EXERCISE - 1 [A]

1. **(B)**

Required equation

$x^2-(1-2)x+(1x-2)=0$

$x^2+x-2=0$

2. **(B)**

Let $3^x=t$

$t+\frac{1}{t}=\frac{10}{3}\qquad \therefore \frac{t^2+1}{t}=\frac{10}{3}$

$3t^2-10t+3=0$

$3t^2-9t-t+3=0$

$3t(t-3)-(t-3)=0$

$(3t-1)(t-3)=0$

$t=\frac{1}{3},3\qquad \therefore 3^x=3^{-1}\qquad \therefore x=-1$

$3^x=3^1\qquad \therefore x=1$

3. **(B)**

Given $16+4m-26+x=0$

$4m+x=10$ .... (i)

$54+9m-39+x=0$

$9m+x=-15$ .... (ii)

From (i) & (ii) $m=-5, x=30$

4. **(C)**

$(x-1)^2+(x-2)^2+(x-3)^2=0$ this is possible only when

$x-1=0$

$x-2=0$

$x-3=0$

But no single value of x for all are zero at a time hence no of solution $=0$

5. **(B)**

Let $3^x=t$

$$\frac{t}{3}+\frac{3}{t}=2$$

$$\frac{t^2+9}{3t}=2$$

$$t^2-6t+9=0$$

$$(t-3)^2=0$$

$$t=3$$

$$\therefore\ 3^x=3^1$$

$$x=1$$

6. **(A)**

$$x^2-x(2i-2i)+(2i)(-2i)=0$$

$$x^2+4=0$$

7. **(D)**

Given, $x=-1$ is the root of given equation

$$1-(p-3)-(3p-5)-(2p-9)+6=0$$

$$7-p+3-3p+5-2p+9=0$$

$$24=6p$$

$$p=4$$

8. (A)

No root

$x=1$ does not satisfy.

9. (B)

As $x+1$ is factor so $x=-1$ will satisfy the expression

$$\Rightarrow (-1)^4+(p-3)(-1)^3-(3p-5)(-1)^2+(2p+q)(-1)+12=0$$

$$\Rightarrow p=2$$

10. **(C)**

Product of roots $=\dfrac{(2m-1)}{m}=-1$

$$2m-1=-m$$

$$3m=1$$

$$m=\frac{1}{3}$$

11. **(B)**

$$\alpha+\beta=0$$

$$-2\frac{(2-a-a^2)}{1}=0$$

$$a^2+a-2=0$$

$$a^2+2a-a-2=0$$
$$a(a+2)-(a+2)=0$$
$$a=1,-2$$

12. **(B)**

$$|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$$
$$=\sqrt{49+4\times 9}$$
$$=\sqrt{49+36}$$
$$=\sqrt{85}$$

13. **(D)**

Given $\alpha+\beta=\dfrac{|\alpha-\beta|}{2}$

$$(\alpha+\beta)^2=\frac{(\alpha-\beta)^2}{4}$$
$$4(\alpha+\beta)^2=(\alpha+\beta)^2-4\alpha\beta$$
$$3(\alpha+\beta)^2=-4\alpha\beta$$
$$3\times\frac{16}{a^{\cancel{2}}}=-4\times\frac{c}{\cancel{a}}$$
$$\frac{3\times 16}{-4}=ac$$
$$\therefore ac=-12$$

14. **(C)**

Given $\alpha+\beta=-1$

$$-\frac{(2a+3)}{(a+1)}=-1$$
$$2a+3=a+1$$
$$a=-2$$
$$\alpha\beta=\frac{3a+4}{a+1}=\frac{-6+4}{-2+1}=2$$

15. **(A)**

$$x^2-x(\text{sum})+\text{product}=0$$
$$\alpha+\beta=-1$$
$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{6}$$
$$\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{6}$$
$$\alpha\beta=-6$$
$$x^2-x(-1)-6=0$$

16. **(C)**

$$\alpha\beta(\alpha+\beta)=\frac{3}{2}\left(\frac{5}{2}\right)=\frac{15}{4}$$

17. **(C)**

$$\alpha\beta+\alpha+\beta+1$$

$$\frac{c}{a}+\frac{b}{a}+1=\frac{a+b+c}{a}$$

18. **(A)**

$$\alpha+\beta=-\sqrt{\alpha}$$

$$\alpha\beta=\beta \qquad \alpha=1$$

$$\beta=-2$$

19. **(A)**

$$a=-(8+2)=-10$$

$$\beta=8\times 2=16$$

$$\alpha=-(3+3)=-6$$

$$b=3\times 3=9$$

$$x^2-10x+9=0$$

$$x=1,9$$

20. **(A)**

Sum of roots $=2-\sqrt{3}+2+\sqrt{3}=4$

Product of roots $=\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)$

$=4-3=1$

Required equation $x^2-4x+1=0$

21. **(A)**

Product of roots $=30$

Sum of roots $=11$

$\therefore$ Required equation $x^2-11x+30=0$

$$x^2-5x-6x+30=0$$

$$x(x-5)-6(x-5)=0$$

$$x=5,6$$

22. **(A)**

Given $\alpha+\beta=-\frac{b}{a}$

$$\alpha\beta=\frac{c}{a}$$

Product of roots $=\frac{\alpha}{\beta}.\frac{\beta}{\alpha}=\frac{\gamma}{p}$

$\therefore p = \gamma$

23. **(C)**

$\alpha + \beta + 2 = 5 + 2 = 7$

$\frac{\alpha\beta}{2} = \frac{16}{2} = 8$

$x^2 - (7+8)x + 56 = 0$

$p = -15,\ q = 56$

24. **(C)**

$(1-p)^2 + p(1-p) + (1-p) = 0$

$(1-p)[1-p+p+1] = 0$

$p = 1$

$x^2 + x + 0 = 0$

$x = 0, -1$

25. **(A)**

$16 + 4p + 12 = 0$

$p = -7$

$p^2 - 4q = 0$

$q = \frac{49}{4}$

26. **(C)**

$p + q = -p \qquad pq = q$

$q = 0$ or $p = 1$

$p = 0 \qquad\qquad q = -2$

$p = 0, 1$

27. **(D)**

$D = 49 - 4.6.K$

$\quad = 49 - 24.K$

If $K = 1$ or 2 D is a perfect square hence roots are rational

28. **(A)**

$D < 0$

$1 - 4m < 0$

$4m > 1$

$m > \frac{1}{4}$

$m \in \left(\frac{1}{4}, \infty\right)$

29. **(D)**

Given $D = 1 - 4ab \ge 0$

$4ab \le 1$ .... (i)

$D = 16ab - 4$

$= 4(ab - 1) \le 0$

Roots of equation $x^2 - 4\sqrt{ab}x + 1 = 0$ will be imaginary

30. **(A)**

Given $16 + 4P + 12 = 0$

$4 + P + 3 = 0$

$P = -7$

$P^2 - 4q = 0$

$49 = 4q$

$\therefore q = \dfrac{49}{4}$

31. **(C)**

Given $4q^2 - 4Pr \ge 0$

$q^2 \ge Pr$ .... (i)

$4Pr - q^2 \ge 0$

$Pr \ge q^2$ .... (ii)

From (i) & (ii)

$q^2 = Pr$

$\dfrac{q}{r} = \dfrac{P}{q}$

32. **(C)**

$D = 100 - 4(21 - m) = 0$

$25 - 21 - m$

$m = -4$

33. **(C)**

$D = 0$

$(k + 2)^2 - 4.2K = 0$

$k^2 + 4 + 4k - 8k = 0$

$(k - 2)^2 = 0$

$k = 2$

34. **(B)**

Sum of roots $= P > 0$

Product of roots $= -q < 0$

Both roots are red and opposite sign

35. **(B)**

Product of roots $= 1$

$$\frac{-2}{\ell} = 1$$
$$\therefore \ell = -2$$

36. **(B)**

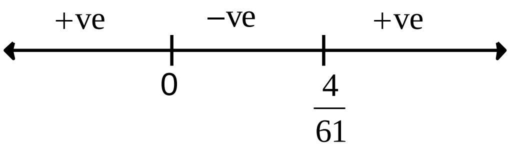
Given $mn^2 - mx + 5m + 1 > 0$
$\forall x \in R$

If $m > 0$

& $81m^2 - 4m(5m+1) < 0$

$81m^2 - 20m^2 - 4m < 0$

$m(61m - 4) < 0$



$$m \in \left(0, \frac{4}{61}\right)$$

37. (B)

$(K-1)^2 - 4(9) < 0$

$K^2 - 2K - 35 < 0$

$K \in (-5, 7)$

38. (C)

$2x^2 + 6x - x - 3 > 0$

$(2x-1)(x+3) > 0$

$$x \in (-\infty, -3) \cup \left(\frac{1}{2}, \infty\right)$$

39. (B)

$2x^2 + 6x - 3x - 9 \le 0$

$(2x-3)(x+3) \le 0$

$$x \in \left[-3, \frac{3}{2}\right]$$

40. (D)

$$\frac{x^2 + 2x + 7}{2x+3} - 6 < 0$$

$$\frac{x^2 - 10x - 11}{2x+3} < 0$$

$$\frac{(x+1)(x-11)}{x + \frac{3}{2}} < 0$$

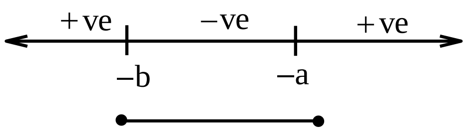
$$x \in \left(-\infty, -\frac{3}{2}\right) \cup (-1, 11)$$

41. (D)

$x^2+ax+bx+ab<0$

$x(x+a)+b(x+a)<0$

$(x+a)(x+b)<0$



42. (C)

$(x+1)^2>5x-1$

$x^2-3x+2>0 \qquad x\in(-\infty,1)\cup(2,\infty)$

$(x+1)^2<7x-3$

$x^2-5x+4<0 \qquad x\in(1,4)$

$x=3$

43. (D)

Replace x by $x-2$ we get required equation

$a(x-2)^2+b(x-2)+c=0$

$ax^2+4a-4ax+bx-2b+c=0$

$ax^2+(b-4a)x+4a-2b+c=0$

44. (C)

If α,β be the roots of equation $2x^2-3x+5=0$

Then equation whose roots are $\frac{1}{\alpha}\,\&\,\frac{1}{\beta}$

$5x^2-3x+2=0$ .... (i)

Given $ax^2+bx+2=0$ .... (ii)

Equation (i) & (ii) are identical

$\frac{a}{5}=\frac{b}{-3}=\frac{2}{2}$

$a=5, b=-3$

45. (A)

$y=\frac{-2}{x}\ \therefore\ x=\frac{-2}{y}$

Replace x by $-2/x$ we get revered equation whose roots are $\frac{-2}{\alpha},\frac{-2}{\beta}$

$2\times\frac{4}{x^2}+\frac{7\times 2}{x}+6=0$

$8+14x+6x^2=0$

46. **(C)**

Let α be common root

$$\alpha^2 - 11\alpha + a = 0$$
$$\alpha^2 - 14x + 2a = 0$$

$$\frac{\alpha^2}{-22a + 14a} = \frac{\alpha}{a - 2a} = \frac{1}{-14 + 11}$$

$$\frac{\alpha^2}{-8a} = \frac{\alpha}{-a} = \frac{1}{-3}$$

$$\therefore \alpha = a/3$$

$\therefore a = 24 \qquad \alpha = 8$

If $a = 0$ then $x = 0$ is common root

47. **(D)**

$$x^2 - x - 2x + 2 = 0$$
$$x(x-1) - 2(x-1) = 0$$
$$x = 1, 2$$
$$x^2 - x + 3x - 3 = 0$$
$$x(x-1) + 3(x-1) = 0$$
$$x = 1, -3$$

Common root $= 1$

$f(1) = 4 + 3 - 7 = 0$

48. **(C)**

$$a\alpha^2 + b\alpha + c = 0$$
$$b\alpha^2 + c\alpha + a = 0$$

$$\frac{\alpha^2}{ab - c^2} = \frac{\alpha}{bc - a^2} = \frac{1}{ac - b^2}$$

$$\therefore \frac{ab - c^2}{bc - a^2} = \frac{bc - a^2}{ac - b^2}$$

$$(bc - a^2)^2 = (ab - c^2)(ac - b^2)$$

$$\cancel{b^2c^2} + a^4 - 2a^2bc = a^2bc - ab^3 - ac^3 + \cancel{b^2c^2}$$

$$a(a^3 + b^3 + c^3 - 3abc) = 0$$

Either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

49. **(B)**

Both equation are identical because roots are imaginary

$$\frac{a}{1} = \frac{b}{3} = \frac{c}{5} = \lambda$$

$$a = \lambda, b = 3\lambda, c = 5\lambda$$

$a + b + c = \lambda.9$

$(a + b + c)_{min} = 9$

50. **(A)**

$$y=\frac{x}{x^2-5x+9}$$

$$yx^2-5yx+9y=x$$

$$yx^2-x(5y+1)+9y=0$$

For read x, $d\ge 0$

$$(5y+1)^2-4.y.9y\ge 0$$

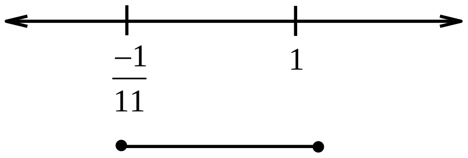
$$25y^2+1+10y-36y^2\ge 0$$

$$11y^2-10y-1\le 0$$

$$11y^2-11y+y-1\le 0$$

$$11y(y-1)+(y-1)\le 0$$

$$(11y+1)(y-1)\le 0$$



$$y_{\max m}=1$$

51. **(D)**

$$y=\frac{x^2+34x-71}{x^2+2x-7}$$

$$yx^2+2xy-7y=x^2+34x-71$$

$$(y-1)x^2+2x(y-17)+(71-7y)=0$$

For real x,

$D\ge 0$

$$4(y-17)^2-4(y-1)(71-7y)\ge 0$$

$$y^2-34y+289-(-7y^2+78y-71)\ge 0$$

$$y^2-34y+289+7y^2-78y+71\ge 0$$

$$8y^2-112y+360\ge 0$$

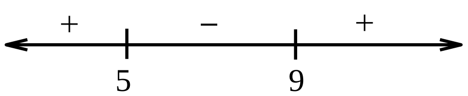
$$2y^2-289+90\ge 0$$

$$y^2-14y+45\ge 0$$

$$y^2-5y-9y+45\ge 0$$

$$y(y-5)-9(y-5)\ge 0$$

$$(y-5)(y-9)\ge 0$$



$$y\in(-\infty+5)\cup[9,\infty)$$

52. **(B)**

$$y_{min} = \frac{-D}{4a} = -\frac{(12^2 - 4.40)}{4}$$

$$y_{min} = -\frac{4 \times \cancel{4}(3^2 - 10)}{\cancel{4}}$$

$$= -4 \times -1 = 4$$

53. **(C)**

$$y = \frac{x^2 - 2x + 1}{x + 1}$$

$$yx + y = x^2 - 2x + 1$$

$$x^2 - x(2 + y) + (1 - y) = 0$$

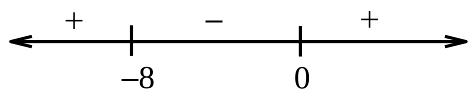
For real $x, D \geq 0$

$$(2 + y)^2 - 4(1 - y) \geq 0$$

$$\cancel{4} + y^2 + 4y - \cancel{4} + 4y \geq 0$$

$$y^2 + 8y \geq 0$$

$$y(y + 8) \geq 0$$



$$y \in (-\infty, -8] \cup [0, \infty)$$

54. **(D)**

$$\Rightarrow 4x^3 + 20x^2 - 23x + 6 = 0$$

Let's take roots are α, α, β

$$\Rightarrow 2\alpha + \beta = \frac{-20}{4} = -5 \qquad \text{……..(1)}$$

$$\Rightarrow \alpha^2 + \alpha\beta + \alpha\beta = \frac{-23}{4}$$

$$\Rightarrow \alpha^2\beta = \frac{-6}{4}$$

$$\Rightarrow \alpha^2(-5 - 2\alpha) = -\frac{6}{4}$$

$$\Rightarrow 5\alpha^2 + 2\alpha^3 - \frac{6}{4} = 0$$

$$\Rightarrow \alpha^3 + 20\alpha^3 - 6 = 0$$

Which gives $\alpha = \frac{1}{2}$

So $2\left(\frac{1}{2}\right) + \beta = -5 \Rightarrow \beta = -6$

So roots are $-6, \frac{1}{2}, \frac{1}{2}$

55. **(D)**

$\Rightarrow 2x^3 - 5x^2 + 3x - 1 = 0$

$\Rightarrow \alpha\beta\gamma = \frac{1}{2}$

$\Rightarrow \alpha + \beta + \gamma = \frac{5}{2}$

$\Rightarrow \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\beta\alpha} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{\left(\frac{5}{2}\right)}{\left(\frac{1}{2}\right)} = 5$

56. **(A)**

$\Rightarrow x^3 - 4x^2 + x + 6 = 0$

Roots are $2\alpha, 3\alpha, \beta$

So, $2\alpha, 3\alpha, \beta = 4$

$\Rightarrow 5\alpha + \beta = 4$ ……….(1)

$\Rightarrow (2\alpha)(3\alpha)(\beta) = -6$

$\Rightarrow \alpha^2\beta = -1$ ………..(2)

By (1) & (2)

$\Rightarrow \alpha^2(4 - 5\alpha) = -1$

$\Rightarrow 5\alpha^3 - 4\alpha^2 - 1 = 0$

By this we get $\alpha = 1$

So roots are $2, 3, -1$

57. **(B)**

$\Rightarrow x^3 + 3x + 2 = 0$

$\Rightarrow a + b + c = 0$

$\Rightarrow ab + bc + ca = 3$

$\Rightarrow abc = -2$

$\Rightarrow a^3 + b^3 + c^3 - 3abx = (a + b + c)(a^2 + b^2 + c^2 + ab + bc + ca) = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3(-2) = -6$

58. **(A)**

$\Rightarrow x^3 + 1 = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = 0$

$\Rightarrow \alpha\beta\gamma = 1$

$\Rightarrow (\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 0$

$\Rightarrow (\alpha^2 + \beta^2 + \gamma^2)^2 = \alpha^4 + \beta^4 + \gamma^4 + 2(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2)$

$$\Rightarrow 0=\alpha^4+\beta^4+\gamma^4+2\left[(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2(\alpha\beta.\beta\gamma+\beta\gamma.\gamma\alpha+\gamma\alpha.\alpha\beta)\right]$$

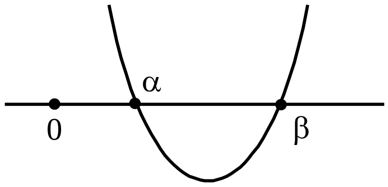
$$=\alpha^4+\beta^4+\gamma^4+2\left[0-2(\beta+\gamma+\alpha)\right] \qquad (\because \alpha\beta\gamma=1)$$

$$\Rightarrow 0=\alpha^4+\beta^4+\gamma^4+2\left[0-2(0)\right]$$

$$\Rightarrow \alpha^4+\beta^4+\gamma^4=0$$

59. **(A)**

Given $D \geq 0$



$1-4.2.K \geq 0$

$8K \leq 1$

$K \leq 1/8$

$f(0)>0 \therefore K>0$

$\frac{+1}{2.2}>0 \qquad \therefore \frac{1}{a}>0$

$\therefore$ roots are the there fore $K>0$

$K \in \left(0, \frac{1}{8}\right]$

60. (A)

Product < 0

$a^2-4a<0$

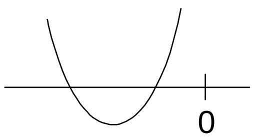
$a \in (0, 4)$

61. (B)

$D \geq 0$

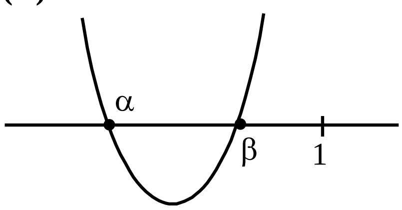
& $-\frac{b}{2a}<0$

& $f(0)>0$



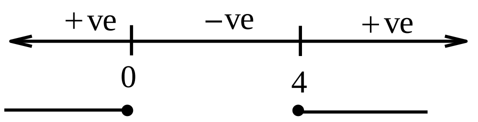
Take intersection of all there inequalities.

62. **(A)**



(i) $D \geq 0 \quad \therefore a^2-4a \geq 0$

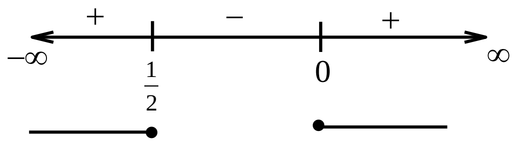
$a(a-4) \geq 0$



$a \in (-\infty, 0] \cup [4, \infty)$ .... (i)

(ii) $a(a + a + 1) > 0$

$a(2a + 1) > 0$



$a \in \left(-\infty, \frac{-1}{2}\right) \cup (0, \infty)$ .... (ii)

$a \in \left(-\infty, \frac{1}{2}\right) \cup [4, \infty)$ (i) & (ii)

EXERCISE - 1 [B]

1. **(C)**

$|\alpha - \beta| > 0$

$(\alpha + \beta)^2 - 4\alpha\beta > 0$

$(P + 2)^2 - 4x2P > 0$

$P^2 + 4 + 4P - 8P > 0$

$(P - 2)^2 > 0$

$\therefore P \in I - \{2\}$

2. **(A)**

Sum = 4 + 3 = 7

Product = 3 × 2 = 6

Roots = 6, 1

3. **(A)**

Given $|\alpha - \beta| = 3$

$(\alpha - \beta)^2 = 9$

$(\alpha + \beta)^2 - 4\alpha\beta = 9$

$\left(\frac{a-4}{a-2}\right)^2 + \frac{8}{(a-2)} = 9$

$(a - 4)^2 + 8(a - 2) = 9(a - 2)^2$

$a^2 - \cancel{8a} + \cancel{16} + \cancel{8a} + \cancel{16} = 9a^2 + 36 - 36a$

$8a^2 - 36a + 36 = 0$

$2a^2 - 9a + 9 = 0$

$$2a^2-3a-6a+9=0$$
$$a(2a-3)-3(2a-3)=0$$
$$(a-3)(2a-3)=0$$
$$a=\frac{3}{2},3$$

4. **(D)**

Given $\alpha+\beta=-p$

$\alpha\beta=q$

$r+\delta=-p$

$r\delta=r$

$$(\alpha-r)(\alpha-\delta)=\alpha^2-\alpha\delta-\alpha r+r\delta$$
$$=\alpha^2-\alpha(\delta+r)+r\delta$$
$$=\alpha^2+p\alpha+r$$
$$=q+r$$

(Given $\alpha^2+p\alpha-q=0$)

$$\alpha^2+p\alpha=q$$

5. **(C)**

Given $\alpha+\beta=\frac{35}{2}$

$\alpha\beta=1$

$$\left[(2\alpha-35)(2\beta-35)\right]^3=\left[4\alpha\beta-70(\alpha+\beta)+35^2\right]^3$$
$$=\left[4-\cancel{70}\times\frac{35}{\cancel{2}}+35^2\right]^3$$
$$=\left(4-\cancel{35^2}+\cancel{35^2}\right)^3=64$$

6. **(B)**

If $ax^2+bx+c=0$ is dx identify the

$a=b=c=0$

$\therefore$ $\lambda^2-3\lambda+2=0$

$\lambda^2-\lambda-2\lambda+2=0$

$\lambda(\lambda-1)-2(\lambda-1)=0$, $(\lambda-1)(\lambda-2)=0$

$\lambda=1,2$

$\lambda^2-2\lambda-3\lambda+6=0$

$\lambda(\lambda-2)-3(\lambda-2)=0$ $\lambda=2,3$

And $\lambda^2=4$

$\lambda=\pm2$

$\therefore$ $\lambda=2$ only common value

7. **(B)**

$\alpha+\beta=-b/a$

$\alpha\beta=c/a$

$$\frac{\beta}{a\left(\alpha+\frac{b}{a}\right)}+\frac{\alpha}{a\left(\beta+\frac{b}{a}\right)}=\frac{\beta}{a\left(\cancel{\alpha}-\cancel{\alpha}-\beta\right)}+\frac{\alpha}{a\left(\cancel{\beta}-\alpha-\cancel{\beta}\right)}$$

$$=\frac{-\beta}{a\beta}-\frac{\alpha}{a\alpha}$$

$$=-\left(\frac{1}{a}+\frac{1}{a}\right)=\frac{-2}{a}$$

8. **(A)**

$\alpha+\beta=\dfrac{-b}{a}$

$\alpha\beta=c/a$

Given $\alpha+\beta=\alpha^2+\beta^2$

$\alpha+\beta=(\alpha+\beta)^2-2\alpha\beta$

$\dfrac{-b}{a}=\dfrac{b^2}{a^2}-2.\dfrac{c}{a}$

$\dfrac{-b}{\cancel{a}}=\dfrac{b^2-2ac}{a^{\cancel{2}}}$

$b^2-2ac=-ab$

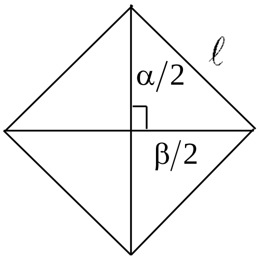
$b^2+ab=2ac$

9. **(A)**

$\ell=\sqrt{\dfrac{\alpha^2}{4}+\dfrac{\beta^2}{4}}$

$=\sqrt{\dfrac{\alpha^2+\beta^2}{2}}$

$=\dfrac{\sqrt{(\alpha+\beta)^2-2\alpha\beta}}{2}$



10. **(D)**

$\alpha+\beta=z$

$\alpha^3+\beta^2=98$

$(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=98$

$8-3.\alpha\beta.z=98$

$-90=6.\alpha\beta$

$\alpha\beta=\dfrac{-30}{2}=-15$

Revered equation $x^2-2x-15=0$

11. **(C)**

$|\alpha-\beta|=1$

$(\alpha+\beta)^2-4\alpha\beta=1$

$p^2-4q=1 \Rightarrow p^2=1+4q$

$p^2+4q^2=1+4q+4q^2$

$=(1+2q)^2$

12. **(D)**

Product of roots = 1

13. **(C)**

Product of roots < 0

14. **(C)**

Other root $=\dfrac{3-5i}{2}$

$\dfrac{c}{2}=$ Product

15. **(A)**

$|\alpha-\beta|=\alpha\beta$

$\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\alpha\beta$

$\alpha+\beta=\dfrac{a+1}{2}$

$\alpha\beta=\dfrac{a-1}{2}$

16. **(A)**

$\alpha+\alpha^2=1$

$\alpha\cdot\alpha^2=\alpha^3=-k$

17. **(B)**

Sum of roots = 0

18. **(C)**

$\alpha+\beta=\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}$

$\alpha+\beta=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$\dfrac{-b}{a}=\dfrac{\frac{b^2}{a^2}-\frac{2.c}{a}}{\frac{c^2}{a^2}}$

$$\frac{-b}{a}=\frac{b^2-2ac}{c^2}$$
$$-bc^2=b^2a-2a^2c$$
$$b^2a+bc^2=2a^2c$$
$$\frac{b^2\cancel{a}}{a^{\cancel{2}}c}+\frac{bc^{\cancel{2}}}{a^2\cancel{c}}=2$$
$$\frac{b^2}{ac}+\frac{bc}{a^2}=2$$

19. **(A)**

$$\alpha-\beta=\frac{\sqrt{D_1}}{a}=\frac{\sqrt{D_2}}{P}$$
$$\frac{D_1}{D_2}=\frac{a^2}{P^2}$$

20. **(A)**

$$\frac{ax-ab+bx-ab}{(x-a)(x-b)}=1$$
$$(a+b)x-2ab=x^2-(a+b)x+ab$$
$$x^2-2(a+b)x+3ab=0 \qquad \text{.......... (i)}$$
Given, sum of roots $=0$
$$2(a+b)=0$$
$$\therefore a+b=0$$

21. **(C)**

Given $\alpha_1-\beta_1=\alpha_2-\beta_2$
$$(\alpha_1-\beta_1)^2=(\alpha_2-\beta_2)^2$$
$$(\alpha_1+\beta_1)^2-4\alpha_1\beta_1=(\alpha_2+\beta_2)^2-4\alpha_2\beta_2$$
$$p^2-4q=q^2-4p$$
$$p^2-q^2+4(p-q)=0$$
$$(p-q)(p+q+4)=0$$
$$\therefore p+q=-4 \quad \because (p\neq a)$$

22. **(B)**

Let roots be $\alpha, 2\alpha$

Sum = $3\alpha=\frac{1-3a}{a^2-5a+3} \Rightarrow \alpha=\frac{1-3a}{3(a^2-5a+3)}$

Product = $\alpha\cdot 2\alpha=2\alpha^2=\frac{2}{a^2-5a+3}$

Solve.

23. **(D)**

$|\alpha-\beta|=1$

$(\alpha+\beta)^2-4\alpha\beta=1$

24. **(A)**

$|\alpha-\beta|<\sqrt{5}$

$(\alpha+\beta)^2-4\alpha\beta<5$

25. **(C)**

$D<0$

$4a^2-4(10-3a)<0$

$a^2+3a-10>0$

$a\in(-5,2)$

26. **(B)**

Given $\mathrm{D}=4(\mathrm{bc}+\mathrm{ad})^2-4(\mathrm{a}^2+\mathrm{b}^2)(\mathrm{c}^2+\mathrm{d}^2)=0$

$\therefore \mathrm{b}^2\cancel{\mathrm{c}^2}+\cancel{\mathrm{a}^2}\mathrm{d}^2+2\mathrm{abcd}-\mathrm{a}^2\mathrm{c}^2-\cancel{\mathrm{a}^2}\mathrm{d}^2-\mathrm{b}^2\cancel{\mathrm{c}^2}-\mathrm{b}^2\mathrm{d}^2=0$

$(\mathrm{ac}-\mathrm{bd})^2$

$\therefore \mathrm{ac}=\mathrm{bd}$

27. **(B)**

Given $\mathrm{D}=\mathrm{b}^2-4\mathrm{ac}<0$

If $\mathrm{b}=0,\mathrm{a}>0,\mathrm{c}>0$

Then $\mathrm{D}<0$

28. **(A)**

$D=0$

29. **(C)**

$D<0$

30. **(C)**

$D=0$

31. **(B)**

$\mathrm{D}<0$

$\mathrm{b}^2-4\mathrm{ac}<0$

$\mathrm{b}^2<4\mathrm{ac}$

32. **(C)**

$\mathrm{f}(1)<0$

$\mathrm{ax}^2+\mathrm{bx}+\mathrm{c}<0\ \ \forall\ \mathrm{x}\in\mathrm{R}$ $\qquad \mathrm{c}<0$

33. (B)

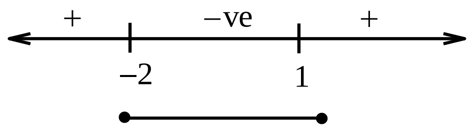
(i) $\left(\lambda^2+\lambda-2\right)x^2+\left(\lambda+2\right)x-1<0 \qquad \forall x \in R$

$\lambda^2+\lambda-2<0$

$\lambda^2+2\lambda-\lambda-2<0$

$\lambda\left(\lambda+2\right)-\left(\lambda+2\right)<0$

$\left(\lambda-1\right)\left(\lambda+2\right)<0$



$\lambda \in \left(-2,1\right)$ .... (i)

(ii) $\left(\lambda+2\right)^2+4\left(\lambda^2+\lambda-2\right)<0$

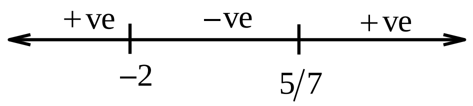
$\lambda^2+4\lambda+4+4\lambda^2+4\lambda-8<0$

$5\lambda^2+8\lambda-4<0$

$5\lambda^2+10\lambda-2\lambda-4<0$

$5\lambda\left(\lambda+2\right)-2\left(\lambda+2\right)<0$

$\left(5\lambda-2\right)\left(\lambda+2\right)<0$



$-2<\lambda<5/2$

From (i) and (ii)

$\lambda \in \left(-2,1\right)$

34. (B)

Product of roots <0 then roots are real and opposite in sign

35. (C)

$D \geq 0$

36. (D)

α, β, γ are roots of $x^3-19x-1=0$

$\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ are roots of $x^3+19x^2-1=0$

(Put $x=\frac{1}{x}$)

37. (C)

$y=x^2+2$

$\therefore x=\sqrt{y-2}$

Replace x by $\sqrt{x-2}$ in the given equation we get revered equation whose roots are

$\alpha^2+2 \,\&\, \beta^2+2$

$2(x-2)-3\sqrt{x-2}-6=0$

$2x-4-6=3\sqrt{x-2}$

$2x-10=3\sqrt{x-2}$

$4x^2+100-40x=9(x-2)$

$4x^2-40x+100=9x-18$

$4x^2-49x+118=0$

38. (A)

$y=\dfrac{x-1}{x+1} \qquad \therefore yx+y=x-1$

$y+1=x(1-y)$

$x=\dfrac{1+y}{1-y}$

Replace x by $\dfrac{1+x}{1-x}$ we get revered equation

$\left(\dfrac{1+x}{1-x}\right)^2-2\dfrac{(1+x)}{(1-x)}+3=0$

$(1+x)^2-2(1+x)(1-x)+3(1-x)^2=0$

$x^2+1+2x-2(1-x^2)+3(1+x^2-2x)=0$

$x^2+1+2x-2+2x^2+3+3x^2-6x=0$

$6x^2-4x+2=0$

$3x^2-2x+1=0$

39. (B)

$x=-\dfrac{1}{x}$

40. (B)

$3\alpha+2, 3\beta+2$

41. (A)

Replace x by $1/x$ we get

$cx^2+bx+a=0$

42. (B)

Given α,β be the roots of

$ax^2+bx+c=0$

$ax^2+bx+c=0$ .... (i)

$a\beta^2+b\beta+c=0$ .... (ii)

From (i) & (ii) $2\alpha, 2\beta$ satisfy equation

$ax^2+2bx+4c=0$

43. **(A)**

$\Rightarrow x^3+3x^2+2x+4=0$

$\Rightarrow \alpha\beta\gamma=-3$

$\Rightarrow \sum\alpha\beta=2$

$\Rightarrow \alpha\beta\gamma=-4$

$\Rightarrow x^3-\left(2\alpha+2\beta+2\gamma\right)x^2+\left(\sum\alpha 2\beta\right)x-\left(2\alpha.2\beta.2\gamma\right)=0$

$\Rightarrow x^3+6x^2+8x+32=0$

44. **(A)**

$\alpha+\beta,\beta+\gamma,\gamma+\alpha$

$\alpha+\beta+\gamma=p$

Hence roots of the equations are $p-\alpha,p-\beta,p-\gamma$

$x=p-\alpha\Rightarrow\alpha=p-x$

Equation is

$\left(p-x\right)^3-p\left(p-x\right)^2+q\left(p-x\right)-r=0$

$\Rightarrow x^3-3px^2+3p^2x-p^3+p^3-2xp^2+px^2-qp+qx+r=0$

$\Rightarrow x^3-2px^2+2\left(p^2+q\right)x+r-pq=0$

45. **(A)**

$x=\dfrac{1+\alpha}{1-\alpha}$

$\Rightarrow\alpha=\dfrac{x-1}{x+1}$

$\left(\dfrac{x-1}{x+1}\right)^3-\left(\dfrac{x-1}{x+1}\right)-1=0$

$\Rightarrow\left(x-1\right)^3-\left(x-1\right)\left(x+1\right)^2-\left(x+1\right)^3=0$

$\Rightarrow -6x^2-2-x^3-x^2+x+1=0$

$\Rightarrow x^3+7x^2-x+1=0$

$\Rightarrow x^3+7x^2-x+1=0$

46. **(B)**

$\left(a_1b_2-a_2b_1\right)\left(b_1c_2-bc_1\right)=\left(c_1a_2-c_2a_1\right)^2$

47. **(B)**

$x^3-2x^2+2x-1=0$

$x^3-x^2-x^2+x+x-1=0$

$x^2\left(x-1\right)-x\left(x-1\right)+\left(x-1\right)=0$

$\left(x-1\right)\left(x^2-x+1\right)=0$ .... (i)

From (i) two common roots are imaginary

$$\frac{a}{1}=\frac{b}{-1}=\frac{a}{1}$$
$$a=-b$$
$$a+b=0$$

48. **(C)**

Use condition for one common root

49. **(D)**

$4+2a+b=0$

$4+2c+d=0$

Subtract $2(a-)c+b-d=0$

$b-d=2(c-a)$

50. **(A)**

Let α be common root

$a\alpha^2+b\alpha+c=0$

$c\alpha^2+b\alpha+a=0$

$$\frac{\alpha^2}{ab-bc}=\frac{\alpha}{c^2-a^2}=\frac{1}{ab-bc}$$
$$\alpha=\frac{(a-c)b}{(c-a)(c+a)}=\frac{-b}{a+c}$$
$$\alpha=\frac{(c-a)(c+a)}{-b(c-a)}=\frac{c+a}{-b}$$
$$\therefore \frac{-b}{a+c}=\frac{(c+a)}{-b}$$
$$(a+c)^2=b^2$$
$$(a+c)^2-b^2=0$$
$$(a+b+c)(a-b+c)=0$$

51. **(C)**

Given $\frac{a}{2}=\frac{b}{-3}=\frac{c}{4}$

$\therefore 6a=-4b=3c$

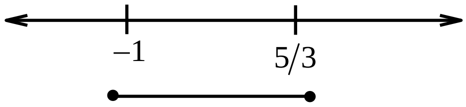
52. **(A)**

$$yx^2+yx+y=x^2+3x+1$$
$$(y-1)x^2+(y-3)x+y-1=0$$

For real x, $D\ge 0$

$$(y-3)^2-4(y-1)(y-1)\ge 0$$
$$(y-3)^2-(2y-2)^2\ge 0$$
$$(y-3+2y-2)(y-3-2y+2)\ge 0$$

$$(3y-5)(-y-1)\ge 0$$
$$(3y-5)(y+1)\le 0$$



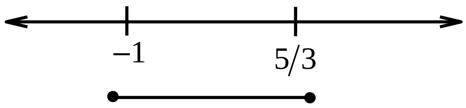
$$y\in[-1,5/3]$$

53. **(A)**

$$yx^2+yx+y=x^2+3x+1$$
$$(y-1)x^2+(y-3)x+y-1=0$$

For real x, $D\ge 0$

$$(y-3)^2-4(y-1)(y-1)\ge 0$$
$$(y-3)^2-(2y-2)^2\ge 0$$
$$(y-3+2y-2)(y-3-2y+2)\ge 0$$
$$(3y-5)(-y-1)\ge 0$$
$$(3y-5)(y+1)\le 0$$



$$y\in[-1,5/3]$$

54. **(A)**

$$Kx^2+Kx+K=x^2-x+1$$
$$(K-1)x^2+(K+1)x+(K-1)=0$$
$$D\ge 0$$

55. **(B)**

$$y_{min}=\frac{-D}{4a}$$
$$=-\frac{(36+4.8.3)}{4\times -8}$$
$$=\frac{-\cancel{12}^{3}(3+8)}{-\cancel{4}\times 8}$$
$$=\frac{3\times 11}{8}=33/8$$

56. **(A)**

$$y=\frac{1}{4\left(x^2+2.\frac{1}{4}x+1\right)}$$

$$= \frac{1}{4\left(x^2 + 2.\frac{1}{4}x + \frac{1}{16} - \frac{1}{16}\right) + 1}$$

$$y = \frac{1}{4\left(x + \frac{1}{4}\right)^2 + \frac{3}{4}}$$

$$y_{maximum} = \frac{4}{3}$$

57. **(C)**

$f(x) = (x+2)^2 - 3$

$f(-4) = 1$

$f(x) \geq 1$

When $x \leq -4$

58. **(D)**

$$y = \frac{x^2 - x + c}{x^2 + x + 2c}$$

$yx^2 + xy + 2cy = x^2 - x + c$

$(y-1)x^2 + (y+1)x + (2y-1)c = 0$

For real x, $(y+1)^2 - 4(y-1)(2y-1)c \geq 0$

$y^2 + 2y + 1 - 4c(2y^2 - 3y + 1) \geq 0$

$(1-8c)y^2 + 2y(1+6c) + (1-4c) \geq 0$ ________(ii)

(ii) Is valid for all y if

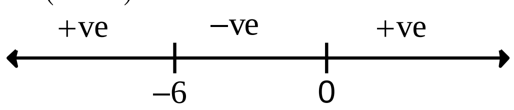
$D \leq 0$

$4(1+6C)^2 - 4(1-8C)(1-4C) \leq 0$

$1 + 36C^2 + 12C - 1 + 12C - 32C^2 \leq 0$

$4C^2 + 24C \leq 0$

$4C(C+6) \leq 0$



$C \in [-6, 0]$ but $C \neq -6, 0$ (as it won't take all real values if $C = -6, 0$)

59. **(A)**

$\alpha^2 + \beta^2 = (a-2)^2 + 2(a+1)$

$= a^2 - 2a + 6 = (a-1)^2 + 5$ is least when $a = 1$

60. **(B)**

Roots are $\sqrt{3} - \sqrt{2}, \sqrt{3} + \sqrt{2}$, $-\sqrt{3} - \sqrt{2}, -\sqrt{3} + \sqrt{2}$.

61. **(A)**

$\Rightarrow x^3+ax+b=0$

$\Rightarrow \alpha+\beta+\gamma=0$

$\Rightarrow \alpha\beta+\beta\gamma+\gamma\alpha=a$

$\Rightarrow \alpha\beta\gamma=-b$

$\Rightarrow \alpha^3+\beta^3+\gamma^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)+\gamma^3$

$=(-\gamma)\left((\alpha+\beta)^2-2\alpha\beta-\alpha\beta\right)+\gamma^3$

$=(-\gamma)(\gamma^2-3\alpha\beta)+\gamma^3$

$=+3\alpha\beta\gamma=-3b$

$\Rightarrow \alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$$\Rightarrow \frac{\alpha^3+\beta^3+\gamma^3}{\alpha^2+\beta^2+\gamma^2}=\frac{-3b}{-2a}=\frac{3b}{2a}$$

62. **(B)**

$\Rightarrow 4x^3-12x^2+11x+k=0$

Sum of roots $(a-d+a+a+d)=\dfrac{12}{4}=3$

$\Rightarrow a=1$

As a is root of equation so it will satisfy if.

$\Rightarrow 4(a)^3-12a^2+11a+k=0$

$\Rightarrow 4-12+11+k=0$

$\Rightarrow k-3$

63. **(B)**

$\Rightarrow x^3-11x^2+37x-35=0$

If root is $3+\sqrt{2}$, second root has to be $3-\sqrt{2}$

Let's take third root α

So, $(3+\sqrt{2})+(3-\sqrt{2})+\alpha=11$

$\Rightarrow \alpha=5$

64. **(A)**

α,β,γ are roots of $x^3-5x^2+5x-3=0$

$\alpha+\beta+\gamma=0$

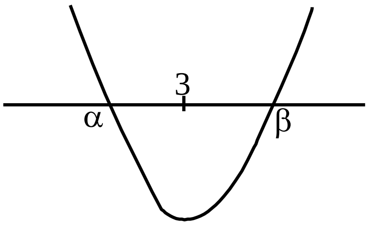
65. **(A)**

$D>0$

$(1-2k)^2-4(k^2-k-2)>0$

$4k^2-4k+1-4k^2+4k+8>0+\ 9>0$

$f(3)<0$



$9+3(1-2k)+\left(k^2-k-2\right)<0$

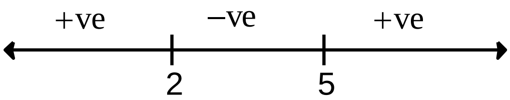
$9+3-6k+k^2-k-2<0$

$k^2-7k+10<0$

$k^2-2k-5k+10<0$

$k(k-2)-5(k-2)<0$

$(k-2)(k-5)<0$



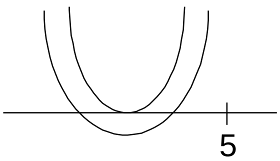
$k\in(2,5)$

66. (C)

$D\geq 0$

& $\dfrac{-b}{2a}<5$

& $f(5)>0$

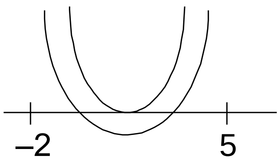


67. (B)

$D\geq 0$

$-2<\dfrac{-b}{2a}<4$

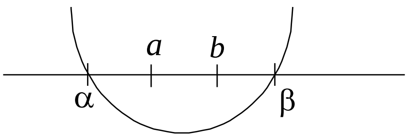
$f(-2)>0,\ \ f(4)>0$



68. (D)

$f(a)<0$

$f(b)<0$



$\alpha<a\ \ \&\ \ \beta<b$

EXERCISE - 1 [C]

1. (1)

$mx^2 - 2x + (2m-1) = 0$

Product = 3

$\frac{2m-1}{m} = 3$

$m = 1$

$m + 2 = 1$

2. (3)

$(k-2)x^2 - (k-4)x = 0$

$(k-2)x^2 - (k-2-2)x - 2 = 0$

$(k-2)x^2 - (k-2)x + 2x - 2 = 0$

$[(k-2)x + 2][x-1] = 0$

$\therefore\ x = \frac{-2}{k-2}$ or $x = 1$

Difference = 3, i.e.

$1 + \frac{2}{k-2} = 3$ or $\frac{-2}{k-2} - 1 = 3$

$k = 3k - 6 \qquad 4k - 8 = -2$

$k = 3 \qquad k = \frac{8}{4}$

The integral value of *k* will be 3

3. (1)

$(a^2 - 1)x^2 + (a-1)x + a^2 - 4a + 3 = 0$

For an quadratic to be an identity.
All coefficients should be 0.

$\therefore\ a^2 - 1 = 0, \quad a - 1 = 0 \quad \& \quad a^2 - 4a + 3 = 0$

$(a-1)(a+1) = 0, \quad a = 1 \quad \& \quad (a-1)(a-3) = 0$

$\therefore\ a = -1, 1, \quad a = 1 \quad \& \quad a = 1, 3$

The only common value which satisfies all 3 is $a = 1$.

4. (1)

$x - 3 = \sqrt{5}$

Squaring on both side

$x^2 - 6x + 9 = 5$

$x^2 - 6x = -4 \qquad ...(1)$

Square on both side

$x^4 - 12x^3 + 36x^2 = 16$

Adding 8 × (1) on both side.

$x^4 - 12x^3 + 36x^2 + 8x^2 - 48x = 16 - 32$

$x^4 - 12x^3 + 44x^2 - 48x = -16$

Add 17 on both side

$x^4 - 12x^3 + 44x^2 - 48x + 17 = 1$

5. (4)

Since both roots are common.

$\frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{5}{10} = \frac{2}{q}$

$q = 4$

6. (15)

Let common root $b \in \alpha$.

$\therefore\ \alpha^2 + a\alpha + 12 = 0 \quad ...(1)$

$\alpha^2 + bd + 15 = 0 \quad ...(2)$

$\alpha^2 + (a+b)\alpha + 36 = 0 \quad ...(3)$

Let's consider (1) + (2) – (3)

$\therefore\ \alpha^2 - 9 = 0$

$\alpha = 3\ \because$ given α is positive

Put $\alpha = 3$ in (3)

$\therefore$ It $(a+b)3 + 36 = 0$

$a + b + 15 = 0$

$\therefore\ a + b + 30 = 15$

7. (7)

Given: $y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$x^2y + 3xy + 4y = x^2 - 3x + 4$

$x^2(y-1) + (3y+3)x + 4y - 4 = 0$

Since, x is real.

$b^2 - 4ac > 0$

$(3y+3)^2 - 4 \times (y-1) \times (4y-4) > 0$

$9y^2 + 18y + 9 - 16(y^2 - 2y + 1) > 0$

$-7y^2 + 50y - 7 \geq 0$

$7y^2 - 50y + 7 \leq 0$

$7y^2 - 49y - y + 7 \leq 0$

$7y(y-7) - 1(y-7) \leq 0$

$(7y-1)(y-7) \leq 0$

For y between $\left[\frac{1}{7}, 7\right]$, the above inequality holds.

$\therefore$ maximum value of $y = 7$.

8. (6)

Given roots of $x^2+2(k-3)x+9=0$

Lie between $(-6,1)$.

$\therefore$ at $x=-6$, quadratic > 0. $\quad (\therefore\ a>0)$

i.e. $(36)+2(k-3)(-6)+9\geq 0$

$12(k-3)\leq 45$

$k-3\leq\dfrac{15}{4}$

$k\leq\dfrac{27}{4}$

Also at $x=1$, quadratic ≥ 0

$1+2(k-3)+9\geq 0$

$2(k-3)\geq -10$

$k-3\geq -5$

$k\geq -2$

$\therefore\ -2<k<\dfrac{27}{4}$

$\therefore$ Min value of $k=-2$.

9. (7)

10. (0)

$3\left(x^2+\dfrac{1}{x^2}\right)-2\left(x+\dfrac{1}{x}\right)+5=0$

$3\left(x^2+\dfrac{1}{x^2}+2-2\right)-2\left(x+\dfrac{1}{x}\right)+5=0$

$3\left(\left(x+\dfrac{1}{x}\right)^2\right)-2\left(x+\dfrac{1}{x}\right)-1=0$

Let $x+\dfrac{1}{x}=t$

$\therefore\ 3t^2-2t-1=0$

$3t^2-3t+t-1=0$

$(3t+1)(t-1)=0$

$t=1,\quad t=-\dfrac{1}{3}$

$\therefore\ x+\dfrac{1}{x}=1$ or $x+\dfrac{1}{x}=-\dfrac{1}{3}$

$x^2-x+1=0$ or

$D=\sqrt{1-4}$ or $D=\sqrt{1-36}$

$D=\sqrt{-3}\qquad D=\sqrt{-35}$

For both the cases no real root is possible as $D<0$.

$\therefore$ No. of real roots = 0.

11. (4)

$$S=\left\{x : x\in R \ \& \ \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10\right\}$$

We have a set S, such that for real x

$$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10$$

& we need to find n (S) = ?

$$\sqrt{3}+\sqrt{2}=\left(\sqrt{3}-\sqrt{2}\right)^{-1} \text{ or } \sqrt{3}-\sqrt{2}=\frac{1}{\sqrt{3}+\sqrt{2}}$$

$\therefore$ Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=t$

We will set $t+\dfrac{1}{t}=10$

$$t^2-10t+1=0$$

$$t=\frac{10\pm\sqrt{96}}{2}$$

$$t=5+\sqrt{24} \text{ or } t=5-\sqrt{24}$$

$$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\sqrt{3}+\sqrt{2}\right)^2 \text{ or } \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\sqrt{3}-\sqrt{2}\right)^2$$

From here or $x^2-4=-2$ $\left(\because \dfrac{1}{\sqrt{3}-\sqrt{2}}\right)=\sqrt{3}+2$

$x^2-4=2$ $\therefore x^2=2$

$x=\pm\sqrt{6}$ $x=\pm\sqrt{2}$

$\therefore$ x satisfying the conditions are $\sqrt{6}, \sqrt{2}, -\sqrt{2}, -\sqrt{6}$.

$\therefore$ $n(S)=4$

12. (58)

$ax^2-2bx+15=0$ has repeated root

i.e. $\alpha=r$

$\therefore$ $2\alpha=\dfrac{2b}{a}$ or $\dfrac{b}{a}=\alpha$

& $\alpha^2=\dfrac{15}{a} \Rightarrow \dfrac{b^2}{a^2}=\dfrac{15}{a} \Rightarrow \alpha=\dfrac{15}{b}$

Now, equation $x^2-2bx+21=0$ has roots α & β

$\alpha+\beta=26$ $\alpha\beta=21$.

Since α is a root.

$$\therefore \left(\frac{15}{b}\right)^2-2b\times\frac{15}{b}+21=0$$

$$\left(\frac{15}{b}\right)^2=9$$

$$b^2=25$$

$$\therefore \alpha^2+\beta^2=\left(\alpha+\beta\right)^2-2\alpha\beta$$

$$\alpha^2+\beta^2=4b^2-42=58$$

13. (24)

$3x^2+\lambda x-1=0 \Rightarrow \alpha+\beta=-\frac{\lambda}{3},\ \alpha\beta=-\frac{1}{3}\ \&\ \frac{1}{\alpha^2}+\frac{1}{\beta^2}=15$

$\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}=15$

$(\alpha+\beta)^2-2\alpha\beta=\frac{5}{3}$

$(\alpha+\beta)^2=\frac{5}{3},-\frac{2}{3}$

$\lambda^2=9$

$\lambda=\pm 3$

Now, $6(\alpha^3+\beta^3)^2=6(\alpha+\beta)^2\left((\alpha+\beta)^2-3\alpha\beta\right)^2$

$=6\cdot 1\cdot(1+1)^2$

$=24$.

JEE Main : PYQ

1. (C)

$x^2+5^{\frac{1}{2}}=-(20)^{\frac{1}{4}}x \Rightarrow \left(x^2+\sqrt{5}\right)^2=\sqrt{20}x^2$

$\Rightarrow x^4+5+2\sqrt{5}x^2=2\sqrt{5}x^2 \Rightarrow x^4=-5 \Rightarrow x^8=25$

So, $\alpha^8+\beta^8=50$

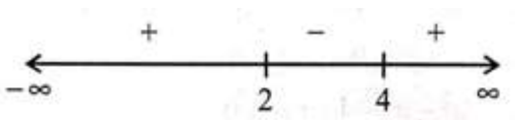
2. (B)

$x^2-2(3k-1)x+8k^2-7>0, \forall x\in R$

$\Rightarrow D<0 \Rightarrow 4(3k-1)^2-4\cdot 1\cdot\left(8k^2-7\right)<0$

$\Rightarrow 9k^2-6k+1-8k^2+7<0 \Rightarrow k^2-6k+8<0$

$\Rightarrow (k-2)(k-4)<0$



$\Rightarrow k\in(2,4)$; So, $k=3$

3. (C)

As, $\left(\alpha^2+\sqrt{3}\right)=-(3)^{1/4}\cdot\alpha$

$\Rightarrow \left(\alpha^4+2\sqrt{3}\alpha^2+3\right)=\sqrt{3}\alpha^2$ (On squaring)

$\therefore \left(\alpha^4+3\right)=(-)\sqrt{3}\alpha^2$

$\Rightarrow \alpha^8+6\alpha^2+9=3\alpha^4$ (Again squaring)

$\therefore \alpha^8+3\alpha^4+9=0$

$\Rightarrow \alpha^8=-9-3\alpha^4$ (Multiple by α^4)

So, $\alpha^{12} = -9\alpha^4 - 2\alpha^8$

$\therefore\ \alpha^{12} = -9\alpha^4 - 3\left(-9 - 3\alpha^4\right)$

$\Rightarrow\ \alpha^{12} = -\cancel{9\alpha^4} + 27 + \cancel{9\alpha^4}$

Hence, $\alpha^{12} = (27)^2$

$\Rightarrow\ \left(\alpha^{12}\right)^8 = (27)^8$

$\Rightarrow\ \alpha^{96} = (3)^{24}$

Similarly, $\beta^{96} = (3)^{24}$

$\therefore\ \alpha^{96}\left(\alpha^{12} - 1\right) + \beta^{96}\left(\beta^{12} - 1\right) = (3)^{24} \times 52$

4. (D)

Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + ...}}}$

$\Rightarrow\ x = 3 + \cfrac{1}{4 + \cfrac{1}{x}}\ \Rightarrow\ (x-3)(4x+1) = x$

$\Rightarrow\ 4x^2 - 11x - 3 = x \Rightarrow\ 4x^2 - 12x - 3 = 0$

$4\left(x - \frac{3}{2}\right)^2 = 12\ \Rightarrow\ x = \sqrt{3} + \frac{3}{2}\left(\sqrt{3} + \frac{3}{2}\text{ rejected}\right)$

5. (A)

$\because\ \alpha + \beta = 64, \alpha\beta = 256$

$\frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}} = \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{\left(2^8\right)^{5/8}} = \frac{64}{32} = 2$

6. (B)

Let α and β be the roots of the given quadratic equation,

$4x^2 + 2x - 1 = 0 \qquad ...(\text{i})$

Then, $\alpha + \beta = -\frac{1}{2}\ \Rightarrow\ -1 = 2\alpha + 2\beta$ and $4\alpha^2 + 2\alpha - 1 = 0$ [$\because \alpha$ is root of eq. (i)]

$\Rightarrow\ 4\alpha^2 + 2\alpha + 2\alpha + 2\beta = 0\ \Rightarrow\ \beta = -2\alpha(\alpha + 1)$

7. (D)

Let α and β be the roots of the quadratic equation

$7x^2 - 3x - 2 = 0$

$\therefore\ \alpha + \beta = \frac{3}{7},\ \alpha\beta = \frac{-2}{7}$

Now, $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$

$= \frac{\alpha - \alpha\beta(\alpha + \beta) + \beta}{1 - \left(\alpha^2 + \beta^2\right) + (\alpha\beta)^2}$

$$=\frac{(\alpha+\beta)-\alpha\beta(\alpha+\beta)}{1-(\alpha+\beta)^2+2\alpha\beta+(\alpha\beta)^2}$$

$$=\frac{\frac{3}{7}+\frac{2}{7}\times\frac{3}{7}}{1-\frac{9}{49}+2\times\frac{-2}{7}+\frac{4}{49}}=\frac{27}{16}$$

8. (A)

$ax^2-2bx+5=0$

If α and β are roots of equation, then sum of roots

$2a=\frac{2b}{a}\Rightarrow\alpha=\frac{b}{a}$ and product of roots $=\alpha^2=\frac{5}{a}\Rightarrow\frac{b^2}{a^2}=\frac{5}{a}$

$\Rightarrow\ b^2=5a\quad(\alpha\neq 0)\qquad ...(\text{i})$

For $x^2-2bx-10=0$

$\alpha+\beta=2b\qquad ...(\text{ii})$

and $\alpha\beta=-10\qquad ...(\text{iii})$

$\alpha=\frac{b}{a}$ is also root of $x^2-2bx-10=0$

$\Rightarrow b^2-2ab^2-10a^2=0$

By equation (i) $\Rightarrow\ 5a-10a^2-10a^2=0$

$\Rightarrow\ 20a^2=5a$

$\Rightarrow a=\frac{1}{4}$ and $b^2=\frac{5}{4}$

$\alpha^2=20$ and $\beta^2=5$

Now, $\alpha^2+\beta^2=5+20=25$

9. (D)

$\alpha\cdot\beta=2$ and $\alpha+\beta=-p$ also $\frac{1}{\alpha}+\frac{1}{\beta}=-q$

$\Rightarrow\ p=2q$

Now, $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$$=\left[\alpha\beta+\frac{1}{\alpha\beta}-\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right]\left[\alpha\beta+\frac{1}{\alpha\beta}+2\right]$$

$$=\frac{9}{2}\left[\frac{5}{2}-\frac{\alpha^2+\beta^2}{2}\right]=\frac{9}{4}\left[5-\left(p^2-4\right)\right]$$

$$=\frac{9}{4}\left(9-p^2\right)\quad\left[\because\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta\right]$$

10. (C)

The given quadratic equation is

$\left(\lambda^2+1\right)x^2-4\lambda x+2=0$

$\because$ One root is in the interval (0, 1)

$\therefore\ f(0)f(1)\le 0.$

$\Rightarrow\ 2\left(\lambda^2+1-4\lambda+2\right)\le 0$

$\Rightarrow\ 2\left(\lambda^2-4\lambda+3\right)\le 0$

$(\lambda-1)(\lambda-3)\le 0\ \Rightarrow\ \lambda\in[1,3]$

But at $\lambda=1$, both roots are 1 so $\lambda\ne 1$

$\therefore\ \lambda\in(1,3]$

11. (C)

Since, α and β are the roots of the equation $5x^2+6x-2=0$

Then, $5\alpha^2+6\alpha-2=0, 5\beta^2+6\beta-2=0$

$5\alpha^2+6\alpha=2$

$$\begin{aligned}5S_6+6S_5&=5\left(\alpha^6+\beta^6\right)+6\left(\alpha^5+\beta^5\right)\\&=\left(5\alpha^4+6\alpha^5\right)+\left(5\beta^6+6\beta^5\right)\\&=\alpha^4\left(5\alpha^2+6\alpha\right)+\beta^4\left(5\beta^2+6\beta\right)\\&=2\left(\alpha^4+\beta^4\right)=2S_4\end{aligned}$$

12. (D)

Since, $2-\sqrt{3}$ is a root of the quadratic equation $x^2+px+q=0$

$\therefore\ 2+\sqrt{3}$ is the other root

$\Rightarrow\ x^2+px+q=\left[x-\left(2-\sqrt{3}\right)\right]\left[x-\left(2+\sqrt{3}\right)\right]$

$=x^2-\left(2+\sqrt{3}\right)x-\left(2-\sqrt{3}\right)x+\left(2^2-\left(\sqrt{3}\right)^2\right)=x^2-4x+1$

Now, by comparing $p=-4,\ q=1$

$\Rightarrow\ p^2-4q-12=16-4-12=0$

13. (C)

Sum of roots $=\dfrac{3}{m^2+1}$

$\because$ sum of roots is greatest, $\therefore\ m=0$

Hence equation becomes $x^2-3x+1=0$

Now, $\alpha+\beta=3,\ \alpha\beta=1\Rightarrow\ \left|-\alpha-\beta\right|=\sqrt{5}$

$\left|\alpha^3-\beta^3\right|=\left|(\alpha-\beta)\left(\alpha^2+\beta^2+\alpha\beta\right)\right|=\sqrt{5}(9-1)=8\sqrt{5}$

14. (B)

Let roots of the quadratic equation are α,β.

Given, $\lambda=\dfrac{\alpha}{\beta}$ and $\lambda+\dfrac{1}{\lambda}=1\Rightarrow\ \dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=1$

$\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=1$ ...(i)

The quadratic equation is, $3mh2x^2+m(m-4)x+2=0$

$\therefore \quad \alpha+\beta=\frac{m(4-m)}{3m^2}=\frac{4-m}{3m}$ and $\alpha\beta=\frac{2}{3m^2}$

Put these values in eq. (i),

$$\frac{\left(\frac{4-m}{3m}\right)^2}{\frac{2}{3m^2}}=3 \Rightarrow (m-4)^2=18 \Rightarrow m=4\pm\sqrt{18}$$

Therefore, least value is $4-\sqrt{18}=4-3\sqrt{2}$

15. (D)

Let α and β be the roots of the equation,

$81x^2+kx+256=0$

Given, $(\alpha)^{\frac{1}{3}}=\beta \Rightarrow \alpha=\beta^3$

$\because \quad$ Product of the roots $=\frac{256}{81}$

$\therefore \quad (\alpha)(\beta)=\frac{256}{81}$

$\Rightarrow \quad \beta^4=\left(\frac{4}{3}\right)^4 \Rightarrow \beta=\frac{4}{3} \Rightarrow \alpha=\frac{64}{27}$

$\because \quad$ Sum of the roots $=-\frac{k}{81}$

$\therefore \quad \alpha+\beta=-\frac{k}{81} \Rightarrow \frac{4}{3}+\frac{64}{27}=-\frac{k}{81}$

$\Rightarrow \quad k=-300$

16. (D)

Consider the quadratic equation

$(c-5)x^2-2cx+(c-4)=0$

Now, $f(0).f(3)>0$ and $f(0).f(2)<0$

$\Rightarrow \quad (c-4)(4c-49)>0$ and $(c-4)(c-24)<0$

$\Rightarrow \ c\in(-\infty,4)\cup\left(\frac{49}{4},\infty\right)$ and $c\in(4,24)$

$\Rightarrow \ c\in\left(\frac{49}{4},24\right)$

Integral values in the interval $\left(\frac{49}{4},24\right)$ are 13, 14, …., 23.

$\therefore \ S=\{13,14,...,23\}$

17. (D)

The given quadratic equation is

$x^2+(3-\lambda)x+2=\lambda$

Sum of roots $=\alpha+\beta=\lambda-3$

Product of roots $=\alpha\beta=2-\lambda$

$\alpha+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(\lambda-3)^2-2(2-\lambda)$

$=\lambda^2-4\lambda+5$

$=(\lambda-2)^2+1$

For least $(\alpha^2+\beta^2)\lambda=2$.

18. (A)

The roots of $6x^2-11x+\alpha=0$ are rational numbers.

$\therefore$ Discriminant D must be perfect square number.

$D=(-11)^2-4\cdot6\cdot\alpha$

$=121-24\alpha$ must be a perfect square

Hence, possible values for α are $\alpha=3,4,5$.

$\therefore$ 3 positive integral values are possible.

19. (B)

Given quadratic equation is : $x^2-mx+4=0$

Both the roots are real and distinct.

So, discriminant $B^2-4AC>0$.

$\therefore$ $m^2-4\cdot1\cdot4>0$

$\therefore$ $(m-4)(m+4)>0$

$\therefore$ $m\in(-\infty,-4)\cup(4,\infty)$ ...(i)

Since, both roots lies in [1, 5]

$\therefore$ $-\frac{-m}{2}\in(1,5)$

$\Rightarrow$ $m\in(2,10)$ ...(ii)

And $1\cdot(1-m+4)>0 \Rightarrow m<5$

$\therefore$ $m\in(-\infty,5)$ ...(iii)

And $1\cdot(25-5m+4)>0 \Rightarrow m<\frac{29}{5}$

$\therefore$ $m\in\left(-\infty,\frac{29}{5}\right)$ ...(iv)

From (i), (ii), (iii) and (iv), $m\in(4,5)$

20. (B)

$\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$

$\frac{x+p+x+q}{(x+p)(x+q)}=\frac{1}{r}$

$(2x+p+q)r=x^2+px+qx+pq$

$x^2+(p+q-2r)x+pq-pr-qr=0$

Let α and β be the roots.

$\therefore\ \alpha+\beta=-(p+q-2r)$ ...(i)

& $\alpha\beta=pq-pr-qr$ ...(ii)

$\because\ \alpha=-\beta$ (given)

$\therefore$ in eq. (i), we get

$\Rightarrow\ -(p+q-2r)=0$ ...(iii)

Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(-(p+q-2r))^2-2(pq-pr-qr)$ ...(from (i) and (ii))

$=p^2+q^2+4r^2+2pq-4pr-4qr-2pq+2pr+2qr$

$=p^2+q^2+4r^2-2pr-2qr$

$=p^2+q^2+2r(2r-p-q)$ ...(from (iii))

$=p^2+q^2$

21. (B)

Let, the roots of the equation, $x^2+(2-\lambda)x+(10-\lambda)=0$ are α and β.

Also roots of the given equation are

$$\frac{\lambda-2\pm\sqrt{4-4\lambda+\lambda^2-40+4\lambda}}{2}=\frac{\lambda-2\pm\sqrt{\lambda^2-36}}{2}$$

The magnitude of the difference of the roots is $\left|\sqrt{\lambda^2-36}\right|$

So, $\alpha^3+\beta^3=\frac{(\lambda-2)^3}{4}+\frac{3(\lambda-2)(\lambda^2-36)}{4}$

$=\frac{(\lambda-2)(4\lambda^2-4\lambda-104)}{4}=(\lambda-2)(\lambda^2-l-26)=f(\lambda)$

As $f(\lambda)$ attains its minimum value at $\lambda=4$.

Therefore, the magnitude of the difference of the roots is $\left|i\sqrt{20}\right|=2\sqrt{5}$

22. (D)

We have

$f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2,\ (x\in R)$

$\Rightarrow\ f(x)=(x+b)^2+2c^2-b^2$ and $g(x)=-(x+c)^2+b^2+c^2$

Now, $f_{\min}=2c^2-b^2$ and $g_{\max}=b^2+c^2$

Given: $\min f(x)>\max g(x)$

$\Rightarrow\ 2c^2-b^2>b^2+c^2$

$\Rightarrow\ c^2>2b^2$

$\Rightarrow\ |c|>|b|\sqrt{2}$

$\Rightarrow\ \frac{|c|}{|b|}>\sqrt{2}\ \Rightarrow\ \left|\frac{c}{b}\right|<\sqrt{2}$

$\Rightarrow\ \left|\frac{c}{b}\right|\in(\sqrt{2},\infty)$

23. (D)

If a and –1 are the roots of the polynomial, then we get

$f(x)=x^2+(1-a)x-a$.

$\therefore \quad f(1)=2-2a$ and $f(2)=6-3a$

As, $f(1)++f(2)=0 \Rightarrow 2-2a+6-3a=0 \Rightarrow a=\frac{8}{5}$

Therefore, the other root is $\frac{8}{5}$

24. (C)

$x^2+bx-1=0 \qquad ...(1)$ and

$x^2+x+b=0 \qquad ...(2)$

Subtracting this to get common root as $x=\frac{b+1}{b-1}$

Substituting the common root in equation (2)

$$\left(\frac{b+1}{b-1}\right)^2+\left(\frac{b+1}{b-1}\right)+b=0$$

$$(b+1)^2+(b+1)(b-1)+b(b-1)^2=0$$

$$b^2+2b+1+b^2-1+b\left(b^2+1-2b\right)=0$$

$$b^3+3b=0$$

$$b=0,\ \pm\sqrt{3}i$$

0 is not possible.

$$|b|=\sqrt{3}$$

25. (B)

$$(a-1)\left(x^4+x^2+1\right)+(a+1)\left(x^2+x+1\right)^2=0$$

$$\Rightarrow (a-1)\left(x^2+x+1\right)\left(x^2-x+1\right)+(a+1)\left(x^2+x+1\right)^2=0$$

$$\Rightarrow \left(x^2+x+1\right)\left[(a-1)\left(x^2-x+1\right)+(a+1)\left(x^2+x+1\right)\right]=0$$

$$\Rightarrow \left(x^2+x+1\right)\left(ax^2+x+a\right)=0$$

For roots to be distinct and real, $a\neq 0$ and $1-4a^2>0$

$$\Rightarrow a\neq 0 \text{ and } a^2<\frac{1}{4} \Rightarrow a\in\left(-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right)$$

26. (B)

$\alpha=2+3i;\ \beta=2-3i,\ \gamma=?$

$\alpha\beta\gamma=\frac{13}{2} \qquad \left[\because \text{product of roots}=-\frac{d}{a}\right]$

$$\Rightarrow (4+9)\gamma=\frac{13}{2} \Rightarrow \gamma=\frac{1}{2}$$

27. (A)

Let $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ be the roots of $ax^2+bx+1=0$

$$\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}=\left(\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}}\right)=-\frac{b}{a}$$

$$\frac{1}{\sqrt{\alpha}\sqrt{\beta}}=\frac{1}{a} \Rightarrow a=\sqrt{\alpha\beta}$$

$$b=-\left(\sqrt{\alpha}+\sqrt{\beta}\right)$$

$$x\left(x+b^3\right)+\left(a^3-3abx\right)=0$$

$$\Rightarrow \quad x^2+\left(b^3-3ab\right)x+a^3=0$$

Putting values of a and b, we get

$$x^2+\left[\left(-\sqrt{a}+\sqrt{\beta}\right)^3+3\left(\sqrt{\alpha\beta}\right)\left(\sqrt{\alpha}+\sqrt{\beta}\right)\right]+\left(\alpha\beta\right)^{3/2}=0$$

$$\Rightarrow \quad x^2-\left[\alpha^{3/2}+\beta^{3/2}+3\sqrt{\alpha\beta}\left(\sqrt{\alpha}+\sqrt{\beta}\right)-x^2--3\sqrt{\alpha\beta}\left(\sqrt{\alpha}+\sqrt{\beta}\right)\right]x+\left(\alpha\beta\right)^{3/2}=0$$

$$\Rightarrow \quad x^2-\left(\alpha^{3/2}+\beta^{3/2}\right)x+\alpha^{3/2}\beta^{3/2}=0$$

Roots of this equation are $\alpha^{3/2}, \beta^{3/2}$

28. (B)

Given $\alpha^3+\beta^3=-p$ and $\alpha\beta=q$

Let $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ be the roots of required quadratic equation

So, $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{-p}{q}$ and $\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=q$

Hence, required quadratic equation is

$$x^2-\left(\frac{-p}{q}\right)x+q=0$$

$$\Rightarrow \quad x^2+\frac{p}{q}x+q=0 \quad \Rightarrow \quad qx^2+px+q^2=0$$

29. (C)

Given quadratic equation is

$$x^2+px+\frac{3p}{4}=0$$

So, $\alpha+\beta=-p, \alpha\beta=\frac{3p}{4}$

Now, given $|\alpha-\beta|=\sqrt{10}$

$$\Rightarrow \quad \alpha-\beta=\pm\sqrt{10}$$

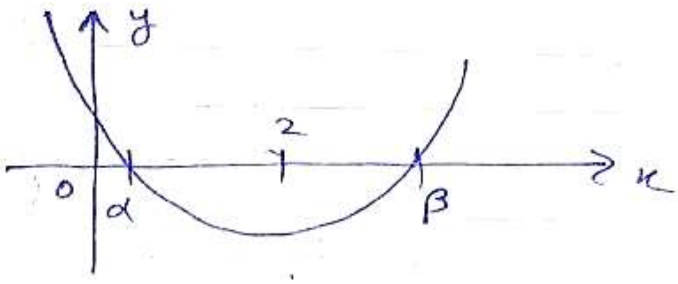
$$\Rightarrow \quad \left(\alpha-\beta^2\right)=10 \Rightarrow \alpha^2+\beta^2-2\alpha\beta=10$$

$$\Rightarrow \quad \left(\alpha+\beta\right)^2-4\alpha\beta=10$$

$$\Rightarrow \quad p^2-4\times\frac{3p}{4}=10 \Rightarrow p^2-3p-10=0$$

$\Rightarrow \quad p = -2, 5 \Rightarrow p \in \{-2, 5\}$

30. (C)



Quadrate equation is: $x^2 - (a + 1)x + a^2 + a - 8 = 0$
One root exceeds 2 and other is lesser than
$\Rightarrow$ 2 lies between the roots.
A rough diagram it as shown below.
Where α and β are its roots.
$\Rightarrow$ For 2 to lie.
between roots.
The only condition is
$f(2) < 0$
$\Rightarrow 2^2 - (a + 1)(2) + a^2 + a - 8 < 0$
$\Rightarrow 4 - 2a - 2 + a^2 + a - 8 < 0$
$\Rightarrow a^2 - a - 6 < 0$
$\Rightarrow a^2 - 3a + 2a - 6 < 0$
$\Rightarrow a(a - 3) + 2(a - 3) < 0$
$\Rightarrow (a + 2)(a - 3) < 0$
$\Rightarrow$ using sign reheme method.
$a \in (-2, 3)$
i.e. $-2 < a < 3$

31. (324)

$x^2 - x - 1 = 0$ roots $= \alpha, \beta$

$\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^{n+1} = \alpha^n + \alpha^{n-1}$

$\beta^2 - \beta - 1 = 0 \Rightarrow \beta^{n+1} = \beta^n + \beta^{n-1}$

$+$

$P_{n+1} = P_n + P_{n-1}$

$29 = P_n + 11$

$P_n = 18$

$P_n^2 = 324$

32. (66)

$$\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$$

$x \in R - \{1, 2\}$

$\Rightarrow k(2x - 4 - x + 1) = 2(x^2 - 3x + 2)$

$\Rightarrow k(x - 3) = 2(x^3 - 3x + 2)$

For $x \neq 3$, $k = 1\left(x - 3 + \frac{2}{x-3} + 3\right)$

$x - 3 + \frac{2}{x-3} \geq 2\sqrt{2}, \forall\, x > 3$ & $x - 3 + \frac{2}{x-3} \leq -2\sqrt{2}, \forall\, x < -3$

$\Rightarrow \left(x - 3 + \frac{2}{x-3} + 3\right) \in \left(-\infty, 6 - 4\sqrt{2}\right] \cup \left[6 + 4\sqrt{2}, \infty\right)$

For no real roots $k \in \left(6 - 4\sqrt{2}, 6 + 4\sqrt{2}\right) - \{0\}$

Integral $k \in \{1, 2, ..., 11\}$

Sum of $k = 66$

33. (18)

Since α is root of both the quadratic equations,

$3\alpha^2 - 10\alpha + 27\lambda = 0 \qquad ...(i)$

$\alpha^2 - \alpha + 2\lambda = 0 \qquad ...(ii)$

On solving equations (i) and (ii)

$-7\alpha + 21\lambda = 0 \Rightarrow \alpha = 3\lambda$

Put $\alpha = 3\lambda$ in equation (i), we get

$9\lambda^2 - 3\lambda + 2\lambda = 0$

$9\lambda^2 = \lambda \Rightarrow \lambda = \frac{1}{9}$ as $\lambda \neq 0$

Now $\alpha = 3\lambda \Rightarrow \lambda = \frac{1}{3}$

Now sum of roots

$\alpha + \beta = 1 \Rightarrow \beta = \frac{2}{3}$

$\alpha + \gamma = \frac{10}{3} \Rightarrow \gamma = 3$

$$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3} \times 3}{\frac{1}{9}} = 18$$

34. (2)

Let $e^x = t, (t > 0)$

$t^4 - t^3 - 4t^2 - t + 1 = 0$

$\Rightarrow t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0$

$\Rightarrow \left(t^2 + \frac{1}{t^2}\right) - \left(t^3 + t\right) - 4 = 0$

$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 6 = 0$

Let $t + \frac{1}{t} = u \qquad (u > 2)$

$\Rightarrow u^2 - u - 6 = 0$

$\Rightarrow (u - 3)(u + 2) = 0$

$\Rightarrow \; u = 3, -1$ (rejected)

$\Rightarrow \; u = 3$

Since, $u = t + \dfrac{1}{t}$

$\Rightarrow \; t + \dfrac{1}{t} = 3 \;\; \Rightarrow \; t^2 - 3t + 1 = 0$

$\Rightarrow \; t = \dfrac{3 \pm \sqrt{5}}{2} = e^x$

$\Rightarrow \; x = \ln \dfrac{3+\sqrt{5}}{2}, \ln \dfrac{3-\sqrt{5}}{2}$

So, the number of real rots is 2.

35. (1)

$x^2 + 5\sqrt{2}x + 10 = 0$ & $P_n = \alpha^n - \beta^n$ (Given)

Now $\dfrac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} = \dfrac{P_{17}\left(P_{20} + 5\sqrt{2}P_{19}\right)}{P_{18}\left(P_{19} + 5\sqrt{2}P_{18}\right)}$

$$\frac{P_{17}\left(\alpha^{20} - \beta^{20} + 5\sqrt{2}\left(\alpha^{19} - \beta^{19}\right)\right)}{P_{18}\left(\alpha^{19} - \beta^{19} + 5\sqrt{2}\left(\alpha^{18} - \beta^{18}\right)\right)}$$

$$\frac{P_{17}\left(\alpha^{19}\left(\alpha + 5\sqrt{2}\right) - \beta^{19}\left(\beta + 5\sqrt{2}\right)\right)}{P_{18}\left(\alpha^{18}\left(\alpha + 5\sqrt{2}\right) - \beta^{18}\left(\beta + 5\sqrt{2}\right)\right)}$$

Since, $\alpha + 5\sqrt{2} = -\dfrac{10}{\alpha}$ ...(1)

& $\beta + 5\sqrt{2} = -\dfrac{10}{\beta}$ ...(2)

Now put there values in above expression

$= -\dfrac{10P_{17}P_{18}}{-10P_{18}P_{17}} = 1$

EXERCISE - 2 [A]

1. (A)

Given $\alpha + \beta = -2a$

$\alpha\beta = b$

$|\alpha - \beta| \le 2m$

$(\alpha + \beta)^2 - 4\alpha\beta \le 4m^2$

$4a^2 - 4b \le 4m^2$

$a^2 - m^2 \le b$ …….(i)

$D > 0$

$4a^2 - 4b > 0$

$\therefore \; a^2 > b$ ……..(ii)

From (i) & (ii)

$b \in \left[a^2 - m^2, a^2\right]$

2. (B)

q and r will be roots of the equation

$a(p+x)^2 + 2bpx + c = 0$

$ax^2 + 2(ap+bp)x + ap^2 + c = 0$

$qr =$ Product of roots

$= \frac{ap^2 + c}{a}$

3. (C)

In both equation coefficient of $x^2 + \text{coefficient of } x + \text{constant term} = 0$

$\therefore$ 1 is a root of both equation hence only one root common

4. (C)

$\alpha + \beta = a$

$\alpha\beta = b$

$\therefore \left.\begin{array}{l} \alpha^2 - a\alpha + b = 0 \\ \beta^2 - a\beta + b = 0 \end{array}\right\}$ ______(i)

$A_n = \alpha^n + \beta^n$

$A_{n+1} = \alpha^{n+1} + \beta^{n+1} = \alpha^{n-1}.\alpha^2 + \beta^{n-1} + \beta^2$

$= \alpha^{n-1}(a\alpha - b) + \beta^{n-1}(a\beta - b)$

$= a(\alpha^n + \beta^n) - b(\alpha^{n-1} + \beta^{n-1})$

$A_{n+1} = a.A_n - bA_{n-1}$

5. (C)

$3x^2 - 2x(a+b+c) + (ab+bc+ac) = 0$

$D = 4(a+b+c)^2 - 4.3(ab+bc+ac)$

$= 4(a^2 + b^2 + c^2 - ab - bc - ac)$

$= 2\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] \ge 0$

6. (D)

$f(n) = (x-a)(x-c) + (x-b)(x-d)$

$f(d) = (b-a)(b-) < 0$

$f(d) = (d-a)(d-c) > 0$

$f(b).f(d) < 0$

One root between $b \,\&\, d$

7. (C)

$\alpha + \beta = -P$ $\therefore \alpha\beta = -2$

$\beta + r = -3P$ $\beta r = -4$

$\gamma + \alpha = -6P$ $\gamma\alpha = 8$

$2(\alpha + \beta + \gamma) = -10P$

$\alpha+\beta+\gamma=-5P$

$\therefore\ \gamma=-4P$ $\qquad\qquad$ $\therefore\ \alpha\beta=-2$

$\alpha=-2P$ $\qquad\qquad$ $-2P^2=-2$

$\beta=P$ $\qquad\qquad$ $P=\pm 1$

8. (C)

$D_1+D_2=p^2-4q+r^2-4s$

$$=p^2+r^2-4^2.\frac{pr}{2}$$

$D_1+D_2=(p-r)^2\geq 0$

$\therefore$ At least one of $D_1\ \&\ D_2\geq 0$

9. (B)

$n-2>0$ $\qquad\qquad$ $\therefore n>2$ $\qquad\qquad$ ________ (i)

$64-4(n-2)(n+4)<0$

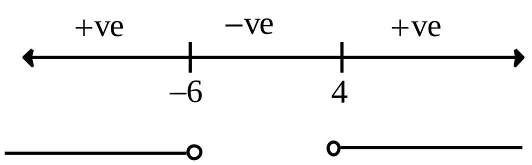
$16-n^2-2n+8<0$

$n^2+2n-24>0$

$n^2+6n-4n-24>0$

$n(n+6)-4(n+6)>0$

$(n-4)(n+6)>0$



$n\in(-\infty,-6)\cup(4,\infty)$ $\qquad\qquad$ ________ (ii)

From (i) & (ii)

$n>4$

10. (A)

$\gamma=\frac{\alpha}{\beta}$

$$\frac{\gamma+1}{\gamma-1}=\frac{\alpha+\beta}{\alpha-\beta}=\frac{\alpha+\beta}{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}$$

$$\frac{\gamma+1}{\gamma-1}=\frac{-\frac{b}{a}}{\sqrt{\frac{b^2}{a^2-4\frac{c}{a}}}}=\frac{-\frac{b}{a}}{\frac{\sqrt{b^2-4ac}}{a}}$$

$$\frac{(\gamma+1)^2}{(\gamma-1)^2}=\frac{b^2}{b^2-4ac}$$

$(b^2-4ac)\gamma^2+2\gamma(b^2-4ac)+(b^2-4ac)$

$$=b^2\gamma^2-2\gamma.b^2+b^2$$

$4ac-\gamma^2-2\gamma(b^2+b^2-4ac)+4ac=0$

$ac\gamma^2 - \gamma\left(b^2 - 2ac\right) + ac = 0$

$acx^2 + \left(2ac - b^2\right)x + ac = 0$

11. (A)

$D = b^2 - 4ac$

$= \frac{\left(4ac + c\right)^2}{4} - 4ac = \frac{16a^2 + c^2 + 8ac - 16ac}{4}$

$D = \frac{\left(4a - c\right)^2}{4} \geq 0$

Hence, roots are real

12. (B)

$y = \frac{x^2 - \left(b + c\right)x + bc}{\left(x - a\right)}$

$yx - ay = x^2 - \left(b + c\right)x + bc$

$x^2 - \left(b + c + y\right)x + bc + ay = 0$

For real x, $D \geq 0$

$\left(b + c + y\right)^2 - 4\left(bc + ay\right) \geq 0$

$y^2 + 2y\left(b + c\right) + \left(b + c\right)^2 - 4bc - 4ay \geq 0$

$y^2 + 2y\left(b + c - 2a\right) + \left(b - c\right)^2 \geq 0$ ________ (i)

Inequation (i) valid only when

$4\left(b + c - 2a\right)^2 - 4\left(b - c\right)^2 \leq 0$

$\left(b + c\right)^2 + 4a^2 - 4a\left(b + c\right) - \left(b - c\right)^2 \leq 0$

$4bc + 4a^2 - 4a\left(b + c\right) \leq 0$

$bc + a^2 - ab - ac \leq 0$

$b\left(c - a\right) - a\left(c - a\right) \leq 0$

$\left(b - a\right)\left(c - a\right) \leq 0$ _________(ii)

From (ii) $b - a \geq 0 \& c - a \leq 0$

$c \leq a \leq b$

$b - a \leq 0 \& c - a \geq 0$

$b \leq a \leq c$

13. (A)

$f\left(n\right) = ax^2 + 2bx - 3c$

Given $4a + 4b - 3c > 0$

Clearly $f\left(2\right) > 0$

It has no real roots and $f\left(2\right) > 0$

$\therefore f\left(x\right) > 0 \ \forall x \in R$

$f\left(0\right) > 0 \Rightarrow -3C > 0$

$C < 0$

Product of roots $= \frac{-3C}{a} > 0$

$\therefore C < 0$
$a > 0$

14. (B)
Coefficient of x^2 + Coefficient of x + Constant term $= 0$
$\therefore$ 1 is the roots of given equation let other roots be β
$\therefore$ Product of roots $= \beta.1 = \dfrac{a-b}{b-c}$
$\therefore \beta = \dfrac{a-b}{b-c}$

15. (B)
Let α, β be the roots of equation
$a_1x^2 + b_1x + c_1 = 0$ and $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$ are the roots of $c_1x^2 + b_1x + a_1 = 0$ ______(i)
Given $a_2x^2 + b_2x + c_2 = 0$ _______(ii)
$\therefore \dfrac{c_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{a_1}{c_2}$

16. (A)
Let α be the common root of given equation
$3\alpha^2 + a\alpha + 1 = 0$
$2\alpha^2 + b\alpha + 1 = 0$

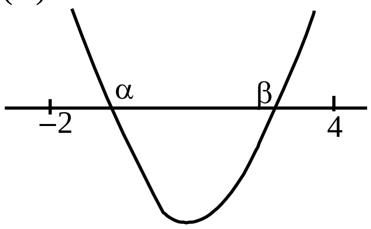
$\dfrac{\alpha^2}{a-b} = \dfrac{\alpha}{2-3} = \dfrac{1}{3b-2a}$
$\therefore \alpha = b - a$
$\& \alpha = 2a - 3b$
$\therefore 2a - 3b = b - a$
$3a = 3b$ $\therefore a = b$
$\therefore 5ab - 2a^2 - 3b^2 = 5a^2 - 5a^2 = 0$

17. (B)
$f(-1) = a - b + c < 0$
$a + c < b$ (Given)
And roots are imaginary then
$f(n) = ax^2 + bx + c < 0$ $\forall x \in R$
$f(-2) = 4a - 2b + c < 0$
$\therefore 4a + c < 2b$

18. (A)
Given $x^2 + 2ax + a < 0$
$\forall x \in [1,2]$
$f(1) < 0$ $\Rightarrow a < -\dfrac{1}{3}$ ________(i)
$f(2) < 0$ $\Rightarrow a < -\dfrac{4}{5}$ _________(ii)

From (i) & (ii) $a \in \left(-\infty, -\frac{4}{5}\right)$

19. (A)



(i) $D \geq 0$

$4m^2 - 4(m^2 - 1) \geq 0$

$1 \geq 0$

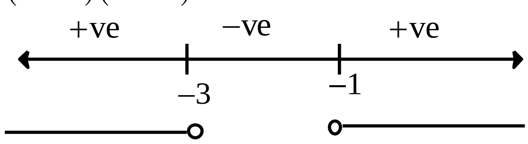
(ii) $f(-2) > 0$

$4 + 4m + m^2 - 1 > 0$

$m^2 + m + 3m + 3 > 0$

$m(m+1) + 3(m+1) > 0$

$(m+1)(m+3) > 0$



$f(4) > 0$

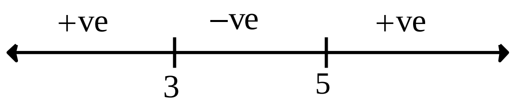
$16 - 8m + m^2 - 1 > 0$

$m^2 - 8m + 15 > 0$

$m^2 - 3m - 5m + 15 > 0$

$m(m-3) - 5(m-3) > 0$

$(m-3)(m-5) > 0$



$m \in (-\infty, 3) \cup (5, \infty)$ __________(ii)

$-2 < +\frac{2m}{2} < 4$

$-2 < m < 4$ __________(iii)

From (i), (ii), & (iii)

$m \in (-1, 3)$

20. (C)

Let $p = 2k + 1$

$q = 2m + 1$

$D = 4p^2 - 4.2q$

$= 4\left[(2k+1)^2 - 2(2m+1)\right]$

$= 4(4k^2 + 4k + 1 - 4m - 2)$

$= 4(4k^2 + 4(k-m) - 1)$

Clearly $4k^2 + 4(k-m) - 1$ is odd if D is a perfect square $4k^2 + 4(k-m) - 1 = (2n+1)^2$

$4k^2+4(k-m)-4n^2-4n=2$

Which is not possible

$\therefore$ D can not be a perfect square hence root can not be a rational

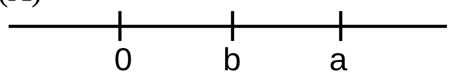
21. (A)

Given $a+b+6c=0$

$D=b^2-4ac=a^2+8ac+36c^2$

$=(a+4c)^2+20c^2>0$

22. (A)



$f(n)=2n^2-an-b^2$

$f(a)=2a^2-a^2-b^2=a^2-b^2>0$

$f\left(-\frac{a}{2}\right)=2.\frac{a^2}{4}+\frac{a^2}{2}-b^2$

$=(a^2-b^2)>0$

$f(0)<0$

$\therefore$ Both roots lies between $-\frac{a}{2}$ & a

23. (C)

$y+yx^2=2x$

$yx^2-2x+y=0$

For real x

$D\ge 0$

$4-4.y^2\ge 0$ $\quad\quad\quad\quad$ $\therefore y^2\le 1$

$|y|\le 1$

$y\in[-1,1]$

Let $\lambda=y^2+y-2=y^2+2.\frac{1}{2}y+\frac{1}{4}-\frac{1}{4}-2$

$=\left(y+\frac{1}{2}\right)^2-\frac{9}{4}$

$\therefore -1\le y\le 1$

$-\frac{1}{2}\le y+\frac{1}{2}\le\frac{3}{2}$

$0\le\left(y+\frac{1}{2}\right)^2\le\frac{9}{4}$

$-\frac{9}{4}\le\left(y+\frac{1}{2}\right)^2-\frac{9}{4}\le 0$

$\therefore y^2+y-2\in\left[-\frac{9}{4},0\right]$

24. (C)

$D \geq 0$

$(a-3)^2 - 4a \geq 0$

$a^2 + 9 - 6a - 4a \geq 0$

$a^2 - 10a + 9 \geq 0$

$a^2 - a - 9a + 9 \geq 0$

$a(a-1) - 9(a-1) \geq 0$

$(a-1)(a-9) \geq 0$

+ve −ve +ve

1 9

$a \in (-\infty, 1] \cup [9, \infty)$ ________(i)

$f(2) > 0$

$\therefore 4 - 2(a-3) + a > 6$

$4 - 2a + 6 + a > 0$

$10 > a$

$2 > \frac{(a-3)}{2}$ $\therefore a < 7$

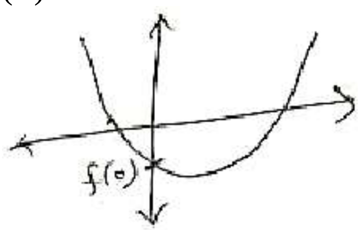
Condition for both root less than 2

$m \in (-\infty, 1]$

Required condition for at least one root one roots greater than 2

$m \in [9, \infty)$

25. (C)



$f(0) < 0$

$a - 3 < 0$

$a < 3$

26. (D)

$\alpha + \beta = m$

$\alpha\beta = n$

$(\alpha^2 + \alpha + 1)(\beta^2 + \beta + 1) = (m\alpha - n + \alpha + 1)(m\beta - n + \beta + 1)$

$= [(1-n) + \alpha(1+m)][(1-n) + \beta(1+m)]$

$= (1-n)^2 + (1+m)(1-n)(\alpha + \beta) + (1+m)^2 \alpha\beta$

$= (1-n)^2 + (1+m)(1-n).m + (1+m)^2 .n$

$= (1-n)^2 + (1+m)(m - mn + n + mn)$

$= 1 + n^2 - 2n + m + n + m^2 + mn$

$= 1 + (m-n) + m^2 + n^2 + mn$

27. (A)

Let root of equation be $\alpha \& \alpha^2$

$\alpha+\alpha^2=-\frac{b}{a}$

$\alpha^3=\frac{c}{a}$

$\left(\alpha+\alpha^2\right)^3=\left(-\frac{b}{a}\right)^3$

$\alpha^3+\alpha^6+3\alpha^3\left(\alpha+\alpha^2\right)=-\frac{b^3}{a^3}$

$\frac{c}{a}+\frac{c^2}{a^2}-3\frac{c}{a}.\frac{b}{a}=-\frac{b^3}{a^3}$

$\frac{ac+c^2-3bc}{a^2}=-\frac{b^3}{a^3}$

$\therefore a^2x+ac^2-3abc=-b^3$

$\therefore b^3+ac^2+a^2c=3abc$

28. (B)

$\alpha+\beta=-\frac{b}{a}$ & $\alpha\beta=\frac{c}{a}$

$x=\frac{-abc\pm\sqrt{a^2b^2c^2-4a^3c^3}}{2a^3}$

$=\frac{-abc\pm ac\sqrt{b^2-4ac}}{2a^3}$

$=\frac{c}{a}\left(\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right)$

$x=\alpha\beta.\alpha,\alpha\beta.\beta$

Required roots $\alpha^2\beta \& \beta^2\alpha$

29. (C)

Given

$\frac{\alpha_1}{\beta_1}=\frac{\alpha_2}{\beta_2}$

$\frac{\alpha_1+\beta_1}{\alpha_1-\beta_1}=\frac{\alpha_2+\beta_2}{\alpha_2-\beta_2}$

$\frac{\left(\alpha_1+\beta_1\right)^2}{\left(\alpha_1+\beta_1\right)^2-4\alpha_1\beta_1}=\frac{\left(\alpha_1+\beta_2\right)^2}{\left(\alpha_2+\beta_2\right)^2-4\alpha_2\beta^2}$

$\frac{b^2}{b^2-4c}=\frac{q^2}{q^2-4r}$

$\therefore b^2q^2-4rb^2=b^2q^2-4cq^2$

$rb^2=cq^2$

30. (B)

$x^2 + abx + c = 0$ has roots α, β

$\alpha\beta = c \qquad ...(1)$

$x^2 + acx + b = 0$ has roots α, γ

$\alpha\gamma = b \qquad ...(2)$

$\alpha^2 + ab\alpha + c = 0$

$\alpha^2 + ax\alpha + b = 0$

Subtract

$a\alpha\cancel{(b-c)} = \cancel{b-c}$

$\alpha = \frac{1}{a} \qquad ...(3)$

Put in (1) & (2)

$\beta = ac \qquad \gamma = ab$

Equation $x^2 - a(b+c)x + a^2bc = 0$

31. (C)

Let roots of equation $x^2 - px + q = 0$ be $\alpha \& \beta$ & roots of equation $x^2 - an + b = 0$ be $\alpha \& \alpha$

$\alpha + \alpha = a$

$\therefore \frac{a}{2} \qquad$ ….. (1)

$a^2 = 4b \qquad$ …... (2)

$\frac{a^2}{4} - p.\frac{q}{2} + q = 0$

$a^2 - 2ap + 4q = 0$

$4b - 2ap + 4q = 0 \qquad$ [from (2)]

$4(b+q) = 2ap$

$\therefore ap = 2(b+q)$

32. (C)

$x^2 - 8x + 4y^2 + 12 = 0$

For real $x, D \geq 0$

$64 - 4(4y^2 + 12) \geq 0$

$16 - 4y^2 - 12 \geq 0$

$4 \geq 4y^2$

$\therefore y^2 \leq 1$

$|y| \leq 1$

$-1 \leq y \leq 1$

33. (D)

Given $\alpha + \beta = p, \alpha\beta = q$

$\alpha + \beta = \left(\alpha^{\frac{1}{2}}\right)^2 + \left(\beta^{\frac{1}{2}}\right)^2 = \left(\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}}\right)^2 - 2\alpha^{\frac{1}{2}}.\beta^{\frac{1}{2}}$

$p = \left(\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}}\right)^2 - 2\sqrt{q}$

$$\alpha^{\frac{1}{2}}+\beta^{\frac{1}{2}}=\left(\alpha^{\frac{1}{4}}\right)^2+\left(\beta^{\frac{1}{4}}\right)^2=\left(\alpha^{\frac{1}{4}}+\beta^{\frac{1}{4}}\right)^2-2\alpha^{\frac{1}{4}}\beta^{\frac{1}{4}}$$

$$\left(\sqrt{p+2\sqrt{q}}+2.q^{\frac{1}{4}}\right)^{\frac{1}{2}}=\alpha^{\frac{1}{4}}+\beta^{\frac{1}{4}}$$

$$\therefore\left(\alpha^{\frac{1}{4}}+\beta^{\frac{1}{4}}\right)=\left(p+2\sqrt{q}+4q^{\frac{1}{2}}+4q^{\frac{1}{4}}\left(p+2\sqrt{q}\right)\right)^{\frac{1}{4}}$$

$$\alpha^{\frac{1}{4}}+\beta^{\frac{1}{4}}=\left[p+6\sqrt{q}+4q^{\frac{1}{4}}\left(p+2\sqrt{q}\right)\right]^{\frac{1}{4}}$$

$$\therefore k=\frac{1}{4}$$

34. (C)

Product of roots $=\dfrac{\alpha+1}{\alpha}\times\dfrac{\alpha}{\alpha-1}=\dfrac{c}{a}$

Solving, $\alpha=\dfrac{c+a}{c-a}$ ...(1)

Sum $=\dfrac{\alpha+1}{\alpha}+\dfrac{\alpha}{\alpha-1}=\dfrac{-b}{a}$ ...(2)

Put value of α from (1) in (2)

35. (D)

Let $x^2+x+1=t$

$$(t+1)^2-(a-3)+(t+1)+(a-4)t^2=0$$

$$t^2(1+a-4-a+3)+t(2-a+3)+1=0$$

$$t(5-a)+1=0$$

$$x^2+x+1=\frac{1}{a-5}$$

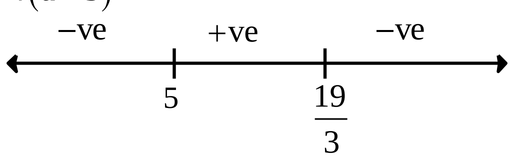
We know minimum value of x^2+x+1 equals $=\dfrac{3}{4}$

$$\therefore\frac{1}{a-5}>\frac{3}{4}$$

$$\frac{1}{a-5}-\frac{3}{4}>0$$

$$\frac{4-3a+15}{4(a-4)}>0$$

$$\frac{(19-3a)}{4(a-5)}>0$$



$$a\in\left(5,\frac{19}{3}\right]$$

36. (C)

$$-1 \le \frac{x^2+nx-2}{x^2-3x+4} \le 2$$

$$x^2+nx-2 \ge -x^2+3n-4$$

$$2x^2+(n-3)x+2 \ge 0$$

$$\therefore D \le 0$$

$$(n-3)^2-4.2.2 \le 0$$

$$(n-3)^2 \le 4^2$$

$$|n-3| \le 4$$

$$-4 \le n-3 \le 4$$

$-1 \le n \le 7$ ________(i)

Similarly

$$x^2+nx-2 \le 2x^2-6x+8$$

$$x^2-x(6+n)+10 \ge 0$$

$$D \le 0$$

$$(6+n)^2-4.10 \le 0$$

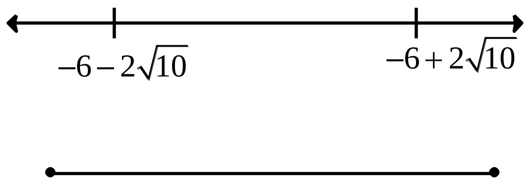
$$36+n^2+12n-40 \le 0$$

$$n^2+12n-4 \le 0$$

$$n = \frac{-12 \pm \sqrt{144+16}}{2}$$

$$n = \frac{-12 \pm 4\sqrt{10}}{2}$$

$$n = -6 \pm 2\sqrt{10}$$



$n \in \left[-6,-2\sqrt{10},-6+2\sqrt{10}\right]$ __________(ii)

From (i) & (ii)

$$n \in \left[-1,\sqrt{40}-6\right]$$

37. (D)

$b\alpha + c = -a^2\alpha^2$ ...(1)

$b\beta + c = a^2\beta^2$ ...(2)

$$f(x) = a^2x^2+2bx+2c$$

$$f(\alpha) = \alpha^2\alpha^2+2 = -a^2\alpha^2 < 0$$

$$f(\beta) = a^2\beta^2+2\left(a^2\beta^2\right) = 3a^2\beta^2 > 0$$

$f(\alpha)$ & $f(\beta)$ have opposite signs

So, $\alpha < \gamma < \beta$

38. (A)

$$m(b)=\frac{-D}{4a}=\frac{-\left(4b^2+4\left(1+b^2\right)\right)}{4\left(1+b^2\right)}$$

$$m(b)=\frac{1}{1+b^2}\in(0,1]$$

39. (C)

Let $P=p$ $\quad q=p+d$ $\quad r=p+2d$ $\quad s=p+3d$

$P+(p+d)=$ sum of roots = 2

$2P+d=2$ $\quad ...(1)$

Similarly $P+2d+p+3d=18$

$2p+5d=18$ $\quad ...(2)$

$d=4 \Rightarrow p=-1$ $\quad r=7$

$q=3$ $\quad s=11$

$A=-3$ $\quad B=77$

40. (A)

Square $x+1+x-1-2\sqrt{x^2-1}=4x-1$

$1-2x=2\sqrt{x^2-1}$

Square $1+4x^2-4x=4x^2-4$ $\quad 4x=5$

$x=\frac{5}{4}$

$1-2x>0$

$x\in\frac{1}{2}$ $\quad x=\frac{5}{4}$ is rejected

So, no solution.

41. (C)

$\alpha+\alpha^2=\frac{-p}{3}$ $\quad ...(1)$

$\alpha\cdot\alpha^2=\frac{3}{3}=1$

$\alpha^3=1 \Rightarrow \alpha^3-1=0$

$(\alpha-1)(\alpha^2+\alpha+1)=0$

$\alpha=1$ or $\alpha^2+\alpha=-1$

Put $\alpha=1$ in (1)

$1+1=\frac{-p}{3}$ $\quad p=-6$ rejected $(p>0)$

Put $\alpha^2+\alpha=-1$ in (1)

$-1=\frac{-p}{3}$ $\quad p=3$

42. (B)

$D>0$

$4(a+b+c)^2>12\lambda(ab+bc+ca)$

$$3\lambda < \frac{(a+b+c)^2}{ab+bc+ca} = \frac{a^2+b^2+c^2}{ab+bc+ca} + \frac{2(ab+bc+ca)}{ab+bc+ca}$$

$$3\lambda < \frac{a^2+b^2+c^2}{ab+bc++ca} + 2 \qquad ...(1)$$

Now, a, b, c are sides of a triangle

$|a-b| < c$ square $a^2+b^2-2ab < c^2$

$a^2+b^2-c^2 < 2ab$

Also $b^2+c^2-a^2 < 2bc$ and $c^2+a^2-b^2 < 2ca$

$\Rightarrow$ Add $a^2+b^2+c^2 < 2(ab+bc+ca)$

$$\frac{a^2+b^2+c^2}{ab+bc+ca} < 2$$

Put in (1)

$3\lambda < 2+2$

$$\lambda < \frac{4}{3}$$

EXERCISE - 2 [B]

1. **(AC)**

According to question

$f(0)f(2) < 0$

$\Rightarrow (k^2+5)(k^2+2k-3) < 0$

$(k+3)(k-1) < 0$

$-3 < k < 1$

Option (A) & (C)

2. (AB)

$x^2 - x + m = 0$ has root α

$\alpha^2 - \alpha + m = 0 \qquad ...(1)$

$x^2 - 3x + 2m = 0$ has root 2α

$4\alpha^2 - 6\alpha + 2m = 0$

$2\alpha^2 - 3\alpha + m = 0 \qquad ...(2)$

Solve (1) & (2)

3. (ABC)

By putting x = 0, x = p & x = q

$\Rightarrow$ We get LHS = RHS

$\Rightarrow$ Expression is an identify in x

$\Rightarrow$ A, B, C

4. (ABC)

(A) $f(x) \geq 0$

$\Rightarrow D \leq 0$

$64k^2 - 16k \leq 0$

$k(4k-1) \leq 0$

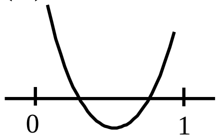
$\left[0, \frac{1}{4}\right]$

(B) $f(0) = k$

$\therefore \ k < 0 \qquad \Rightarrow \ f(0) < 0$

$\Rightarrow \ 0$ lies between roots

(C) Both roots between (0, 1)



(i) $D > 0 \ \Rightarrow k \in (-\infty, 0] \cup \left[\frac{1}{4}, \infty\right)$

(ii) $f(0) > 0 \Rightarrow k > 0$

(iii) $f(1) > 0 \Rightarrow k < \frac{4}{7}$

$\therefore \text{(i)} \cap \text{(ii)} \cap \text{(iii)} \Rightarrow \left(\frac{1}{4}, \frac{4}{7}\right)$

(D) min value is $\frac{-D}{4a} \Rightarrow k - \frac{64k^2}{16}$

$k(1-4k)$

5. (CD)

$x = 2 = \frac{-b}{2a}$

$10 = -\frac{D}{4a} = 8 - \frac{b^2}{4a}$

Solve to get $a = \frac{-1}{2}, \ b = 2$

6. (AC)

(A) $y = \frac{x^2 + 14x + 9}{x^2 + 2x + 3}$

$yx^2 + 2yx + 3y = x^2 + 14x + 9$

$(y-1)x^2 + 2(y-7)x + 3y - 9 = 0$

$x \in R \Rightarrow D \ge 0$

$4(y-7)^2 - 4(g-1)(3y-9) \ge 0$

$y^2 - 14y + 49 - (39^2 - 12y + 9) \ge 0$

$-2y^2 - 2y + 40 \ge 0$

$y^2 + y - 20 \le 0$

y can 10 interval values

$y \in [-5, 4]$

(B) there are 17 integral values

By graph

(C) put y = – 5 we get

$x^2 + 4x + 4 = 0$

$\Rightarrow x = -2$

(D) 2017 is not possible.

7. (CD)

If y Real $\quad D \geq 0$

$16x^2 - 4 \times 4(x+6) \geq 0$

$x^2 - x - 6 \geq 0$

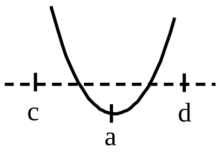
$(x-3)(x+2) \geq 0$

$x \in (-\infty, -2] \cup [3, \infty)$

8. (ABC)

Vertex is $(a, b-a^2)$

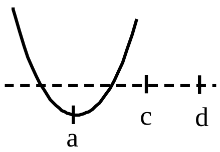
(a) of c < a < d $\Rightarrow$ min is $b - a^2$



(b) of a < c < d

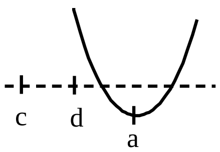
$\therefore$ min is f (c)

$= c^2 - 2ac + b$



(c) $\therefore f(\min) = f(d)$

$d^2 - 2ad + b$



(d) No comment

9. **(AC)**

α, β roots $\Rightarrow x^2 - px + 9 = 0$

Of $\frac{1}{\alpha}, \frac{1}{\beta}$ roots $\Rightarrow 9x^2 - px + 1 = 0$

$\Rightarrow (x^2 - px + 9)(9x^2 - px + 1) = 0$

has roots $\frac{1}{\alpha}, \frac{1}{\beta}, \alpha, \beta$

but equation is given as $x^4 - ax^3 + bx^2 - ax + 1 = 0$

comparing

$$\frac{q}{1} = \frac{-p(q+1)}{-a} = \frac{p^2+q^2+1}{b} = \frac{p(q+1)}{a} = \frac{q}{1}$$

$\Rightarrow a = \frac{p(q+1)}{q}$

& $\quad b = \frac{p^2+q^2+1}{q}$

10. (ABC)

Clearly a < 0 & c > 0 (y intercept)

Also $-\dfrac{b}{2a} > 0$

$\Rightarrow\ b > 0$

(a) correct

Hence $f(2) > 0 \Rightarrow 4a + 2b + c > 0$ …(1)

$\Rightarrow$ (c) correct

Also $f(1) > 0 \Rightarrow a + b + c > 0$ …(2)

Add (i) & (ii)

$\Rightarrow 4a + 3b + 2c > 0$ (c)

11. (ABD)

(A) $f(1) + f(2) + f(3) = 0$

$\Rightarrow$ at least one out of f (1), f(2) or f(3) is negative or zero.

$\Rightarrow$ (A) option correct

(B) $b + 2a = 0 \Rightarrow f(2) = c = f(0)$

$\Rightarrow$ by symmetry about vector $f(1) = f(3)$

$f(2) = 0 \quad \Rightarrow f(2) + f(3) = 0$

$f(2)$ & $f(3)$ opp signed

$\therefore$ one root between 2 & 3

(C) wrong

(D) $f(3) = 0 \Rightarrow f(1)$ & $f(2)$ opp signed

$\therefore$ other root between 1 & 2

(D) correct

12. (AB)

Apply one root common condition & get the answer

13. (ABC)

$x^2 - (b+1)x + b - 2 = 0$

$D = (b+1)^2 - 4(b-2)$

$D = (b-1)^2 + 8 > 0$

$\therefore$ roots real

Now, if b > 2

$\therefore$ S_2 is positive & $S_1 > 0$

$\Rightarrow$ both roots positive

Of b = 2 are root is 0

Of b < 2 $\therefore$ $S_2 < 0$

$\Rightarrow$ roots opp singed

$\therefore$ Any value of b at least one root is non-negative

$\Rightarrow$ A & B also correct

14. (ABC)

Clearly $\sin\theta$ and $\cos\theta$ roots $\Rightarrow$ they are real $\Rightarrow$ roots are real $\Rightarrow$(a) correct

Now

$\sin\theta + \cos\theta = \dfrac{-b}{a}$ $\qquad\qquad$ $\sin\theta\cos\theta = \dfrac{c}{a}$

Or $\quad 1 + 2\sin\theta\cos\theta = \dfrac{b^2}{a^2}$ $\qquad\qquad$ $\sin 2\theta = \dfrac{2c}{a}$ $\Rightarrow$ b correct

$1 + \dfrac{2c}{a} = \dfrac{b^2}{a^2}$

$\Rightarrow \dfrac{a+2c}{\sqrt{2}b} = \dfrac{b}{\sqrt{2}a} = \dfrac{(\sin\theta + \cos\theta)}{-\sqrt{2}}$

= C also correct

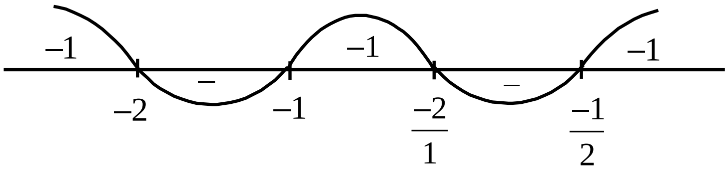
15. (BC)

$\dfrac{2x}{2x^2+5x+2} - \dfrac{1}{x+1} > 0$

$\dfrac{2x^2+2x-2x^2-5x-2}{(x+1)(2x+1)(x+2)} > 0$

$-\dfrac{(3x+2)}{(x+1)(x+2)(2x+1)} > 0$

$\Rightarrow$



$\Rightarrow (-2,-1) \cup \left(-2/3, -1/2\right)$

16. (ABC)

$x^2 + px + qr = 0 \qquad \ldots(1)$

$\alpha, \beta \qquad \Rightarrow \alpha + \beta = -p$

$x^2 + qx + rp = 0 \qquad \ldots(2)$

$\beta, \gamma \qquad \Rightarrow \beta + \gamma = -q$

$x^2 + rx + pq = 0 \qquad \ldots(3)$

$\gamma, \alpha \qquad \Rightarrow \gamma + \alpha = -r$

Apply 1 root common condition

$(rp - qr)^2 = (q-p)(p^2 - q^2)r$

$\Rightarrow r^2(p-q)^2 = -(p-q)^2(p+q)r$

$\Rightarrow p + q + r = 0$

$\therefore \alpha + \beta + \gamma = -\dfrac{(p+q+r)}{2} = 0$

17. (ABC)

$(p^2 - 4p + 5)x^2 + (2p-1)x - 3p = 0$

Coefficient of x^2 is > 0

So, product of roots < 0

$\Rightarrow$ Both roots opp. signed

18. (ABCD)

Clearly, sum of coefficient is 0

$\Rightarrow$ Exactly are root is 1

Let other root be α

Here $b+c-2a<0$ $\quad [a>b>c]$

$\Rightarrow S_2<0 \quad \Rightarrow$ (A) correct & (D)

(B) let $-1<\alpha<0 \Rightarrow \alpha+1>0$

$\Rightarrow \dfrac{a+c-2b}{a+b-2c}>0 \quad (a+b-2c>0)$

$\Rightarrow a+c-2b>0$

(B) correct

(C) if $\alpha<-1$

$\Rightarrow \alpha+1<0 \Rightarrow a+c<2b$

(C) correct

19. (ABCD)

$x^2-2ax+ab=0$

(roots real & positive)

$D = 4a^2-4ab$

$\Rightarrow 4a(a-b)>0 \Rightarrow a>b$

$S_1>0 \quad \& \quad S_2>0 \quad \Rightarrow \dfrac{b}{a}<1$

$\Rightarrow a>0, \; b>0$

$\Rightarrow$ option A correct

(B) $f(0)f(b)=ab(b^2-ab)<0$

B correct

(C) $f(2a)f(2a-b)=ab(b^2-ab)<0$

$\Rightarrow$ also correct

(D) since $a \neq b$ D is positive

Both roots distinct.

20. (AC)

Let I be integral root

$(I+2010)(I+9)=-1$

$\Rightarrow$ I + 2010 = 1 & I + a = – 1 …(i)

Or I + 2010 = – 1 & I + a = 1 …(ii)

$\Rightarrow a=2008$ or 2012

21. (BC)

Use transformation

22. (AB)

Put D = 0 & get answer

23. (AC)

$y=\dfrac{x^2+2x-11}{x-3}$

$yx-3y=x^2+2x-11$

$x^2+(2-y)x+3y-11=0$

$D \geq 0 \quad (y-2)^2 \, 4(3y-11) \geq 0$

$$y^2-4y+4-12y+44\ge 0$$
$$y^2-16y+48\ge 0$$
$$(-\infty,4]\cup[12,\infty)$$
a = 4, b = 12

24. (AB)
Put values & verify

25. (BD)
$a+b+c=f(1)$
$c=f(0)$
a and *c* will have same signs

26. (ABC)
Simplify the equation
$f(x)=a(x-b)(x+c)+b(x-a)(x+c)-c(x-a)(x-b)$
Here $S_2=\dfrac{-3abc}{a+b+c}<0$
$\Rightarrow$ roots real & opp. signed
Also $f(b)\cdot f(a)$
$=b(b-a)(b+a)(a)(a-b)(a+c)$
Which is positive

27. **(AB)**
Clearly a b c < d (a < b < c < d)
$\therefore S_2=\dfrac{abc-d}{a}<0$
(c) $f(a)f(b)$ no comment sign
(d) $f(b)f(c)$ no comment sing
$\Rightarrow$ can't say for sure if C & D correct

28. (B)
$x^2+(a-b-1)x-(a+b)=0$
$x\in \text{real}\Rightarrow D>0$
$(a-b-1)^2+4(a+b)>0\quad \forall\in R$
$a^2+b^2+1-2ab-2a+2b+4a+4b>0\qquad \forall b\in R$
$b^2-2b(a-3)+a^2+2a+1>0$
$\Rightarrow$ only possible of D < 0
$4(a-3)^2-4(a^2+2a+1)<0\Rightarrow a>2$

29. **(AC)**
$x^2-4x+3+9y^2=0$
$\Rightarrow D\ge 0$ (x Real)
$16-4(3+9y^2)\ge 0$
$\Rightarrow -\dfrac{1}{3}\le y\le \dfrac{1}{3}$

Also, $y^2 = -\dfrac{\left(x^2-4x+3\right)}{9}$

y real $\quad x^2-4x+3\le 0$

$x\in[1,3]$

30. **(BC)**

Observe c > 0 (y intercept)

a < 0 (concave down)

& $\dfrac{-b}{2a}<0 \Rightarrow \dfrac{b}{a}>0$

$\Rightarrow\ b<0$

Passage - 1

31. (B) 32. (B)

Let α,β be the roots of equation $x^2 + ax + b = 0$

We have to find equation whose roots are $\alpha+k,\beta+k$ replace x by x – k we get

$(x-k)^2 + a(x-k) + b = 0$

$x^2 + x(a-2k) + k^2 - ak + b = 0$ …(i)

Given,

$x^2 + bx + a = 0$ … (ii)

Equation (i) & (ii) are identical

$\dfrac{1}{1}=\dfrac{a-2k}{b}=\dfrac{k^2-ak+b}{a}$

$\therefore k=\dfrac{a-b}{2}\ \&\ k^2-ak+b=a$

$\dfrac{(a-b)^2}{4}-a\dfrac{(a-b)}{2}=(a-b)$

$(a-b)[(a-b)-2a-4] = 0$

For $\quad a = b$, $a + b + 4 = 0$ but $a\ne b$ so $a+b+4=0$

$k = 0$

$ab=\lambda$

$\lambda=a(-4-a)$

$\lambda=-4a-a^2$

$\lambda=4-(a+2)^2$

$(a + b)^2$ always positive values of $(a + 2)^2 = 0$

At a = –2

$\lambda_{max}=4$

$\lambda\in(-\infty,4)$

Passage - 2

33. (C)

$D=2^2-4.3<0$

Equation $x^2 + 2x + 3 = 0$ have both roots imaginary and hence both roots common

$\frac{a}{1}=\frac{b}{2}=\frac{c}{3}$

$\therefore$ a : b : c = 1 : 2 : 3

34. (C)

$x^3+3x^2+3x+1+1=0$

$(x+1)^3=-1$

$\therefore$ x = –2

$\therefore x^3+2x^2+x^2+2x+x+2=0$

$x^2(x+2)+x(x+2)+(x+2)=0$

$(x+2)(x^2+x+1)=0$

Equation $ax^2+bx+c=0$ and $x^2 + x + 1 = 0$ have both roots common.

$\therefore \frac{a}{1}=\frac{b}{1}=\frac{c}{1}$ $\qquad\therefore$ a = b = c

35. (A)

$(6k + 2)x^2 + rx + 3k – 1 = 0$ …(i)

$(12k + 4)x^2 + px + 6k – 2 = 0$ …(ii)

Equation (i) & (ii) have both roots common

$\therefore \frac{4(\cancel{3k+1})}{2(\cancel{3k+1})}=\frac{p}{r}=\frac{4(\cancel{3k-1})}{2(\cancel{3k-1})}$

P = 2r $\qquad\therefore$ 2r – P = 0

36. (D)

Let α be common root

$\alpha^2-a\alpha+b=0$

$\underline{\alpha^2+b\alpha-a=0}$

$\frac{\alpha^2}{a^2-b^2}=\frac{\alpha}{b+a}=\frac{1}{b+a}$

$\alpha=\frac{(a-b)(a+b)}{(a+b)}$ $\qquad\therefore \alpha=1$

$\alpha=a-b$

$\therefore$ a – b $\qquad\therefore$ a – b = 1

Passage – 3

37. (A)

$x^2-\frac{3ax}{a+1}+\frac{4a}{a+1}=0$

Use $D\geq 0,\ f(1)>0$, $\frac{-B}{2A}>1$

and take common values of *a* satisfying the three inequalities.

38. (B)

$f(6) < 0$

39. (B)

$x^2 + \frac{2a}{1-a^2} x \frac{-1}{1-a^2} = 0$

$D \geq 0, 0 < \frac{-B}{2A} < 1, f(0) > 0 \;\&\; f(1) > 0$

40. (D)

$D \geq 0, \frac{-\beta}{2a} > 2, f(2) > 0$

Passage – 4

41. (A)

$x^2 - px - (p+c) = 0$

$\alpha + \beta = p \qquad \alpha\beta = -p - c$

42. (B)

$$\frac{(\alpha+1)^2}{(\alpha+1)^2 + c - 1} + \frac{(\beta+1)^2}{(\beta+1)^2 + c - 1}$$

Put $c - 1 = -(\alpha+1)(\beta+1)$ from Q.41

Passage – 5

43. (D)

$\because\ AC = 4\sqrt{2}$

$\therefore\ AB = BC = \frac{4\sqrt{2}}{\sqrt{2}} = 4$ units

$OB = \sqrt{4^2 - \left(2\sqrt{2}\right)^2} = 2\sqrt{2}$

$\therefore\ A\left(-2\sqrt{2}, 0\right), B\left(2\sqrt{2}, 0\right), C\left(0, -2\sqrt{2}\right)$

Since, $y = ax^2 + bx + c$ passes through *A*, *B* and *C*, we get

$y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$

44. (C)

Minimum values of $y = \frac{x^2}{\left(2\sqrt{2}\right) - 2\sqrt{2}}$ is $-2\sqrt{2}$ at $x = 0$.

45. (C)

$f(x) = 0$

$\Rightarrow\ \frac{x^2}{\left(2\sqrt{2}\right) - 2\sqrt{2}} = 0$ **or** $x = \pm 2\sqrt{2}$

Therefore, number of integral values of k for which k lies in $\left(-2\sqrt{2}, 2\sqrt{2}\right)$ is 5.

46. (A) Q ; (B) R ; (C) S ; (D) P
(A) $|\alpha-\beta|=1 \qquad (\alpha+\beta)^2-4\alpha\beta=1 \qquad a^2=4b+1$
(B) Use (A)
(C) $\alpha+2\alpha=a \qquad \alpha\cdot 2\alpha=b$
(D) $\alpha+\beta=\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}$

47. (A) P, Q, R, S ; (B) P, Q, R, S ; (C) P, Q, R, S ; (D) S
Put values of x from column 2 and check.

48. (A) S ; (B) R ; (C) Q ; (D) P
(A) $\dfrac{a-4}{a-5}>0$
(B) Product $\dfrac{a-4}{a-5}<0$
(C) $D\ge 0$ & $\dfrac{a-4}{a-5}>0$
(D) $D\ge 0$

49. (A) PQRS ; (B) R ; (C) P ; (D) RS
Trial and error method.

50. (A) P; (B) P; (C) Q; (D) Q
From old theory

EXERCISE - 2 [C]

1. **(4)**

$$\mathrm{k}x^2+(1-\mathrm{k})x+5=0 \quad \text{............ (i)}$$

Given $\alpha+\beta=\dfrac{\mathrm{k}-1}{\mathrm{k}}$

$$\alpha\beta=\frac{5}{\mathrm{k}}$$

Given $\dfrac{\alpha^2+\beta^2}{\alpha\beta}=4/5$

$$\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=4/5$$

$$\frac{\dfrac{(\mathrm{k}-1)^2}{\mathrm{k}^2}-\dfrac{2.5}{\mathrm{k}}}{5/\mathrm{k}}=4/5$$

$$\frac{(\mathrm{k}-1)^2-10\mathrm{k}}{\cancel{5}\mathrm{k}}=4/\cancel{5}$$

$$\mathrm{k}^2-2\mathrm{k}-10\mathrm{k}+1=4\mathrm{k}$$

$$\mathrm{k}^2-16\mathrm{k}+1=0$$

Given $k_1+k_2=16$

$k_1k_2=1$

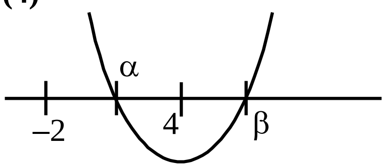
$$\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}+2\right)^{1/4}=\left(\frac{k_1^2+k_2^2}{k_1k_2}+2\right)^{1/4}$$

$$=\left(\frac{(k_1+k_2)^2-2k_1\cancel{k}_2+2\cancel{k}_1k_2}{k_1k_2}\right)^{1/4}$$

$$=(16\times16)^{1/4}$$

$$=\left(4^4\right)^{1/4}=4$$

2. **(4)**



Given

(i) $D>0$

$4m^2-4\left(m^2-1\right)>0$

$\cancel{4m^2}-\cancel{4m^2}+4>0$

(ii) $f(-2)f(4)<0$

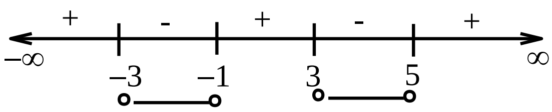
$\left(4+4m+m^2-1\right)\left(16-8m+m^2-1\right)<0$

$\left(m^2+4m+3\right)\left(m^2-8m+15\right)<0$

$\left(m^2+m+3m+3\right)\left(m^2-3m-5m+15\right)<0$

$\left(m(m+1)+3(m+1)\right)\left(m(m-3)-5(m-3)\right)<0$

$(m+3)(m+1)(m-3)(m-5)<0$



$m\in(-3,-1)\cup(3,5)$

Largest $m=4$

3. **(2)**

(i) $D\geq0$

$4-4\times4a\geq0$

$1-4a\geq0 \qquad \therefore 4a\leq1$

$a\leq\frac{1}{4}$ .... (i)

(ii) $4f(-1)>0$

$(4+2+a)>0 \ \therefore a>-6$ ...... (ii)

$4f(1)>0$

$4(4-2+a)>0 \qquad \therefore a>-2$ .... (iii)

Given from (i), (ii), (iii)

$a \in \left(-2, \frac{1}{4}\right]$

$a = -1 \text{ and } 0 \text{ only}$

4. **(6)**

$3x^2 - 7x + 8 \geq x^2 + 1$

$2x^2 - 7x + 7 \geq 0$

$x \in R$ .... (i)

$\therefore$ $3x^2 - 7x + 8 \leq 2x^2 + 2$

$x^2 - 7x + 6 \leq 0$

$x^2 - 7x + 6 \leq 0$

$x^2 - x - 6x + 6 \leq 0$

$x(x-1) - 6(x-1) \leq 0$

$(x-1)(x-6) \leq 0$

$x \in [1, 6]$

Total number of integral value of $x = 6$

5. **(2)**

Given α be the root of equation $x^2 - x - a = 0$ and

2α be the root of equation $x^2 - x - 3a = 0$

$$\frac{\alpha^2 - \alpha - a = 0.....(i)}{4\alpha^2 - 2\alpha - 3a = 0......(ii)}$$

$$\frac{\alpha^2}{3a - 2a} = \frac{\alpha}{-4a + 3a} = \frac{1}{-2 + 4}$$

$$\frac{\alpha^2}{a} = \frac{\alpha}{-a} = \frac{1}{2}$$

$\therefore \alpha = -a/2$

$4\alpha = -1$

$\therefore \frac{-a}{2} = -1$

$a = 2$

6. **(7)**

Let $y = \frac{ax^2 + 3x - 4}{a + 3x - 4x^2}$

$ay + 3xy - 4x^2y = ax^2 + 3x - 4$

$(a + 4y)x^2 + 3x(1 - y) - (4 + ay) = 0$ .... (i)

For real x, $D \geq 0$

$9(1-y)^2 + 4(a + 4y)(4 + ay) \geq 0$

$9 + 9y^2 - 18y + 4\left(4a + a^2y + 16y + 4ay^2\right) \geq 0$

$(9 + 16a)y^2 + \left(4a^2 + 64\right)y + 16a + 9 - 18y \geq 0$

$(9 + 16a)y^2 + \left(4a^2 + 46\right)y + 16a + 9 \geq 0$ .... (ii)

(ii) Satisfy only when

$9 + 16a > 0 \quad \therefore a > \frac{-9}{16}$ &

$$D \le 0 \left(4a^2+46\right)^2 - 4\left(10a+9\right)^2 \le 0$$

$$\left(4a^2+46\right)^2 - 4\left(9+16a\right)\left(16a+9\right) \le 0$$

$$16a^4 + 2116 + 368a^2 - 4\left(256a^2+288a+81\right) \le 0$$

$$16a^4 + 2116 + 368a^2 - 1024a^2 - 1152a - 324 \le 0$$

$$\cancel{16}a^4 - \underset{41}{\cancel{656}}\,a^2 - \underset{72}{\cancel{1152}}\,a + \underset{112}{\cancel{1792}} \le 0$$

$$a^3(a-1) + a^2(a-1) - 40a(a-1) - 112(a-1) \le 0$$

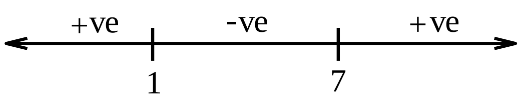
$$(a-1)\left(a^3+a^2-40a-112\right) \le 0$$

$$(a-1)\left(a^3-7a^2+8a^2-56a+16a-112\right) \le 0$$

$$(a-1)\left[a^2(a-7)+8a(a-7)\right] \le 0$$

$$(a-1)(a-7)\left(a^2+8a+16\right) \le 0$$

$$(a-1)(a-7)(a+4)^2 \le 0$$



Maximum $a = 7$

7. **(1)**

Given $\alpha, 2\alpha$ be the root of equation

$$(\ell - m)x^2 + \ell x + 1 = 0$$

$\therefore$ $$3\alpha = \frac{-\ell}{(\ell - m)} \quad \text{.......... (i)}$$

$$2\alpha^2 = \frac{1}{\ell - m} \quad \text{.......... (ii)}$$

From (i) & (ii)

$$\frac{1}{\cancel{(\ell - m)}} = 2.\frac{\ell^2}{-(\ell - m)^{\cancel{2}}}$$

$$(\ell \pm m)$$

$$9(\ell - m) = 2\ell^2$$

$$2\ell^2 - 9\ell + 9m = 0 \quad \text{........... (iii)}$$

For real ℓ_1 $\quad D \ge 0$

$$81 - 4.2.9m \ge 0$$

$$8m \le 9$$

$$m \le 9/8$$

Greatest $[m] = 1$

8. **(4)**

If both roots are integer then sum of roots and product of roots are integer

$$\alpha + \beta = \frac{-2(a+1)}{(a+2)} \quad \text{........... (i)}$$

$$\alpha\beta = \frac{a}{a+2} \quad \text{.............. (ii)}$$

From (i) & (ii)

$a=0,-3,-1,-4$ only for which both are integer

9. **(0)**

Condition for a common root

$\left(3(-4m)-1(-2m)\right)\left(-2m(2)-(-4m)(-4)\right)=\left(-4(1)-2(3)\right)^2$

$(-10m)(-20m)=100$

$200m^2=100 \qquad m=\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}$

Sum = 0

10. **(3)**

$(a-1)\left(x^2+x+1\right)^2=(a+1)\left(x^2+x+1\right)\left(x^2-x+1\right)$

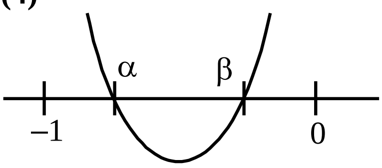
$ax^2+ax+a-x^2-x-1=ax^2-ax+a+x^2-x+1$

$2x^2-2ax+2=0 \qquad x^2-ax+1=0$

$D>0 \qquad a^2-4>0 \qquad a\in(-\infty,-2)\cup(2,\infty)$

$a_{\min}=3$

11. **(4)**



$b^2-4ac\geq 0$ .... (i)

$a(a-b+c)>0$ .... (ii)

$ac>0$ .... (iii)

$-1<\frac{-b}{2a}<0$ .... (iv)

Given a, b, c, are natural number which satisfy all four condition above and whose product is least then only $a=4, b=4, c=1$

Then $\sqrt{abc}=\sqrt{16}=4$

12. **(4)**

Divided by $x^2 \qquad x^2+\frac{1}{x^2}+4\left(x+\frac{1}{x}\right)+a=0$

$x+\frac{1}{x}=t \qquad t^2-2+4t+a=0$

$t\leq 2$ or $\leq -2 \qquad (t+2)^2=6-a$

$t+2\geq 4$ or ≤ 0

13. **(2)**

Let $P(x)=x^4+ax^3+bx^2+cx+d$

$=(x-1)(x-2)(x-3)(x-4)+x^3$ ...... (i)

Given $P(1)=1$

$P(2)=2^3$

$P(3)=3^3$

$$P(4) = 4^3$$
$$P(10) = 9\times8\times7\times6+10^3 = 4024$$
$$\frac{P(10)}{2012} = 2$$

14. (1)

$x^2+2\lambda x+\lambda^2+1=0$ has imaginary roots so both roots are common.

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$
$$\frac{a}{1}=\frac{b}{2\lambda}=\frac{c}{\lambda^2+1}$$

Now use $a+b>c$, $b+c>a$ & $c+a>b$

Solve the inequalities for λ.

15. (1)

$a^2+b^2=(a+b)^2-2ab=K^2-2K=(K-1)^2-1$ is minimum when $K=1$

Sum of roots = 1

16. (2)

$p^2>4q$ & $q^2>4p$

Square $p^4>16q^2>64p$ $\quad p^3>64 \quad p>4 \quad q>4$

$p=5 \quad q=6$

$p=6 \quad q=5$

$x^2-5x+6=0 \quad x=2,3$

$x^2-6x+5=0 \quad x=1,5$

2 possible pairs of p and q

17. (4)

$x^2+19x+46=y^2 \Rightarrow x^2+19x+46-y^2=0$

$$x=\frac{-19\pm\sqrt{361-4(46-y^2)}}{2}=\frac{-19\pm\sqrt{177+4y^2}}{2}$$

$177+4y^2$ has to square of an odd integer

$177+4y^2=(2n+1)^2 \qquad (2n+1)^2-4y^2=177$

$(2n+1+2y)(2n+1-2y)=177$

$$\begin{aligned}(2n+1+2y)(2n+1-2y)=177&=59\times3\\&=-59\times-3\\&=177\times1\\&=-177\times-1\end{aligned}$$

$2n+1+2y=59$

$2n+1-2y=3$

Add n = 15, y = 14

$$x=\frac{-199\pm31}{2}$$

$x=-84,\ -115$

Similalrly, check other factors

18. (0)

$ax^2-2a^2x+6x-12a\ge 0$ $\quad\quad$ $x^2-4a\ge 0$

$ax(x-2a)+6(x-2a)\ge 0$ $\quad\quad$ $x\in(-\infty,-7]\cup[7,\infty)$

$(ax+6)(x-2a)\ge 0$

Case 1 : $a>0 \quad x\in\left(-\infty,-\frac{6}{a}\right]\cup[2a,\infty)$

$-\frac{6}{a}\le -7$ & $2a\ge 7$

$a\le\frac{6}{7}$ & $a\ge\frac{7}{2} \quad\quad a\in\phi$

Case 2 : $a<0 \quad \left(x+\frac{6}{a}\right)(x-2a)\le 0 \quad\quad x\in\left[2a,-\frac{6}{a}\right] \quad a\in\phi$

19. (3)

$|x|=t \quad\quad (2t-3)^2-t-6=0 \quad\quad 4t^2-13t+3=0$

$4t^2-12t-t+3=0 \quad t=3,\frac{1}{4}=|x|$

$x=3,-3,\frac{1}{4},-\frac{1}{4}$

$4x+1\ne 0 \quad\quad x\ne -\frac{1}{4}$

So, $x=3,-3,\frac{1}{4}$

20. (3)

We have $P(x)=\frac{5}{3}-6x-9x^2=-(3x+1)^2+\frac{8}{3}$

$\Rightarrow P_{\max}=\frac{8}{3}$

Similarly, $Q(y)=-4y^2+4y+\frac{13}{2}=-(2y-1)^2+\frac{15}{2}$

$\Rightarrow Q_{\max}=\frac{15}{2}$

Now, $P_{\max}\times Q_{\max}=\frac{8}{3}\times\frac{15}{2}=20$

So, $(x,y)\equiv\left(-\frac{1}{3},\frac{1}{2}\right)$

Hence, $6x+10y=6\left(\frac{-1}{3}\right)+10\left(\frac{1}{2}\right)=-2+5=3$

21. (2)

We have $x_1+x_2+x_3=8$

$x_1x_2x_3=d$

$x_1x_2+x_2x_3+x_3x_1=c$

Possible roots 1, 2, 5 or 1, 3, 4

$\therefore\ d=10$ or $d=12$

$\Rightarrow c=2+10+5=17$ or $3+12+4=19$

Hence, $d=10$ and $c=17$ or $d=12$ and $x=19$

22. (3)

$2x^2+4x(y-3)7y^2-2y+t=0$

$D=0$ (for one solution)

$\Rightarrow 16(y-3)^2-8(7y^2-2y+t)=0$

$\Rightarrow 2(y-3)^2-(7y^2-2y+t)=0$

$\Rightarrow 2(y^2-6y+9)-(7y^2-2y+t)=0$

$\Rightarrow -5y^2-10y+t-18=0$

Again D = 0 (for one solution) m

$\Rightarrow 100-20(t-18)=0$

$\Rightarrow 5-t+18=0$

$\Rightarrow t=23$

For $t=23; 5y^3+10y+5=0$

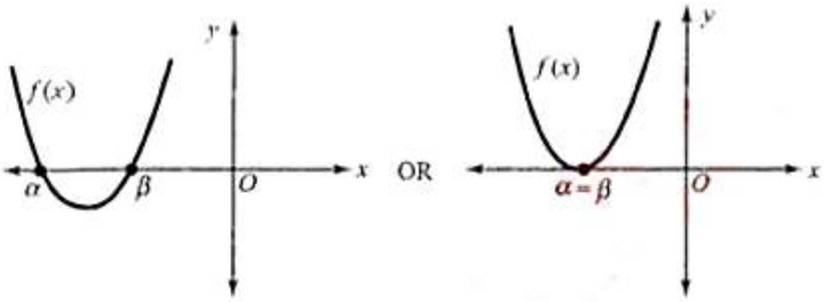
$(y+1)^2=0 \Rightarrow y=-1$

For $y=-1; 2x^2-16x+32=0$

$x^2-8x+16=0$

$x=4 \Rightarrow x+y=3$

23. (6)



Let $f(x)=x^2 2(\lambda+1)x+\lambda^2+\lambda+7$

If both roots of $f(x)=0$ are negative, then

$D=b^2-4ac=4(\lambda+1)^2-4(\lambda^2+\lambda+7)\geq 0 \Rightarrow \lambda-6\geq 0$

$\Rightarrow \lambda\in[6,\infty)$ ...(1)

Sum of roots $=-2(\lambda+1)<0$

$\Rightarrow \lambda\in(-1,\infty)$ ...(2)

and product of roots $=\lambda^2+\lambda+7>0 \ \forall \ \lambda\in R$ ...(3)

$\therefore$ From (1), (2), (3), we get $\lambda\in[6,\infty)$ (As (1), (2), (3) must be satisfied simultaneously)

Hence, the leat value of $\lambda=6$.

24. (4)

$y=\dfrac{3x^2+mx+n}{x^2+1}$

$x^2(y-3)-mx+y-n=0$

$x\in R$

$D \geq 0$

$\Rightarrow m^2 - 4(y-3)(y-n) \geq 0$

$\Rightarrow m^2 - 4\left(y^2 - ny - 3y + 3n\right) \geq 0$

$4y^2 - 4y(n+3) + 12n - m^2 \leq 0$ ...(1)

Also given $(y+4)(y-3) \leq 0$

$y^2 + y - 12 \leq 0$ ...(2)

Compare (1) and (2), we get $\frac{4}{1} = -\frac{4(n+3)}{1} = \frac{12n - m^2}{-12}$

$\Rightarrow m = 0$ and $n = -4$

25. (3)

$x^2 + x(y-a) + y^2 - ay + 1 \geq 0 \; x \in R$

$\Rightarrow (y-a)^2 - 4\left(y^2 - ay + 1\right) \leq 0$

$\Rightarrow -3y^2 + 2ay + a^2 - 4 \leq 0$

$\therefore 3y^2 - 2ay + 4 - a^2 \geq 0 \quad y \in R$

$D \leq 0$

$\Rightarrow 4a^2 - 4.3\left(4 - a^2\right) \leq 0 \Rightarrow a^2 - 3\left(4 - a^2\right) \leq 0 \Rightarrow 4a^2 - 12 \leq 0$

$\therefore$ range of $a \in \left(-\sqrt{3}, \sqrt{3}\right) \Rightarrow$ number of integer $\{-1, 0, 1\}$

26. (5)

$P(x) = \left(x^2 - 1\right)Q(x) + ax + b$

$P(1) = 10 \qquad a + b = 10$

$P(-1) = 10 \qquad -a + b = 10$

$a = 0, \quad b = 10$

$\frac{3(0) + 10}{2} = 5$

27. (0)

$\left(x^2 - 4x - 8x + 32\right)(x-1) - 10(x-8) > 0$

$\left[x(x-4) - 8(x-4)\right](x-1) - 10(x-8) > 0$

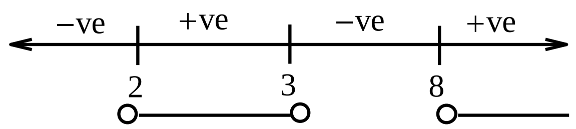
$(x-8)(x-4)(x-1) - 10(x-8) > 0$

$(x-8)\left(x^2 - 5x - 6\right) > 0$

$(x-8)\left(x^2 - 2x - 3x - 6\right) > 0$

$(x-8)\left[x(x-2) - 3(x-2)\right] > 0$

$(x-8)(x-2)(x-3) > 0$ .... (i)



No negative integer satisfy equation (i)

28. (2)

Let $x+\frac{1}{x}=t \quad \therefore t\geq 2$

Or $t\leq -2$

$t^2-7t+6=0$

$t^2-t-6t+6=0$

$t(t-1)-6(t-1)=0$

$(t-1)(t-6)=0 \qquad t\neq 1$

$\therefore t=6$

$x+\frac{1}{x}=6$

$x^2-6x+1=0$

$D>0$

Therefore two value of x satisfy given equation

29. (0)

$\frac{\sqrt{x-1}+1}{\sqrt{x-1}-1}=3 \quad \therefore x\geq 1 \,\&\, x\neq 2$

$\sqrt{x-1}+1=3\sqrt{x-1}-3$

$4=2\sqrt{x-1}$

$\therefore \sqrt{x-1}=2$

$x-1=4$

$x=5$

$x=5$ do not satisfy equation (2) hence no common solution of given equation

30. (0)

$D=4\cos^2\theta-4(3+\sin\theta)(2-\sin\theta)$

$=4\left[\cos^2\theta-6+\sin\theta+\sin^2\theta\right]$

$D=4[\sin\theta-5]<0$

Hence no real value of x satisfy given equation

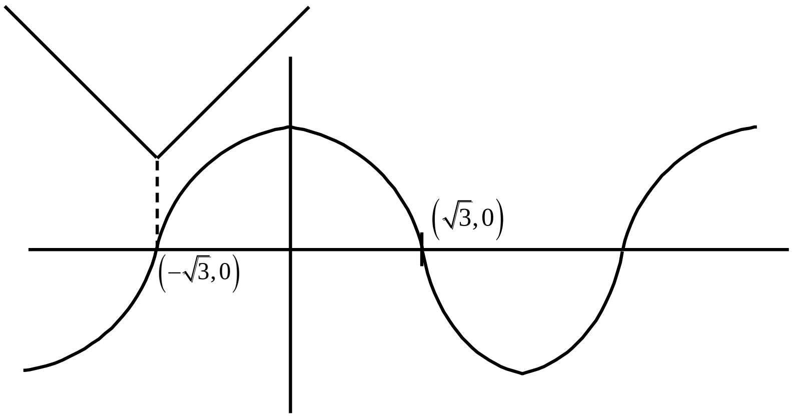
31. (0)

Draw the graph of $\quad y=\cos\left(\frac{\pi x}{2\sqrt{3}}\right)\&$

$y=x^2+2\sqrt{3}x+4$

$y=\left(x+\sqrt{3}\right)^2+1$

$\left(x+\sqrt{3}\right)^2=(y-1)$



From graph it is clear that two equation have no inter section point hence no common solution

32. **(1)**

$1-\sin^2 x-\sin x+a=0$

$\sin^2 x+2.\frac{1}{2}\sin x+\frac{1}{4}\frac{-1}{4}=a+1$

$\left(\sin x+\frac{1}{2}\right)^2=a+\frac{5}{4}$ .... (i)

$0<\sin x<1 \quad \therefore x\in(0,\pi/2)$

$\frac{1}{2}<\sin x+\frac{1}{2}<\frac{3}{2}$

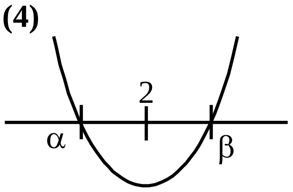
$\frac{1}{4}<\left(\sin x+\frac{1}{2}\right)^2<9/4$

$\frac{1}{4}<a+\frac{5}{4}<\frac{9}{4}$ (from (i))

$-1<a<1$

Hence only one integer between $-1\&1$

33. **(4)**



Given 2 lies between roots of given equation therefore

(i) $D>0$

$(k+1)^2-4(k^2+k-8)>0$

$k^2+2k+1-4k^2-4k+32>0$

$3k^2+2k-31<0$

$\frac{-1-\sqrt{94}}{3} \qquad \frac{-1+\sqrt{94}}{3}$

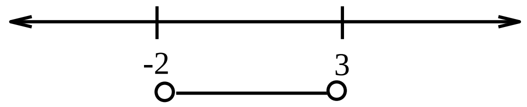
$k\in\left(\frac{-1-\sqrt{94}}{3},\frac{-1+\sqrt{94}}{3}\right)$ .... (i)

(ii) $1.f(2)<0$

$4-(k+1).2+(k^2+k-8)<0$

$4-2k-2+k^2+k-8<0$

$k^2 - k - 6 < 0$
$k^2 - 3k + 2k - 6 < 0$
$k(k-3) + 2(k-3) < 0$
$(k+2)(k-3) < 0$



$k \in (-2, 3)$ .... (ii)

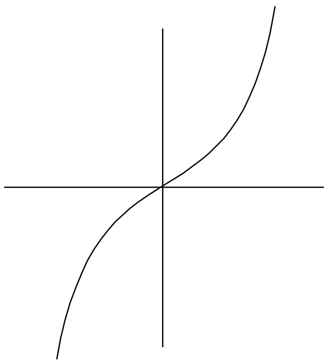
From (i) & (ii)

$k \in \left(-2, \frac{-1+\sqrt{94}}{3}\right)$ .... (iii)

$k = -1, 0, 1, 2$ only satisfy equation (iii)

34. **(0)**

$2x^3 + 3x^2 + 6x = -k$
$y = 2x^3 + 3x^2 + 6x$
$= x(2x^2 + 3x + 6)$



The graph can never intersect the line $y = -k$ at 3 points.
So, no value of k possible

35. **(3)**

Let the roots of equation be
$x, x+1, x+2$
$\therefore x + x + 1 + x + 2 = -a$
$3(x+1) = -a$ .... (a)
$x(x+1) + x(x+2) + (x+1)(x+2) = b$
$3x^2 + 6x + 2 = b$ .... (ii)
$3(x^2 + 2x + 1 - 1) + 2 = b$
$3(x+1)^2 = b + 1$
$3\frac{a^2}{9} = b + 1$
$\therefore \frac{a^2}{b+1} = 3$

36. **(4)**

$yx^2 + 3xy + cy = x^2 - 3x + c$
$(y-1)x^2 + 3x(y+1) + c(y-1) = 0$

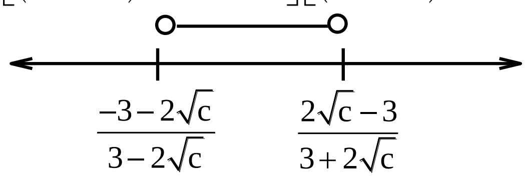
For real $x, D \geq 0$

$9(y+1)^2 - 4(y-1)^2 . C \geq 0$

$(3y+3)^2 - \left(2\sqrt{c}(y-1)\right)^2 \geq 0$

$\left[3y+3+2\sqrt{c}(y-1)\right]\left[3y+3-2\sqrt{c}(y-1)\right] \geq 0$

$\left[\left(3+2\sqrt{c}\right)y+3-2\sqrt{c}\right]\left[\left(3-2\sqrt{c}\right)y+3+2\sqrt{c}\right] \geq 0$



Given $\frac{-3-2\sqrt{c}}{3-2\sqrt{c}} = 7$

$-3-2\sqrt{c} = 21-14\sqrt{c}$

$\cancel{12}\sqrt{c} = \cancel{24}$

$c = 4$

37. **(0)**

Given $(a+4)x^2 - 2ax + (2a-6) \leq 0$

$D \leq 0$

$4a^2 - 4(a+4)(2a-6) \leq 0$

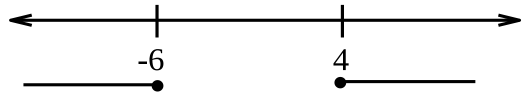
$a^2 - 2a^2 - 2a + 24 \leq 0$

$a^2 + 2a - 24 \geq 0$

$a^2 + 6a - 4a - 24 \geq 0$

$a(a+6) - 4(a+6) \geq 0$

$(a+6)(a-4) \geq 0$



$a \in (-\infty, -6] \cup [4, \infty)$ .... (ii)

$a + 4 < 0$

$a < -4$ .... (ii)

38. **(4)**

$x^2 + p_1x + q_1 = 0 \rightarrow \alpha, \beta \qquad \alpha + \beta = -p_1$

$x^2 + p_2x + q_2 = 0 \rightarrow \beta, \gamma \qquad \beta + \gamma = -p_2$

$x^2 + p_3x + q_3 = 0 \rightarrow \alpha, \gamma \qquad \gamma + \alpha = -p_3$

$(p_1 + p_2 + p_3)^2 = \left(-2(\alpha+\beta+\gamma)\right)^2 = 4(\alpha+\beta+\gamma)^2$

$p_1p_2 + p_2p_3 + p_3p_1 - q_1 - q_2 - q_3$

$= (\alpha+\beta)(\beta+\gamma) + (\beta+\gamma)(\gamma+\alpha) + (\gamma+\alpha)(\alpha+\beta) - \alpha\beta - \beta\gamma - \gamma\alpha$

$= (\alpha+\beta+\gamma)^2$

$4(\alpha+\beta+\gamma)^2 = K(\alpha+\beta+\gamma)^2$

$K = 4$

39. **(2)**

Given $\tan\alpha+\tan\beta=p$

$\tan\alpha\tan\beta=q$

$\cot\alpha+\cot\beta=r, \cot\alpha\cot\beta=s$

$$\frac{1}{\tan\alpha}+\frac{1}{\tan\beta}=r$$

$$\frac{\tan\alpha+\tan\beta}{\tan\alpha.\tan\beta}=r$$

$\therefore \frac{p}{q}=r$ .... (i)

$+\alpha+\in+\beta=p$

$\frac{\cot\alpha+\cot\beta}{\cot\alpha\cot\beta}=p$ $\therefore \frac{r}{s}=p$ .... (ii)

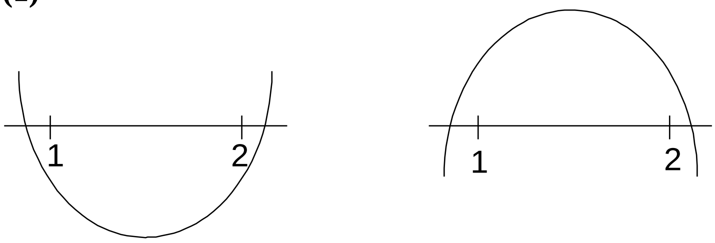
Form (i) & (ii)

$$\frac{p}{q}=ps$$

$qs=1$

$$\left(\frac{p}{r}+q\right)s=\frac{ps}{p}+qs=1+1=2$$

40. **(1)**



Given

(i) $D>0$

$4k^2-4(k-5)(k-4)>0$

$\cancel{k^2}-\cancel{k^2}+9k-20>0$

$k>20/9$ .... (i)

(ii) $(k-5)f(1)<0$

$(k-5)\left(\cancel{k}-5-\cancel{2k}+\cancel{k}-4\right)<0$

$(k-5)(x-9)<0$ $\therefore k-5>0$

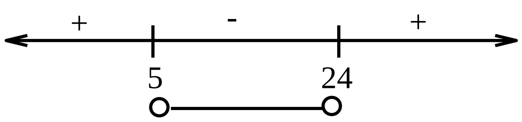
$k>5$ .... (ii)

$(k-5)f(2)<0$

$(k-5)\left(4(k-5)-4k+k-4\right)<0$

$(k-5)\left(\cancel{4k}-20-\cancel{4k}+k-4\right)<0$

$(k-5)(k-24)<0$



$k\in(5,24)$

$\therefore a=5$

$b=24$
$a^2-b=25-24=1$

JEE Advanced : PYQ

1. (D)
Consider the quadratic polynomials in the form of equation
$x^2+20x-2020=0$ ... (i)
$x^2-20x+2020=0$ ... (ii)
Since, a and b are roots of the equation (i), then
$a+b=-20,\ ab=-2020$
$\because\ c$ and d are the roots of the equation (ii), then
$c+d=20, cd=2020$
Now, $ac(a-c)+ad(a-d)+bc(b-c)+bd(b-d)$
$=a^2c-ac^2+a^2d-ad^2+b^2c-bc^2+b^2d-bd^2$
$=a^2(c+d)+b^2(c+d)-c^2(a+b)-d^2(a+b)$
$=(c+d)\left((a+b)^2-2ab\right)-(a+b)\left((c+d)^2-2cd\right)$
$=20\left[(20)^2+4040\right]+20\left[(20)^2-4040\right]$
$=20\times 800=16000$

2. (D)
Quadratic equation with real coefficients and purely imaginary roots can be considered as
$p(x)=x^2+a=0$ where $a>0$ and $a\in R$
The $p\left[p(x)\right]=0 \Rightarrow \left(x^2+a\right)^2+a=0$
$\Rightarrow\ x^4+2ax^2+\left(a^2+a\right)=0$
$$\Rightarrow\ x^2=\frac{-2a\pm\sqrt{4a^2-4a^2-4a}}{2}$$
$\Rightarrow\ x^2=-a\pm\sqrt{a}\,i$
$\Rightarrow\ x=\sqrt{-a\pm\sqrt{a}\,i}=\alpha=i\,\beta$, where $\alpha,\beta\neq 0$
$\therefore\ p\left[p(x)\right]=0$, has complex roots which are neither purely real nor purely imaginary.

3. (C)
α,β are roots of $x^2-6x-2=0$
$\Rightarrow \alpha^2-6\alpha-2=0\ \&\ \beta^2-6\beta-2=0$
$$\frac{a_0-2a_8}{2a_0}=\frac{\alpha^{10}-\beta^{10}-2\left(\alpha^8-\beta^8\right)}{2\left(\alpha^9-\beta^9\right)}$$
$$=\frac{\alpha^8\left(\alpha^2-2\right)-\beta^8\left(\beta^2-2\right)}{2\left(\alpha^9-\beta^9\right)}$$
$$=\frac{\alpha^8.6\alpha-\beta^8.6\beta}{2\left(\alpha^9-\beta^9\right)}=3$$

4. (B)

$$\frac{x^2}{b^2+1}=\frac{-x}{b+1}=\frac{1}{1-b}$$

$$\Rightarrow x=\frac{b+1}{b-1} \qquad .....(i)$$

$$\& \ x^2=\frac{b^2+1}{1-b} \qquad .....(ii)$$

From (i) & (ii)

$$\left(\frac{b+1}{b-1}\right)^2=\frac{b^2+1}{1-b}$$

$$\Rightarrow \left(b^2+1\right)\left(1-b\right)=\left(b+1\right)^2 \ \Rightarrow -b^2+1+b^2-b=b^2+1+2b$$

$$\Rightarrow -b^2-3b=0 \ \Rightarrow b\left(b^2+3\right)=0$$

$$\Rightarrow b=0, b=\sqrt{3}i$$

5. (B)

$$\alpha^3+\beta^3=q$$

$$\Rightarrow \left(\alpha+\beta\right)^3-3\alpha\beta\left(\alpha+\beta\right)=q$$

$$\Rightarrow \alpha\beta=\frac{p^3+q}{3p}$$

Sum of the roots

$$=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$=\frac{p^2-2\frac{\left(p^3+q\right)}{3p}}{\frac{p^3+q}{3p}}=\frac{p^3-2q}{p^3+q}$$

Product of the roots = 1

Required equation is $\left(p^3+q\right)x^2-\left(p^3-2q\right)x+\left(p^3+q\right)=0$

6. (D)

Since, α and β are the roots of $x^2-px+r=0$

$$\therefore \ \alpha+\beta=p \qquad ...(i)$$

$$\text{and} \ \ \alpha\beta=r \qquad ...(ii)$$

Also $\frac{\alpha}{2}$ and 2β are the roots of $x^2-qx+r=0$

$$\therefore \ \frac{\alpha}{2}+2\beta=q \ \Rightarrow \ \alpha+4\beta=2q \qquad ...(iii)$$

Solving (i) and (iii) for α and β, we get

$\beta=\frac{1}{3}\left(2q-p\right)$ and $\alpha=\frac{2}{3}=\left(2q-q\right)$

On substituting the values of α and β, in equation (ii),

We get $\frac{2}{9}\left(2p-q\right)\left(2q-p\right)=r$.

7. (A, D)

$\alpha x^2 - x + \alpha = 0$

$D = 1 = 4\alpha^2$

distinct real roots $D > 0$

$\Rightarrow \alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ ....(i)

given, $|x_1 - x_2| < 1$

$\Rightarrow \frac{\sqrt{1-4\alpha^2}}{|\alpha|} < 1$

$\Rightarrow 1 - 4\alpha^2 < \alpha^2$

$\Rightarrow \alpha \in \left(-\infty, \frac{-1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$ ....(ii)

From (i) & (ii)

$\alpha \in \left(\frac{-1}{2}, \frac{-1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

8. (C)

$\alpha^2 = \alpha + 1 \Rightarrow \alpha^4 = 3\alpha + 2$

$\therefore a_2 = 28 \Rightarrow p\alpha^4 + q\beta^4 = p(3\alpha + 2) + q(3\beta + 2) = 28$

$\Rightarrow p(3\alpha + 2) + q(3 - 3\alpha + 2) = 28$

$\Rightarrow \alpha(3p - 3q) + 2p + 5q = 28$

$\Rightarrow p = q, 2p + 5q = 28 \Rightarrow p = q = 4$

$\therefore p + 2q = 12$

9. (C)

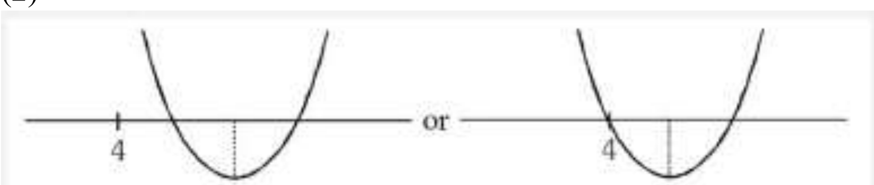
$\alpha^2 = \alpha + 1 \Rightarrow \alpha^n = \alpha^{n-1} + \alpha^{n-2}$

$\Rightarrow p\alpha^n + q\beta^n = p\left(\alpha^{n-1} + \alpha^{n-2}\right) + q\left(\beta^{n-1} + \beta^{n-2}\right)$

$a_n = a_{n-1} + a_{n-2}$

$\Rightarrow a_{12} = a_{11} + a_{10}$

10. (2)



$f(x) = x^2 - 8kx + 16\left(k^2 - k + 1\right) = 0$

$\because$ roots are real $D > 0$

$64k^2 - 64\left(k^2 - k + 1\right) > 0$

$k > 1 \Rightarrow k \in (1, \infty)$

At both the roots ≥ 4

$$\Rightarrow \begin{cases} f(4) \geq 0 \\ \dfrac{-b}{2a} > 4 \end{cases} \Rightarrow \begin{cases} k^2 - 3k + 2 \geq 0 \\ k > 1 \end{cases}$$

$$\Rightarrow k[2, \infty) \Rightarrow \text{least value of } k = 2$$