Binomial Theorem

EXERCISE - 1 [A]

1. (b)

$S=\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} 2^m$

$\Rightarrow S=(x-3+2)^{100}=(x-1)^{100}$

Now, coefficient of x^{53} in S, $(x-1)^{100}$ is $(-1)^{53}$

${}^{100}C_{53}$

2. (b)

Let $E=(\sqrt{5}+1)^5-(\sqrt{5}-1)^5$

$\Rightarrow E=2\left\{{}^5C_1(\sqrt{5})^4+{}^5C_3(\sqrt{5})^2+{}^5C_3(\sqrt{5})^0\right\}$

$\Rightarrow E=2\{125+50+1\}=352$

3. (a)

Since, coefficient of $(3r)^{\text{th}}$ term in $(1+x)^{2n}$ equals coefficients of $(r+2)^{\text{th}}$ term in $(1+x)^{2n}$.

$\therefore\ {}^{2n}C_{3r-1}={}^{2n}C_{r+1}$

$\Rightarrow 3r-1+r+1=2n$

$\therefore\ r=\frac{n}{2}$

4. (c)

$t_{r+1}={}^9C_r\left(\frac{4}{3}x^2\right)^{9-r}\left(\frac{-3}{2x}\right)^r$

$\Rightarrow t_{r+1}={}^9C_r\left(\frac{4}{3}x^2\right)^{9-r}\left(\frac{-3}{2x}\right)^r$

$\Rightarrow t_{r+1}={}^9C_r\,\frac{2^{18-3r}}{3^{9-2r}}(-1)^r\,x^{18-3r}$

For term independent of x, we get

$18-3r=0$

$r=6$

5. (b)

$$\frac{n(n-1)}{2!}x^2 = \frac{-1}{8}x^2$$

$$\Rightarrow n(n-1) = -\frac{1}{4}$$

$$\Rightarrow 4n^2 - 4n + 1 = 0$$

$$\therefore n = \frac{1}{2}$$

6. (d)

$$t_{n+1} = {}^nC_n\left(2^{\frac{1}{3}}\right)^{n-n}\left(\frac{-1}{\sqrt{2}}\right)^n$$

$$\Rightarrow \left(\frac{1}{3^{3^{\frac{2}{3}}}}\right)^{\log_3 8} = (-1)^n\, 2^{\left(\frac{-n}{2}\right)}$$

$$\Rightarrow 3^{\frac{-5}{3}\log_3 8} = (-1)^n\, 2^{\left(\frac{-n}{2}\right)}$$

$$\Rightarrow \frac{1}{32} = \frac{(-1)^n}{2^{\frac{n}{2}}}$$

$$\therefore n = 10$$

Now, $t_5 = {}^{10}C_4\left(2^{\frac{1}{3}}\right)^{10-4}\dfrac{(-1)^4}{\left(2^{\frac{1}{2}}\right)^4} \Rightarrow t_5 = {}^{10}C_4$

7. (a)

$$t_5 + t_6 = 0 \Rightarrow {}^nC_4(a)^{n-4}b^4 - {}^nC_5(a)^{n-5}b^5 = 0$$

$$\Rightarrow {}^nC_4\,a = {}^nC_5\,b$$

$$\Rightarrow \frac{a}{b} = \frac{{}^nC_5}{{}^nC_4} = \frac{n-4}{5}$$

8. (a)

General term, $t_{r+1} = {}^{4n-2}C_r\,(i)^r\,x^r$

$r = 2, 6, \ldots\ldots, 4n-2$

$\therefore$ n terms

9. (a)

Coefficient of t^{32} in $\left(1+t^{12}\right)\left(1+t^{24}\right)\left(1+t^2\right)^{12}$

$\Rightarrow$ Coefficient of t^{32} in $\left(1+r^{12}+t^{24}+t^{36}\right)\left(\sum_{r=0}^{12}{}^{12}C_r\, t^{2r}\right)$

$\Rightarrow$ Coefficient of t^{32} is ${}^{12}C_{10}+{}^{12}C_4=561$

10. (c)

$E=\left(1-2x^3+3x^5\right)\left(\sum{}^8C_r x^{-r}\right)$

Coefficient of x in e is $(-2)\,{}^8C_2+3\,{}^8C_4=154$

11. (a)

General term, $t_{r+1}={}^{15}C_r\, 2^{\frac{1}{2}(15-r)}3^{\frac{1}{2}r}$

Now, $r\in w$ and $0\le r\le 15$

Also, $15-r$ is EVEN and r is EVEN

$\Rightarrow$ r is ODD and r is EVEN

$\Rightarrow \therefore\ e\in\phi$

Hence, no radical term exist in the given expansion $\Rightarrow$ all terms are irrational.

$\therefore$ Number of irrational terms = 16

12. (a)

General form, $t_{r+1}={}^{55}C_r\, x^{\frac{1}{5}(55-r)}y^{\frac{1}{10}r}$

$r\in w; 0\le r\le 55$

$\therefore$ $55-r$ is a multiple of 5 and r is a multiple of 10

$\Rightarrow r=0,10,20,30,40,50$

$\therefore$ 6 terms are rational.

13. (a)

$\Rightarrow 2\,{}^nC_5={}^nC_4+{}^nC_6$

$\Rightarrow \dfrac{{}^nC_4}{{}^nC_5}+\dfrac{{}^nC_6}{{}^nC_5}=2$

$\Rightarrow \dfrac{5}{n-4}+\dfrac{n-5}{6}=2$

$\Rightarrow n^2-21n+108=0$

$\Rightarrow (n-9)(n-12)=0$

$\therefore n=9,12$

14. (a)

${}^5C_2\, x^3 x^{2\log_{101}x}=10^6 \Rightarrow x^{3+2\log_{10}x}=10^5$

$\therefore 3\log_{10}x+2\left(\log_{10}x\right)^2-5=0$

$\Rightarrow 2\left(\log_{10}x\right)^2+5\log_{10}x-2\left(\log_{10}x\right)-5=0$

$\Rightarrow \log_{10}x=\dfrac{-5}{2},1$

$\therefore\ x = 10^{\frac{-5}{2}}, 10$

15. (d)

$${}^8C_5 \frac{1}{x^8} x^{10} (\log_{10} x)^5 = 5600 \Rightarrow x^2 (\log_{10} x)^5 = 100$$

$\therefore\ x = 10$

16. (a)

$$t_3 = {}^nC_2\, 2^{x(n-2)} \left(\frac{1}{4^x}\right);\ t_2 = {}^nC_1\, 2^{x(n-1)} \left(\frac{1}{4^x}\right)^1$$

$$\frac{t_3}{t_2} = 7 \Rightarrow \frac{{}^nC_2}{{}^nC_1} \frac{2^{x(n-2)}}{2^{x(n-1)}} \frac{\left(\frac{1}{4^x}\right)^2}{\left(\frac{1}{4^x}\right)^1} = 7$$

$$\Rightarrow \frac{n-1}{2} \frac{1}{2^x} \left(\frac{1}{4}\right)^x = 7$$

Also, ${}^nC_1 + {}^nC_2 = 36 \Rightarrow n^2 + 2n - 72 = 0$

$\Rightarrow n = 8$

$\therefore\ 2^{3x+1} = 2^0$

$$\Rightarrow x = \frac{-1}{3}$$

17. (d)

$$(1+x)^{131} (1-x+x^2)^{130} = (1+x)(1+x^3)^{130}$$

Now each term in the expansion of $(1+x^3)^{130}$ will be of type x^{3r}.

Hence in $(1+x)(1+x^3)^{130}$ terms will be of type x^{3x} and x^{3r+1}.

But no number of type $3r$ and $3r+1$ can be 65.

Hence coefficient of $x^{65} = 0$.

18. (c)

We can write it as

$$(1+x)\left((1+x)(1-x+x^2)\right)^{100}$$

$$(1+x)(1+x^3)^{100}$$

$$(1+x)\left({}^{100}C_0 + {}^{100}C_1 x^3 + \ldots .{}^{100}C_{100} x^{300}\right)$$

Clearly, This multiplication we can't get power of form $3r+2$.

19. (d)

$$(1+0.0001)^{10000} = \left(1+\frac{1}{10^4}\right)^{10^4} = 1 + {}^{10^4}C_1 \frac{1}{10^4} + {}^{10^4}C_2 \frac{1}{10^8} + {}^{10^4}C_3 \frac{1}{10^{12}} + \ldots .. + {}^{10^4}C_{10^4} \frac{1}{10^{10^4}}$$

$$(1+0.0001)^{10000}=1+1+\frac{9999}{10^4\times 2!}+\frac{9999\times 9998}{10^8\times 3!}+...+\frac{1}{10^{10^4}}$$

Clearly except the first two terms all the rest are non integers

So, the positive integer just greater than $(1+0.0001)^{10000}$ is 3.

20. (b)

Coefficient of x^{-1} in $(1+x)^n\left(1+\frac{1}{x}\right)^n$

= Coefficient of x^{-1} in $\frac{(1+x)^{2n}}{x^n}$

= Coefficient of x^{n-1} in $(1+x)^{2n}$

$= {}^{2n}C_{n-1}$

$=\frac{(2n)!}{(n-1)!(n+1)!}$

21. (c)

$101^{100}-1$

$\Rightarrow\ (1+100)^{100}-1$

$\Rightarrow\ {}^{100}C_0+{}^{100}C_1 100+{}^{100}C_2 100^2+.......+{}^{100}C_{100}(100)^{100}-1$

$\Rightarrow\ (100)^2\left[1+{}^{100}C_2+.....\right]$

22. (c)

$${}^9C_4(2a)^5\left(\frac{a^2}{5}\right)^4;-{}^9C_5(2a)^4\left(\frac{a^2}{4}\right)^5$$

23. (a)

$$t_3<t_4>t_5\ \Rightarrow\ {}^{10}C_2(2)^8\left(\frac{3}{8}|x|\right)^2<{}^{10}C_3(2)^7\left(\frac{3}{8}|x|\right)^3>{}^{10}C_4(2)^6\left(\frac{3}{8}|x|\right)^4$$

$$\Rightarrow\ {}^{10}C_2\,2\left(\frac{3}{8}|x|\right)^{-1}<{}^{10}C_3>{}^{10}C_4\,\frac{1}{2}\left(\frac{3}{8}|x|\right)$$

$\Rightarrow\ \frac{{}^{10}C_2}{{}^{10}C_3}2\cdot\frac{8}{6}<\lvert x\rvert$	$\frac{{}^{10}C_3}{{}^{10}C_4}2\cdot\frac{8}{3}>\lvert x\rvert$
$\Rightarrow\ \frac{3}{10-25}\cdot 2\cdot\frac{8}{3}<\lvert x\rvert$	$\frac{4}{10-3}\cdot 2\cdot\frac{8}{3}>\lvert x\rvert$
$\therefore\ \lvert x\rvert>2$	$\lvert x\rvert<\frac{64}{21}$

$$\Rightarrow\ 2<|x|<\frac{64}{21}$$

24. (b)

Let t_{r+1} is the greater term, then $t_{r+1} > t_r$

$\Rightarrow\ {}^nC_r\left(2x\right)_{n-r}\ 7^r > {}^nC_{r-1}\left(2x\right)^{n-r+1}\ 7^{r-1}$

$\Rightarrow\ \dfrac{{}^nC_r}{{}^nC_{r-1}}\ 7 > 2x$

$\Rightarrow\ \dfrac{n-r+1}{r}7 > 6$

$\Rightarrow\ 7\left(n+1\right) > 13r$

$\Rightarrow r < \dfrac{77}{13}$

$\therefore$ 6^{th} terms the greatest terms.

25. (b)

$\left(1+2\right)^n = 6561$

$\Rightarrow\ n = 8$

$\therefore$ Greatest term is 5^{th} term.

26. (d)

Let $P = 5^{99} = 5\times 5^{98} = 5(25)^{49} = 5(26-1)^{49}$

$= 5[{}^{49}C_0(26)^{49} - {}^{49}C_1(26)^{48} + {}^{49}C_2(26)^{47} - ..... + {}^{49}C_{48}(26) - {}^{49}C_{49}.1]$

$= 5\times 26k - 5$ when k is an integer

$\therefore \dfrac{P}{13} = 10k - \dfrac{5}{13} = 10k - 1 + \dfrac{8}{13}$

Hence, the remainder is 8

27. (b)

Now, $\dfrac{3^{2003}}{28} = \dfrac{3^2\times 3^{2001}}{28} = \dfrac{9}{28}(3^3)^{667} = \dfrac{9}{28}(28-1)^{667}$

$= \dfrac{9}{28}\{(28)^{667} - {}^{667}C_1(28)^{666} + {}^{667}C_2(28)^{665} - ..... + {}^{667}C_{666}(28) - 1\}$

$= 9k - \dfrac{9}{28}$ where k is an integer

$(9k-1) + \dfrac{19}{98}$

Or $\left\{\dfrac{3^{2003}}{28}\right\} = \left\{(9k-1) + \dfrac{19}{28}\right\} = \dfrac{19}{28}$

28. (c)

Write $\left(23\right)^{14} = \left(20+3\right)^{14}$ and see last two digit.

29. (b)

Write $\left(3\right)^{100} = \left(9\right)^{50} = \left(10-1\right)^{50}$ and expand.

30. (b)

Write $3^{37} = 3\cdot\left(3^4\right)^9 = 3\cdot(80-1)^9$ and expand.

31. (a)

$$1+(1+x)+\left(1+x^2\right)+.....+(1+x)^n = \frac{(1+x)^{n+1}-1}{x}$$

Coefficient of x^k is ${}^{n+1}C_{k+1}$

32. (d)

$$S = \sum_{r=0}^{n}(2r+1)\ {}^{n}C_r$$

$$S = 2\sum_{r=0}^{n} n^{n-1}C_{r+1} + \sum_{r=0}^{n} {}^{n}C_r = 2\cdot 2^{n-1} + 2^n$$

$$= (n+1)2^n$$

33. (c)

Coefficient of x^0 in $(1+x)^n\left(1+\frac{1}{x}\right)^n = {}^{2n}C_n$

Coefficient of x^0 in $(1+x)^n\left(1-\frac{1}{x}\right)^n = 0$

$$\therefore\ C_1^2 + C_3^2 + ... + C_n^2 = \frac{{}^{2n}C_n}{2} = \frac{(2n)!}{(n!)^2\,2}$$

34. (a)

Use ${}^{n}C_r = {}^{n}C_{n-r}$

${}^{n-1}C_{n-1} + {}^{n}C_{n-1} + {}^{n+1}C_{n-1} + {}^{n+2}C_{n-1} + ... + {}^{n+r-1}C_{n-r}$

$= {}^{n+r}C_n$

35. (a)

Clearly it is van denounced 2

Rewrite the caprenian as ${}^{n}C_n \cdot {}^{n}C_r + {}^{n}C_{n-1}\,{}^{n}C_{r+1} + ...$ By vandermont 2

${}^{2n}C_{n+r}$

36. (a)

$(n_1-n_2)(n_1-n_2-1) = 30$ ...(1)

$(n_1+n_2)(n_1+n_2-1) = 90$ ...(2)

$\therefore\ n_1 = 8$ and $n_2 = 2$

37. (d)

$${}^{n-1}C_r = \left(k^2-3\right)\frac{n}{r+1}\,{}^{n-1}C_r$$

$k^2 - 3 = \frac{r+1}{n}$

I: $k^2 - 3 > 0$

II: $k^2 - 3 < 1$

38. (c)

$^nC_r \cdot r! = 5040\left(^{n-1}C_{r-1} + ^{n-1}C_r \right) \Rightarrow ^nC_r \cdot r! = 5040 \cdot ^nC_r$

$\Rightarrow r = 7$

39. (a)

$^{35}C_8 + \sum_{r=1}^{7} {}^{42-r}C_7 + \sum_{r=1}^{5} {}^{47-r}C_{40-r}$

$\Rightarrow {}^{35}C_8 + \left(^{41}C_7 + ^{42}C_7 + ^{43}C_7 + ... + ^{35}C_7 \right) + \left(^{46}C_7 + ^{45}C_7 + ^{44}C_7 + ... + ^{42}C_7 \right)$

By $^{n-1}C_{r-1} + ^{n-1}C_r = ^nC_r$

$^{35}C_8 + ^{35}C_7 + ^{36}C_7 + ... + ^{41}C_7 = ^{42}C_8$ & $^{42}C_8 + ^{42}C_7 + ^{43}C_7 + ... + ^{46}C_7 = ^{47}C_8$

40. (b)

$\sum_{r=0}^{49} {}^{50}C_r \cdot {}^{50}C_{r+1} = \sum_{r=0}^{49} {}^{50}C_{50-r} \cdot {}^{50}C_{r+1}$

$= {}^{100}C_{51}$

41. (b)

Put the values to get answer.

42. (c)

$(1+x)^{21} + (1+x)^{22} + ... + (1-x)^{20}$

$= (1+x)^{21}\left[\frac{(1+x)^{10} - 1}{(1+x) - 1}\right] = \frac{1}{x}\left[(1+x)^{31} - (1+x)^{21}\right]$

$\Rightarrow$Coefficient of x^5 in the given expression

$=$Coefficient of x^5 in $\left\{\frac{1}{x}\left[(1+x)^{31} - (1+x)^{21}\right]\right\}$

= Coefficient of x^6 in $[(1 + x)^{31} – (1 + x)^{21}]$

$= {}^{31}C_6 - {}^{21}C_6$

43. (b)

$\frac{1}{n!}\left[\frac{n!}{1!(n-1)!} + \frac{n!}{3!(n-3)!} + ...\right] = \frac{1}{n!}\left(^nC_1 + ^nC_3 + ...\right)$

$= \frac{1}{n!} 2^{n-1}$

44. (a)

We know that

$$(1-1)^{20} = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} - {}^{20}C_{11} + {}^{20}C_{12} - .... + {}^{20}C_{20} = 0$$

$$2({}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9) + {}^{20}C_{10} = 0 \quad \left[\because {}^{20}C_{20} = {}^{20}C_0, {}^{20}C_{19} = {}^{20}C_1\ etc\right]$$

$$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9 + {}^{20}C_{10}$$

$$= -\frac{1}{2}\,{}^{20}C_{10} + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$$

45. (a)

$$\because (1+x)^m + (1+x)^{m+1} + ... + (1+x)^n$$

$$= \frac{(1+x)^m\{(1+x)^{n-m+1} - 1\}}{(1+x)-1} = \frac{(1+x)^{n+1} - (1+x)^m}{x}$$

$\therefore$ Coefficient of x^m in

$(1+x)^m + (1+x)^{m+1} + ... + (1+x)^n$

Or coefficient of x^m in $\dfrac{(1+x)^{n+1} - (1+x)^m}{x}$

Or coefficient of x^{m+1} in $(1+x)^{n+1} - (1+x)^m$

$$= {}^{n+1}C_{m+1} - 0 = {}^{n+1}C_{m+1}$$

46. (c)

We have $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + ... + {}^{20}C_{20} x^{20}$

On dividing by x, we get

$$\frac{(1+x)^{20}}{x} = \frac{{}^{20}C_0}{x} + {}^{20}C_1 + {}^{20}C_2 x + {}^{20}C_3 x^2 + ... + {}^{20}C_{20} x^{19}$$

On differentiating w.r.t. x, we get

$$\frac{20(1+x)^{19}.x - (1+x)^{20}}{x^2} = \frac{-{}^{20}C_0}{x^2} + 0 + {}^{20}C_2 + 2.{}^{20}C_3 x + ... + 19.\,{}^{20}C_{20} x^{18}$$

On putting x = 1, we get

$$20(2)^{19} - (2)^{20} = -\frac{1}{1} + {}^{20}C_2 + 2.{}^{20}C_3 + ... + 19.\,{}^{20}C_{20}$$

$$\therefore {}^{20}C_2 + 2.{}^{20}C_3 + ... + 19.{}^{20}C_{20} = 1 + 9.2^{20}$$

47. (b)

We have $T_{r+1} = {}^{29}C_r 3^{29-r}(7x)^r = ({}^{29}C_r \times 3^{29-r} \times 7^r)x'$

Coefficient of (r + 1)th term is ${}^{29}C_r \times 3^{29-r} \times 7^r$

And coefficient of rth term is ${}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1}$

From given condition,

$${}^{29}C_r \times 3^{29-r} \times 7^r = {}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1}$$

$$\frac{{}^{29}C_r}{{}^{29}C_{r-1}} = \frac{3}{7} \Rightarrow \frac{30-r}{r} = \frac{3}{7}\ or\ r = 21$$

48. (b)

Let term be $\frac{30!}{n_1!\,n_2!\,n_3!}\left(x^3\right)^{n_2}\left(-x^6\right)^{n_3}$

$n_1+n_2+n_3=30$

$3n_2+6n_3=28$

LHS is multiple of 3 but RHS is not cell 10

49. (a)

Number of ways to distribute 8 identical objects in 3 distinct groups $={}^{8+2}C_2=\frac{10\cdot 9}{2!}=45$.

50. (b)

Number of ways to distribute n identical objects in 5 distinct groups $={}^{n+4}C_4$

51. (a)

$(1-ax)^{-1}(1-bx)^{-1}(1-cx)^{-1}$

$=(1+ax+....)(1+bx+....)(1+cx+....)$

Hence coefficient of $x=(a+b+c)$.

52. (a)

$$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{3}{2}x\right)^3=\frac{27}{128}x^3$$

53. (c)

$(abcd)^{10}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^{10}$

So instead find cell of $\frac{1}{a^2}\frac{1}{b^6}\cdot\frac{1}{c}\frac{1}{d}$ in $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^1$

$\frac{10!}{2!6!}=2520$

54. (d)

Let term be $\frac{5!}{n_1!\,n_2!\,n_3!}\left(x^2\right)^{n_1}(-x)^{n_2}(-2)^{n_3}$

Now $n_1+n_2+n_3=5$ and $2n_1+n_2=5$

Put $n_1=0 \Rightarrow n_2=5$ & $n_3=0$

Coefficient is –1

If $n_1=1 \Rightarrow n_2=3, n_3=1$

Coefficient is $\frac{5!}{3!}\times 2=40$

If $n_1=2 \Rightarrow n_2=1,\ n_3=2$

Coefficient is $-\frac{5!}{2!2!}\times 4 \Rightarrow -120$

Add all coefficient –81

55. (d)

Let term be $\frac{20!}{n_1!\,n_2!\,n_3!}(1)^{n_1}(-x)^{n_2}(y)^{n_3}$

$n_1+n_2+n_3=20$

$n_2=2$

$n_3=3$

$n_1=15$

Coefficient $\frac{20!}{15!\,3!\,2!}$

56. (d)

$\left(1+3x+2x^2\right)^6=[1+x(3+2x)]^6$

$=1+{}^6C_1x(3+2x)+{}^6C_2x^2(3+2x)^2+{}^6C_3x^3(3+2x)^3+{}^6C_4x^4(3+2x)^4+{}^6C_5x^5(3+2x)^5$

$+{}^6C_6x^6(3+2x)^6$

We get x^{11} only from ${}^6C_6x^6(3+2x)^6$. Hence, coefficient of x^{11} is ${}^6C_5\times 3\times 2^5=576$

1. (a)

$$\frac{n^2+n-14}{2}=\frac{n(n+1)}{2}-7$$

$$\Rightarrow\ x^{\frac{n^2+n-14}{2}}=\frac{x\cdot x^2\cdot x^3\cdot\ldots\ldots x^n}{x^7}$$

Hence, we need to find those terms which are product of all the x^r terms in each bracket except those in which the sum of powers is 7.

Such terms are $(x^7),(x\cdot x^6),(x^2\cdot x^5),(x^3x^4),(x\cdot x^2\cdot x^4),(x\cdot x^3\cdot x^3)$

Hence required coefficient is $-(7)+(1\cdot 6)+(2\cdot 5)+(3\cdot 4)-(1\cdot 2\cdot 4)=13$

2. (a)

Conceptual

3. (b)

${}^{\min}C_m=A\ \&\ B={}^{m+n}C_n$

Clearly $A=B$

4. (a)

General $\tan^{100}(r(i))^r$

$A=1,3,5,7,\ldots\ldots..,99\Rightarrow 50$ terms

5. (c)

Expand

6. (c)

Let $E=(\sqrt{5}+1)^5-(\sqrt{5}-1)^5$

$$\Rightarrow\ E=2\left\{{}^5C_1(\sqrt{5})^4+{}^5C_3(\sqrt{5})^2+{}^5C_3(\sqrt{5})^0\right\}$$

$$\Rightarrow\ E=2\{125+50+1\}=352$$

7. (b)

Expand

8. (b)

General term, $t_{r+1}={}^{15}C_r\,2^{\frac{1}{2}(15-r)}3^{\frac{1}{2}r}$

Now, $r\in w$ and $0\le r\le 15$

Also, $15-r$ is EVEN and *r* is EVEN

$\Rightarrow\ r$ is ODD and *r* is EVEN

$\Rightarrow\ \therefore\ e\in\phi$

Hence, no radical term exist in the given expansion $\Rightarrow$ all terms are irrational.

$\therefore$ Number of irrational terms = 16

9. (c)

$\left(x^3 - \frac{1}{x^2}\right)^n$

General terms $T_{r+1} = \frac{n!}{r!(n-r)!}(-1)^{n-r}\ x^{5r-2n}$

If $5r - 2n = 5,$ then 5r = 2n + 5 or $r = \frac{2n}{5} + 1$

If $5r - 2n = 10,$ then $5r = 2n + 10$ or $r = \frac{2n}{5} + 2$

x^5 *and* x^{10} terms occurs if n = 5k

Given that sum of x^5 *and* x^{10} is zero

$\Rightarrow \frac{5k!}{(2k+1)!(3k-1)!} - \frac{5k!}{(2k+2)!(3k-2)!} = 0$

Or $\frac{1}{3k-1} - \frac{1}{2k+2} = 0$

Or k = 3 $\Rightarrow$ n = 15

10. (b)

$(1-x)(1-x)^n$

$= (1-x)[1 + n(-x) + ... + {}^nC_{n-1}(-x)^{n-1} + {}^nC_n(-x)^n]$

Therefore, coefficient of x^n is

${}^nC_n(-1)_n - {}^nC_{n-1}(-1)^{n-1} = (-1)^n + (-1)^n n$

$= (-1)^n(1+n)$

11. (b)

Put x = i

$(1+i)^5 = (a_0 - a_2 + a_4) + i(a_1 - a_3 + a_5)$

$\Rightarrow |1+i|5 = |(a_0 - a_2 + a_4) + i(a_1 - a_3 + a_5)]$

$\Rightarrow (a_0 - a_2 + a_4)^2 + (a_1 - a_3 + a_5)^2 = 2^5 = 32$

12. (c)

For $\left(ax^2 + \left(\frac{1}{bx}\right)\right)^{11}, T_{r+1} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$

$= {}^{11}C_r a^{11-r}\frac{1}{b^r}x^{22-3r}$

For x^7 22- 3r = 7

Or $3r = 15$

Or r = 5

$\Rightarrow T_6 = {}^{11}C_5 a^6 \frac{1}{b^5}x^7$

$\Rightarrow$Coefficient of x^7 is ${}^{11}C_5\ \frac{a^6}{b^5}$

Similarly, coefficient of x^{-7} in $\left(ax-\left(\frac{1}{bx^2}\right)\right)^{11}$ is ${}^{11}C_6\,\frac{a^5}{b^6}$

Given that ${}^{11}C_5\,\frac{a^6}{b^5}={}^{11}C_6\,\frac{a^5}{b_6}$

$\Rightarrow a=\frac{1}{b}$

Or $ab=1$

13. (b)

$t_{r+1}={}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r={}^{10}C_r x^{5-5n2}(-k)^r$

For this to be independent of x, r must be 2, so that ${}^{10}C_2k^2=405\ or\ k\pm3$

14. (b)

$T_5={}^nC_4a^{n-1}(-2b)^4$

And $T_6={}^nC_5a^{n-5}(-2b)^5$

As $T_5+T_6=0$, we get

${}^nC_4 2^4\,a^{n-4}b^4={}^nC_5 2^5a^{n-5}b^5$

Or $\frac{a^{n-4}b^4}{a^{n-5}b^5}=\frac{n!2^5}{5!(n-5)!}.\frac{4!(n-4)!}{n!2^4}$

Or $\frac{a}{b}=\frac{2(n-4)}{5}$

15. (a)

We have $S=\frac{1(2^{2000}-1)}{2-1}=2^{2000}-1=(2^2)^{1000}-1$

$=(5-1)^{1000}-1$

$=(5^{1000}-{}^{1000}C_1.5^{999}+{}^{1000}C_2.5^{998}.....+{}^{1000}C_{998}.5^2-{}^{1000}C_{999}.5+1)-1$

$=5(5^{999}-{}^{1000}C_1.5^{998}+{}^{1000}C_2.5^{997}-....-{}^{1000}C_{999})$

$\therefore$ Remainder is 0

16. (c)

$6^{83}+8^{83}=(7-1)^{83}+(7+1)^{83}$

$=2(7^{83}+{}^{83}C_2.7^{81}+{}^{83}C_4.7^{79}+.....+{}^{83}C_{80}7^3+{}^{83}C_{82}7)$

$=2\{49m+{}^{83}C_{82}.7\}$

Where m is an integer

$=98m+2.{}^{83}C_1.7=98m+2.83.7$

$=98m+2(77+6).7=49(2m+22)+84$

$=49(2m+22)+49+35$

$= 49(2m+23)+35 = 49n+35$
Where n is an integer
Hence, the remainder is 35

17. (c)
Middle term of $(1+\alpha x)^4$ is T_3
Its coefficient is ${}^4C_2(\alpha)^2 = 6\alpha^2$
Middle term of $(1-\alpha x)^6$ is T_4
Its coefficient is ${}^6C_3(-\alpha)^3 = -20\alpha^3$
According to question
$6\alpha^2 = -20\alpha^3$
Or $3\alpha^2 + 10\alpha^3 = 0$
Or $\alpha^2(3+10\alpha) = 0$
Or $\alpha = -\frac{3}{10}$

18. (d)
$3^{400} = 81100 = (1+80)^{100}$
$= {}^{100}C_0 + {}^{100}C_1 80 + ... + {}^{100}C_{100} 80^{100}$
Thus, the last two digits are 01.

19. (a)
$\frac{2^{4n}}{15} = \frac{(15+1)^n}{15}$
$= \frac{({}^nC_0 15^n + {}^nC_1 15^{n-1} + ... + {}^nC_{n-1} 15 + {}^nC_n)}{15}$
= Integer $+\frac{1}{15}$
Hence, the fractional part of $\frac{2^{4n}}{15}$ *is* $\frac{1}{15}$

20. (c)
Use concept of middle term.

21. (c)

$$(103)^{86} - (86)^{103}$$

$$\Rightarrow (1 + 102)^{86} - (1 + 85)^{103}$$

$$\Rightarrow \left[{}^{86}C_0 (102)^0 + {}^{86}C_1 102 + {}^{86}C_2 102^2 + \ldots\right]$$

$$- \left[{}^{103}C_0 85^0 + {}^{103}C_1 85 + {}^{103}C_2 85^2 + \ldots\right]$$

$$\Rightarrow \left[1 + {}^{86}C_1 102 + {}^{86}C_2 102^2 + \ldots\right] - \left[1 + {}^{103}C_1 85 + {}^{103}C_2 (85)^2 + \ldots\right]$$

$$\Rightarrow \left[{}^{86}C_1 102 + {}^{86}C_2 102^2 + \ldots\right] - \left[{}^{103}C_1 85 + {}^{103}C_2 85^2 + \ldots\right]$$

$$\Rightarrow 17\left\{\left({}^{86}C_1 6 + {}^{86}C_2 6(102) + \ldots\right) - \left({}^{103}C_1 5 + {}^{103}C_2 5(85) + \ldots\right)\right\}$$

Thus, it is divisible by 17.

Hence, C is correct option.

22. (c)

The correct option is **C**

$\frac{8}{31}$

$2^{78} + 2^3.2^{75} = 8\ 25^{15} = 8(1 + 31)^{15} = 8$

${}^{15}C_0 + {}^{15}C_1 31 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + {}^{15}C_{15}(31)^{15}$

$2^{78} = 8$ + an integer multiple of 31

$\frac{2^{78}}{31} = \frac{8}{31}$ + an integer

23. (a)

Correct option is A)

The above expression can be rewritten as

$(20-3)^{1983}+(1+10)^{1983}-(10-3)^{1983}$

Let n = 1983

Therefore

$(20-3)^n+(1+10)^n-(10-3)^n$

$=(20^n-3\,{}^nC_1 20^{n-1}...-3^n)+(1+\ {}^nC_1 10+$

$nC_2 10^2...10^n)-(10^n-3\,{}^nC_1 10^{n-1}...-3^n)$

Now in the above expansion.

The units digits will be given by

$-3^n+1-(-3^n)$

$=1$

24. (b)

$23^{23}=23\times 23^{22}=23\times(23^2)^{11}$

$=23(529)^{11}=23(530-1)^{11}$

$=23(53m-1), m\in N$

$=23\times 53m-23=53(23m-1)+$

30

$\therefore$ Remainder is 30.

25. (c)

The term independent of x will be

$^{10}C_5 \sin(\alpha)^5 \cos(\alpha)^5$

$= \frac{10!}{32(5!)(5!)}[32 \sin(\alpha)^5 \cos(\alpha)^5]$

$= \frac{10!}{32(5!)(5!)}[\sin^5(2\alpha)]$

Hence maximum value is for $\alpha = 45^0$

Maximum value being $\frac{10!}{32(5!)(5!)}$.

26. (c)

$^{50}C_{25} = \frac{50!}{(25!)(25!)}$

$18 = 3 \times 2 \times 2$

We have to find numbers of 2's and 3's in 50! and 25!

2's in 50! :

$= \left[\frac{50}{2}\right] + \left[\frac{50}{4}\right] + \left[\frac{50}{8}\right] + \left[\frac{50}{16}\right] + \left[\frac{50}{32}\right]$

$= 25 + 12 + 6 + 3 + 1$

$= 47$

3's in 50!

$= \left[\frac{50}{3}\right] + \left[\frac{50}{9}\right] + \left[\frac{50}{27}\right]$

$= 16 + 5 + 1$

$= 22$

Similarly no. of 2's in 25 = 22 and no. of 3's in 25 = 10

$\therefore$ In $\frac{50!}{(25!)(25!)}$ → no. of 2's = 47 − 44 = 3 and no. of 3's = 22 − 20 = 2.

So, only 18^1 is possible.

Hence the answer is 1.

27. (c)

$(99)^n + 1$

$= (100 - 1)^n + 1$

$= 1 - (1 - 100)^n$ ...(since n is odd)

$= 1 - [1 - {}^nC_1 100 + ...(-1)^r \, {}^{100}C_r 100^r + ...]$

$= 1 - [1 - 100 + 100(k)]$ where K is a integer.

$= 100 - 100k$

Hence the ending digits have 2 zeros, and k becomes a negative integer since $99^n + 1$ is positive.

28. (c)

$E = 3^{2003} = 3^{2001} \times 3^2 = 9(27)^{667} = 9(28-1)^{667}$

$\Rightarrow E = 9[{}^{667}C^{o}28^{667} - {}^{667}C^{1}(28)^{666} + \ldots - {}^{667}$

$C_{667}] = 9 \times 28k - 9$ or $\frac{E}{28} = 9k - \frac{9}{28} = 9k -$

$1 + \frac{19}{28}$

That means if we dived 3^{2003} by 28, the remainder is 19. Thus,

$\left\{\frac{3^{2003}}{28}\right\} = \frac{19}{28}$

29. (a)

Let $(\sqrt{3}+1)^{2m} = I + f$

and $(\sqrt{3}-1)^{2m} = f'$ where $0 < f' < 1$

and $0 < f' < 1$ and I is an integer.

Thus $I + f + f' = (\sqrt{3}+1)^{2m} + (\sqrt{3}-1)^{2m}$

$= (4+2\sqrt{3})^m + (4-2\sqrt{3})^m$

$= 2^m[(2+\sqrt{3})^m + (2-\sqrt{3})^m]$

$= 2^{m+1}[2^m + {}^mC_2 .2^{m-2}.(\sqrt{3})^2 + \ldots..]$

Now as in part (a) $f + f' = 1$ and so $I + f + f'$ is an integer next above $(\sqrt{3}+1)^{2m}$ which by (1) contains 2^{m+1} as a factor.

30. (a)

Write $(101)^{100} - 1 = (1+100)^{100} - 1$, then expand.

31. (a)

We have, $\sum_{r=0}^{n-1}\frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}=\sum_{r=0}^{n-1}\frac{{}^nC_r}{{}^{n+1}C_{r+1}}$

$=\sum_{r=0}^{n-1}\frac{{}^nC_r}{\frac{n+1}{r+1}{}^nC_r}\sum_{r=0}^{n-1}\frac{r+1}{n+1}$

$=\frac{1}{n+1}[1+2+....+n]=\frac{n(n+1)}{2(n+1)}=\frac{n}{2}$

32. (b)

We have ${}^{39}C_{3r-1}+{}^{39}C_{3r}={}^{39}C_{r^2}+{}^{39}C_{r^2-1}$

$\Rightarrow {}^{40}C_{3r}={}^{40}C_{r^2}$

$\Rightarrow 3r=r^2\ or\ 40-3r=r^2$

$\Rightarrow r=0,\ 3\ or\ r^2+3r-40=0$

$\Rightarrow (r+8)(r-5)=0\Rightarrow r=0,3,5,-8$

But r = 0, -8 do not satisfy the given equation $\therefore r=3,5$

33. (d)

$\sum_{r=0}^{20}r(20-r)({}^{20}C_r)^2$

$\sum_{r=0}^{20}r\times{}^{20}C_r(20-r)\times{}^{20}C_{20-r}=\sum_{r=0}^{20}20.\ {}^{19}C_{r-1}\times 20\times\ {}^{19}C_{19-r}$

$=400\sum_{r=0}^{20}{}^{19}C_{r-1}\times{}^{19}C_{19-r}$

$=400\sum_{r=0}^{20}{}^{19}C_{r-1}\times{}^{19}C_{19-r}$

$=400\times\ coefficient\ of\ x^{18}\ in\ (1+x)^{19}(1+x)^{19}$

$=400\times{}^{38}C_{18}=400\times{}^{38}C_{20}$

34. (c)

As we know that ${}^nC_0-{}^nC_1^2+{}^nC_2^2-{}^nC_3^2+.....+(-1)^n\ {}^nC_n^2=0$ (if n is odd) and in the question n = 15 (odd). Hence sum of given series is 0.

35. (a)

$\sum_{r=0}^{40}r\,{}^{40}C_r\,{}^{30}C_r$

$=40\sum_{r=0}^{40}{}^{39}C_{r-1}\ {}^{30}C_r$

$=40\sum_{r=0}^{40}{}^{39}C_{r-1}\ {}^{30}C_{30-r}$

$=40\,{}^{39+30}C_{r-1+30-r}$

$=40\ {}^{69}C_{29}$

36. (a)

$$\frac{r\times 2r}{(r+2)!}=\frac{(r+2-2)2^r}{(r+2)!}$$

$$=\frac{2^r}{(r+1)!}-\frac{2^{r+1}}{(r+2)!}$$

$$=-\left(\frac{2^{r+1}}{(r+2)!}-\frac{2^r}{(r+1)!}\right)$$

= (V(r) – V(r – 1))

$$\Rightarrow \sum_{r=1}^{15}\frac{r\times 2^r}{(r+2)!}=-(V(15)-V(0))$$

$$=-\left(\frac{2^{16}}{17!}-\frac{2}{2!}\right)$$

$$=1-\frac{2^{16}}{(17)!}$$

37. (b)

Put $x=1$

$$a_0+a_1+a_2+a_3..........a_{12}=0$$

Put $x=-1$

$$1-a_1+a_2-a_3.........+a_{12}=2^6$$

Add $2\left(1+a_2+a_4+.....a_12\right)=2^5$

$$a_2+a_4+.......a_{12}=2^5$$

38. (c)

$$\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3}k.{}^{k}C_r$$

$$\sum_{k=1}^{\infty}\frac{1}{3^k}\left(2^k\right)$$

Which is a G.P.

$$\frac{\frac{2}{3}}{1-\frac{2}{3}}=2$$

39. (c)

Writing the general term, we get

$$1 + \frac{^nC_r}{^nC_{r-1}}$$

$$= 1 + \frac{n - r + 1}{r} \quad ...(i)$$

Hence the above summation can be written as

$$(1 + n) + (1 + \frac{n-1}{2}) + (1 + \frac{n-2}{3}) + (1 + \frac{n-3}{4})...(1 + \frac{1}{n})$$

$$= n + [n + \frac{n-1}{2} + \frac{n-2}{3} ... \frac{1}{n}]$$

$$= \frac{(n+1)^n}{n!}$$

40. (a)
Do yourself.

41. (c)

Here $a_i = {}^{10}C_i$

ie, $(a_0 - a_2 + a_4 - a_6 + a_8 - a_{10})^2 + (a_1 - a_3 + a_5 - a_7 + a_9)^2$

$$= ({}^{10}C_0 - {}^{10}C_2 + {}^{10}C_4 - {}^{10}C_6 + {}^{10}C_8 - {}^{10}C_{10})^2 + ({}^{10}C_1 - {}^{10}C_3 + {}^{10}C_5 - {}^{10}C_7 + {}^{10}C_9)^2$$

$\Rightarrow$

$$(({}^{10}C_0 - {}^{10}C_{10}) + ({}^{10}C_8 - {}^{10}C_2) + ({}^{10}C_4 - {}^{10}C_6))^2$$

$$((-1)(-2)^{\frac{10}{2}})^2 = 2^{10}$$

Since $\sum_{i=0}^{[n/2]} {}^{n}C_{2i} = \begin{cases} 0, & \text{If } \frac{n+2}{4} \in \text{Integers} \\ (-1)^{[(n+2)/4]} 2^{[n/2]} \end{cases}$

and $\sum_{i=0}^{[n/2]} {}^{n}C_{2i+1} = \begin{cases} 0, & \text{If } \frac{n}{4} \in \text{Integers} \\ (-1)^{[n/4]} 2^{[n/2]} \end{cases}$

42. (a)

$C_0 + (C_0 + C_1) + (C_0 + C_1 + C_2) + ... + (C_0 + C_1 +$

$C_2 + ... + C_{n-1})$

$= nC_0 + (n-1)C_1 + (n-2)C_2 + ... + 1.C_{n-1}$

$= n[C_0 + C_1 + C_2 + ... + C_n] - [0.C_0 + 1.C_1 +$

$2.C_2 + ... + n.C_n]$

$= n[2^n] - [n\, 2^{n-1}]$

$= n[2^n - 2^{n-1}]$

$= n\, 2^{n-1}$

43. (c)

$$\sum_{r=0}^{n} r^2 \ {}^nC_r x^r y^{n-r}$$

$$= \sum_{r=0}^{n}[r(r-1)+r]\ {}^nC_r x^r y^{n-r}$$

$$= \sum_{r=0}^{n}(r(r-1)) \cdot {}^nC_r x^r y^{n-r} + \sum_{r=0}^{n} r \cdot {}^nC_r x^r y^{n-r}$$

$$\sum_{r=0}^{n}(r(r-1)) \cdot {}^nC_r x^r y^{n-r} = x^2 \frac{d^2}{dx^2}(\sum_{r=0}^{n}$$

$${}^nC_r x^r y^{n-r}) = x^2 \frac{d^2}{dx^2}(x+y)^n = n(n-1)x^2$$

$$\sum_{r=0}^{n} r \cdot {}^nC_r x^r y^{n-r} = x\frac{d}{dx}(\sum_{r=0}^{n} {}^nC_r x^r y^{n-r}) =$$

$$x\frac{d}{dx}(x+y)^n = nx$$

$$\therefore \sum_{r=0}^{n}(r(r-1)) \cdot {}^nC_r x^r y^{n-r} + \sum_{r=0}^{n} r \cdot {}^nC_r x^r y^{n-r}$$

$$= n(n-1)x^2 + nx$$

$$= nx[x(n-1)+1]$$

$= nx[nx - x + (x+y)]$ ...it is given in the question that $x + y = 1$

$$= nx[nx + y]$$

44. (a)

$= n\,2^{n-2}\ [2 + n - 1] = n(n + 1)2^{n-2}$

keeping in view 2^3C_2 or 3^3C_3

Differentiate and multiply by x

$n(1 + x)^{n-1}x = C_1x + 2C_2x^2 + 3C_3x^3 + ...$

Again differentiate and multiply by x

$nx\left\{(n - 1)(1 + x)^{n-2}\cdot x + 1(1 + x)^{n-1}\right\}$

$= C_1x + 2^2C_2x^2 + 3^2C_3x^3 + ...$

or $n(n - 1)(1 + x)^{n-2}x^2 + nx(1 + x)^{n-1} =$

$C_1x + 2^2C_2x^2 + 3^2C_3x^3 + ...$

Differentiate again w.r.t. x.

$n(n -$

$1)\left\{(n - 2)\cdot(1 + x)^{n-3}\cdot x^2 + (1 + x)^{n-2}\cdot 2x\right\}$

$+n\left\{(n - 1)x(1 + x)^{n-2} + 1(1 + x)^{n-1}\right\}$

$= C_1 + 2^3C_2x + 3^3C_3x^2 + ...$

Now put x = 1

$n(n - 1)\left\{(n - 2)2^{n-3} + 2^{n-2}\cdot 2\right\} + n(n -$

$1)2^{n-2} + n\cdot 2^{n-1}$

$1)2^{n-2} + n\cdot 2^{n-1}$

$2^{n-3}\ \{n(n - 1)(n - 2) + 4n(n - 1) + 2n(n -$

$=$

$2^{n-3}n\left(n^2 - 3n + 2 + 4n - 4 + 2n - 2 + 4\right)$

$= 2^{n-3}n(n^2 + 3n) = n^2(n + 3)2^{n-3}$

45. (b)

$$\sum_{r=0}^{n} r^3\left(\frac{n-r+1}{r}\right)^2 = \sum_{r=0}^{n} r\left(n-r+1\right)^2$$

$$= \sum_{r=0}^{n}\left(n+1\right)^3 r - \sum_{r=0}^{n} 2\left(n+1\right)r^2 + \sum_{r=0}^{n} r^3$$

$$= \frac{1}{12} n\left(n+1\right)^2\left(n+2\right)$$

46. (a)

$$\sum_{r=0}^{n} (-1)^r {}^nC_r \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \ldots\right] \text{ upto m terms}$$

$$= \sum_{r=0}^{n} (-1)^r \left(\frac{1}{2}\right)^r + \sum_{r=0}^{n} (-1)^r {}^nC_r \left(\frac{3}{4}\right)^r + \sum_{r=0}^{n} (-1)^r {}^nC_r \left(\frac{7}{8}\right)^r + \ldots \text{ upto m terms}$$

$$= \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + \ldots \text{ upto m terms}$$

using $\{\sum_{r=0}^{n} (-1)^r {}^nC_r x^r = (1-x)^n\}$

$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n + \left(\frac{1}{8}\right)^n + \ldots \text{ upto m terms}$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1-\left(\frac{1}{2^n}\right)^m}{1-\frac{1}{2^n}}\right] = \frac{2^{mm}-1}{2^{mm}(2^n-1)}$$

47. (b)

$$S = \sum_{i=0}^{r} {}^{n_1}C_{r-i}\ {}^{n_2}C_i$$

Coefficient of x^r in $\left(1+x\right)^{n_1}\left(1+x\right)^{n_2} = \left(1+x\right)^{n_1+n_2}$

$= {}^{n_1+n_2}C_r$

48. (c)

The r_{th} term in the given expansion is

$$T_r = \frac{{}^nC_{2r-1}}{2r}$$

$$\text{Since } \frac{1}{r+1} \cdot {}^nC_r = \frac{1}{n+1} \cdot {}^{n+1}C_{r+1}$$

$$\therefore T_r = \frac{{}^nC_{2r-1}}{2r} = \frac{1}{n+1} \cdot {}^{n+1}C_{2r}$$

$$\therefore \frac{{}^nC_1}{2} + \frac{{}^nC_3}{4} + \frac{{}^nC_5}{6} + \ldots\ldots = \frac{1}{n+1}[{}^{n+1}C_2 + {}^{n+1}C_4 + \ldots..]$$

$$= \frac{1}{n+1}[2^{n+1-1} - {}^nC_0] = \frac{2^n - 1}{n+1}$$

49. (c)

50. (d)

Use formula

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

51. (d)

$\left(1+2x+3x^2\right)^{10} = a_0 + a_1x + ...a_{20}a^{20}$ …………(1)

Diff $10\left(1+2x+3x^2\right)^9\left(2+6m\right) = a_1 + 2a_2x + ...$

Put $x = 0$

$20 = a_1$ In equation (1)

Put $x = 1$

$a_0 + a_1 + a_2 + ...a_{20} = 6^{10}$

Clearly coefficient of x^{20} is 3^{10}

Use PNC to find coefficient of x^2

$3.{}^{10}C_1 + 4.\frac{10}{2}$

$30 + 180 = 210$

52. (c)

$\Rightarrow 1 + x + x^2 + x^3 = [1\ (1+x) + x^2\ (1+x\ )\ ]$

$then\ (1 + x + x^2 + x^3\ )^5 = [\ (1+x)(1+x^2)\ ]^5 = (1+x)^5\ (1+x^2)^5$

$= (1+x)^5 \times [\ {}^5C_0\ 1^5 + {}^5C_1\ 1^4\ x^2 + {}^5C_2\ 1^3 x^4 + {}^5C_3\ 1^2\ x^6 + {}^5C_4\ 1^1\ x^8 + {}^5C_5\ 1^0\ x^{10}\ ]$

$a_{10} = Coefficient\ of\ x^{10} = (\ {}^5C_0\ 1^5 \times {}^5C_5\ 1^0) + ({}^5C_2\ 1^3 \times {}^5C_4\ 1^1) + ({}^5C_4\ 1^1 \times {}^5C_3\ 1^2)$

$= 1 + 50 + 50$

$So,\ a_{10} = 101$

53. (b)

The correct option is **C**

$4^{1/3}$

$$S = 1 + \frac{2}{3}.\frac{1}{2} + \frac{2.5}{3.6}\left(\frac{1}{2}\right)^2 + \frac{2.5.8}{3.6.9}\left(\frac{1}{2}\right)^3 + \ldots.$$

$$= 1 + \frac{\frac{2}{3}}{1}\left(\frac{1}{2}\right) + \frac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)}{2!}\left(\frac{1}{2}\right)^2 + \frac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)\left(\frac{8}{3}\right)}{3!}\left(\frac{1}{2}\right)^3 + \ldots\ldots$$

$$= \left(1 - \frac{1}{2}\right)^{-\frac{2}{3}} = 2^{\frac{2}{3}} = 4^{\frac{1}{3}}$$

54. (b)

Let, $\alpha = \frac{5}{2!3} + \frac{5.7}{3!3^2} + \frac{5.7.9}{4!3^3} + \ldots\ldots\ldots$

or, $\alpha = \frac{3.5}{2!3^2} + \frac{3.5.7}{3!3^3} + \frac{3.5.7.9}{4!3^4} + \ldots\ldots\ldots$

or, $\alpha = \frac{3.(3+2)}{2!3^2} + \frac{3.(3+2).(3+4)}{3!3^3} +$

$\frac{3.(3+2).(3+4).(3+6)}{4!3^4} + \ldots\ldots\ldots$

or, $\alpha =$

$$\left\{1 + \frac{\frac{3}{2}}{1!}\left(\frac{2}{3}\right) + \frac{\frac{3}{2}.(\frac{3}{2}+1)2^2}{2!3^2} + \frac{\frac{3}{2}.(\frac{3}{2}+1).(\frac{3}{2}+2)2^3}{3!3^3} + \frac{\frac{3}{2}.(\frac{3}{2}}{}\right.$$

2

or, $\alpha =$

$$\left\{1 + \frac{\frac{3}{2}}{1!}\left(\frac{2}{3}\right) + \frac{\frac{3}{2}.(\frac{3}{2}+1)}{2!}\left(\frac{2}{3}\right)^2 + \frac{\frac{3}{2}.(\frac{3}{2}+1).(\frac{3}{2}+2)}{3!}\left(\frac{2}{3}\right)\right.$$

2

or, $\alpha = \left(1 - \frac{2}{3}\right)^{-\frac{3}{2}} - 2 = \left(\frac{1}{3}\right)^{-\frac{3}{2}} - 2 = 3^{\frac{3}{2}} - 2.$

55. (a)

Let $s = (1 + x + 2x^2 + 3x^3 \dots \dots + nx^n)^2$

$\Rightarrow s = 1 + x + 2x^2 + 3x^3 + \dots \dots + nx^n$

$\Rightarrow x.s = x^o + x^2 + 2x^3 + 3x^4 + \dots\dots + n.x^{n+1}$

$\Rightarrow (1 - x)s = 1 + x + x^2 + x^3 \dots \dots + x^n$

$\Rightarrow -nx^{n+1} - x = \dfrac{1 - x^{n+1}}{1 - x} - nx^{n+1} - x$

$\Rightarrow s = \dfrac{1}{(1 - x)^2} - \dfrac{x}{1 - x} = \dfrac{1 - x + x^2}{(1 - x)^2}$

Ignoring terms we have powers of x greater than x^n

coefficient of x^n in

$(1 + x + 2x^2 + 3x^3 + \dots\dots + nx^n)^2$

$\Rightarrow$ Coefficient of x^n in $(1 - x + x^2)(1 - x^{-4})$

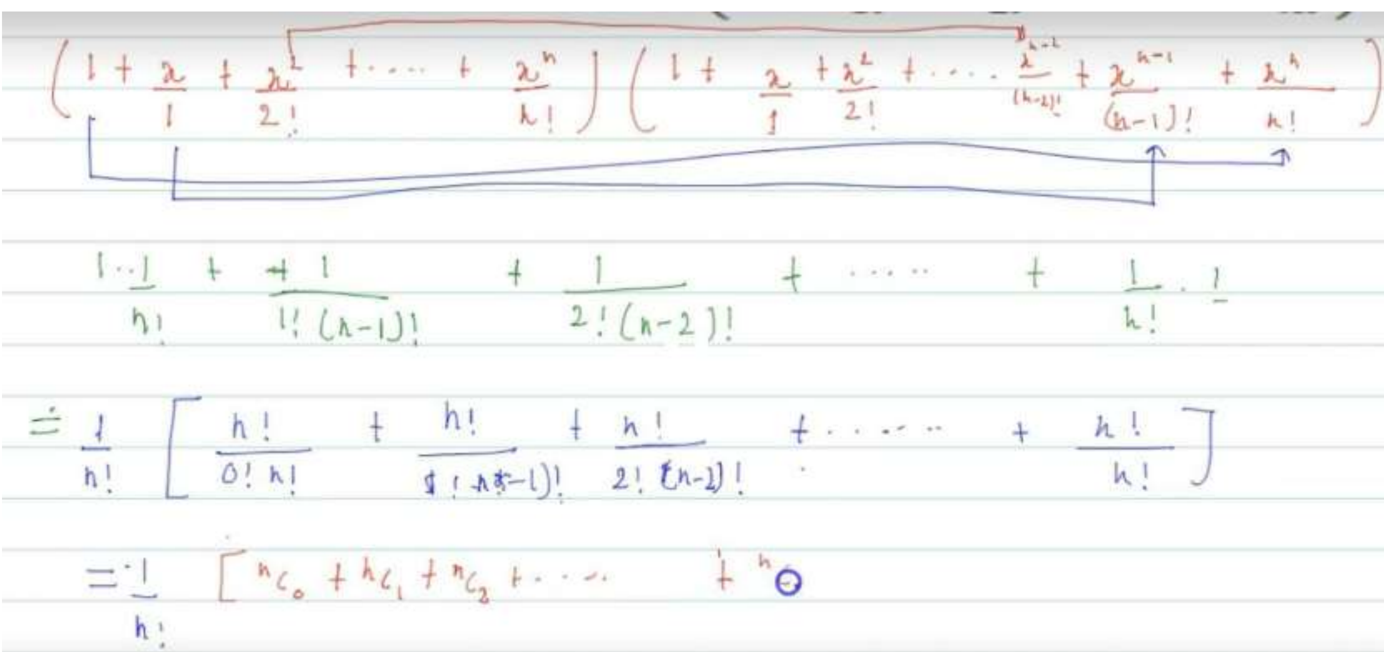
$\Rightarrow$ Such coefficient is clearly

$$2n + \sum_{k=1}^{n-1} k(n - k)$$

$$\Rightarrow \frac{n(n^2 + 11)}{6}$$

$\therefore$ So the answer is A= $\frac{n(n^2 + 11)}{6}$.

56. (b)



57. (c)
A.G.P.

58. (b)

$$(abc)^{12}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^{12}$$

$\Rightarrow$ Find coefficient of $\frac{1}{a^4}\frac{1}{b^2}\frac{1}{c^6}$ in $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^{12}$

Then $\frac{12!}{4!2!6!}$

59. (d)

$\left(1+2x+3x^2+...\right)^{-3/2}=\left[\left(1-x\right)^{-2}\right]^{-3/2}$

$=(1-x)^3=1-3x+3x^2-x^3$

Therefore, coefficient of x^5 is 0.

60. (d)

$(1+x+x^2+x^3+....)^2=\left(\left(1-x\right)^{-1}\right)^2=\left(1-x\right)^{-2}$

$=1+2x+3x^2+.....$

Therefore, coefficient of x^n is n + 1

61. (d)

$$\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$$

$$=\frac{\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right)-\left(1+\frac{3}{2}x+3\frac{x^2}{4}\right)}{(1-x)^{1/2}}$$

$$=\frac{-3}{8}x^2(1-x)^{-1/2}$$

$$=-\frac{3}{8}x^2\left(1+\frac{x}{2}\right)$$

$$=-\frac{3}{8}x^2$$

62. (d)

$$\frac{1}{(1-ax)(1-bx)}=a_0+a_1x+a_2x^2+......+a_nx^n+...$$

But $(1-ax)^{-1}(1-bx)^{-1}=(1+ax+a^2x^2+....)(1+bx+b^2x^2+....)$

$\Rightarrow$ Coefficient of x^n is $b^n+ab^{n-1}+a^2b^{n-2}+....+a^{n-1}b+a^k=\frac{b^{n+1}-a^{n-1}}{b-a}$

$$\Rightarrow a_n=\frac{b^{n+1}-a^{n+1}}{b-a}$$

EXERCISE - 1 [C]

1. (15)

$$\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}=\sum_{i=0}^{m}{}^{10}C_i\,{}^{20}C_{m-i}$$

$$= {}^{10}C_0\cdot{}^{20}C_m+{}^{10}C_1\cdot{}^{20}C_{m-1}+{}^{10}C_2\cdot{}^{20}C_{m-2}+\ldots+{}^{10}C_m\cdot{}^{20}C_0$$

= Coefficient of x^m in the expansion of product $(1+x)^{10}(1+x)^{20}$

= Coefficient of x^m in the expansion of $(1+x)^{30}={}^{30}C_m$

To get maximum value of the given sum, ${}^{30}C_m$ should be maximum.

Which is so, when $m=\frac{30}{2}=15$

2. (141)

$\because\ \left(x+\frac{1}{x}+1\right)^6=\sum_{r=0}^{6}{}^6C_r\left(x+\frac{1}{x}\right)^r$ for constant term r must be even integer.

$$\therefore\ a_0={}^6C_0+{}^6C_2\times{}^2C_1+{}^6C_4\times{}^6C_2+{}^6C_6\times{}^6C_3$$

$$=1+30+90+20=141$$

3. (210)

Since, last term in the expansion of $\left(\sqrt[3]{2}-\frac{1}{\sqrt{2}}\right)^n$

$$=\left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}\Rightarrow{}^nC_n\cdot\left(-\frac{1}{\sqrt{2}}\right)^n=\left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}$$

$$\Rightarrow\ (-1)^n\cdot\left(\frac{1}{2}\right)^{n/2}=\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}=\left(3^{-5/3}\right)^{\log_2 2^3}$$

$$=3^{-\frac{5}{3}\times3\times\log_3 2}=3^{-5\log_3 2}=3^{\log_3 2^{-5}}=2^{-5}=\left(\frac{1}{2}\right)^5$$

$$\Rightarrow\ (-1)^n\cdot\left(\frac{1}{2}\right)^{n/2}=\left(\frac{1}{2}\right)^5$$

$$\therefore\ n=10$$

Now, 5th term beginning $={}^{10}C_4\left(\sqrt[3]{2}\right)^6\left(-\frac{1}{\sqrt{2}}\right)^4$

$$={}^{10}C_4\cdot2^2\cdot\frac{1}{2^2}={}^{10}C_4={}^{10}C_6$$

4. (5)

Here, $f(x)=\sum_{r=1}^{n}\left\{r^2\left({}^nC_r-{}^nC_{r-1}\right)+(2r+1)\,{}^nC_r\right\}$

$$=\sum_{r=1}^{n}\left(r^2+2r+1\right){}^nC_r-r^2\cdot{}^nC_{r-1}$$

$$=\sum_{r=1}^{n}\left((r+1)^2\cdot{}^nC_r-{}^nC_r-r^2\cdot{}^nC_{r-1}\right)$$

$$=(n+1)^2\cdot{}^nC_n-1^2\cdot{}^nC_0$$

$$=(n+1)^2-1=\left(n^2+2n\right)$$

$\therefore \quad f(30)=(30)^2+2(30)=960$

$$=30\times32=30(2)^5=30(2)^\lambda$$

Hence, $\lambda=5$

5. (2)

$\because\ 9^{100}=(2\cdot4+1)^{100}=4n+1$ [say] [Where n is positive integer]

$\therefore\ 2^{9^{100}}=2^{4n+1}=2^{4n}\cdot2=(16)^n\cdot2$

The digit at unit's place in $(16)^n\cdot2=2$

$\therefore$ The digit at unit's place in $(16)^n\cdot2=2$

6. (5)

Here, $a_r={}^nC_r$

$\therefore\ b_r=1+\dfrac{a_r}{a_{r-1}}=1+\dfrac{{}^nC_r}{{}^nC_{r-1}}$

$$=1+\frac{n-r+1}{r}=\frac{(n+1)}{r}$$

$\Rightarrow\ \prod_{r=1}^{n}b_r=\prod_{r=1}^{n}\dfrac{(n+1)}{r}$

$$=\frac{(n+1)}{1}\cdot\frac{(n+1)}{2}\cdot\frac{(n+1)}{3}\ldots\frac{(n+1)}{n}=\frac{(n+1)^n}{n!}$$

$$=\frac{(101)^{100}}{100!}\quad\text{[Given]}$$

$\therefore\ n=100\Rightarrow\ \dfrac{n}{20}=5$

7. (8)

We have, $2^{2006}=2^2\left(2^3\right)^{668}$

$$=4(1+7)^{668}=4(1+7k)=4+28k$$

$\therefore\ 2^{2006}+2006=4+28k+7\times(286)+4$

Hence, remainder is 8.

8. (6)

In the expansion of $(1 + x)^{18}$

coefficient of $T_{2r+4} = {}^{18}C_{2r+3}$

coefficient of $T_{r-2} = {}^{18}C_{r-3}$

Given,

${}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$\Rightarrow 2r - 3 = r + 3$ or $2r + 3 + r - 3 = 18$

$\Rightarrow r = -6$ or $3r = 18 \Rightarrow r = 6$

but r cannot be negative

$\therefore r = 6$

9. (9)

Since, coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in $(1 + x)^{14}$ are in A.P.

$\Rightarrow 2({}^{14}C_r) = {}^{14}C_{r-1} + {}^{14}C_{r+1}$

$\frac{{}^{14}C_{r-1}}{{}^{14}C_r} + \frac{{}^{14}C_{r+1}}{{}^{14}C_r} = 2$

$\frac{r}{14 - r + 1} + \frac{14 - r}{r + 1} = 2$

$\Rightarrow \frac{r^2 + r + (15 - r)(14 - r)}{(15 - r)(r + 1)} = 2$

$\Rightarrow 2r^2 - 18r + 210 + 2(15 - r)(r + 1)$

$\Rightarrow 4r^2 - 56r + 180 = 0$

$\Rightarrow r^2 - 14r + 45 = 0$

$\Rightarrow r = 5, 9$

10. (3)

$$(1 + 0.00002)^{50000} = \left(1 + \frac{1}{50000}\right)^{50000}$$

Now we know that $2 \leq \left(1 + \frac{1}{n}\right)^{n} < 3 \forall n \geq$

$1 \Rightarrow$ Least integer is 3

11. (0)

Middle term is $\left(\frac{n}{2} + 1\right)^{th}$, i.e., $(4 + 1)^{th}$, i.e.,

T_5

$\therefore\ T_5 = {}^8C_4 \left(\frac{x}{2}\right)^4 .2^4 = 1120 \Rightarrow x^4 =$

$\frac{8.7.6.5}{1.2.3.4} x^4 = 1120$

$\Rightarrow\ x\$ = \frac{1120}{70} = 16$

$\Rightarrow\ (x^2 + 4)(x^2 - 4) = 0$

$\therefore\ x = \pm 2$ only as $x \in R$

12. (5)

We know ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

${}^{23}C_r + {}^{23}C_{r+1} + {}^{23}C_{r+1} + {}^{23}C_{r+2} \geqslant {}^{25}C_{15}$

$\Rightarrow {}^{23}C_{r+1} + {}^{23}C_r + {}^{23}C_{r+2} + {}^{23}C_{r+1} \geqslant {}^{25}C_{15}$

$\Rightarrow {}^{24}C_{r+1} + {}^{24}C_{r+2} \geqslant {}^{25}C_{15}$

$\Rightarrow {}^{24}C_{r+2} + {}^{24}C_{r+1} \geqslant {}^{25}C_{15}$

$\Rightarrow {}^{25}C_{r+2} \geqslant {}^{25}C_{15}$

The Inequity holds when
r = 8, 9, 10, 11, 12, 13
Hence, number of value of 'r' which
satisfies
the given Equation is '6'.

13. (4)

$$\left(5^{\frac{1}{5}\log\sqrt{4^x+44}} + \frac{1}{5^{\log 5\sqrt{2^{x-1}+7}}}\right)$$

$$= \left((\sqrt{4^x+444})^{2/5} + \left(\frac{1}{\sqrt[3]{2^{x-1}+7}}\right)\right)^8$$

$$= \left((4^x+444)^{2/5} + \left(\frac{1}{(2^{x-1}+7)^{1/3}}\right)\right)^8$$

Now $T_4 = T_{3+1} = {}^8C_3((4^x +$

$44)^{1/5})^{8-3}\frac{1}{((2^{x-1}+7)^{1/3})^3}$

Given $336 = {}^8C_3\left(\frac{4^x+44}{2^{x-1}+7}\right)$

Let $2^x = y$

$\Rightarrow 336 = {}^8C_3\left(\frac{y^2+44}{(y/2)+7}\right)$

$\Rightarrow 366 = \frac{8\times7\times6}{3\times2\times1}\left(\frac{2(y^2+44)}{y+14}\right)$

$\Rightarrow y^2 - 3y + 2 = 0 \Rightarrow y = 0, 2$

14. (4)

We have b = coefficient of x^3 in

$((1 + x + 2x^2 + 3x^3) + 4x^4)^4$

= coefficient of x^3 in $[{}^4C_0(1 + x + 2x^2 +$

$3x^3)^4(4x^4)^0 + {}^4C_1(1 + x + 2x^2 + 3x^3)^3(4x^4)^1 +$

$...]$

= coefficient of x^3 in $(1 + x + 2x^2 + 3x^3)^4$ =

Hence, 4a/b = 4.

15. (1)

$$\sum_{k=0}^{4}\left(\frac{3^{4-k}}{(4-k)!}\right)\left(\frac{x^k}{k!}\right)$$

$$=\sum_{k=0}^{4}\left(\frac{4!}{(4-k)!k!}3^{4-k}.\,x^4.\,\frac{1}{4!}\right)$$

$$=\sum_{k=0}^{4}\frac{.^4\,C_k.3^{4-k}.\,x^4}{04!}$$

$$=\frac{(3+x)^4}{4!}$$

According to the question ,

$$\frac{(3+x)^4}{4!}=\frac{32}{3}$$

or $(3+x)^4=256$

or $x+3=256$

or $x=1$

1. (c)
We are given that $7^{2022} + 3^{2022}$
$=(49)^{1011}+(9)^{1011}=(50-1)^{1011}+(10-1)^{1011}$
$=5\lambda-1+5K-1=5m-2$
$\Rightarrow$ Remainder = 5 – 2 = 3

2. (c)
$(202)^{2023}=(7K-2)^{2023}$
$= {}^{2023}C_0(7\lambda)^{2023}-...{}^{2023}C_{2023}2^{2023}=7\mu-2^{2023}$
$\therefore\ -2^{2023}=-2\times2^{2022}$
$=-2\times\left(2^3\right)^{674}=-2(1+7\gamma)^{674}=-(7\beta+2)$
$\Rightarrow$ remainder = –2 or +5

3. (a)
Given expression is
$(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+...+x^{500}$
Take $(5+x)^{500}$ common, then it forms G.P.
$\left[(5+x)^{500}\left(\left(\frac{x}{5+x}\right)^{501}-1\right)\right]\left(\frac{x}{5+x}\right)-1$
$=\frac{(5+x)^{501}-x^{501}}{5}$
$\Rightarrow$ Coefficient x^{101} in given expression is calculate by
$\left(\frac{1}{5}(x+5)^{501}\right)$ only $=\frac{{}^{501}C_{101}5^{400}}{5}={}^{501}C_{101}5^{399}$

4. (d)
$(1+x)^{20}=C_0+C_1x+C_2x^2+....+C_{20}x^{20}$
Diff. w.r.t. x
$20(1+x)^{19}=C_1+2C_2x+...+20C_{20}x^{19}$
Multiple by x both side
$20x(1+x)^{19}=C_1x+2C_2x^2+...+20C_{20}x^{20}$
Diff. w.r.t. x
$20(1+x)^{19}+20.x.19(1+x)^{18}$
$=C_1+4C_2x+....+20^2C_{20}x^{19}$
Let $x=1$
$\Rightarrow\ 20\times19.2^{18}+20.2=C_1+4C_2+....+20^2C_{20}$

$= 420 \times 2^{18}$.

5. (d)

$$\frac{3^{200}}{8} = \frac{1}{8}\left(9^{100}\right)$$

$$= \frac{1}{8}(1+8)^{100} = \frac{1}{8}\left[1 + n\cdot 8 + \frac{n(n+1)}{2}\cdot 8^2 + \ldots\right] = \frac{1}{8} + \text{Integer}$$

$$\therefore \quad \left\{\frac{3^{200}}{8}\right\} = \left\{\frac{1}{8} + \text{integer}\right\} = \frac{1}{8}$$

6. (a)

$$\left(x^2 + \frac{1}{x^3}\right)^n$$

General term $T_{r+1} = {}^nC_r\left(x^2\right)^{n-r}\left(\frac{1}{x^3}\right)^r = {}^nC_r \cdot x^{2n-5r}$

To Find coefficient of x, $2n - 5r = 1$

Given ${}^nC_r = {}^nC_{23} \Rightarrow r = 23$ or $n - r = 23$

$\therefore$ $n = 58$ or $n = 38$, Minimum value is $n = 38$

7. (d)

$$\left[\frac{1}{\sqrt{5x^3+1} - \sqrt{5x^3-1}}\right]^8 + \left[\frac{1}{\sqrt{5x^3+1} + \sqrt{5x^2-1}}\right]^8$$

After rationalize the polynomial we get

$$\left[\frac{1}{\sqrt{5x^3+1} - \sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1} - \sqrt{5x^3-1}}{\sqrt{5x^3+1}\sqrt{5x^3-1}}\right]^8 + \left[\frac{1}{\sqrt{5x^3+1} - \sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1} - \sqrt{5x^3-1}}{\sqrt{5x^3+1}\sqrt{5x^3-1}}\right]^8$$

$$= \left[\frac{\sqrt{5x^3+1} + \sqrt{5x^3-1}}{\left(5x^3+1\right) - \left(5x^3-1\right)}\right]^8 + \left[\frac{\sqrt{5x^3+1} + \sqrt{5x^3-1}}{\left(5x^3+1\right) - \left(5x^3-1\right)}\right]^8$$

$$= \frac{1}{2^8}\left[\left(\sqrt{5x^3+1} + \sqrt{5x^3-1}\right)^8 + \left(\sqrt{5x^3+1} - \sqrt{5x^3-1}\right)^8\right]$$

$$= \frac{1}{2^8}\Big[{}^8C_0\left(\sqrt{5x^3+1}\right)^8 + {}^8C_2\left(\sqrt{5x^3+1}\right)^6\left(\sqrt{5x^3-1}\right)^2 + {}^8C_4\left(\sqrt{5x^3+1}\right)^4\left(\sqrt{5x^3-1}\right)^4$$
$$+ {}^8C_6\left(\sqrt{5x^3+1}\right)^2\left(\sqrt{5x^3-1}\right)^6 + {}^8C_8\left(\sqrt{5x^3-1}\right)^8\Big]$$

$$= \frac{1}{2^8}\Big[{}^8C_0\left(5x^3+1\right)^4 + {}^8C_2\left(5x^3+1\right)^3\left(5x^3-1\right) + {}^8C_4\left(5x^3+1\right)^2\left(5x^3-1\right)^2$$
$$+ {}^8C_6\left(5x^3+1\right)\left(5x^3-1\right)^3 + 8C_8\left(5x^3-1\right)^4\Big]$$

So, the degree of polynomial is 12,

Now, coefficient of x^{12}

$$= \left[{}^{8}C_0\, 5^4 + {}^{8}C_2\, 5^4 + {}^{8}C_4\, 5^4 + {}^{8}C_6\, 5^4 + {}^{8}C_8\, 5^4 \right]$$

$$= 5^4 \times \frac{2^8}{2} = 5^4 \times 2^4 \times \frac{2^4}{2} = 10^4 \times 2^3 = 8(10)^4$$

8. (c)

$$\frac{{}^{n}C_r}{1} = \frac{{}^{n}C_{r+1}}{7} = \frac{{}^{n}C_{r+2}}{42}$$

By solving we get $r = 6$

So, it is 7^{th} term.

9. (c)

General term of $\left(ax^{1/8} + bx^{-1/12}\right)^{10}$ is

$$= {}^{10}C_r \left(ax^{1/8}\right)^{10-r} \left(bx^{-1/12}\right)^r$$

On solving $r = 6$

$$\Rightarrow \left(a^4 b^6\right)^{\frac{1}{4}} \ge \frac{}{2\left(\frac{1}{a^2} + \frac{1}{b^2}\right)} \Rightarrow a^4 b^6 \ge \left(\frac{1}{2}\right)^4$$

$$\Rightarrow {}^{10}C_6 \left(\frac{1}{2}\right)^4 = \frac{210}{16} \Rightarrow \frac{105}{8}$$

10. (b)

General term :

$$T_{r+1} = {}^{60}C_r \cdot \left(3^{\frac{1}{4}}\right)^{60-r} \cdot \left(5^{\frac{1}{8}}\right)^r = {}^{60}C_r \cdot 3^{15\frac{r}{-}} = \frac{r}{-}$$

Term will be rational if r divisible by 8.

$\therefore\ r = 0, 8, 16, 24, 32, 40, 48, 56$

So, total number of irrational terms = n = 61 – 8 = 53

Hence, n – 1 = 52 is divisible by 26.

11. (c)

General term $= T_{r+1} = {}^{10}C_r \left(\sqrt{x}\right)^{10-r} \cdot \left(-\frac{k}{x^2}\ ^{r}\right)$

$$= {}^{10}C_r (-k)^r \cdot x^{\frac{10-r}{2} - 2r} = {}^{10}C_r (-k)^r \cdot x^{\frac{10-5r}{2}}$$

Since, it is constant term, then

$$\frac{10 - 5r}{2} = 0 \Rightarrow r = 2 ; \therefore\ {}^{10}C_2 (-k)^2 = 405$$

$$\Rightarrow k^2 = \frac{405 \times 2}{10 \times 9} = \frac{81}{9} = 9$$

$$\therefore\ |k| = 3$$

12. (c)

Given expression can be written as

$$\left[\frac{\left(x^{1/3}\right)^2+1^3}{x^{2/3}-x^{1/3}+1}-\frac{\left(\sqrt{x}\right)^2-1^2}{\sqrt{x}\left(\sqrt{x}-1\right)}\right]^{10}$$

$$=\left(\left(x^{1/3}+1\right)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}=\left(x^{1/3}+1-1-\frac{1}{\sqrt{x}}^{10}\right)$$

$$=\left(x^{1/3}-x^{-1/2}\right)^{10}$$

General term $=T_{r+1}$

$$={}^{10}C_r\left(x^{1/3}\right)^{10-r}\left(-x^{-1/2}\right)^r={}^{10}C_r\ x^{\frac{10-r}{3}}\cdot(-1)^r\cdot x^{-\frac{r}{2}}$$

$$={}^{10}C_r(-1)^r\cdot x^{\frac{10-r}{3}-\frac{r}{2}}$$

Term will be independent of x when $\frac{10-r}{3}-\frac{r}{2}=0$

$\Rightarrow r=4$

So, required term $=T_5={}^{10}C_4=210$

13. (b)

Given expression is $\sum_{t=1}^{20}\left(r^2+1\right)(r!)$

$$\Rightarrow \sum_{x=1}^{20}\left((r+1)^2-2r\right)r!$$

$$\Rightarrow \sum_{x=1}^{20}\left((r+1)(r+1)!r.r!\right)-\sum_{r=1}^{20}r.r!$$

$$\Rightarrow \sum_{x=1}^{20}\left((r+1)(r+1)!-r.r!\right)-\sum_{r=1}^{20}\left((r+1)!-r!\right)$$

$$=\left(21.\underline{|21}-1\right)-\left(\underline{|21}-1\right)=20.21!=22!-2.21!$$

14. (a)

Given expansion is $\sum_{\substack{i,\,j=0\\ i\neq j}}^{n}{}^nC_i\ {}^nC_j$

$$=\sum_{i=0}^{n}{}^nC_i\cdot\sum_{j=0}^{n}{}^nC_j-\sum_{i=j=0}^{n}\left({}^nC_i\right)^2$$

$$=\left(2^n\right)\left(2^n\right)-{}^{2n}C_n=2^{2n}-{}^{2n}C_n$$

15. (d)

Constant term in $\left(3x^3-2x^2+\frac{5}{x^5}\right)^{10}$ to make one term without x

Let, $x^{50}\left(3x^{8}-2x^{7}+5\right)^{10}$

How general term of the expression is

$\frac{10!}{p!q!r!}\left(3x^{8}\right)^{p}\left(-2x^{7}\right)^{q}(5)^{r}$

Here $8p+7q=50$ and $p+q+r=10$

$\Rightarrow$ $p=1, q=6, r=3$ in only valid solution.

$\therefore$ $\frac{10!}{1!6!r!}3^{1}2^{6}\cdot 5^{3}=2^{K}\cdot l \Rightarrow K=9$

16. (a)

Given expression is $\sum_{K=1}^{31} {}^{31}C_{K}\cdot {}^{31}C_{K-1}$

$= {}^{31}C_{1}\cdot {}^{31}C_{0}+{}^{31}C_{2}\cdot {}^{31}C_{1}+....+{}^{31}C_{31}\cdot {}^{31}C_{30}$

Here, ${}^{n}L_{r}={}^{n}L_{n-r}$

$= {}^{31}C_{0}\cdot {}^{31}C_{30}+{}^{31}C_{1}\cdot {}^{31}C_{29}+.....+{}^{31}C_{30}\cdot {}^{31}C_{0}={}^{62}C_{30}$

Similarly

$\sum_{K=1}^{30}\left({}^{31}C_{K}\cdot {}^{31}C_{K-1}\right)={}^{60}C_{29}$

$=1\ {}^{62}C_{30}-{}^{60}C_{29}=\frac{62!}{30!32!}-\frac{60!}{29!31!}$

$=\frac{60!}{29!31!}\left\{\frac{62.61}{30.32}-1\right\}=\frac{60!}{30!31!}\left(\frac{2822}{32}\right)$

Compare above equation with $\frac{\alpha(60!)}{(30!)(31!)}$

So, $\alpha=\frac{2822}{32}$

$\therefore$ $16\alpha=16\times\frac{2822}{32}=1411$

17. (b)

We are given that the expression is

$\left(1-x^{2}+3x^{3}\right)\left(\frac{5}{2}x^{3}-\frac{1}{5x^{2}}\right)^{11}$; $x\neq 0$

$\because$ General term of $\left(\frac{5}{2}x^{3}-\frac{1}{5x^{2}}\right)^{11}$

${}^{11}C_{r}\left(\frac{5}{2}x^{3}\right)^{11-r}\left(-\frac{1}{5x^{2}}\right)^{r}$ [$\because$ General term of $(x+y)^{n}$ is ${}^{n}C_{r}(x)^{n-r}\cdot y^{r}$]

Now general term of ${}^{11}C_{r}\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^{r}x^{33-5r}$

$\therefore$ Term independent of x is

$1\times$ coefficient of x^0 in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ $+-1\times$coefficient of x^{-2} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

For coefficient of x^0 $\quad 33 - 5r = 0$ not possible

For coefficient of x^{-2} $\quad 33 - 5r = -2$

$\Rightarrow 35 = 5r \Rightarrow r = 7$

For coefficient of x^{-3} $\quad 33 - 5r = -3$

$\Rightarrow 36 = 5r$ not possible

So term independent of x is

$$(-1)\,{}^{11}C_7\left(\frac{5^4}{2}\right)\left(-\frac{1}{5}\right)^7 = \frac{33}{200}$$

18. (c)

$$\sum_{k=0}^{20} {}^{20}C_k \cdot {}^{20}C_{20-k}$$

$$= \left({}^{20}C_0\right)^2 + \left({}^{20}C_1\right)^2 + \left({}^{20}C_2\right)^2 + \ldots + \left({}^{20}C_{20}\right)^2$$

$$= {}^{20}C_0 \cdot {}^{20}C_{20} + {}^{20}C_1 \cdot {}^{20}C_{19} \ldots {}^{20}C_{20} \cdot {}^{20}C_0 \qquad \left(\because\ {}^nC_r = {}^nC_{n-r}\right)$$

= coefficient of x^{20} in $(1+x)^{20}\cdot(1+x)^{20}$

= coefficient of x^{20} in $(1+x)^{40} = {}^{40}C_{20}$

19. (a)

The given series, $\sum_{r=0}^{20} {}^{50-r}C_6$

$$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + {}^{47}C_6 + \ldots + {}^{32}C_6 + {}^{31}C_6 + {}^{30}C_6$$

$$= \left({}^{30}C_7 + {}^{30}C_6\right) + {}^{31}C_6 + {}^{32}C_6 + \ldots + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$$

$$= \left({}^{31}C_7 + {}^{31}C_6\right) + {}^{32}C_6 + \ldots + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$$

$$= \left({}^{31}C_7 + {}^{32}C_6\right) + \ldots + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$$

....

....

....

$$= {}^{51}C_7 - {}^{30}C_7$$

20. (b)

Given, ${}^{20}C_1 + 2^2\cdot{}^{20}C_2 + 3^2\cdot{}^{20}C_3 + \ldots + 20^2\cdot{}^{20}C_{20}$

$= A\left(2^{\beta}\right)$ Taking L.H.S., $= \sum_{r=1}^{20} r^2\cdot{}^{20}C_r = 20\sum_{r=1}^{20} r\cdot{}^{19}C_{r-1}$

$$= 20\left[\sum_{r=1}^{20}(r-1)\,{}^{19}C_{r-1} + \sum_{r=1}^{20}{}^{19}C_{r-1}\right]$$

$$= 20\left[19\sum_{r=2}^{20} {}^{18}C_{r-2} + 2^{19}\right] = 20\left[19\cdot 2^{18} + 2^{19}\right] = 420\times 2^{18}$$

Now, compare it with R.H.S., A = 420 and $\beta = 18$

21. (a)

Let $a = \left(\left(1+2x+3x^2\right)^6 + \left(1-4x^2\right)^6\right)$

$\therefore$ Coefficient of x^2 in the expansion of the product $\left(2-x^2\right)\left(\left(1+2x+3x^2\right)^6 + \left(1-4x^2\right)^6\right)$

= 2 (Coefficient of x^2 in a) – 1 (Constant of expansion)

In the expansion of $\left(\left(1+2x+3x^2\right)^6 + \left(1-4x^2\right)^6\right)$.

Constant = 1 + 1 = 2

Coefficient of $x^2 =$ [Coefficient of x^2 in $\left({}^6C_0\left(1+2x\right)^6\left(3x^2\right)^0\right)$]

+ [Coefficient of x^2 in $\left({}^6C_1\left(1+2x\right)^5\left(3x^2\right)^1\right) - \left[{}^6C_1\left(4x^2\right)\right]$

= 60 + 6 × 3 – 24 = 54

$\therefore$ The coefficient of x^2 in $\left(2-x^2\right)\left(\left(1+2x+3x^2\right)^6 + \left(1-4x^2\right)^6\right)$

$= -2\times 54 - 1(2) = 108 - 2 = 106$

22. (a)

We have $\left({}^{21}C_1 + {}^{21}C_2 .... + {}^{21}C_{10}\right) - \left({}^{10}C_1 + {}^{10}C_2 .... {}^{10}C_{10}\right)$

$= \frac{1}{2}\left[\left({}^{21}C_1 + ... + {}^{21}C_{10}\right) + \left({}^{21}C_{11} + ... {}^{21}C_{20}\right)\right] - \left(2^{10} - 1\right)$

$\left(\because\ {}^{10}C_1 + {}^{10}C_2 + ... + {}^{10}C_{10} = 2^{10} - 1\right)$

$= \frac{1}{2}\left[2^{21} - 2\right] - \left(2^{10} - 1\right) = \left(2^{20} - 1\right) - \left(2^{20} - 1\right) = 2^{20} - 2^{10}$

23. (c)

We know that $\left(a+b\right)^n + \left(a-b\right)^n$

$= 2\left[{}^nC_0\, a^n\, b^0 + {}^nC_2\, a^{n-2} b^2 + {}^nC_4 a^{n-4} b^4 ...\right]$

$\left(1-2\sqrt{x}\right)^{50} + \left(1+2\sqrt{x}\right)^{50}$

$2\left[{}^{50}C_0 + {}^{50}C_2\left(2\sqrt{x}\right)^2 + {}^{50}C_4\left(2\sqrt{x}\right)^4 ...\right]$

$= 2\left[{}^{50}C_0 + {}^{50}C_2 2^2 x + {}^{50}C_4 2^4 x^2 + + ...\right]$

Putting $x = 1$, we get,

$${}^{50}C_0 + {}^{50}C_2\, 2^2 + {}^{50}C_4\, 2^4 .... = \frac{3^{50} + 1}{2}$$

24. (24)

Given binomial expression is

$\left(3+6x\right)^{n}={}^{n}C_{0}3^{n}+{}^{n}C_{1}3^{n-1}\left(6x\right)^{1}+....$

General term is shown below.

$T_{r+1}{}^{n}C_{r}3^{n-r}.\left(6x\right)r={}^{n}C_{r}3^{n-r}.6^{r}.x^{r}$

$={}^{n}C_{r}3^{n-r}.3^{r}.2^{r}.\left(\frac{3}{2}\right)^{r}={}^{n}C_{r}3^{n}.3^{r}$ $\left[\text{for } x=\frac{3}{2}\right]$

T_{9} is greatest of $x=\frac{3}{2}$

So, $T_{9}>T_{10}$ and $T_{9}>T_{8}$

Here, $\frac{T_{9}}{T_{10}}>1$ and $\frac{T_{9}}{T_{8}}>1\Rightarrow\frac{{}^{n}C_{8}3^{n}.3^{8}}{{}^{n}C_{9}3^{n}.3^{9}}>1$ and $\frac{{}^{n}C_{8}3^{n}.3^{8}}{{}^{n}C_{7}3^{n}.3^{7}}>1$

So, $\frac{{}^{n}C_{8}}{{}^{n}C_{7}}>\frac{1}{3}$ and $\frac{n-7}{8}>\frac{1}{3}\Rightarrow\frac{29}{3}<n<11\Rightarrow n=10=n_{0}$

So, in $\left(3+6x\right)^{n}$ for $n=n_{0}=10$

Now, Take $\left(3+6x\right)^{10}$, here $T_{r+1}={}^{10}C_{r}\ 310-r\ 6^{r}\ x^{r}$

$T_{7}={}^{10}C_{6}3^{4}.6^{6}.x^{6}=210.3^{10}.2^{6}x^{6}$

$T_{4}={}^{10}C_{6}3^{7}.6^{3}.x^{3}=120.3^{10}.2^{3}x^{3}$

Ratio of coefficients of x^{6} and coefficient of $x^{3}=k\ \therefore\ k=\frac{210.3^{10}.2^{6}}{120.3^{10}.2^{3}}=\frac{7}{4}\times2^{3}=14$

Therefore, $k=n_{0}=14+10=24$.

25. (23)

Given expression is $\left(1+x\right)^{p}\left(1-x\right)^{q}$ and coefficients of x and x^{2} is –3 and –5 respectively.

$\left(1+x\right)^{p}\left(1-x\right)^{q}=\left(1+{}^{p}C_{1}x+{}^{p}C_{2}x^{2}...\right)\left(1-{}^{q}C_{1}x+{}^{q}C_{2}x^{2}...\right)$

According to questions,

${}^{-q}C_{1}+{}^{p}C_{1}=-3\Rightarrow\ p-q=-3$ ...(i)

${}^{q}C_{2}+{}^{p}C_{2}-{}^{p}C_{1}{}^{q}C_{2}=-5$

$\frac{q\left(q-1\right)}{2}-pq+\frac{p\left(p-1\right)}{2}=-5$

$q^{2}-q-2pq+p^{2}-p=-10$

$\left(p-q\right)^{2}-\left(p+q\right)=-10$

From (i),

$\left(-3\right)^{2}-\left(p+q\right)=-10$

$9+10=\left(p+q\right)=19$

Add (i) & (ii), p = 8, q = 11.

Coefficient of $x^3 = (1+x)^8 (1-x)^{11} = \left(1-x^2\right)^8 (1-x)^3$

$= 1\times(-1) + (-8)\times(-3) = -1 + 24 = 23$

26. (5)

$T_{r+1} = (-1)^r . {}^{15}C_r . 2^{15-r} x^{\frac{15-2r}{5}} \Rightarrow m = {}^{15}C_{10} 2^5$

For coefficient of x^{-1}

$\frac{15-r}{5} - \frac{r}{5} = -1 \Rightarrow r = 10 \Rightarrow n = -1$

Given, $mn^2 = {}^{15}C_5 2^5$

27. (83)

Given binomial expansion is $\left(2x^3 + \frac{3}{x}\right)^{10}$.

$T_{r+1} = {}^{10}C_r \left(2x^3\right)^{10-r} \left(\frac{3}{x}\right)^r = 10C_r 2^{10-r} 3^r \; x^{30-4r}$

Put $r = 0, 1, 2, ...7$

$= {}^{10}C_0 2^{10} 3^0 + {}^{10}C_1 2^9 3 + {}^{10}C_2 2^8 3^2 + ... + {}^{10}C_{10} 2^0 3^{10} - \left({}^{10}C_8 2^2 3^8 + {}^{10}C_9 \cdot 2 \cdot 3^9 + {}^{10}C_0 3^{10}\right)$

Use $(a+b)^n$ expansion,

$= (2+3)^{10} - \left(3\times5\times4\times3^9 + 2\times5\times2\times3^9 + 3\cdot3^9\right)$

$= (5)^{10} - 3^9 (60+20+3) = 5^{10} - 8^3 9^3$

Compare with given equation.
Then, $B = 83$.

28. (13)

$T_{r+1} = {}^{22}C_r \cdot \left(x^m\right)^{22-r} \cdot \left(\frac{1}{x^2}\right)^r$

$T_{r+1} = {}^{22}C_r \cdot x^{22m-mr-2r}$

$\because \; 22m - mr - 2r = 1 \Rightarrow r = \frac{22m-1}{m+2} \Rightarrow r = 22 - \frac{3\cdot3\cdot5}{m+2}$

So, possible value of m = 1, 3, 7, 13, 43 $(\because \; r \in W)$

But ${}^{22}C_r = 1540$

$\therefore$ Only possible value of m = 13.

29. (84)

$\frac{T_5}{T_{n-3}} = \frac{{}^nC_4 \left(2^{1/4}\right)^{n-4} \left(3^{-1/4}\right)^4}{{}^nC_{n-4} \left(2^{1/4}\right)^4 (3-1/4)^{n-4}} = \frac{\sqrt[4]{6}}{1}$

$\Rightarrow\ (6)^{\frac{n-8}{4}} = 6^{1/4} \Rightarrow 6^{n-8} = 6$

$\Rightarrow n-8=1 \Rightarrow n=9$

$T_6 = {}^9C_5\left(2^{1/4}\right)^4\left(3^{-1/4}\right)^5 = \frac{84}{\sqrt[4]{3}}$

$\therefore\ \alpha = 84$

30. (57)

Coefficient of middle term

${}^4C_2 \times \frac{\beta^2}{6}, -6\beta, -{}^6C_3 \times \frac{\beta^3}{8}$ are in A.P.

$2(-6\beta) = {}^4C_2\frac{\beta^2}{6} - {}^6C_3\frac{\beta^3}{8}$

$\beta^2 - \frac{5}{2}\beta h3 = -12\beta$

$\beta = \frac{12}{5}$ or $\beta = -2$

$\therefore\ \beta = \frac{12}{5}$

Common difference

$d = \frac{72}{5} - \frac{144}{25} = -\frac{504}{25}$

$\therefore\ 50 - \frac{2d}{\beta^2} = 57$

31. (924)

Sum of coefficient of $(x+y)^n = 2^n \Rightarrow 2^n = 4096$

$\Rightarrow 2^n = 2^{12} \Rightarrow n = 12$

Greatest coefficient $= {}^{12}C_6 = \frac{12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1}$

$= 11\times3\times4\times7 = 924$

32. (1)

Middle term of $(1+x)^{20} = T_{11}$

$\therefore\ T_{11} = T_{10+1} = {}^{20}C_{10}x^{10}$

Coefficient $= {}^{20}C_{10}$

Middle terms of $(1+x)^{19} = T_{10}$ and T_{11}

$\therefore\ T_{10} = T_{9+1} = {}^{19}C_{10}x^{10}$ and $T_{11} = T_{10+1} = {}^{19}C_{10}x^9$

Sum of coefficient $= {}^{19}C_9 + {}^{19}C_{10} = {}^{20}C_{10}$

So, required ratio $= \frac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$.

33. (99)

From given expressional $1+\left(1+2^{49}\right)\left(2^{49}-1\right)=2^{98}$

$\Rightarrow\ m=1, n=98\ \Rightarrow\ m+n=99$

34. (102)

${}^{40}C_0 + {}^{41}C_1 + {}^{41}C_2 + \ldots + {}^{59}C_{19} + {}^{60}C_{20}$

$= {}^{40}C_{40} + {}^{41}C_{40} + {}^{42}C_{40} + \ldots. + {}^{60}C_{40}$

35. (286)

Given expansion is

$(1+x)^{10} = C_0 + C_1x + C_2x^2 + \ldots.. + C_{10}x^{10}$

Differentiating

$10(1+x)^9 = C_1 + 2C_2x + 3C_3x^2 + \ldots. + 10C_{10}x^9$

Replace *x* by x^2

$10\left(1+x^2\right)^9 = C_1 + 2C_2x^2 + 3C_3x^4 + \ldots. + 10C_{10}x^{18}$

$10x\left(1+x^2\right)^9 = C_1x + 2C_2x^3 + 3C_3x^5 + \ldots. + 10C_{10}x^{19}$

Differentiate w.r.t. *x*.

$10\left(\left(1+x^2\right)^9.1 + x.9\left(1+x^2\right)^8 2x\right)$

$= C_1x + 2X_2.3x^3 + 3.5.C_3x^4 + \ldots.. + 10.19C_{10}x^{18}$

Put $x=1$,

$10\left(2^9 + 18.2^8\right)$

$= C_1 + 3.2.C_2 + 5.3.C_3 + \ldots. + 19.10C_{10}$

$C_1 + 3.2.C_2 + \ldots.. + 19.10C_{10} = 10.2^9.10 = 100.^{2^9}$

Take, $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots. + \frac{C_9}{11} + \frac{C_{10}}{11} = \frac{2^{11}-1}{11}$

↑ ↑

10th term 11th term

$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots. + \frac{C_9}{11} = \frac{2^{11}-2}{11}$

Now, $100.2^9 = \frac{\alpha.2^{11}}{2^\beta - 1}\left(\frac{2^{11}-2}{11}\right)$;

$25.2^{11} = \frac{\alpha.2^{11}}{2^\beta - 1}\cdot\left(\frac{2^{11}-2}{11}\right)$

Compare the above equation,

$\alpha = 25\times 11 = 275\ \&\ \beta = 11 \Rightarrow \alpha+\beta = 275+11 = 286$

36. (315)

General term:

$$\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha}(2b)^{\beta}.(4ab)^{\gamma}=\frac{10!}{\alpha!\beta!\gamma!}a^{\alpha+\gamma}b^{\beta+\gamma}.2^{\beta}.4^{\gamma}$$

$\Rightarrow \quad \alpha+\beta+\gamma=10 \qquad (i)$

$\alpha+\gamma=7 \qquad (ii)$

$\beta+\gamma=8 \qquad (iii)$

Solving equations (i), (ii) and (iii), we get

$\gamma=5, \alpha=2, \beta=3$

So, coefficients $=\frac{10!}{2!3!5!}2^{3}.2^{10}$

$$=\frac{10\times9\times8\times7\times6\times5!}{2\times3\times2\times5!}\times2^{13}=315\times2^{16}\Rightarrow k=315$$

1. (a)

$$S=\sum_{r=0}^{n} .^{n} C_r \sin rx.\, \text{Cos}(n-r)x$$
$$= .^{n} C_0 \sin 0x \cos nx +$$
$$.^{n} C_1 \sin x \cos(n-1)x$$

$$+ .^{n} C_2 \sin 2x \cos(n-2)x. \ldots .. +$$
$$.^{n} C_{n-1} \sin(n-1)x \cos x$$

$$+ .^{n} C_n \sin nx \cos 0x$$

Writing the sum in reverse order, we get

$$S = .^{n} C_n \sin nx \cos 0x +$$
$$.^{n} C_{n-1} \sin(n-1)x \cos x$$

$$+ .^{n} C_{n-2} \sin(n-2)x \cos 2x + \ldots\ldots.$$
$$+ .^{n} C_1 \sin x \cos(n-1)x +$$
$$.^{n} C_0 \sin 0x \cos nx$$

$$\therefore S = .^{n} C_0 \sin nx \cos 0x +$$
$$.^{n} C_1 \sin(n-1)x \cos x$$

$$+ .^{n} C_2 \sin(n-2)x \cos 2x + \ldots\ldots$$
$$+ .^{n} C_{n-1} \sin x \cos(n-1)x +$$
$$.^{n} C_n \sin 0x \cos xnx$$

Adding (1) and (2), we get

$2S = .^n \sin(0x + nx) +$
$.^n C_1 \sin(x + (n-1)x)$

$+ .^n C_2 \sin(2x + (n-2)x) + ... +$
$.^n C_n \sin(nx + 0x)$

$= (.^n C_0 + .^n C_1 + .^n C_2 + .^n C_3$
$+ ...... + .^n C_n)\sin nx$

$= 2^n \sin nx$
$\therefore S = 2^{n-1} \sin nx$

2. (a)

Given: An expression $\sum_{r=0}^{n} ({}^nC_r) a^r . b^{n-r} . \cos(rB - (n-r)A)$ in a triangle ABC.

To find: The value of the given expression.

Solution: We know that real part of the above given expression

$= \sum_{r=0}^{n} {}^nC_r a^r b^{n-r} e^{i(rB-(n-r)A)}$

$= \sum_{r=0}^{n} {}^nC_r \cdot (ae^{iB})^r (b \cdot e^{-iA})^{n-r}$

$= (ae^{iB} + be^{-iA})^n$

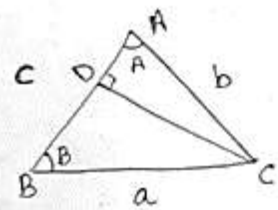
$$= (a\cos B + ia\sin B + b\cos A - ib\sin A)^n$$

$$\left[\because \frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow b\sin A = a\sin B\right] \text{ — (i)}$$

$$= (a\cos B + ia\sin B + b\cos A - ib\sin A)^n \quad \text{(using i)}$$

$$= (a\cos B + b\cos A)^n$$

$$= c^n \quad \text{(ans)} \quad [\text{From eq ii}]$$



$BD = a\cos B$ $\qquad AD + BD = c$

$AD = b\cos A$ $\qquad \therefore c = a\cos B + b\cos A$ — (ii)

Hence option A is correct.

3. (b)

$$\left(n.n.n \ldots n \text{ times}\right)\left(\left(\frac{n+1}{2}\right)\left(\frac{n+1}{2}\right) \ldots 2n \text{ times}\right)$$

$$\frac{N}{\left(\frac{n+1}{2}\right)^3} = n^n\left(\frac{n+1}{2}\right)^{2n-3} \geq 1$$

4. (a)

Given $p = (8 + 3\sqrt{7})^n$

and $p = [p] + f$ (where[.] =GIF)

Sol. : Let $N = (8 - 3\sqrt{7})^n$

$\Rightarrow$ Now, $(a + b)^n + (a - b)^n =$

$2\left[{}^nC_0 a^n + {}^nC_2 a^{n-2} - b^2 + \ldots b^n\right]$

$\Rightarrow$ Take $a = 8$ & $b = 3\sqrt{7}$

$\therefore p + N = 2\left[{}^nC_0 8^n + {}^nC_2 8^{n-2} - (3\sqrt{7})^2 + \ldots\right]$

$= 2.K$ (where k &I)

Now, $0 < f < 1$ & $0 < N < 1$ $\because N =$

$(\sqrt{64} - \sqrt{63})^n$

$\therefore p + N = [p] + f + N = 2K$

$[P]$ is always integer & $p + N$ is an integer

$= f + N$ must be an integer.

But, $0 < (f + N) < 2$ $\rightarrow f + N = 1$

$\Rightarrow p(1 - f) = p.N = (8 + 3\sqrt{7})^n (8 - 3\sqrt{7})^n$

$\Rightarrow (8^2 + (3\sqrt{7})^2)^n = 1^n = 1.$

5. (a)

As $0 < 2 - \sqrt{3} < 1$ we get $0 < F = (2-\sqrt{3})^{2n} < 1$

We have $R + F = (2+\sqrt{3})^{2n} + (2-\sqrt{3})^{2n}$

$$= 2\left[{}^{2n}C_0 2^{2n} + {}^{2n}C_2 2^{2n-2}(\sqrt{3})^2 + {}^{2n}C_4 (2^{2n-4})(\sqrt{3})^4 + \ldots + {}^{2n}C_{2n}(\sqrt{3})^{2n}\right]$$

$\Rightarrow R + F$ is an even integer

$\Rightarrow [R] + f + F$ is an even integer.

$\Rightarrow f + F$ is an integer.

But $0 \le f < 1$ and $0 < F < 1 \Rightarrow 0 < f + F < 2$

But the only integer between 0 and 2 is 1.

Thus, $f + F = 1 \Rightarrow 1 - f = F$

Now, $R(1-f) = RF = (2+\sqrt{3})^{2n}(2-\sqrt{3})^{2n}$

$= (4-3)^{2n} = 1^{2n} = 1$

So, correct option is A

6. (c)

We have given

$n = 1 + P!$; P is prime.

$\Rightarrow n + r = (r+1) + P!$

If $1 \le r \le P-1$ then $2 \le r+1 \le P$

Clearly $(n+r)$ is divisiby by $r+1$.

That mean $(n+r)$ can't be a prime.

Hence, there is no prime in the given list $n+1, n+2, \ldots n+P-1$.

$\Rightarrow$ Option c. zero. is the correct Answer.

7. (b)

$$f(x) = x^n$$

$$\Rightarrow f^r\left(x\right) = \frac{n!}{(n-r)!}x^{n-r} \Rightarrow f^r\left(1\right) = \frac{n!}{(n-r)!}$$

Now,

$$f\left(1\right) + \frac{f^1(1)}{1} + \frac{f^2(1)}{2!} + \frac{f^3(1)}{3!} + \ldots + \frac{f^n(1)}{n!}$$

$$= \sum_{r=0}^{n} \frac{f^r(1)}{r!} = \sum_{r=0}^{n} \frac{n!}{(n-r)!r!} = \sum_{r=0}^{n} {}^nC_r = 2^n$$

8. (c)

We have $(1+x)^{101}(1-x+x^2)^{100} = (1+x)((1+x)(1-x+x^2))^{100}$

$= (1+x)(1+x^3)^{100}$

$= (1+x)\{C_0 + C_1x^3 + C_2x^6 + ... + C_{100}x^{300}\}$

$(1+x)\sum_{r=0}^{n} {}^nC_r x^{3r} = \sum_{r=0}^{n} {}^nC_r x^{3r} + \sum_{r=0}^{n} {}^nC_r x^{3r+1}$

Hence, there will be no term containing 3r +2.

9. (c)

It is given that 6th term in the expansion of $\left(\frac{1}{x^{8/3}} + x^2 \log_{10} x\right)^8$ is 5600 therefore

${}^8C_5\left(x^2 \log_{10} x\right)^5 \left(\frac{1}{x^{8/3}}\right)^3 = 5600$

Or $56x^{10}(\log_{10} x)^5 \frac{1}{x^8} = 5600$

Or $x^2(\log_{10} x)^5 = 100$

Or $x^2(\log_{10} x)^5 = 10^2(\log_{10} 10)^5$

Or x = 10

10. (a)

Last term of $\left(2^{1/3} - \frac{1}{\sqrt{2}}\right)^n$ is

$T_{n+1} = {}^nC_n(2^{1/3})^{n-n}\left(-\frac{1}{\sqrt{2}}\right)^n = {}^nC_n(-1)^n \frac{1}{2^{n/2}} = \frac{(-1)^n}{2^{n/2}}$

Also we have

$\left(\frac{1}{3^{5/3}}\right)^{\log_3 8} = \frac{1}{(3^{5/3})^{3\log 2}} = 3^{-(5/3)\log_3 2^3} = {}^{2-5}$

Thus, $\frac{(-1)^n}{2^{n/2}} = 2^{-5}$

Or $\frac{(-1)^n}{2^{n/2}} = \frac{(-1)^{10}}{2^5}$

Or $n/2 = 5$

Or n = 10

Now, $T_5 = T_{4+1} = {}^{10}C_4 (2^{1/3})^{10-4}\left(-\frac{1}{\sqrt{2}}\right)^4$

$= \frac{10!}{4!6!}(2^{1/3})^6(-1)^4(2^{-1/2})^4$

$= 210(2^2)(1)(2^{-2}) = 210$

11. (c)

$(1+x+x^2+x^3)^5 = a_0 + a_1x + a_2x^2 + a_3x^4 + ... + a_{15}x^{15}$

Putting x = 1 and x = -1 alternatively, we have

$a_0 + a_1 + a_2 + a_3 + ... + a_{15} = 4^5$ …..(1)

$a_0 - a_1 + a_2 - a_3 ..... - a_{15} = 0$ ….(2)

Adding (1) and (2) we have

$2(a_0 + a_2 + a_4 + ... + a_{14}) = 4^5$

Or $a_0 + a_2 + a_3 + ... + a_{14} = 2^9 = 512$

12. (c)

Sum of coefficients in $(1 - x\sin\theta + x^2)^n$ *is* $(1 - \sin\theta + 1)^n$ (putting x = 1)

This sum is greatest when $\sin\theta = -1$, then maximum sum is 3^n

13. (b)

$(1-x)^{30} = {}^{30}C_0x^0 - {}^{30}C_1x^1 + {}^{30}C_2x^2 + ... + (-1)^{30}\,{}^{30}C_{30}x^{30}$ (1)

$(x+1)^{30} = {}^{30}C_0x^{30} + {}^{30}C_1x^{29} + {}^{30}C_2x^{28} + ... + {}^{30}C_{10}x^{20} + ... + {}^{30}C_{30}x^0$ (2)

Multiplying (1) and (2) and equating the coefficient of x^{20} on both sides, we get required sum is equal to coefficient of x^{20} in $(1-x^2)^{30}$, which is given by ${}^{30}C_{10}$

14. (b)

Given series is ${}^{20}C_0 + {}^{20}C1 + {}^{20}C_2 + ... + {}^{20}C_8$

$= \frac{1}{2}(2.\,{}^{20}C_0 + 2\,{}^{20}C_1 + ...2.\,{}^{20}C_8)$

$= \frac{1}{2}\left[\left({}^{20}C_0 + {}^{20}C_1 + ... + {}^{20}C_8 + {}^{20}C_9 + {}^{20}C_{10} + {}^{20}C_{11} + ... + {}^{20}C_{20}\right) - \left({}^{20}C_9 + {}^{20}C_{10} + {}^{20}C_{11}\right)\right]$

$= \frac{1}{2}[2^{20} - 2.\,{}^{20}C_9 - {}^{20}C_{10}]$

$= 2^{19} - \frac{(2.\,{}^{20}C_9 + {}^{10}C_{10})}{2}$

$$=\frac{(2^{20}-{}^{20}C_{10})}{2}-{}^{20}C_9=2^{19}-\frac{({}^{20}C_{10}+2\times{}^{20}C_9)}{2}$$

15. (d)

$$\sum_{r=1}^{n}(-1)^{r+1}\frac{{}^nC_r}{r+1}=\frac{1}{n+1}\sum_{r=1}^{n}(-1)^{r+1}\,{}^{n+1}C_{r+1}$$

$$=\frac{1}{n+1}(0-1+(n+1))=\frac{n}{n+1}$$

16. (c)

$$(1+x)^n=C_0C_1x+C_2x^2+C_3x^3+.....+C_{n-1}x^{n-1}+C_nx^n \qquad .....(1)$$

$$(x+1)^n=C_0x^n+C_1x^{n-1}+C_2x^{n-2}+.....+C_{n-1}x+C_n \qquad ....(2)$$

Multiplying Eqs(1) and (2) and equating the coefficient of x^{n-2}, we get

$$C_0C_2+C_1C_3+C_2C_4+....+C_{n-2}C_n$$

=Coefficient of x^{n-2} in $(1 + x)^{2n}$

$$={}^{2n}C_{n-2}$$

$$=\frac{(2n)!}{(n-2)!(n+2)!}$$

17. (d)

$$\sum_{r=0}^{10}r\,{}^{10}C_r3^r(-2)^{10-r}$$

$$=10\sum_{r=0}^{10}{}^9C_{r-1}3^r(-2)^{10-r}$$

$$=10\times3\sum_{r=0}^{10}{}^9C_{r-1}3^{r-1}(-2)^{10-r}$$

$$=30(3-2)^{10}$$

$$=30$$

18. (d)

General term in the expansion of $\left(\sqrt{2}+\sqrt[3]{3}+\sqrt[6]{5}\right)^{10}$ is $\frac{10!}{a!b!c!}\left(\sqrt{2}\right)^a\left(\sqrt[3]{3}\right)^b\left(\sqrt[6]{5}\right)^c$ where a + b +c = 10.

For rational, term, we have the following

Value of a, b, c	Value of term
a = 4, b =0, c = 6	$\frac{10!}{4!0!6!}\left(\sqrt{2}\right)^4\left(\sqrt[3]{3}\right)^0\left(\sqrt[6]{5}\right)^6=4200$
a=10, b =0, c =0	$\frac{10!}{10!0!0!}\left(\sqrt{2}\right)^{10}\left(\sqrt[3]{3}\right)^0\left(\sqrt[6]{5}\right)^0=32$
a= 4, b = 6, c = 0	$\frac{10!}{4!6!0!}\left(\sqrt{2}\right)^4\left(\sqrt[3]{3}\right)^6\left(\sqrt[6]{5}\right)^0=7560$

19. (b)

$$f(x) = 1 - x + x^2 - x^3 + ... - x^{15} + x^{16} - x^{17} = \frac{1 - x^{18}}{1 + x}$$

$$\Rightarrow f(x-1) = \frac{1-(x-1)^{18}}{x}$$

Therefore required coefficient of x^2 is equal to coefficient of x^3 in $1-(x-1)^{18}$, which is given by $^{18}C_3 = 816$.

20. (a)

$$\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n} {}^kC_{r-1}\right)$$

$$= \sum_{r=1}^{n+1}\left(\sum_{k=1}^{n}\left({}^{k+1}C_r - {}^kC_r\right)\right)$$

$$= \sum_{r=1}^{n+i}\left({}^{n+1}C_r - {}^1C_r\right)$$

$$= 2^{n+1} - 2$$

21. (c)

$= (1 + x^2)^4(1 + x^3)^7(1 + x^4)^{12}$

$= ({}^4C_0 + {}^4C_1x^2 + {}^4C_2x^4 + {}^4C_3x^6 + {}^4C_4x^8)({}^7C_0 +$

${}^7C_1x^3 + {}^7C_2x^6 + {}^7C_3x^9 + ... + {}^7C_7x^{21})$

$\times({}^{12}C_0 + {}^{12}C_1x^4 + {}^{12}C_2x^8 + ... + {}^{12}C_{12}x^{48})$

required power of x is 11

$\Rightarrow x^{(2l+3m+4n)} = x^{11}$

$\Rightarrow (2l, 3m, 4n) \equiv$

$(0, 3, 8); (2, 9, 0); (4, 3, 4); (8, 3, 0)$

$\Rightarrow (l, m, n) \equiv (0, 1, 2); (1, 3, 0); (2, 1, 1); (4, 1, 0)$

Calculation of coefficient of x^{11}

$= ({}^4C_0 \times {}^7C_1 \times {}^{12}C_2) + ({}^4C_1 \times {}^7C_3 \times {}^{12}C_0) +$

$({}^4C_2 \times {}^7C_1 \times {}^{12}C_1) + ({}^4C_4 \times {}^7C_1 \times 1)$

$= (1 \times 7 \times 66) + (4 \times 35 \times 1) + (6 \times 7 \times 12) +$

$(1 \times 7) = 462 + 140 + 504 + 7 = 1113.$

22. (c)

Let,

$$b = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$$

$$= \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} \text{ (we can replace r by n − r)}$$

$$= \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$$

Adding (1) and (2), we get

$$2b = \sum_{r=0}^{n} \frac{r}{{}^nC_r} + \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$$

$$= n\sum_{r=0}^{n} \frac{1}{{}^nC_r}$$

$$= na_n$$

$$\Rightarrow = \frac{n}{2}a_n$$

23. (d)

$$\sum_{r=1}^{n} \frac{{}^nC_r}{r+2} = \frac{{}^nC_1}{3} + \frac{{}^nC_2}{4} + \ldots$$

$$(x+1)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + {}^nC_4 x^4 + \ldots$$

$$\int_0^x x(x+1)^n = \int_0^x {}^nC_0 x + \int {}^nC_1 x^2 + {}^nC_2 \int x^3 + {}^nC_3 \int x^4 + \ldots$$

$$\Rightarrow \int_0^x x(1+x)^n dx = {}^nC_0 \frac{x^2}{2} + {}^nC_1 \frac{x^3}{3} + {}^nC_2 \frac{x^4}{4} + \ldots$$

$$I = \int_0^x x(1+x)^n dx = \int (t-1)\, t^n dt$$

$$= \int t^{n+1} dt - \int t^n \cdot dt$$

$$= \frac{t^{n+2}}{n+2} - \frac{t^{n+1}}{n+1}$$

$$= \frac{(1+x)^{n+2}}{n+2} - \frac{(1+x)^{n+1}}{n+1}$$

$$\therefore \text{required} = \frac{2^{n+2}}{n+2} - \frac{2^{n+1}}{n+1} - \frac{1}{2}$$

$$= 2^{n+1}\left[\frac{2n+2-n-2}{n+2}\right] - \frac{1}{2} = \frac{2^{n+1} \cdot n}{n+2} - \frac{1}{2}$$

24. (a)

25. (d)

26. (c)

27. (b)

28. (d)

$(n+2)2^{n-1}-1$

$$\sum_{0\le i<j\le n}\sum ({}^nC_i + {}^nC_j)$$

$$\begin{cases} C_0 + C_1 + C_2 + \ldots + C_n \\ \quad + C_1 + C_2 + \ldots + C_n \\ \qquad + C_2 + C_3 + \ldots + C_n \\ \qquad \ldots \quad \ldots \quad \ldots \\ \qquad\qquad + C_{n-1} + C_n. \end{cases}$$

$= (C_0 + 2C_1 + 3C_2 + \ldots + {}^nC_{n-1} + {}^{n+1}C_n - C_n \ldots\ldots (1$

$C_0x + C_1x^2 + C_2x^3 + \ldots\ldots + {}^nC_{n-1} + C_nx^{n+1} = x(1+$

Diff. both sides , we get

$C_0 + 2C_1 + 3C_+ \ldots + (n+1)C_nx^n$

$= (1+x)^n + xn(1+x)^{n-1}$

Putting x = 1 ,

$C_0 + 2C_1 + 3C_2 + \ldots\ldots + (n+1)C_n = 2^n + n2^{n-1}$

$= (n+2)2^{n-1}$

Hence , $\sum_{0\le i<j\le n}\sum (C_i + C_j) = (n+2)2^{n-1} - 1$

29. (b)

30. (a)

31. (b)

$$\sum_{0\le i}\sum_{\le j\le n}(C_i + C_j)^2 i = 0, 1, 2, ..., (n-1) \text{ where}$$

$j = 1, 2, 3, ..., n$ and $i < j$

$$= n\left(C_0^2 + C_1^2 + ... + C_n^2\right) +$$

$$2\sum\sum C_i C_j 0 \le i < j \le n$$

$$= n.{}^{2n}C_n +$$

$$\left[(C_0 + C_1 + ... + C_n)^2 - \left(C_0^2 + C_1^2 + ... + C_n^2\right)\right]$$

$$= n.{}^{2n}C_n + (2n)^2 - {}^{2n}C_n = (n-1).{}^{2n}C_n + 2^{2n}.$$

32. (d)

33. (d)

$$S = \sum_{r=0}^{n} {}^nC_r.\frac{1 + rx}{(1 + nx)^r}(-1)^r$$

$$= (-1)^r \sum_{r=0}^{n} {}^nC_r \frac{1}{(1 + nx)^r} +$$

$$(-1)^r \sum_{r=0}^{n} r\, {}^nC_r \frac{1}{(1 + nx)^r}$$

$$=$$

$$\sum_{r=0}^{n} {}^nC_r \left(\frac{-1}{(1 + nx)}\right)^r + x(-1)^r \sum_{r=1}^{n} n\, {}^{n-1}C_{r-1} ($$

$$= \left(1 - \frac{1}{(1 + nx)}\right)^n +$$

$$nx.\sum_{r=1}^{n} {}^{n-1}C_{r-1}\left(\frac{-1}{(1 + nx)}\right)^r$$

$$nx.\sum_{r=1}^{n} {}^{n-1}C_{r-1}\left(\frac{-1}{(1+nx)}\right)^{r}$$

$$= \left(\frac{nx}{1+nx}\right)^{n} +$$

$$nx\left(\frac{-1}{1+nx}\right)\sum_{r=1}^{n} {}^{n-1}C_{r-1}\left(\frac{-1}{(1+nx)}\right)^{r}$$

$$= \left(\frac{nx}{1+nx}\right)^{n} +$$

$$\left(\frac{-nx}{1+nx}\right)\left[\left(1-\frac{1}{1+nx}\right)^{n-1}\right]$$

$$= \left(\frac{nx}{1+nx}\right)^{n} - \left(\frac{nx}{1+nx}\right)^{n}$$

$$= 0.$$

EXERCISE - 2 [B]

1. (a, b, d)

Let $x^2 + 1 = a, x^2 - 1 = b$

Therefore the given expression can be written as,

$$\frac{2}{\sqrt{a}+\sqrt{b}} = \sqrt{a} - \sqrt{b}$$

$(\sqrt{a}+\sqrt{b})^6 + (\sqrt{a}-\sqrt{b})^6 = 2 \times (a^3 + b^3 + 15 \times a \times b \times (a + b))$

so,required solution is $64 \times x^6 - 48 \times x^2$

so,options a,b,d are correct

2. (b, c)

Soln let $x-3=y$ $x-2=y+1$

$$\sum_{r=0}^{2n} a_r(y+1)^r = \sum_{r=0}^{2n} b_r y^r.$$

$\Rightarrow \quad a_0 + a_1(1+y) + \ldots a_{n-1}(1+y)^{n-1} + (1+y)^n + \ldots (1+y)^{2n} = \sum_{r=0}^{2n} b_r y^r.$

$\Rightarrow \quad {}^{n}C_n + {}^{n+1}C_n + \ldots + {}^{2n}C_n = b_n.$ $\quad \Big| \; {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r.$

$\Rightarrow {}^{n+1}C_{n+1} + {}^{n+1}C_n + {}^{n+2}C_n + \ldots {}^{2n}C_n = b_n.$

$\Rightarrow \quad {}^{n+2}C_{n+1} + {}^{n+2}C_n + \ldots + {}^{2n}C_n = b_n.$

$\Rightarrow \quad {}^{n+2}C_{n+1} + {}^{n+2}C_n + \ldots {}^{2n}C_n = b_n$

$b_n = {}^{2n+1}C_{n+1} + {}^{2n}C_n = {}^{2n+1}C_{n+1}$

$\therefore \quad \boxed{b_n = {}^{2n+1}C_{n+1}}$

3. (c, d)

$$\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$$

Put $x = t^6$ in the given binomial

$$=\left(\frac{t^6+1}{t^4-t^2+1}-\frac{t^6-1}{t^6-t^3}\right)^{10}$$

$$=\left(\frac{(t^2)^3+1^3}{t^4-t^2+1}-\frac{(t^3)^2-1^2}{t^6-t^3}\right)^{10}$$

$$=\left(\frac{(t^2+1)(t^4-t^2+1)}{t^4-t^2+1}-\frac{(t^3-1)(t^3+1)}{t^3(t^3-1)}\right)^{10}$$

$$=\left(t^2+1-\frac{(t^3+1)}{t^3}\right)^{10}$$

$$=\left(t^2-\frac{1}{t^3}\right)^{10}$$

$$= \left(t^2 - \frac{1}{t^3}\right)^{10}$$

Let T_{r+1} is independent term of t

$$T_{r+1} = (-1)^r\, {}^{10}C_r (t^2)^{10-r} (t^{-3})^r$$

$$T_{r+1} = (-1)^r\, {}^{10}C_r t^{20-5r} \quad ....(1)$$

So $20 - 5r = 0$ ..... [Since the term is independent of t]

$\Rightarrow r = 4$

Put this value in (1), we get

$$T_5 = {}^{10}C_4 = 210$$

4. (a, d)

Let (r + 1)th term is independent of x in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$

$T_{r+1} = {}^{2n}C_r x^{2n-2r}$

This term is independent of x.

$\therefore 2n - 2r = 0$

$\Rightarrow r = n$

Thus, $T_{r+1} = {}^{2n}C_n$

Thus, $T_{r+1} = {}^{2n}C_n$

$$= \frac{(2n)!}{n!n!} = \frac{1.2.3..4...(2n-1)(2n)}{n!n!}$$

$$= \frac{(1.3.5...(2n-1)(1.2...n)\cdot 2^n}{n!n!}$$

$$= \frac{1.3.5...(2n-1)}{n!}\cdot 2^n.$$

5. (a, b, c)

It is given that,

$(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots\ldots\ldots\ldots +$

$a_{2n} x^{2n}$ ....(1)

Substitute x = 1 in equation(1), we get

$a_0 + a_1 + a_2 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{2n} = 3^n$ ....(2)

Now substitute x = w in equation (1),

(where w is the non-real cube root of unity)

we get,

$(1 + w + w^2)^n = a_0 + a_1 w + a_2 w^2 + a_3 w^3 +$

$\ldots\ldots\ldots\ldots\ldots\ldots + a_{2n} w^{2n}$

According to the property of the cube root of unity, $1 + w + w^2 = 0$

$\Rightarrow a_0 + a_1 w + a_2 w^2 + a_3 w^3 + \ldots\ldots\ldots\ldots\ldots\ldots +$

$a_{2n} w^{2n} = 0$ ....(3)

Now, substitute $x = w^2$ in equation (1),

(where w is the non-real cube root of unity)

We get,

$(1 + w^2 + w^4)^n = a_0 + a_1 w^2 + a_2 w^4 + a_3 w^6 +$

$\ldots\ldots\ldots\ldots + a_{2n} w^{4n}$

Now, $(1 + w^2 + w^4)$

$= (1 + w^2 + ww^3)$

$= (1 + w^2 + w) \quad (\because w^3 = 1)$

= 0 (as per the property of the cube root of unity)

$\Rightarrow a_0 + a_1 w^2 + a_2 w^4 + a_3 w^6 + \ldots\ldots\ldots\ldots +$

$a_{2n} w^{4n} = 0$ ....(4)

Adding equations (2),(3) and (4), we get

$3a_0 + a_1(1 + w + w^2) + a_2(1 + w^2 + w^4) +$

$a_3(1 + w^3 + w^6) + \ldots\ldots\ldots\ldots = 3^n$

Now, apply properties of the cube root of unity i.e. $1 + w + w^2 = 0$ and $w^3 = 1$

Now, apply properties of the cube root of unity i.e. $1 + w + w^2 = 0$ and $w^3 = 1$

We get,

$$3a_0 + 3a_3 + 3a_6 + \ldots\ldots\ldots\ldots = 3^n$$

$$\Rightarrow 3(a_0 + a_3 + a_6 + \ldots\ldots\ldots\ldots) = 3^n$$

$$\Rightarrow a_0 + a_3 + a_6 + \ldots\ldots\ldots\ldots = 3^{n-1}$$

6. (a, b, c)

We know $T_2 = {}^nC_1 a^{n-1} b$

$na^{n-1}.b = 135$ ...(i)

$T_3 = {}^nC_2 a^{n-2} b^2$

$\frac{n(n-1)}{2} a^{n-2} b^2 = 30$ ...(ii)

Taking ratio, we get

$$\frac{a}{b(n-1)} = \frac{9}{4}$$

$$\frac{a}{b} = \frac{9(n-1)}{4} \text{ ... (A)}$$

$$T_4 = {}^nC_3 a^{n-3} b^3 = \frac{10}{3} \text{ ...(iii)}$$

Hence, $\frac{T_3}{T_4}$

$$\frac{6(a)}{2(n-2).b} = 9$$

$$\frac{a}{b} = \frac{9(n-2)}{3} \text{ ...(B)}$$

Comparing A and B, we get

$$\frac{9(n-2)}{3} = \frac{9(n-1)}{4}$$

$$4n - 8 = 3n - 3$$

$$n = 5$$

Hence $\frac{a}{b} = 3(n-2)$

$= 9$

7. (a, d)

Since, coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in $(1+x)^{14}$ are in A.P.

$$\Rightarrow 2(^{14}C_r) = ^{14}C_{r-1} + ^{14}C_{r+1}$$

$$\frac{^{14}C_{r-1}}{^{14}C_r} + \frac{^{14}C_{r+1}}{^{14}C_r} = 2$$

$$\frac{r}{14-r+1} + \frac{14-r}{r+1} = 2$$

$$\Rightarrow \frac{r^2 + r + (15-r)(14-r)}{(15-r)(r+1)} = 2$$

$$\Rightarrow 2r^2 - 18r + 210 + 2(15-r)(r+1)$$

$$\Rightarrow 4r^2 - 56r + 180 = 0$$

$$\Rightarrow r^2 - 14r + 45 = 0$$

$$\Rightarrow r = 5, 9$$

8. (a, b)

Given: $P_n = {}^nC_0 . {}^nC_1 . {}^nC_2 ... {}^nC_n$

Now, $\frac{P_{n+1}}{P_n} = \frac{{}^{n+1}C_0 . {}^{n+1}C_1 . {}^{n+1}C_2 ... {}^{n+1}C_{n+1}}{{}^nC_0 . {}^nC_1 . {}^nC_2 ... {}^nC_n}$

$\Rightarrow \frac{P_{n+1}}{P_n} = {}^{n+1}$

$C_0 . \left(\frac{{}^{n+1}C_1}{{}^nC_0}\right) . \left(\frac{{}^{n+1}C_2}{{}^nC_1}\right) ... \left(\frac{{}^{n+1}C_n}{{}^nC_n}\right) . {}^{n+1}C_{n+1}$

Since ${}^{n+1}C_{r+1} = \frac{n+1}{r+1} . {}^nC_r$

$\Rightarrow \frac{P_{n+1}}{P_n} = 1\left(\frac{n+1}{1}\right)\left(\frac{n+2}{2}\right) ... \left(\frac{n+1}{n}\right)1$

$\therefore \frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n!}$

9. (c, d)

10. (c)

11. (b)

12. (d)

13. (a, c, d)

14. (b, c)

15. (a, b)

16. (a, b)

17. (b, c, d)

18. (a, b)

19. (a, c)

20. (b, c)

21. (a, b, c)

22. (a, b, c, d)

23. (a, b, c, d)

24. (b, c)

25. (c, d)

26. (c, d)

27. (a, b, c, d)

28. (a, b, d)

29. (a, c)

30. (a, b, c, d)

31. (c)

32. (c)

33. (d)

34. (a)

35. (a)

36. (c)

37. (b)

38. (d)

39. (d)

40. (a)

41. (c)

42. (A) $\rightarrow$ (Q) (B) $\rightarrow$ (Q) (C) $\rightarrow$ (Q) (D) $\rightarrow$ (Q), (R), (S)

43. (A) $\rightarrow$ (R) (B) $\rightarrow$ (S) (C) $\rightarrow$ (P) (D) $\rightarrow$ (Q)

44. (A) $\rightarrow$ (S) (B) $\rightarrow$ (P) (C) $\rightarrow$ (Q) (D) $\rightarrow$ (R)

45. (A) $\rightarrow$ (P) (B) $\rightarrow$ (Q) (C) $\rightarrow$ (R) (D) $\rightarrow$ (P)

EXERCISE - 2 [C]

1. (00)
2. (16)
3. (06)
4. (20)
5. (91)
6. (84)
7. (20)
8. (08)
9. (08)
10. (03)
11. (3)
12. (4)
13. (5)
14. (5)
15. (5)
16. (7)
17. (0)
18. (6)
19. (0)
20. (1)

1. (d)

Clearly $A_r = {}^{10}C_r, B_r = {}^{20}C_r, C_r = {}^{30}C_r$

Now, $\sum_{r=1}^{10} A_r\left(B_{10}B_r - C_{10}A_r\right)$

$= \sum_{r=1}^{10} {}^{10}C_r\left({}^{20}C_{10}\ {}^{20}C_r - {}^{30}C_{10}\ {}^{10}C_r\right)$

$= {}^{20}C_{10}\sum_{r=1}^{10} {}^{10}C_r\ {}^{20}C_r - {}^{30}C_{10}\sum_{r=1}^{10} {}^{10}C_r \times {}^{10}C_r$

$= {}^{20}C_{10}\left({}^{10}C_1\,{}^{20}C_1 + {}^{10}C_2\,{}^{20}C_2 + \ldots + {}^{10}C_{10}\,{}^{20}C_{10}\right) -$

$\quad {}^{30}C_{10}\left({}^{10}C_1 \times {}^{10}C_1 + {}^{10}C_2 \times {}^{10}C_2 + \ldots + {}^{10}C_{10}\,{}^{10}C_{10}\right)$ … (i)

Now on expanding $(1+x)^{10}$ and $(1+x)^{20}$ and comparing the coefficients of x^{20} in their product on both sides, we get

${}^{10}C_0\,{}^{20}C_0 + {}^{10}C_1\,{}^{20}C_1 + {}^{10}C_2\,{}^{20}C_2 + \ldots + {}^{10}C_{10}\,{}^{20}C_{10}$

= coeff. Of x^{20} in $(1+x)^{30} = {}^{30}C_{20} = {}^{30}C_{10}$

$\therefore\ {}^{10}C_1\,{}^{20}C_1 + {}^{10}C_2\,{}^{20}C_2 + \ldots + {}^{10}C_{10}\,{}^{20}C_{10} = {}^{30}C_{10} - 1$ … (ii)

Again on expanding $(1+x)^{10}$ and $(x+1)^{10}$ and comparing the coefficients of x^{10} in their product on both sides, we get

$\therefore\ \left({}^{10}C_0\right)^2\left({}^{10}C_1\right)^2 + \left({}^{10}C_2\right)^2 + \ldots. + \left({}^{10}C_{10}\right)^2$

= coeff. Of x^{10} in $(1+x)^{20} = 20C_{10}$

$\therefore\ \left({}^{10}C_1\right)^2 + \left({}^{10}C_2\right)^2 + \ldots + \left({}^{10}C_{10}\right)^2 = {}^{20}C_{10} - 1$ … (iii)

Now, from equations (i), (ii) and (iii), we get

Required value = ${}^{20}C_{10}\left({}^{30}C_{10} - 1\right) - {}^{30}C_{10}\left({}^{20}C_{10} - 1\right)$

$= {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$

2. (c)

3. (a)

4. (5)

$(1+x)^2 + (1+x)^3 + \ldots + (1+x)^{49} + (1+mx)^{50}$

$= (1+x)^2\left[\frac{(1+x)^{48} - 1}{(1+x) - 1}\right] + (1+mx)^{50}$

$$=\frac{1}{x}\left[(1+x)^{50}-(1+x)^{2}\right]+(1+mx)^{50}$$

Coeff. Of x^2 in the above expansion

= Coeff. Of x^3 in $(1+x)^{50}$ + Coeff. Of x^2 in $(1+mx)^{50}$

$$= {}^{50}C_3 + {}^{50}C_2 m^2$$

$$\therefore\ (3n+1)\,{}^{51}C_3 = {}^{50}C_3 + {}^{50}C_2 m^2$$

$$\Rightarrow\ (3n+1)=\frac{{}^{50}C_3}{{}^{51}C_3}+\frac{{}^{50}C_2}{{}^{50}C_3}m^2$$

$$\Rightarrow\ 3n+1=\frac{16}{17}+\frac{1}{17}m^2\ \Rightarrow\ n=\frac{m^2+1}{51}$$

$\therefore$ Least position integer m for which n is an integer is m = 16 and then n = 5

5. (6)

Let the coefficients of three consecutive terms of $(1+x)^{n+5}$ be ${}^{n+5}C_{r-1}$, ${}^{n+5}C_r$, ${}^{n+5}C_{r+1}$, then we have

$${}^{n+5}C_{r-1} : {}^{n+5}C_r : {}^{n+5}C_{r+1} = 5:10:14$$

$$\frac{{}^{n+5}C_{r-1}}{{}^{n+5}C_r}=\frac{5}{10}\ \Rightarrow\ \frac{r}{n+6-r}=\frac{1}{2}$$

$$\Rightarrow\ n-3r+6=0 \qquad ...(i)$$

Also $\frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}}=\frac{10}{14}\ \Rightarrow\ \frac{r+1}{n-r+5}=\frac{5}{7}$

$$\Rightarrow\ 5n-12r+18=0 \qquad ...(ii)$$

Solving (i) and (ii), we get $n=6$.

6. (646)

$$\sum_{r=0}^{n} r\left({}^{n}C_r\right)r=0^{2n} = n\sum_{r=0}^{n} {}^{n}C_r\ {}^{n-1}C_{r-1}$$

$$=n\sum_{r=1}^{n} {}^{n}C_{n-r}\ {}^{n-1}C_{r-1} = n^{2n-1}C_{n-1}$$

Now, $X=\left({}^{10}C_1\right)^2+2\left({}^{10}C_2\right)^2+3\left({}^{10}C_3\right)^2+...+10\left({}^{10}C_{10}\right)^2$

$$=\sum_{n=0}^{10} r\left({}^{10}C_r\right)^2 = 10^{19}C_9$$

$$\therefore\ \frac{X}{1430}=\frac{1}{143}\ {}^{19}C_9 = 646$$

7. $(101)^{50}$

$$(101)^{50}-\left\{(99)^{50}+(100)^{50}\right\}$$

$$=(100+1)^{50}-(100-1)^{50}-(100)^{50}$$

$$=(100)^{50}\left[(1+0.01)^{50}-1(1-0.01)^{50}-1\right]$$

$$=(100)^{50}\left[2\left({}^{50}C_1(0.01)+{}^{50}C_3(0.01)^3+...\right)-1\right]$$

$$=(100)^{50}\left[2\times 50\times\frac{1}{100}+2\left({}^{50}C_3(00.01)^3+....\right)-1\right]$$

$$=(100)^{50}\left[2\left({}^{50}C_3(0.01)^3+....\right)\right]>0$$

$$\therefore\ (101)^{50}>(99)^{50}+(100)^{50}$$

$\therefore\ (101)^{50}$ is integer.

8. (6.20)

Here, $\sum_{k=0}^{n}k=\frac{n(n+1)}{2}$

$$\sum_{k=0}^{n}{}^{n}C_k\,k^2=\sum_{k=1}^{n}\frac{n}{k}\cdot{}^{n-1}C_{k-1}.k^2=\sum_{k=1}^{n}n.{}^{n-1}C_{k-1}.k$$

$$=n\sum_{k=1}^{n}{}^{n-1}C_{k-1}(k-1+1)=n\left[\sum_{k=2}^{n}\frac{n-1}{k-1}\,{}^{n-2}C_{k-2}(k-1)+\sum_{k=1}^{n}{}^{n-1}C_{k-1}\right]$$

$$=n(n-1)2^{n-2}+2\times 2^{n-1}$$

$$\sum_{k=0}^{n}{}^{n}C_k\times k=\sum_{k=1}^{n}{}^{n-1}C_{k-1}\times k=n\times 2^{n-1}\ \text{and}\ \sum_{k=0}^{n}{}^{n}C_k 3^k=4^n$$

$$\therefore\ \det\begin{bmatrix}\sum_{k=0}^{n}k & \sum_{k=0}^{n}{}^{n}C_k k^2\\ \sum_{k=0}^{n}{}^{n}C_k k & \sum_{k=0}^{n}{}^{n}C_k 3^k\end{bmatrix}=0$$

$$\Rightarrow\begin{vmatrix}\frac{n(n+1)}{2} & n(n-1)2^{n-2}+n\times 2^{n-1}\\ n\times 2^{n-1} & 4^n\end{vmatrix}=0$$

$$\Rightarrow\ n(n+1)\times 2^{2n-1}-n^2\left[(n-1)2^{2n-3}+2^{2n-2}\right]=0$$

$$\Rightarrow\ 2^{2n-3}\times n\left[4(n+1)-n[n-1+2]\right]=0$$

$$\Rightarrow\ 2^{2n-3}\times n\left[4n+4-n^2-n\right]=0$$

$$\Rightarrow\ n^2-3n-4=0\ \Rightarrow\ n=4$$

$$\therefore\ \sum_{k=0}^{n}\frac{{}^{n}C_k}{k+1}=\sum_{k=0}^{4}\frac{{}^{4}C_k}{k+1}={}^{4}C_0+\frac{{}^{4}C_1}{2}+\frac{{}^{4}C_2}{3}+\frac{{}^{4}C_3}{4}+\frac{{}^{4}C_4}{5}$$

$$=1+2+2+1+\frac{1}{5}=6.20$$

9. prove that ques.

10. prove that ques.

11. prove that ques.