sqrt{(1-a^2)}\times\sqrt{\{(1-a^2+1)/(1-a^2)\}}$

$=\sqrt{(2-a^2)}+(1-a^2)\times\sqrt{(2-a^2)}$

$=\sqrt{(2-a^2)}\times(1+1-a^2)$

$=\sqrt{(2-a^2)}\times(2-a^2)$

$=(2-a^2)^{1/2+1}$

$=(2-a^2)^{3/2}$

=RHS (Proved)

6. $\tan\theta+\sin\theta=m \quad \tan\theta-\sin\theta=n$,

$\Rightarrow 2tan\theta=m+n, \quad \Rightarrow 2sin\theta=m-n$

mn$=tan^2\theta-sin^2\theta=sin^4\theta/cos^2\theta$,

$\sqrt{mn}=sin^2\theta/cos\theta=tan\theta sin\theta$

$\Rightarrow m^2 - n^2 = (m-n)(m+n)$

$\Rightarrow m^2 - n^2 = 4tan\theta sin\theta = 4\sqrt{mn}$

7. $(\tan x + \sec x)(\tan y + \sec y)(\tan z + \sec z) = (\sec x - \tan x)(\sec y - \tan y)(\sec z - \tan z) = K,$

$\Rightarrow$ (tanx+secx)(tany+secy)(tanz+secz)=(secx-tanx)(secy-tany)(secz-tanz)

$\Rightarrow 1 = (secx - tanx)^2 (secy - tany)^2 (secz - tanz)^2$

$\Rightarrow \pm 1 = (secx - tanx)(secy - tany)(secz - tanz) = k$

8. $a\cos^3\theta + 3a\cos\theta\sin^2\theta = m \quad a\sin^3\theta + 3a\cos^2\theta\sin\theta = n,$

$m + n = a(cos^3\theta + Sin^3\theta + 3sin\theta\ Cos\theta(cos\theta + sin\theta))$

$= a((cos\theta + sin\theta)(cos^2 + sin^2\theta - sin\theta cos\theta) + 3sin\theta cos\theta(cos\theta + sin\theta))$

$= a(cos\theta + sin\theta)(cos^2\theta + sin^2\theta + 2sin\theta\cos\theta)$

$m - n = a(cos^3\theta - Sin^3\theta + 3sin\theta\ Cos\theta(sin\theta - cos\theta))$

$= a((cos\theta - sin\theta)(cos^2 + sin^2\theta + sin\theta cos\theta) - 3sin\theta cos\theta(cos\theta - sin\theta))$

$= a(cos\theta - sin\theta)(cos^2\theta + sin^2\theta - 2sin\theta\cos\theta)$

$a(cos\theta - sin\theta)^3$

$(m+n)^{2/3} + (m-n)^{2/3} =$

$a^{2/3}((cos\theta + sin\theta)^2 + (cos\theta - sin\theta)^2)$

$= a^{2/3}(1 + 2sin\theta cos\theta + 1 - 2sin\theta cos\theta)$

$2a^{2/3}$

9. $\theta = \frac{11}{3}\pi, \ (\cos\theta - \sin\theta)$

$\Rightarrow \cos(4\pi - \pi/3) - \sin(4\pi - \pi/3)$

$\Rightarrow cos(\pi/3) + sin(\pi/3)$

$\Rightarrow 1/2 - \sqrt{3}/2$

10. $\sin^2\alpha\cos^2\beta + \cos^2\alpha\sin^2\beta + \sin^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta$

$\Rightarrow sin^2\alpha(1 - sin^2\beta) + (1 - sin^2\alpha)sin^2\beta + sin^2\alpha sin^2\beta + (1 - sin^2\alpha)(1\text{-}\sin\text{\textasciicircum}2\,\beta)$

$\Rightarrow sin^2\alpha - sin^2\alpha sin^2\beta + sin^2\beta - sin^2\alpha sin^2\beta + sin^2\alpha sin^2\beta + 1 - sin^2\alpha - sin^2\beta + sin^2\alpha sin^2\beta$

$\Rightarrow 1$

11. $\tan\theta = \frac{a}{b},$

$b\cos 2\theta + a\sin 2\theta$

$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$ and $\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

$$LHS = a(\frac{2\tan\theta}{1+\tan^2\theta}) + b(\frac{1-\tan^2\theta}{1+\tan^2\theta})$$

Substitute, $\tan\theta = \frac{a}{b}$

$$LHS = a(\frac{2(\frac{a}{b})}{1+(\frac{a}{b})^2}) + b(\frac{1-(\frac{a}{b})^2}{1+(\frac{a}{b})^2})$$

$$LHS = a(\frac{\frac{2a}{b}}{\frac{b^2+a^2}{b^2}}) + b(\frac{\frac{b^2-a^2}{b^2}}{\frac{b^2+a^2}{b^2}})$$

$$LHS = a(\frac{2a}{b} \times \frac{b^2}{b^2+a^2}) + b(\frac{b^2-a^2}{b^2} \times \frac{b^2}{b^2+a^2})$$

$$LHS = \frac{2a^2b}{b^2+a^2} + \frac{b^3-a^2b}{b^2+a^2}$$

$$LHS = \frac{2a^2b+b^3-a^2b}{b^2+a^2}$$

$$LHS = \frac{a^2b+b^3}{b^2+a^2}$$

$$LHS = \frac{b(a^2+b^2)}{b^2+a^2}$$

$$LHS = b$$

12. $(tanx - 1)^2 = 0$

tanx=1, x=45∘

13. $\sqrt{\frac{1+\sin x}{1-\sin x}}$

$\Rightarrow \sqrt{\frac{(1-sinx)^2}{1-sin^2x}}$

$\Rightarrow \frac{1-sinx}{cosx}$

$\Rightarrow secx - tanx$

14. $\sin^3\alpha(1+\cot\alpha)+\cos^3\alpha(1+\tan\alpha)$

(sin x)^3*(1+cot x)+(cos x)^3*(1+tan x)=sin x+cos x [To be proved]

LHS= (sin x)^3*(1+cot x)+(cos x)^3*(1+tan x)

[Replacing tan x by (sinx /cos x) and cot x by (cos x/sin x)]

[(sin x)^3*(1+(cos x/sin x))]+[(cos x)3*(1+(sinx /cos x))]

[(sin x)^3*(sin x+cos x)/sin x]+[(cos x)3*(sin x+cos x)/cos x]

(sin x)^2*(sin x+cos x)+(cos x)^2*(sin x+cos x)

[Taking (sin x+cos x) as common]

we obtain,

(sin x+cos x)*((sin x)^2+(cos x)^2)

[we know that ((sin x)^2+(cos x)^2)=1

(sin x+cos x)=RHS

15. $A+B+C=\pi$

$\cos A=\cos B\cos C$

$\Rightarrow cos(\pi - B - C) = cosBcosC$

$\Rightarrow$ -cos(B+C)=cosBcosC

$\Rightarrow cosBcosC - sinBsinC =- cosBcosC$

$\Rightarrow 1 - tanBtanC =- 1,\ tanBtanC = 2$

$\Rightarrow cotBcotC = 1/2$

16. $\tan\frac{2\pi}{5}-\tan\frac{\pi}{15}-\sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}$

$\Rightarrow tan(\frac{2\pi}{5} - \frac{\pi}{15)} = \frac{tan\frac{2\pi}{5}-tan\frac{\pi}{15}}{1+tan\frac{2\pi}{5}tan\frac{\pi}{15}}$

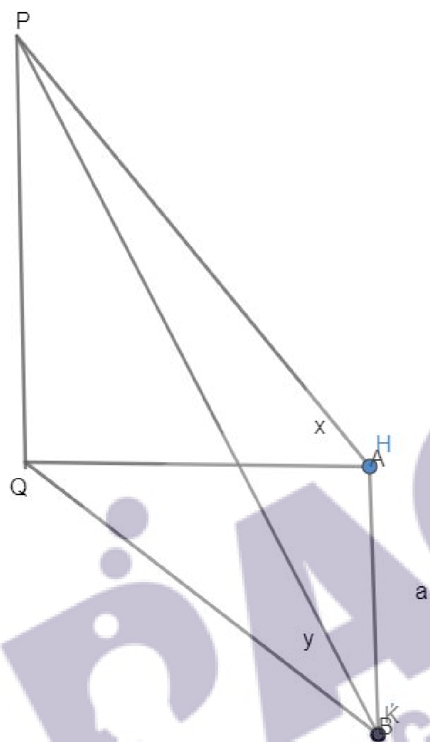
$\Rightarrow tan\frac{\pi}{3} = \frac{tan\frac{2\pi}{15} - tan\frac{\pi}{15}}{1+tan\frac{2\pi}{5} tan\frac{\pi}{15}}$

$\Rightarrow \sqrt{3} = \frac{tan\frac{2\pi}{15} - tan\frac{\pi}{15}}{1+tan\frac{2\pi}{5} tan\frac{\pi}{15}}$

$\sqrt{3} + \sqrt{3}tan\frac{2\pi}{5} tan\frac{\pi}{15} = tan\frac{2\pi}{15} - tan\frac{\pi}{15}$

$\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15} = \sqrt{3}$

17.



tanx= $\frac{PQ}{AQ}$, $AQ = PQcotx$

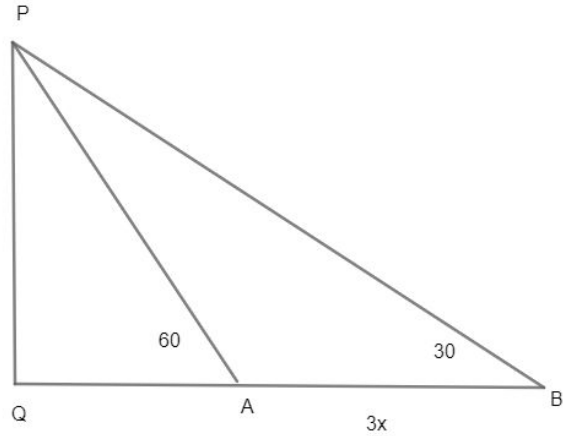
tany= $\frac{PQ}{BQ}$, $BQ = PQcoty$

$BQ^2 = AB^2 + AQ^2 = a^2 + PQ^2cot^2x$

$PQ^2cot^2y - PQ^2cot^2x = a^2$

PQ= $\frac{a}{\sqrt{cot^2x - xot^2y}}$

18. Let the speed of car=xm/min



tan60= $\frac{PQ}{AQ}$,PQ= $\sqrt{3}AQ$

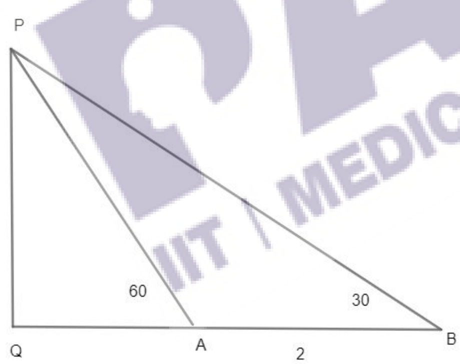
tan30= $\frac{PQ}{BQ}, PQ = BQ/\sqrt{3}$

$\Rightarrow \sqrt{3}AQ = (AQ + 3x)/\sqrt{3}$

$\Rightarrow 2AQ = 3x$

Time taken by car to reach foot= $\frac{3x/2}{x} = 1.5\ mins$

19.



tan60= $\frac{PQ}{AQ}, tan30 = \frac{PQ}{BQ}$

$\Rightarrow AQ = \frac{1}{\sqrt{3}}PQ,$

BQ= $\sqrt{3}PQ$

$\Rightarrow BQ = \sqrt{3}PQ$

$\Rightarrow AB + AQ = \sqrt{3}PQ$

$\Rightarrow 2 + PQ/\sqrt{3} = \sqrt{3}PQ,$

$\Rightarrow \frac{2}{\sqrt{3}PQ} = 2, PQ = \sqrt{3}$

20. $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

tan2A= $\frac{2tanA}{1-tan^2A}$ = $\frac{1}{1-1/4}$ = 4/3

$\Rightarrow$ tan(2A+B)= $\frac{tan2A+tanB}{1-tan2AtanB}$

$\Rightarrow \frac{4/3 + 1/3}{1-4/3.1/3}$

$\Rightarrow 3$