INCHAPTER EXERCISE

Q.1 $y=[x^2+2\sqrt{x}]x=x^3+2(x)^{1/2+1}=x^3+2(x)^{3/2}$

$\Rightarrow \frac{dy}{dx}=3x^2+2\left[\frac{3}{2}(x)^{3/2-1}\right]=3x^2+3\sqrt{x}$

Q.2 $y=\underset{u}{\underset{\uparrow}{x^2}}\,.\,\underset{v}{\underset{\uparrow}{\cos x}}$, $\quad u=x^2\Rightarrow\frac{du}{dx}=2x$

$v=\cos x\Rightarrow\frac{dv}{dx}=-\sin x$

$\Rightarrow \frac{dy}{dx}=v\frac{du}{dx}+u\frac{dv}{dx}=\cos x(2x)+x^2(-\sin x)$

$= 2x\cos x - x^2\sin x$

Q.3 $y=(\underset{u}{\underset{\uparrow}{4x^2-7x+5}})\underset{v}{\underset{\uparrow}{\sec x}}$ $\quad \overset{4(2x)}{\downarrow}$

$u=4x^2-7x+5\Rightarrow\frac{du}{dx}=8x-7$

$v=\sec x\Rightarrow\frac{dv}{dx}=\sec x\ \tan x$

$\frac{dy}{dx}=v\frac{du}{dx}+u\frac{dv}{dx}=\sec x\,(8x-7)+(4x^2-7x+5)(\sec x\tan x)$

Q.4 $y=\underset{u}{\underset{\downarrow}{x^4}}(\underset{v}{\underset{\downarrow}{5\sin x-3\cos x}})$

$u=x^4\Rightarrow\frac{du}{dx}=4x^3$

$v=5\sin x-3\cos x\Rightarrow\frac{dv}{dx}=5[\cos x]-3(-\sin x)$

$= 5\cos x + 3\sin x$

$\Rightarrow\frac{dy}{dx}=(5\sin x-3\cos x)4x^3-x^4(5\cos x+3\sin x)$

Q.5 $y=\frac{\sqrt{x}+1}{\sqrt{x}-1}\begin{matrix}\leftarrow u\\ \leftarrow v\end{matrix}$

$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

$u=\sqrt{x}+1=(x)^{1/2}+1$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2}(x)^{\frac{1}{2}-1} + 0 = \frac{1}{2}(x)^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$v = \sqrt{x} - 1 = (x)^{1/2} - 1$$

$$\frac{dv}{dx} = \frac{1}{2}(x)^{\frac{1}{2}-1} - O = \frac{1}{2}(x)^{-1/2} = \frac{2}{2\sqrt{x}}$$

$$\underset{\downarrow}{v} \quad \underset{\downarrow}{du/dx} \quad \underset{\downarrow}{u}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(\sqrt{x}-1\right)\frac{1}{2\sqrt{x}} - \left(\sqrt{x}+1\right)\left(\frac{1}{2\sqrt{x}}\right)}{\left(\sqrt{x}-1\right)^2}$$

$$= \frac{\sqrt{x}-1-\sqrt{x}+1}{2\sqrt{x}\left(\sqrt{x}-1\right)^2} = \frac{-2}{2\sqrt{x}\left(\sqrt{x}-1\right)^2}$$

$$= \frac{-1}{\sqrt{x}\left(\sqrt{x}-1\right)^2}$$

Q.6 $y = \frac{x \ln x}{e^x} \begin{matrix} \leftarrow u \\ \leftarrow v \end{matrix}$, $v = e^x \Rightarrow \frac{dv}{dx} = e^x$

$$u = \underset{\substack{\uparrow \\ P}}{(x)}\ \underset{\substack{\uparrow \\ Q}}{(\text{In}\, x)}, \qquad P = x \Rightarrow \frac{dp}{dx} = 1, \qquad Q = \text{Ln}x \Rightarrow \frac{dq}{dx} = \frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = P\frac{dQ}{dx} + Q\frac{dp}{dx}$$

$$= (x)\left(\frac{1}{x}\right) + (1nx)(1) = 1 + \text{Ln}x$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(e^x)(1+\text{Ln}x) - (\text{Ln}x)(e)^x}{(e^x)^2}$$

$$= \frac{e^x[1+\text{Ln}x - x\text{Ln}x]}{e^x} = \frac{1+\text{Ln}x - x\text{Ln}x}{e^x}$$

INCHAPTER EXERCISE

Q.1 **(i)** $y = \frac{x^3}{3} - x$

Step – 1 Find $\frac{dy}{dx}, \frac{dy}{dx} = \frac{1}{\cancel{3}}[\cancel{3}x^2] - 1 = x^2 - 1$

Step – 2 Equate $\frac{dy}{dx} = 0$

$\Rightarrow x^2 - 1 = 0$

i.e. $x^2 = 1 \Rightarrow x = 1$ or $x = -1$

Step -3 Find $\frac{d^2y}{dx^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{d(x^2-1)}{dx}$

$\frac{d^2y}{dx^2} = 2(x) - 0 = 2x$

Step-4 Substitute x = 1and x = -1 in $\frac{d^2y}{dx^2}$ and compare it with zero

For x = 1in $\frac{d^2y}{dx^2} = (2x) = 2(1) = 2 > 0 \; x = 1$

$\Rightarrow x = 1 = \text{minima}$

And [y_{min}]= $\frac{[x=1]^3}{3} - (x=1)$

Local

$\left(\because y = \frac{x^3}{3} - x\right)$

$\Rightarrow y_{min} = \frac{1}{3} - 1 = \frac{-2}{3}$

For x = -1 in $\frac{d^2y}{dx^2}(= 2x) = 2(-1) = -2 < 0$

$\Rightarrow x = -1 = \text{maxima}$

And $[Y_{max}]_{local} = \frac{(x=-1)^3}{2} - (x=-1)$

= $\frac{-1}{3} + 1 = 2/3$

Q.1 **(ii)** $y = 4x^3 - 21x^2 + 18x + 5$

$S^{-1} \Rightarrow 4(3x^2) - 21(2x) + 18(1) + 0$

$= 12x^2 - 42x + 18$

$S - 2 = \frac{dy}{dx} = 0$

$\Rightarrow 12x^2 - 42x + 18 = 0$

$\Rightarrow 6(2x^2 - 7x + 3) = 0$

Or $(2x^2 - 7x + 3) = 0$

$\Rightarrow 2x^2 - 6x - x + 3 = 0$

$2x(x-3) - 1(x-3) = 0$

$\Rightarrow (x-3)(2x-1) = 0$

$\Rightarrow$ x = 3 or x = ½

$S - 3 \; \frac{d^2y}{dx^2} = 12(2x) - 42(1) + 0$

$= 24x - 42$

S – 4 x = 3 in $\frac{d^2y}{dx^2} = 24(3) - 42 = 30 > 0$

$\Rightarrow x = 3$ is minima.

$x = \frac{1}{2}$ in $\frac{d^2y}{dx^2} = 24\left(\frac{1}{2}\right) - 42 = -30 > 0$

$\Rightarrow x = \frac{1}{2} = \text{maxima}$

Q.1 **(iii)** y = sin x

S – 1 $\frac{dy}{dx} = \cos x$

S -2 $\frac{dy}{dx} = \cos x = 0$

$\Rightarrow x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$

S – 3 $\frac{d^2y}{dx^2} = -\sin x$

S – 4 $x = \frac{\pi}{2}$ in $\frac{d^2y}{dx^2} = -\sin\left(\frac{\pi}{2}\right) = -1 < 0$

$\Rightarrow x = \pi / 2 = \text{maxima}$

$x = \frac{3\pi}{2}$ in $\frac{d^2y}{dx^2} = -\sin\left(\frac{3\pi}{2}\right) = -(-1) = 1 > 0$

$\Rightarrow x = \frac{3\pi}{2} = \text{minima}$

Q.2 **(i)** $y = x^3 - 3x + 10$

S – 1 $\frac{dy}{dx} = 3x^2 - 3(1) - 0 = 3x^2 - 3$

S – 2 $\frac{dy}{dx} = 3x^2 - 3 = 0$

$\Rightarrow \cancel{3}x^2 = \cancel{3}$

$x^2 = 1$

$\Rightarrow x = 1$ or $x = -1$

S – 3 $\frac{d^2y}{dx^2} = 3(2x) - 0 = 6x$

S – 4 * x = 1 in $\frac{d^2y}{dx^2} (= 6x) = 6(1) = 6 > 0$

$\Rightarrow x = 1 = \text{minima}$

$y_{min} = (1)^3 - 3(1) + (10) = 8$

$\left(\because y = x^3 - 3x + 10\right)$

- x = -1 in $\frac{d^2y}{dx^2} = 6x = 6(-1) = -6 < 0$

$\Rightarrow x = -1 = \text{maxima}$

$\Rightarrow y_{max} = (-1)^3 - 3(-1) + 10$

$y_{max} = -1 + 3 + 10 = 12$

Q.2 **(ii)** $y=\frac{x^2}{2}+\frac{1}{x}$

$x^{-1}-1(x)=\frac{-1}{x^2}$

S –1 2 $\frac{dy}{dx}=\frac{1}{2}(2x)-\frac{1}{x^2}=x-\frac{1}{x^2}$

S – 2 $\frac{dy}{dx}=x-\frac{1}{x^2}=0\Rightarrow x=\frac{1}{x^2}$

$\Rightarrow x^3=1\Rightarrow x=1$ only real value s of x

We only get 1 value of x hence either we'll get maxima or we'll get minima.

S – 3 $\frac{d^2y}{dx^2}=1-\frac{(-2)}{x^3}=1+\frac{2}{x^3}$

S – 4 $x=1$ in $\frac{d^2y}{dx^2}=1+\frac{2}{(x=1)^3}=3>0$

$\Rightarrow x=1=\text{minima}\Rightarrow y_{min}\ \frac{(1)^2}{2}+\frac{1}{1}=\frac{3}{2}$

$y=\frac{x^2}{2}+\frac{1}{x}$

INCHAPTER EXERCISE

Q.1 $\int\left(x^5+\frac{2}{x^2}-\frac{1}{x}-\frac{4}{x}-\frac{4}{\sqrt{x}}+10\right)dx$

$=\int x5dx+\int\frac{2}{x^2}dx-\int\frac{1}{x}dx-\int\frac{4}{\sqrt{x}}dx+10\int dx$

$=\frac{x^6}{6}\frac{-2}{x}=\text{Lnx}-8\sqrt{x}+10x+c$

Q.2 $\int\left(7e^x+4\sin x-\frac{9}{x^3}+e\right)dx$

$7\int e^x+4\int\sin x-9\int\frac{dx}{x^3}+e\int dx$

$7e^x-4\cos x+\frac{9}{2x^2}+ex+c$

INCHAPTER EXERCISE

Q.1 $\int x\sin(1+x^2)dx$

Let P = 1 + x^2

$\Rightarrow\frac{dp}{dx}=0+2x\Rightarrow\frac{dp}{dx}=2x$

Or $\frac{dp}{2}=xdx$

$$\Rightarrow \int x\sin(1+x^2)dx = \int \sin(1+x^2)xdx = \int \sin p\left(\frac{dp}{2}\right)$$

$$= \frac{1}{2}(-\cos p) + c$$

$$= -\frac{\cos(1+x^2)}{2} + c$$

Q.2 $\int \frac{2x}{1+x^2}dx$

Let $1+x^2 \Rightarrow \frac{dp}{dx} = 2x \Rightarrow dp = 2xdx$

$$\Rightarrow \int \frac{2xdx}{1+x^2} = \int \frac{dP}{P} = \text{LnP} + C$$

Ans. $\ln(1+x^2) + C$

INCHAPTER EXERCISE

Q.1 $\int_1^5 (3+2t)dt = \int_1^5 3dt + \int_1^5 2tdt$

$$= 3[t]^5 + 2\left(\frac{t^2}{2}\right)_1^5$$

$= 3(5 - 1) + [(5)^2 - (1)^2]$

$= 12 + 24 = 36$

Q.2 $\int_{\pi/2}^{\pi/2} (2\sin x - \cos x)dx$

$$\int_{\pi/4}^{\pi/2} 2\sin x\,dx - \int_{\pi/4}^{\pi/2} \cos xdx$$

$$= 2[-\cos x]_{\pi/4}^{\pi/2} - [\sin x]_{\pi/4}^{\pi/2}$$

$$= 2\left[\left(-\cos\frac{\pi}{2}\right) - \left(-\cos\frac{\pi}{4}\right)\right] - \left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right]$$

$$= 2\left[0 + \frac{1}{\sqrt{2}}\right] - \left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}} - 1$$

INCHAPTER EXERCISE

Q.1 S = $(2t + 4t^2)$m

$$v = \frac{ds}{dt} = 2 + 8t \rightarrow \text{at } t = 0 \Rightarrow v = 2$$

$$\text{at } t = 2 \Rightarrow v = 18$$

$$\text{at } t = 10 \Rightarrow v = 82$$

$$a = \frac{dv}{dt} = 8 = \text{constant.}$$

Hence same value at all times

Q.2 v = $3t^2$

$$* \int ds = \int v dt = \int 3t^2 dt = 3\left(\frac{t^3}{3}\right)^t = [t^3]$$

$$[s]_0^s = [t^3]_0^t \Rightarrow (s - 0) = (t^3 - 0)$$

* At t = 0 $\Rightarrow$ s = 0 m
* At t = 2 $\Rightarrow$ s = 8m
* At t = 10 $\Rightarrow$ S = 1000m
* $a = \frac{dv}{dt} = 6t \leftarrow$ variable acceleration
* acceleration at t = 0 $\Rightarrow a = 0 m/s^2$
* acceleration at t = 2 $\Rightarrow$ a = 12 m/s^2
* acceleration at t = 10 $\Rightarrow a = 60 m/s^2$

Q3. If v = u + at

$$\text{As } S = \int_0^t v dt$$

$$= \int_0^t (u + at) dt \quad a = \text{constant}$$

$$= u\int_0^t dt + a\int_0^t t \, dt$$

$$= u[t]_0^t + a\left(\frac{t^2}{2}\right)_0^t$$

$$= ut + \frac{1}{2}at^2$$

Q.4 $a = \frac{dV}{ds}\frac{ds}{dt} = v$

$$a = v\frac{dv}{ds}$$

$$\Rightarrow ads = vdv$$

$$\Rightarrow \int ads = \int vdv$$

$$a(s)_0^s = \left[\frac{v^2}{2}\right]_u^v$$

$$\Rightarrow a(S-0) = \frac{v^2-u^2}{2}$$

$$\Rightarrow v^2 - u^2 = 2as \Rightarrow v^2 = u^2 + 2as$$

EXERCISE # I

1. $y = (x^2 - 3x + 3)(x^2 + 2x - 1)$

$$\underset{u}{\uparrow} \qquad \underset{v}{\uparrow}$$

$$u = x^2 - 3x + 3 \Rightarrow \frac{du}{dx} = 2x - 3(1) + 0 = 2x - 3$$

$$v = x^2 + 2x - 1 \Rightarrow \frac{dv}{dx} = 2x + 2(1) - 0 = 2x + 2$$

$$\frac{dy}{dx} = v\frac{dy}{dx} + u\frac{dv}{dx} = (x^2 + 2x - 1)(2x - 1) + (x^2 - 3x + 3)(2x + 2)$$

$$= 2x^3 + 4x^2 - 2x - 3x^2 - 6x + 3 + 2x^3 - 6x^2 + 6x + 2x^2 - 6x + 6$$

$$= 4x^3 - 3x^2 - 8x + 9$$

2. $y = \frac{x+1 \leftarrow u}{x-1 \leftarrow v}\Bigg\} \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$u = x + 1 \Rightarrow \frac{du}{dx} = (1)$$

$$v = x - 1 \Rightarrow \frac{dv}{dx} = 1$$

3. $y = \frac{x}{x^2+1} \begin{matrix} \leftarrow 4 \\ \leftarrow v \end{matrix}$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$u = x \Rightarrow \frac{du}{dx} = 1$$

$$v = x^2 + 1 \Rightarrow \frac{dv}{dx} = 2x + 0 = 2x$$

Substitute and solve

4. $y=\dfrac{ax+b\leftarrow u}{cx+d\leftarrow v}$, a, b, c and d are constants

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$$u=ax+b\Rightarrow\frac{du}{dx}=a(1)=a$$

$$v=cx+d\Rightarrow\frac{dv}{dx}=C(1)=C$$

Substitute values and solve

5. $z=\dfrac{x^2+1}{3(x^2-1)}+(x^2-1)(1-x)$

$\downarrow$ P $\quad\quad$ $\downarrow$ Q

$$\frac{dz}{dx}=\frac{dP}{dx}+\frac{dQ}{dx}$$

$$P=\frac{x^2+1\to u}{3(x^2-1)\to v},\ \frac{dP}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$$u=x^2+1\Rightarrow\frac{du}{dx}=2x$$

$$v=3(x^2-1)\Rightarrow\frac{dv}{dx}=3(2x-0)=6x$$

$$\Rightarrow\frac{dp}{dx}=\frac{3(x^2-1)(2x)-(x^2+1)6x}{[3(x^2-1)^2]^2}=\frac{6x(x^2-1-x^2-1)}{3(x^2-1)^2}$$

$$\frac{dP}{dx}=\frac{-12x}{9(x^2-1)^2}=\frac{-4x}{3(x^2-1)^2}$$

$$Q=(x^2-1)(1-x),\ \frac{dQ}{dx}=b\frac{dQ}{dx}+a\frac{db}{dx}$$

$\uparrow$ a $\quad$ $\uparrow$ b

$$\left.\begin{aligned}a=x^2-1\Rightarrow\frac{dq}{dx}=2x\\ b=1-x\Rightarrow\frac{db}{dx}=-1\end{aligned}\right\}\frac{dQ}{dx}=(1-x)(2x)+(x^2-1)(-1)$$

$$=2x-2x^2-x^2+1$$

$$\frac{dQ}{dx}=-3x^2+2x+1$$

$$\Rightarrow\frac{dz}{dx}=\frac{dp}{dx}+\frac{dQ}{dx}=\frac{-4x}{3(3x^2-1)^2}-3x^2+2x+1$$

6. $y = \dfrac{1-x^3}{1+x^3}$ $\begin{matrix} \leftarrow u \rightarrow \dfrac{du}{dx} = -3x^2 \\ \leftarrow v \rightarrow \dfrac{du}{dx} = 3x^2 \end{matrix}$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

Substitute to get the answer

7. $y = \dfrac{2}{x^3-1}$ $\begin{matrix} \leftarrow u \Rightarrow \dfrac{du}{dx} = 0 \\ \leftarrow v \Rightarrow \dfrac{du}{dx} = 3x^2 \end{matrix}$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

8. $y = \dfrac{x^2 - x + 1}{a^3 - 3}$,

Here a is constant hence $a^3 - 3$ = constant

$$\Rightarrow y = \left[\frac{1}{a^3-3}\right](x^2 - x + 1)$$

$$\frac{dy}{dx} = \left[\frac{1}{a^3-3}\right]\frac{d(x^2 - x + 1)}{dx} = \left(\frac{1}{a^3-3}\right)(2x-1)$$

9. $y = \dfrac{1-x^3}{\sqrt{\pi}} = \dfrac{1}{\sqrt{\pi}}[1-x^3]$ constant

$$\frac{dy}{dx} = \frac{1}{\sqrt{\pi}}\frac{d(1-x^3)}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\pi}}(0 - 3x^2) = \frac{-3x^2}{\sqrt{\pi}}$$

10. $y = \underset{\downarrow}{\sin x} + \underset{\downarrow}{\cos x}$

$$\frac{dy}{dx} = \cos x + (-\sin x) = \cos x - \sin x$$

11. $y = \dfrac{2}{1-\cos x}$ $\begin{matrix} \leftarrow u \Rightarrow \dfrac{du}{dx} = 1 \\ \leftarrow v \Rightarrow \dfrac{dv}{dx} = 0 - (\sin x) = \sin x \end{matrix}$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$ Solve further

12. $y=\dfrac{\tan x}{x}$ $\begin{array}{l}\to u \Rightarrow \dfrac{du}{dx} = \sec^2 x \\ \to v \Rightarrow \dfrac{dv}{dx}=1\end{array}$

$\dfrac{dy}{dx}=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$ Solve further

13. $y=[\underset{P}{\underset{\downarrow}{(x)}}\underset{Q}{\underset{\downarrow}{(\sin x)}}]+[\cos x]$

$\dfrac{dy}{dx}=\left[P\dfrac{dQ}{dx}+Q\dfrac{dP}{dx}\right]+\dfrac{d(\cos x)}{dx}$

$=[x(\cos x)+\sin x(1)]+[-\sin x]$

= xcosx + sinx – sinx

= x cosx

14. $y = \underset{P}{\underset{\downarrow}{\dfrac{\sin x}{x}}}+\underset{Q}{\underset{\downarrow}{\dfrac{x}{\sin x}}} \Rightarrow \dfrac{dy}{dx}=\dfrac{dP}{dx}+\dfrac{qQ}{dx}+\dfrac{dQ}{dx}$

$P = \dfrac{\sin x}{x}\ \dfrac{\to a}{\to b}\Rightarrow \dfrac{da}{dx}=\cos x$

$\Rightarrow \dfrac{d_b}{d_x}=1$

$\dfrac{dP}{dx}=\dfrac{b\dfrac{dq}{dx}-a\dfrac{db}{dx}}{b^2}\Leftarrow$ division rule

$\dfrac{dP}{dx}=\dfrac{(x)\cos x-\sin x(1)}{x^2}=\dfrac{x\cos x-\sin x}{x^2}$

$Q=\dfrac{x}{\sin x}\to u\Rightarrow \dfrac{du}{dx}=1$

$v\Rightarrow \dfrac{dv}{dx}=\cos x$

$\dfrac{dQ}{dx}=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}=\dfrac{\sin x(1)-(x)\cos x}{(\sin x)^2}$

$\dfrac{dQ}{dx}=\dfrac{\sin x-x\cos x}{(\sin x)^2}$

$\dfrac{dy}{dx}=\dfrac{dP}{dx}+\dfrac{dQ}{dx}$

$=\dfrac{x\cos x-\sin x}{x^2}+\dfrac{\sin x-x\cos x}{(\sin x)^2}$

$\dfrac{dy}{dx}=x\cos x-\sin x\left[\dfrac{1}{x^2}-\dfrac{1}{(\sin x)^2}\right]$

15. $y = \dfrac{\sin x}{1+\cos x}$ $\rightarrow u \Rightarrow \dfrac{du}{dx} = \cos x$

$\rightarrow v \Rightarrow \dfrac{dv}{dx} = 0 + (-\sin x) = -\sin x$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(1+\cos x)(\cos x) - (\sin x)(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{1+\cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{(1+\cos x)}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

16. $y = \dfrac{x}{\underbrace{\sin x + \cos x}_{=v}}$ $\rightarrow u \Rightarrow \dfrac{du}{dx} = 1$

$\Rightarrow \dfrac{dv}{dx} = \cos x - \sin x$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

17. $y = \dfrac{[(x)(\sin x)] \leftarrow u}{[1+\tan x] \leftarrow v}$

$u = [\underset{\uparrow}{x}][\underset{\uparrow}{\sin(x)}], \dfrac{dp}{dx} = 1, \dfrac{dQ}{dx} = \cos x$

$$\frac{du}{dx} = Q\frac{dP}{dx} + P\frac{dQ}{dx} = \sin x(1) + (x)\cos x$$

$$\frac{du}{dx} = \sin x + x\cos x$$

$$v = 1 + \tan x \Rightarrow \frac{dv}{dx} = 0 + \sec^2 x = \sec^2 x$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(1+\tan x)(\sin x)(x\cos x) - (x\sin x)\sec^2 x}{(1+\tan x)^2}$$

18. $y = \cos^2 x = \underset{\underset{u}{\uparrow}}{\cos x}\ \underset{\underset{v}{\uparrow}}{\cos x}$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = \cos x(-\sin x) + \cos x(-\sin x)$$

$$= -2\sin x\cos x = -\sin 2x$$

19. $y = \dfrac{1}{4}\tan^4 x$

Let $P = \tan x \Rightarrow y = \dfrac{1}{4}(P)^4$

$$\frac{dP}{dx} = \sec^2 x,\ \frac{dy}{dP} = \frac{1}{4}(4P^3)$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = (P)^3(\sec^2 x)$$

$$= (\tan x)^3\ (\sec^2 x)$$

20. $y = \cos x - \frac{1}{3}\cos^3 x$

$$\frac{dy}{dx} = \frac{d(\cos x)}{dx} - \frac{1}{3}\frac{d(\cos^3 x)}{dx}$$

Let Z = $\cos^3 x.,\ \frac{dZ}{dx} = \frac{d(\cos^3 x)}{dx} = ?$

Let P = $\cos x \Rightarrow z = p^3$

$$\frac{dP}{dx} = -\sin x, \frac{dZ}{dP} = 3P^2$$

$$\Rightarrow \frac{dZ}{dx} = \frac{dZ}{dP} \times \frac{dP}{dx} = -3P^2 \times \sin x$$

$$\frac{dZ}{dx} = -3(\cos x)^2 \sin x$$

$$\Rightarrow \frac{dy}{dx} = -\sin x - \frac{1}{3}[-3(\cos x)2\sin x)$$

$$= -\sin x[\underbrace{1 - \cos^2 x}_{\sin^2 x}]$$

$= -\sin^3 x$

21. $y = 3\underset{\substack{\downarrow \\ P}}{\sin^2 x} - \underset{\substack{\downarrow \\ Q}}{\sin^3 x},\ \frac{dy}{dx} = \frac{3dP}{dx} - \frac{dQ}{dx}$

P = $\sin^2 x$

Let $z = \sin x \Rightarrow p = z^2$

$$\Rightarrow \frac{dz}{dx} = \cos x,\ \frac{dp}{dz} = 2z$$

$$\frac{dP}{dx} = \frac{dP}{dz} \times \frac{dz}{dx} = 2z_x \cos x$$

$$= 2z\cos x = 2(\sin x)\cos x = \sin 2x$$

Q= $\sin^3 x$

Let y = sinx, Q = y^3

$$\Rightarrow \frac{dy}{dx} = \cos x,\ \frac{dQ}{dy} = 3y^2$$

$$\frac{dQ}{dx} = \frac{dQ}{dy} \times \frac{dy}{dx} = (3y^2) \times \cos x = 3y^2 \cos x = 3(\sin x)^2 \cos x$$

$$\Rightarrow \frac{dy}{dx} = 3\frac{dp}{dx} - \frac{dQ}{dx} = 3(\sin 2x) - 3\sin^2 x \cos x$$

$$= 6\sin x\cos x - 3\sin^2 x\cos x$$

22. $y = \frac{1}{3}\underset{a}{\underset{\downarrow}{\tan^3 x}} - \underset{b}{\underset{\downarrow}{\tan x}} + \underset{c}{\underset{\downarrow}{x}}$

$$\frac{dy}{dx} = \frac{1}{3}\frac{da}{dx} - \underset{\sec^2 x}{\underset{\downarrow}{\frac{db}{dx}}} + \frac{dc}{dx}$$

$a = \tan^3 x$

Let $P = \tan x \Rightarrow a \Rightarrow p^3$

$\frac{dP}{dx} = \sec^2 x, \frac{da}{dP} = 3P^2$

$\Rightarrow \frac{da}{dx} = \frac{da}{dP} \times \frac{dP}{dx} = 3P^2 \sec^2 x$

$= 3(\tan x)(\sec^2 x)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{3}[\tan^2 x \sec^2 x] - \sec^2 x + 1$

$= \tan^2 x \sec^2 x + [\sec^2 x - \tan^2 x] - \sec^2 x$

$= \tan^2 x(\sec^2 x - 1)$

$= (\tan^2 x)(\tan^2 x) = \tan^4 x$

23. $y = \underbrace{(\sec^2 x)}_{u} - \underbrace{(\tan x)}_{v} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx} \Rightarrow \frac{dv}{dx} = \sec^2 x$

$u = (\underset{a}{\underset{\downarrow}{x}})(\underset{b}{\underset{\downarrow}{\sec^2 x}}), a = x \Rightarrow \frac{da}{dx} = 1$

$b = \sec^2 x$

Let $P = \sec x \Rightarrow b = p^2$

$\frac{dP}{dx} = \sec x \tan x, \frac{db}{dP} = 2P$

$\frac{db}{dx} = \frac{dp}{dP} \times \frac{dP}{dx} = 2P \times \sec x \tan x$

$= 2\sec x \sec x \tan x$

$\frac{db}{dx} = 2\sec^2 x \tan x$

$\frac{du}{dx} = b\frac{da}{dx} + a\frac{db}{dx} = \sec^2 x(1) + (x)2\sec^2 x \tan x$

$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx} = (\sec^2 x + 2x\sec^2 x \tan x) - \sec^2 x$

$= 2x\sec^2 x \tan x$

24. $y = \underset{u}{\underset{\downarrow}{x}}\,\underset{v}{\underset{\downarrow}{\ln x}}$

$\frac{du}{dx} = 1, \frac{dv}{dx} = \frac{1}{x}$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

25. $y = Ln^2 x = (Lnx)^2$

Let $P = Lnx \Rightarrow y = p^2$

$$\frac{dP}{dx} = \frac{1}{x},\ \frac{dy}{dP} = 2P$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = (2p)\left(\frac{1}{x}\right) = \frac{2Lnx}{x}$$

26. $y = Ln(x^2)$

Let $p = x^2 \Rightarrow y = L\,n\,P$

$$\frac{dP}{dx} = 2x, \frac{dy}{dP} = \frac{1}{P}$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = \frac{1}{P} \times 2x = \frac{2x}{x^2} = \frac{2}{x}$$

27. $y = \sqrt{Lnx} = (Lnx)^{1/2}$

Let $P = Lnx \Rightarrow y = p^{1/2}$

$$\frac{dP}{dx} = \frac{1}{x}, \frac{dy}{dP} = \frac{1}{2}(P)^{\frac{1}{2}-1} = \frac{(P)^{-1/2}}{z} = \frac{1}{2\sqrt{P}}$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = \frac{1}{2\sqrt{P}} \times \frac{}{x} = \frac{1}{2\sqrt{Lnx}} \times \frac{1}{x}$$

28. $\int \sqrt{x}dx = \int (x)^{1/2}dx$

$$= \frac{(x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{2(x)^{3/2}}{3} + C$$

29. $\int \sqrt[m]{x^n}dx = \int [x]^{\frac{n}{m}}dx$

$$\frac{[x]^{\frac{n}{m}+1}}{\frac{n}{m}+1} + C = \frac{(x)^{\frac{n+m}{m}}}{\frac{n+m}{m}} + C$$

$$= \frac{m}{n+m}(x)^{\frac{n+m}{m}} + C$$

30. $\int \frac{dx}{x^2} = \int x^{-2}dx$

$$= \frac{(x)^{-2+1}}{-2+1} + C = \frac{(x)^{-1}}{-1} + C$$

$$= \frac{-1}{x} + C$$

31. $\int \frac{dx}{2\sqrt{x}} = \frac{1}{2}\int (x)^{-1/2}dx$

$$= \frac{1}{2}\frac{(x)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} = \frac{1}{2}\frac{(x)^{1/2}}{\frac{1}{2}} + C$$

$$= \sqrt{x} + C$$

32. $\int (1-2u)du = \int du - 2\int udu$

$$= \int u^0 du - 2\frac{(u)^{1+1}}{1+1}$$

$$= \frac{u^{0+1}}{0+1} - 2\frac{u^2}{2} + C$$

$$= u - u^2 + C$$

EXERCISE # II

1. **(B)**

$$y = \frac{1}{\sqrt{x}} = (x)^{-1/2}$$

$$\frac{dy}{dx} = \frac{-1}{2}(x)^{\frac{-1}{2}-1} = \frac{-1}{2}(x)^{-3/2}$$

2. **(A)**

$$y = \frac{1}{(ax+b)^2} = (ax+b)^{-2} \quad (0, 6 = \text{constant})$$

$$P = (ax+b) \Rightarrow y = (P)^{-2}$$

$$\frac{dP}{dx} = a(1) + 0 = a \quad \frac{dy}{dP} = -2(P)^{-2-1} = -2P - 3 = \frac{-2}{P^3}$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = (a)\left(\frac{-2}{P^3}\right) = \frac{-2a}{(ax+b)^3}$$

3. **(C)**

$$y = x^3 + \frac{1}{x^3} + 8 = x^3 + (x)^{-3} + 8$$

$$\frac{dy}{dx} = 3x^2 + -3(x)^{-4} + 0 = 3x^2 - 3x^{-4}$$

4. **(D)**

$y = \sin x^3$

Let $P = x^3 \Rightarrow y = \sin P$

$\frac{dP}{dx} = 3x^2 \Rightarrow \frac{dy}{dP} = \cos P$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dP} \times \frac{dp}{dx} = (\cos P)(3x^2) = (3x^2)\cos(x^3)$

5. **(D)**

$y = \sqrt{4x^3 - 5} = [4x^3 - 5]^{1/2}$

Let $P = 4x^3 - 5 \Rightarrow y = [P]^{1/2}$

$\frac{dp}{dx} = 4(3x^2) - 0, \frac{dy}{dP} = \frac{1}{2}(P)^{\frac{1}{2}-1}$

$= 12x^2 \quad = \frac{1}{2}(P)^{-1/2} = \frac{1}{2\sqrt{P}}$

$\frac{dy}{dx} = \frac{dy}{P} \times \frac{dP}{dx}$

$= \frac{1}{2\sqrt{P}} \times 12x^2 = \frac{6x^2}{\sqrt{4x^3 - 5}}$

6. **(D)**

$y = \sin(Lnx)$

$P = Lnx \Rightarrow y = \sin P$

$\Rightarrow \frac{dP}{dx} = \frac{1}{x}, \frac{dy}{dP} = \cos(p)$

$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = \cos(P) \times \frac{1}{x} = \frac{\cos(Lnx)}{x}$

7. **(B)**

$y = \sqrt{2x^2 + 1} = (2x^2 + 1)^{1/2}$

Let $P = (2x^2 + 1) \Rightarrow y = (P)^{1/2}$

$\frac{dP}{dx} = 2(2x) + 0 = 4x, \frac{dy}{dP} = \frac{1}{2}(P)^{1/2-1} = \frac{1}{2\sqrt{P}}$

$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = \frac{1}{\cancel{2}\sqrt{p}} \times \overset{2}{\cancel{4}}x = \frac{2x}{\sqrt{2x^2 + 1}}$

8. **(A)**

$y = (e)^{\sqrt{2x}}$

Let $P = \sqrt{2x} = \left(\sqrt{2}\right)(x)^{1/2} \Rightarrow y = e^P \Rightarrow \frac{dy}{dP} = e^P$

$\frac{dP}{dx} = \sqrt{2}\frac{1(x)}{2\sqrt{2}} = \frac{1}{\sqrt{2}}(x)^{-1/2} = \frac{1}{\sqrt{2}\sqrt{x}}$

$$\frac{dy}{dx}=\frac{dy}{dP}\times\frac{dP}{dx}=(e)^{P}\times\frac{1}{\sqrt{2x}}=\frac{(e)^{\sqrt{2x}}}{\sqrt{2x}}$$

9. **(D)**

$$y=x^4-\sin x+3\cos x$$

$$\frac{dy}{dx}=4x^3-2(\cos x)+3(-\sin x)=4x^3-2\cos x-3\sin x$$

10. **(A)**

$$y=\underbrace{(x^2)(\sin x)}_{u}\;\underset{v}{(Lnx)}$$

$$\frac{dy}{dx}=v\frac{du}{dx}+u\frac{dv}{dx}$$

$$= Lnx[2x\sin x+x^2\cos x]+(x^2\sin x)\left(\frac{1}{x}\right)$$

$$v=Lnx\Rightarrow\frac{dv}{dx}=\frac{1}{x}$$

$$u=(\underset{\substack{\uparrow\\P}}{x^2})(\underset{\substack{\uparrow\\Q}}{\sin x})$$

$$\frac{du}{dx}=Q\frac{dP}{dx}+P\frac{dQ}{dx}$$

$$=(\sin x)(2x)+x^2(\cos x)$$

$$\frac{du}{dx}=2\sin x+x^2\cos x$$

11. **(A)**

$$y=\frac{x^2+1\leftarrow u}{x+1\ \leftarrow v}\Rightarrow\frac{du}{dx}=2x$$

$$\Rightarrow\frac{dv}{dx}=1$$

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

Solve further

12. **(D)**

$$v=\frac{4}{3}\pi r^3$$

$$\Rightarrow\frac{dV}{dr}=\left(\frac{4}{3}\pi\right)\left(3r^2\right)=4\pi r^2$$

13. **(D)**

$xy = c^2$

$$\Rightarrow y=\frac{c^2}{x}\Rightarrow y=c^2(x)^{-1}$$

$$\frac{dy}{dx} = c^2(-1)(x)^{-1-1} = \frac{-C^2}{x^2}$$

$$\frac{dy}{dx} = \frac{-1}{x^2}(xy)$$

$$= \frac{-y}{x}$$

14. **(B)**

$$y = \frac{1}{\sqrt{2x+1}} = (2x+1)^{-1/2}$$

Let P = 2x +1 , $\Rightarrow y = (P)^{-1/2}$

$$\frac{dP}{dx} = 2,\ \frac{dy}{dP} = -\frac{1}{2}(P)^{-1/2-1} = -\frac{1}{2\left(P^{3/2}\right)}$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = -\frac{1}{2\left(P^{3/2}\right)} \times 2 = -\frac{1}{P^{3/2}} = -\frac{1}{\left(2x+1\right)^{3/2}}$$

15. **(B)**

x = at^2,y = 2at

$$\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

16. **(A)**

$$\int \sqrt[5]{x}\ dx = \int (x)^{1/5} x$$

$$= \frac{[x]^{\frac{1}{5}+1}}{\frac{1}{5}+1} + C\ = \frac{5}{6}(x)^{6/5} + c$$

17. **(C)**

$$\int \frac{1}{(ax+b)^2} dx = \int (ax+b)^{-2} dx$$

We’ll use integration by substitution

Let P = ax + b

$$\Rightarrow \frac{dP}{dx} = a \Rightarrow dx = \frac{dP}{a}$$

$$\Rightarrow \int (ax+b)^{-2} dx = \int (p)^{-2} \frac{dP}{a}$$

$$= \left(\frac{1}{a}\right) \int P^{-2} dP = \frac{1}{a}\left[\frac{P}{-2+1}\right]^{-2+1} + C$$

$$= \left(\frac{1}{a}\right)\left(\frac{P^{-1}}{-1}\right) + C = \frac{-1}{a} \times \frac{1}{P} + C$$

$$= \frac{1}{a(ax+b)} + C$$

18. **(A)**

$$\int \sin x \cos x dx$$

Let P = sinx $\Rightarrow \frac{dP}{dx} = \cos x \Rightarrow dP = \cos x dx$

$$\Rightarrow \int \sin x \cos x dx = \int P\, dP = \frac{P^2}{2} + C'$$

But options are in terms of cos2x, sin2x

So lets convert $\sin^2 x$ into cos2x

$$\cos 2x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\frac{\left[\frac{1-\cos 2x}{2}\right]}{2} + c' = \frac{1-\cos 2x}{4} + c'$$

$$= \frac{-\cos 2x}{4} + \left[c' - \frac{1}{4}\right] \text{ new const} = c$$

$$= -\frac{\cos 2x}{4} + c$$

19. **(B)**

$$\int \frac{x}{x^2+a^2} dx$$

Let P = $x^2 + a^2$

$$\frac{dP}{dx} = 2x + 0 \Rightarrow \frac{dP}{2} = x dx$$

$$\Rightarrow \int \frac{x}{x^2+a^2} dx = \int \frac{1}{p}\frac{dP}{2} = \frac{1}{2}\int \frac{dP}{P}$$

$$\Rightarrow \frac{1}{2}\text{Ln}(p) + C = \frac{1}{2}\text{Ln}(x^2+a^2) + C$$

20. **(C)**

$$\int_{-\pi/2}^{\pi/2} \cos x dx = [\sin x]_{-\pi/2}^{\pi/2}$$

$$= \left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right]$$

=[1 – (-1)] = 2

21. **(A)**

$$\int_0^{\pi/2} \sqrt{1+\cos x}\, dx$$

Half angle formula

$\cos x = 2\cos^2\left(\frac{x}{2}\right) - 1$

$\int_0^{\pi/2} \sqrt{1 + 2\cos 2\left(\frac{x}{2}\right) - 1} = \int_0^{\pi/2} \sqrt{2\cos^2\left(\frac{x}{2}\right)}$

$\int_0^{\pi/2} \sqrt{2} = \cos\left(\frac{x}{2}\right)dx$

$\int_0^{\pi/2} \sqrt{2} = \cos\left(\frac{x}{2}\right)dx$

$\Rightarrow \text{Let } P = \frac{x}{2} \Rightarrow \frac{dP}{dx} = \frac{1}{2}$

$\Rightarrow dx = 2dP$

$\Rightarrow \sqrt{2}\int_0^{\pi/2} \cos\left(\frac{x}{2}\right)dx = \sqrt{2}\int_0^{\pi/2} \cos(P)(2dP)$

$= 2\sqrt{z}\int_0^{\pi/2} \cos P dP$

$= 2\sqrt{z}\int_0^{\pi/2} \cos P\, dP$

$= 2\sqrt{2}[\sin P]_0^{\pi/2} = 2\sqrt{2}\left[\sin\left(\frac{x}{2}\right)\right]_0^{\pi/2}$

$= 2\sqrt{2}\left[\sin\left(\frac{\pi/2}{2}\right) - \sin\left(\frac{0}{2}\right)\right]$

$= 2\sqrt{2}\left[\sin\left(\frac{\pi}{4}\right) - 0\right] = 2\sqrt{z}\left(\frac{1}{\sqrt{2}} - 0\right)$

$= 2$

22. **(D)**

$\int (1-x)\sqrt{x}dx$

$\int \sqrt{x}dx - \int x\sqrt{x}dx$

$\int (x)^{1/2}dx - \int \underbrace{(x)'(x)^{1/2}}_{x^{3/2}} dx$

$= \frac{[x]^{1/2+1}}{\frac{1}{2}+1} - \frac{(x)^{3/2+1}}{\frac{3}{2}+1} + C$

$= \frac{2}{3}(x)^{3/2} - \frac{2}{5}(x)^{5/2} + C$

23. **(B)**

$$\int \frac{1}{\sin^2 x \cos 2x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \sec^2 x dx + \int \cos ec^2 x dx$$

$$= \tan x + (-\cot x) + C$$

= tanx – cotx + C

24. **(A)**

$$\int \frac{1}{1+e^{-x}} dx = \int \frac{1}{1+\frac{1}{e^x}} dx$$

$$= \int \frac{1}{\frac{e^x+1}{e^x}} dx = \int \frac{e^x dx}{e^x+1}$$

Let p = e^x + 1

$$\frac{dP}{dx} = e^x \Rightarrow dP = e^x dx$$

$$= \int \frac{e^x dx}{e^x+1} = \int \frac{dP}{P}$$

$$= Ln(P) + C = Ln(1+e^x) + c$$

25. **(A)**

$$\int \frac{\cos ec^2 x}{1+\cot x} dx$$

$$P = 1 + \cot x \Rightarrow \frac{dP}{dx} = 0 + [-\cos ec^2 x]$$

$$\Rightarrow dP = -\cos ec^2 x dx \Rightarrow \cos ec^2 x dx = -dp$$

$$\Rightarrow \int \frac{\cos ec^2 x dx}{1+\cot x} = \int \frac{-dP}{P} = -1 \int \frac{dP}{P}$$

$$= -Ln(P) + C = -Ln(1+\cot x) + C$$

26. **(B)**

$$\int \frac{Lnx}{x} dx$$

Let $P = Lnx \Rightarrow \frac{dP}{dx} = \frac{1}{x}$

$$\Rightarrow dP = \left[\frac{dx}{x}\right]$$

$$\int \frac{Lnx}{d} dx = \int P \ dP = \left[\frac{P^2}{2}\right] + C$$

$$=\frac{(Lnx)^2}{2}+C$$

27. **(A)**

$$\int_0^{\pi/2}[\sin x+\cos x]dx$$

$$\int_0^{\pi/2}\sin x\ dx+\int_0^{\pi/2}\cos x\ dx$$

$$[-\cos x]_0^{\pi/2}+[\sin x]_0^{\pi/2}$$

$$\left[\left(-\cos\frac{\pi}{2}\right)-(-\cos 0)\right]+\left[\left(\frac{\pi}{2}\right)-\sin 0\right]$$

= 0 + 1 +(1) -0

= 2

28. **(A)**

$$\int_0^{\infty}e^{-x}\ dx$$

$$P=-x\Rightarrow\frac{dP}{dx}=-1\Rightarrow dx=-dP$$

$$\int_0^{\infty}e^{-x}dx=\int_0^{\infty}e^{P}(-dP)=\int_0^{\infty}e^{p}dP$$

$$=-\left[e^{P}\right]_0^{\infty}=-[e^{-x}]_0^{\infty}$$

$$=-[e^{-\infty}-e^{-0}]$$

$$=-\left[\underset{=\infty}{\underset{\downarrow}{\frac{1}{e^{\infty}}}}-\underset{-1}{\underset{\downarrow}{\frac{1}{e^{0}}}}\right]$$

$$=-\left[\frac{1}{\infty}-\frac{1}{1}\right]=1$$

EXERCISE # III

1. **$0.320\pi\ cm^2/s$**

$$\frac{dr}{dt} = 0.05 cm/s$$

$$A = \pi r^2$$

$$\frac{dA}{dr} = (\pi)(2r) = 2\pi r$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = (2\pi r)(0.05)$$

$$= 0.1\pi r = 0.01 \times 3.2\pi$$

$$= 0.32\pi$$

2. **$900\ cm^3/s$**

$$\frac{dl}{dt} = 3 cm/s$$

$$v = \ell^3 \Rightarrow \frac{dV}{d\ell} = 3\ell^2$$

$$\frac{dv}{dt} = \frac{dv}{d\ell}\frac{d\ell}{dt} = 3\ell^2 \times 3$$

$$= 9\ell^2 = 9(10)^2$$

$$= 900\ cm^3$$

3. **$\frac{1}{\pi}\ cm/s$**

$$\frac{dv}{dt} = 300 cm^3/s$$

$$\frac{d_r}{dt} = ?$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr}\left[\frac{4}{\cancel{3}}\pi\right](\cancel{3}r^2) = 4\pi r^2$$

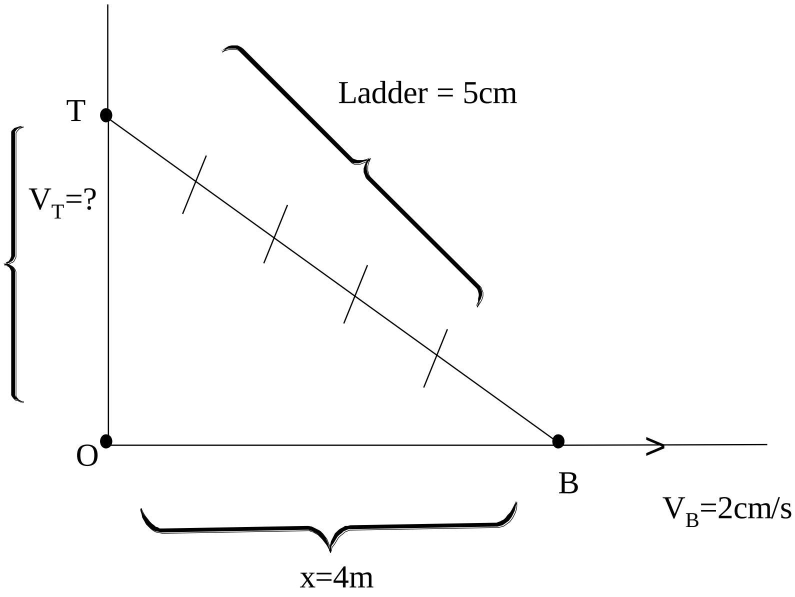
$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$300 = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{900}{4\pi(r=15)^2} = \frac{\cancel{900}\ \cancel{60}\ \cancel{4}}{\cancel{4}\pi \times \cancel{15} \times 15}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{\pi} cm/s$$

4. $\frac{8}{3}$ **cm/s**



$x^2 + y^2 = 5^2$

$y^2 = 25 - x^2 \Rightarrow y = \sqrt{25 - x^2}$

Let $P = 25 - x^2 \Rightarrow y = \sqrt{p} = (P)^{1/2}$

$\Rightarrow \frac{dP}{dx} = 0 - 2x, \frac{dy}{dP} = \frac{1}{2}(P)^{-1/2}$

$= - 2x$

$\frac{dy}{dx} = \left(\frac{dy}{dP}\right)\left(\frac{dP}{dx}\right) = \frac{1}{\cancel{2}}(P)^{-1/2}(-\cancel{2}x)$

$\frac{dy}{dx} = -\frac{x}{(P)^{1/2}} = \frac{-x}{\sqrt{25 - x^2}}$

$dy = -\frac{x}{\sqrt{25 - x^2}}dx$

$\Rightarrow \frac{dy}{dt} = \frac{-x}{\sqrt{25 - x^2}}\frac{dx}{dt}$

$V_T = \frac{dy}{dt}$ * Rate of change of y

$\frac{dx}{dt} = V_B$ * Rate of change of x

$\Rightarrow V_T = \frac{-x}{\sqrt{25 - x^2}}V_B$

$= \frac{-(4)}{\sqrt{25 - (4)^2}}(2)$

$V_T = \frac{-8}{\sqrt{9}} = \frac{-8}{3}$ -ve indicates that V_T is –ve and it should be as y is ↓sing

5. $\mathbf{\frac{1}{48\pi}\,cm/s}$

$$\frac{dV}{dt} = 12cm^3/s$$

$$V_{cone} = \frac{1}{3}\pi R^2 h,\ h_{cone} = \frac{R}{6} \Rightarrow R = 6h$$

$$V = \frac{1}{3}\pi(R = 6h)^2(h) = \frac{1}{3}\pi 36h^3 = 12\pi h^3$$

$$\frac{dv}{dh} = 12\pi(3h^2) = 36\pi h^2$$

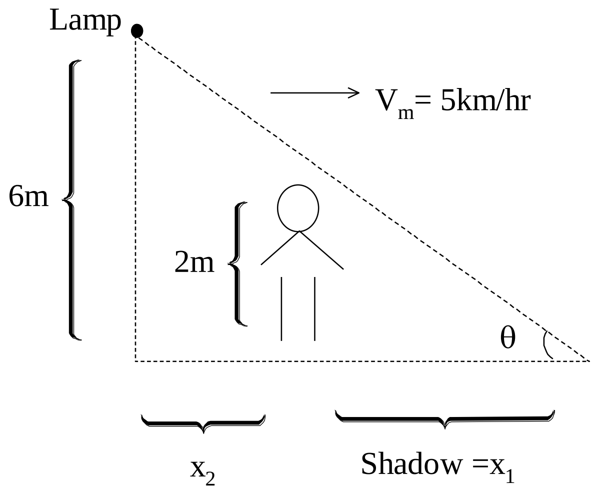
$$dv = 36\pi h^2 dh$$

$$\Rightarrow \frac{dv}{dt} = 36\pi h^2 \frac{dh}{dt}$$

$$12 = 36\pi(h = 4)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{48\pi}$$

6. $\mathbf{\frac{5}{2}\,km/h}$



$$\tan\theta = \frac{6}{x_1 + x_2} = \frac{2}{x_1} \Rightarrow 3x_1 = x_1 + x_2$$

$$\Rightarrow 2x_1 = x_2$$

$$\frac{d(x_2)}{d(x_1)} = 2$$

$$\Rightarrow dx_2 = 2dx_1 \Rightarrow \frac{dx_2}{dt} = 2\frac{dx_1}{dt}$$

$$\frac{dx_2}{dt} = V_m = 5 = 2\left[\frac{dx_1}{dt} = V_{shadow}\right]$$

$$\frac{5}{2} = V_{shadow}$$

7. $\mathbf{\frac{15}{2}, \frac{15}{2}}$

x + y = 15

$y = 15 - x$

Let, $z = x^2 + y^2 = x^2 + (15 - x)^2$

$z = x^2 + 225 + x2 - 30x$

$z = 2x^2 - 30x + 225$

Minimize z.

S – 1 $\frac{dZ}{dx} = 2(2x) - 30(1) + 0$

= 4x – 30

S – 2 $\frac{dz}{dx} = 4x - 30 = 0 \Rightarrow x = 7.5$

$\Rightarrow$y = 15 – x

= 7.5

8. **3 cm**

Box $\Rightarrow$Base = square of length $18 - 2\ell$

ht = l

Volume $(V) = (18 - 2\ell)^2 \ (\ell)$

$= [324 + 4\ell^2 - 72\ell](\ell)$

V = $4\ell^3 - 72\ell^2 + 324\ell$

$S - 1 \ \frac{dV}{d\ell} = 4(3\ell^2) - 72(2\ell) + 324$

$12\ell^2 - 144\ell + 324$

S -2 $\frac{dv}{d\ell} = 12\ell^2 - 144\ell + 324 = 0$

$12[\ell^2 - 12\ell + 27] = 0$

$\Rightarrow \ell^2 - 12\ell + 27 = 0$

$\ell^2 - 9\ell - 3\ell + 27 = 0$

$\ell(\ell - 9) - 3(\ell - 9)$

$(\ell - 9)(\ell - 3) = 0$

$\Rightarrow \ell = 3$ or $\ell = 9$

S – 3 $\frac{d^2v}{d\ell^2} = 24(\ell) - 144 = 24\ell - 144$

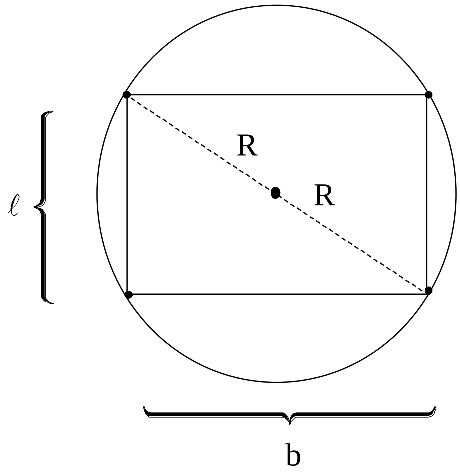
S – 4 for $\ell = 3$ $\frac{d^2v}{dQ^2} = 24(3) - 144 = -72 < 0$

$\Rightarrow \ell = 3$maxima

(ii) For $\ell = g$ $\frac{d^2v}{d\ell^2} = 24(9) - 144 = 216 - 144$=72> 0

$\Rightarrow \ell = 9 =$ minima

9.



$\ell^2 + b^2 = (2R)^2$

$\ell^2 + b^2 = 4R^2$

$\ell = \sqrt{4R^2 - b^2}$

Area $= \ell \times b$

$A = \sqrt{4R^2 - b^2}\, b$

S – 1 $\frac{dA}{dB} = b\frac{d\sqrt{4R^2 - b^2}}{db} + \sqrt{4R^2 - b^2}\,\frac{d(b)}{db}$

$\frac{dA}{db} = b\left[\frac{-b}{\sqrt{4R^2 - b^2}}\right] + \sqrt{4R^2 - b^2}(1)$

S – 2 $\frac{dA}{dB} = 0$

$\frac{dA}{dB} = \frac{-b^2}{\sqrt{4R^2 - b^2}} + \sqrt{4R^2 - b^2} = 0$

$\sqrt{4R^2 - b^2} = \frac{b^2}{\sqrt{4R^2 - b^2}}$

$\sqrt{4R^2 - b^2}\sqrt{4R^2 - b^2} = b^2$

$4R^2 - b^2 = b^2$

$\Rightarrow 4R^2 = 2b^2$ or $b = R\sqrt{2}$

As $\ell = \sqrt{4R^2 - b^2} \Rightarrow \ell\sqrt{4R^2 - \left(\sqrt{2}R\right)^2}$

$\ell = \sqrt{4R^2 - 2R^2}$

$\ell = R\sqrt{2}$

$\Rightarrow$Area is maximum $b = \ell = R\sqrt{2}$

10. **44.1 m**

$V = 29.4 - 9.8t$

$V = \frac{ds}{dt} = 29.4 - 9.8t = 0 \Rightarrow 29.4 = 9.8t \Rightarrow t = 3$

$$\frac{dS}{dt} = 29.4 - 9.8t$$

$$\int dS = 29.4\int dt - 9.8\int tdt$$

$$= 29.4\left[\frac{t^0+1}{0+1}\right]_0^3 - 9.8\left[\frac{t^2}{2}\right]_0^3$$

$$= 29.4[3-0] = 4.9[(3^2)-(0)^2]$$

$$= 88.2 - 4.9\times 9$$

= 88.2 – 44.1

= 44.1 m

EXERCISE # IV

1. $S = \frac{1}{4}t^4 - 4t^3 + 16t^2$

(i) S = 0 according to question

$$\Rightarrow \frac{t^4}{4} - 4t^3 + 16t^2 = 0$$

$$t^2\left(\frac{t^2}{4} - 4t + 16\right) = 0$$

$$\Rightarrow t^2 = 0 \text{ or } \frac{t^2}{4} - 4t + 16 = 0$$

$$\Rightarrow t = 0\ ;\ \frac{t^2}{4} - 2t - 2t + 16 = 0$$

$$t\left(\frac{t}{4} - 2\right) - 8\left(\frac{t}{4} - 2\right) = 0$$

$$\Rightarrow \left(\frac{t}{4} - 2\right)(t-8) = 0$$

$$\Rightarrow \frac{t}{4} - 2 = 0 \Rightarrow \frac{t}{4} = 2 \text{ or } t = 8 \text{ or } t - 8 = 0$$

$$\Rightarrow t = 8$$

(ii) $v = \frac{ds}{dt}, S = \frac{1}{4}t^4 - 4t^3 + 16t^2$

$$\Rightarrow \frac{ds}{dt} = \frac{1}{4}(4t^3) - 4(3t^2) + 16(2t)$$

$$v = t^3 - 12t^2 + 32t$$

$v = 0 \Rightarrow t^3 - 12t^2 + 32t = 0$

$$t[t^2 - 12t + 32] = 0$$

$$\Rightarrow t = 0 \text{ or } t^2 - 12t + 32 = 0$$

$$t^2 - 8t - 4t + 32 = 0$$

t(t – 8) – 4(t – 8) =0

(t – 8) (t – 4) = 0

$$\Rightarrow t = 8 \text{ or } t = 4$$

2. m = 3kg, s= $(1 + t + t^2)$cm

$v = \frac{ds}{dt} = 0 + 1 + 2t \frac{cm}{s}$

$V = 2t + 1 \frac{cm}{s}$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}(3)\underset{(t=5)}{\underset{\downarrow}{(2t+1)^2}}$

$= \frac{3}{2}(kg)(2 \times 5 + 1 = 11)^2 \left(\frac{cm}{S}\right)^2$

$= \frac{3}{2} 121 kg \frac{cm^2}{s^2}$

$= \frac{363}{2} \frac{kgm^2}{S^2} \times \frac{1}{104} \because 1cm^2 \frac{1}{10^4} m^2$

$= 0.01815 \frac{kgm^2}{s^2}$

3. $S = t^3 - 4t^2 - 3t$, Find acc^n when v = 0

$V = \frac{ds}{dt} = 3t^2 - 4(2t) - 3$

$= 3t^2 - 8t - 3 = 0$

$3t^2 - 9t + t - 3 = 0$

$3t[t - 3] + 1[t - 3] = 0$

(t – 3((3t + 1) = 0

$t = 3,\ t = -\frac{1}{3} \times$ not possible

t = 3

$a = \frac{dv}{dt} = 3(2t) - (8) = 6t - 8$

= 6t – 8 = 6(3) – 8

= 18 – 8 =10m/s^2

4. $t = \sqrt{x} + 3 \Rightarrow \sqrt{x} = t - 3$ squaring both sides

$x = (t - 3)^2$

$x = (t^2 + 9 - 6t)$

$v = \frac{dx}{dt} = 2t - 6$

$v = 2t - 6 = 0 \Rightarrow t = 3$

$X = (3)^2 + 9 - 1(t = 3)$

= 9 + 9 – 18 = 0

5. $V = 3t^2 + 2t + 1$

$\frac{ds}{dt} = 3t^2 + 2t + 1$

$ds = 3t^2dt + 2tdt + dt$

$\int ds = 3\int t^2dt + 2\int tdt + \int t^0dt$

$= 3\left[\frac{t^3}{3}\right]_0^{10} + 2\left[\frac{t^2}{2}\right]_0^{10} + \left[\frac{t^{0+1}}{0+1}\right]_0^{10}$

$= [(10)^3 - 0^3] + [(10)^2 - 0^2] + (10)' - (0)]$

$= 1000 + 100 + 10$

$= 1110$

6. $V = 9t^2 - 8t$

$\frac{ds}{dt} = 9t^2 - 8t \Rightarrow ds = 9\int t^2dt - 8\int tdt$

$S = \not{9}^3\left[\frac{t^3}{\not{3}}\right] - \not{8}^4\left[\frac{t^2}{\not{2}}\right]$

$S = [3t^3]_3^4 - [4t^2]_3^4$ 4^{th} second means 3 to 4

$= 3[(4)^3 - (3)^3] - 4[(4)^2 - (3)^2]$

$= 3[64 - 27] - 4[16 - 9]$

$= 3(37) - 4(7)$

$= 111 - 28 = 83$

7. $V = 6t^2 + 4$

$\frac{ds}{dt} = 6t^2 + 4 \Rightarrow ds = 6t^2dt + 4dt$

$\int ds = 6\int t^2dt + 4\int t^0dt$

$[S]_0^5 = 2\not{6}\left[\frac{t^3}{3}\right]_0^5 + 4\left[\frac{t^{0+1}}{0+1}\right]_0^5$

$S - 0 = 2(5^3 - 0) + 4(5 - 0)$

$= 2(125) + 20$

$S = 270$

8. $S = s_0 + v_0t + \frac{1}{2}gt^2$

$\frac{ds}{dt} = 0 + v_0(1) + \frac{9}{2}(2t)$

$= v_0 + gt$

9. $x = t^4 - 12t^2 - 40$

(i) at $t = 2 \Rightarrow x = (2)^4 - 12(2)^2 - 40$

$= 16 - 48 - 40$

$= -72$

(ii) $v = \frac{dx}{dt} = 4t^3 - 12(2t) - 0$

$V = 4t^3 - 24t$

At $t = 2$, $v = 4(2)^3 - 24(2)$

$= 32 - 48 = -16$

(iii) $a = \frac{dv}{dt} = 4(3t^2) - 24$

$\Rightarrow a = t - 24$

At t = 2 $\Rightarrow a = 12(2)^2 - 24$

A = 24