Quadratic Equation

EXERCISE - 1 [A]

1. **(B)**

Required equation

$x^2-(1-2)x+(1x-2)=0$

$x^2+x-2=0$

2. **(B)**

Let $3^x=t$

$t+\frac{1}{t}=\frac{10}{3}$ $\therefore \frac{t^2+1}{t}=\frac{10}{3}$

$3t^2-10t+3=0$

$3t^2-9t-t+3=0$

$3t(t-3)-(t-3)=0$

$(3t-1)(t-3)=0$

$t=\frac{1}{3},3$ $\therefore 3^x=3^{-1}$ $\therefore x=-1$

$3^x=3^1$ $\therefore x=1$

3. **(B)**

Given $16+4m-26+x=0$

$4m+x=10$ .... (i)

$54+9m-39+x=0$

$9m+x=-15$ .... (ii)

From (i) & (ii) $m=-5, x=30$

4. **(C)**

$(x-1)^2+(x-2)^2+(x-3)^2=0$ this is possible only when

$x-1=0$

$x-2=0$

$x-3=0$

But no single value of x for all are zero at a time hence no of solution $=0$

5. **(B)**

Let $3^x=t$

$\frac{t}{3}+\frac{3}{t}=2$

$\frac{t^2+9}{3t}=2$

$t^2-6t+9=0$

$(t-3)^2=0$

$t=3$

$\therefore\ 3^x=3^1$

$x=1$

6. **(A)**

$x^2-x(2i-2i)+(2i)(-2i)=0$

$x^2+4=0$

7. **(D)**

Given, $x=-1$ is the root of given equation

$1-(p-3)-(3p-5)-(2p-9)+6=0$

$7-p+3-3p+5-2p+9=0$

$24=6p$

$p=4$

8. **(A)**

No root

$x=1$ does not satisfy.

9. **(B)**

As $x+1$ is factor so $x=-1$will satisfy the expression

$\Rightarrow(-1)^4+(p-3)(-1)^3-(3p-5)(-1)^2+(2p+q)(-1)+12=0$

$\Rightarrow p=2$

10. **(C)**

Product of roots $=\frac{(2m-1)}{m}=-1$

$2m-1=-m$

$3m=1$

$m=\frac{1}{3}$

11. **(B)**

$\alpha+\beta=0$

$-2\frac{(2-a-a^2)}{1}=0$

$a^2+a-2=0$

$a^2+2a-a-2=0$

$a(a+2)-(a+2)=0$

$a=1,-2$

12. (B)

$|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

$=\sqrt{49+4\times 9}$

$=\sqrt{49+36}$

$=\sqrt{85}$

13. (D)

Given $\alpha+\beta=\frac{|\alpha-\beta|}{2}$

$(\alpha+\beta)^2=\frac{(\alpha-\beta)^2}{4}$

$4(\alpha+\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$3(\alpha+\beta)^2=-4\alpha\beta$

$3\times\frac{16}{a^{\not 2}}=-4\times\frac{c}{\not a}$

$\frac{3\times 16}{-4}=ac$

$\therefore ac=-12$

14. (C)

Given $\alpha+\beta=-1$

$-\frac{(2a+3)}{(a+1)}=-1$

$2a+3=a+1$

$a=-2$

$\alpha\beta=\frac{3a+4}{a+1}=\frac{-6+4}{-2+1}=2$

15. (A)

$x^2-x(\text{sum})+\text{product}=0$

$\alpha+\beta=-1$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{6}$

$\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{6}$

$\alpha\beta=-6$

$x^2-x(-1)-6=0$

16. **(C)**

$$\alpha\beta(\alpha+\beta)=\frac{3}{2}\left(\frac{5}{2}\right)=\frac{15}{4}$$

17. **(C)**

$$\alpha\beta+\alpha+\beta+1$$

$$\frac{c}{a}+\frac{b}{a}+1=\frac{a+b+c}{a}$$

18. **(A)**

$$\alpha+\beta=-\sqrt{\alpha}$$

$$\alpha\beta=\beta \qquad \alpha=1$$

$$\beta=-2$$

19. **(A)**

$$a=-(8+2)=-10$$

$$\beta=8\times2=16$$

$$\alpha=-(3+3)=-6$$

$$b=3\times3=9$$

$$x^2-10x+9=0$$

$$x=1,9$$

20. **(A)**

Sum of roots $=2-\sqrt{3}+2+\sqrt{3}=4$

Product of roots $=(2-\sqrt{3})(2+\sqrt{3})$

$=4-3=1$

Required equation $x^2-4x+1=0$

21. **(A)**

Product of roots $=30$

Sum of roots $=11$

$\therefore$ Required equation $x^2-11x+30=0$

$$x^2-5x-6x+30=0$$

$$x(x-5)-6(x-5)=0$$

$$x=5,6$$

22. **(A)**

Given $\alpha+\beta=-\frac{b}{a}$

$$\alpha\beta=\frac{c}{a}$$

Product of roots $=\frac{\alpha}{\beta}.\frac{\beta}{\alpha}=\frac{\gamma}{p}$

$\therefore p=\gamma$

23. **(C)**

$\alpha+\beta+2=5+2=7$

$\frac{\alpha\beta}{2}=\frac{16}{2}=8$

$x^2-(7+8)x+56=0$

$p=-15,\ q=56$

24. **(C)**

$(1-p)^2+p(1-p)+(1-p)=0$

$(1-p)[1-p+p+1]=0$

$p=1$

$x^2+x+0=0$

$x=0,-1$

25. **(A)**

$16+4p+12=0$

$p=-7$

$p^2-4q=0$

$q=\frac{49}{4}$

26. **(C)**

$p+q=-p \qquad pq=q$

$q=0$ or $p=1$

$p=0 \qquad q=-2$

$p=0,1$

27. **(D)**

$D=49-4.6.K$

$\quad =49-24.K$

If $K=1$ or 2 D is a perfect square hence roots are rational

28. **(A)**

$D<0$

$1-4m<0$

$4m>1$

$m>\frac{1}{4}$

$m\in\left(\frac{1}{4},\infty\right)$

29. **(D)**

Given $D=1-4ab\geq 0$

$4ab\leq 1$ .... (i)

$D=16ab-4$

$= 4(ab-1) \le 0$

Roots of equation $x^2 - 4\sqrt{ab}x + 1 = 0$ will be imaginary

30. **(A)**

Given $16 + 4P + 12 = 0$

$4 + P + 3 = 0$

$P = -7$

$P^2 - 4q = 0$

$49 = 4q$

$\therefore q = \frac{49}{4}$

31. **(C)**

Given $4q^2 - 4Pr \ge 0$

$q^2 \ge Pr$ .... (i)

$4Pr - q^2 \ge 0$

$Pr \ge q^2$ .... (ii)

From (i) & (ii)

$q^2 = Pr$

$\frac{q}{r} = \frac{P}{q}$

32. **(C)**

$D = 100 - 4(21-m) = 0$

$25 - 21 - m$

$m = -4$

33. **(C)**

$D = 0$

$(k+2)^2 - 4.2K = 0$

$k^2 + 4 + 4k - 8k = 0$

$(k-2)^2 = 0$

$k = 2$

34. **(B)**

Sum of roots $= P > 0$

Product of roots $= -q < 0$

Both roots are red and opposite sign

35. **(B)**

Product of roots $= 1$

$\frac{-2}{\ell} = 1$

$\therefore \ell = -2$

36. **(B)**

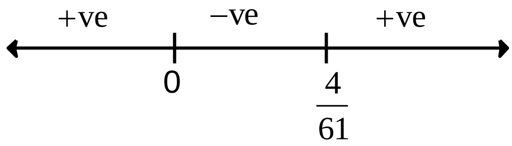
Given $mn^2 - mx + 5m + 1 > 0$
$\forall x \in R$

If $m > 0$

& $81m^2 - 4m(5m+1) < 0$

$81m^2 - 20m^2 - 4m < 0$

$m(61m - 4) < 0$



$m \in \left(0, \frac{4}{61}\right)$

37. **(B)**

$(K-1)^2 - 4(9) < 0$

$K^2 - 2K - 35 < 0$

$K \in (-5, 7)$

38. **(C)**

$2x^2 + 6x - x - 3 > 0$

$(2x-1)(x+3) > 0$

$x \in (-\infty, -3) \cup \left(\frac{1}{2}, \infty\right)$

39. **(B)**

$2x^2 + 6x - 3x - 9 \le 0$

$(2x-3)(x+3) \le 0$

$x \in \left[-3, \frac{3}{2}\right]$

40. **(D)**

$\frac{x^2 + 2x + 7}{2x + 3} - 6 < 0$

$\frac{x^2 - 10x - 11}{2x + 3} < 0$

$\frac{(x+1)(x-11)}{x + \frac{3}{2}} < 0$

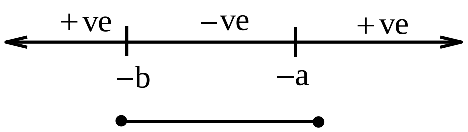
$x \in \left(-\infty, -\frac{3}{2}\right) \cup (-1, 11)$

41. **(D)**

$x^2+ax+bx+ab<0$

$x(x+a)+b(x+a)<0$

$(x+a)(x+b)<0$



42. **(C)**

$(x+1)^2>5x-1$

$x^2-3x+2>0 \qquad x\in(-\infty,1)\cup(2,\infty)$

$(x+1)^2<7x-3$

$x^2-5x+4<0 \qquad x\in(1,4)$

$x=3$

43. **(D)**

Replace x by $x-2$ we get required equation

$a(x-2)^2+b(x-2)+c=0$

$ax^2+4a-4ax+bx-2b+c=0$

$ax^2+(b-4a)x+4a-2b+c=0$

44. **(C)**

If α,β be the roots of equation $2x^2-3x+5=0$

Then equation whose roots are $\frac{1}{\alpha}$ & $\frac{1}{\beta}$

$5x^2-3x+2=0$ .... (i)

Given $ax^2+bx+2=0$ .... (ii)

Equation (i) & (ii) are identical

$$\frac{a}{5}=\frac{b}{-3}=\frac{2}{2}$$

$a=5, b=-3$

45. **(A)**

$$y=\frac{-2}{x} \therefore x=\frac{-2}{y}$$

Replace x by $-2/x$ we get revered equation whose roots are $\frac{-2}{\alpha},\frac{-2}{\beta}$

$$2\times\frac{4}{x^2}+\frac{7\times 2}{x}+6=0$$

$8+14x+6x^2=0$

46. **(C)**

Let α be common root

$$\alpha^2 - 11\alpha + a = 0$$
$$\alpha^2 - 14x + 2a = 0$$

$$\frac{\alpha^2}{-22a+14a} = \frac{\alpha}{a-2a} = \frac{1}{-14+11}$$

$$\frac{\alpha^2}{-8a} = \frac{\alpha}{-a} = \frac{1}{-3}$$

$$\therefore \alpha = a/3$$

$\therefore a = 24 \qquad \alpha = 8$

If $a = 0$ then $x = 0$ is common root

47. **(D)**

$$x^2 - x - 2x + 2 = 0$$
$$x(x-1) - 2(x-1) = 0$$
$$x = 1, 2$$
$$x^2 - x + 3x - 3 = 0$$
$$x(x-1) + 3(x-1) = 0$$
$$x = 1, -3$$

Common root $= 1$

$f(1) = 4 + 3 - 7 = 0$

48. **(C)**

$$a\alpha^2 + b\alpha + c = 0$$
$$b\alpha^2 + c\alpha + a = 0$$

$$\frac{\alpha^2}{ab - c^2} = \frac{\alpha}{bc - a^2} = \frac{1}{ac - b^2}$$

$$\therefore \frac{ab - c^2}{bc - a^2} = \frac{bc - a^2}{ac - b^2}$$

$$(bc - a^2)^2 = (ab - c^2)(ac - b^2)$$

$$\cancel{b^2c^2} + a^4 - 2a^2bc = a^2bc - ab^3 - ac^3 + \cancel{b^2c^2}$$

$$a(a^3 + b^3 + c^3 - 3abc) = 0$$

Either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

49. **(B)**

Both equation are identical because roots are imaginary

$$\frac{a}{1} = \frac{b}{3} = \frac{c}{5} = \lambda$$

$$a = \lambda, b = 3\lambda, c = 5\lambda$$

$a + b + c = \lambda.9$

$(a + b + c)_{min} = 9$

50. **(A)**

$$y=\frac{x}{x^2-5x+9}$$

$$yx^2-5yx+9y=x$$

$$yx^2-x(5y+1)+9y=0$$

For read x, $d\geq 0$

$$(5y+1)^2-4.y.9y\geq 0$$

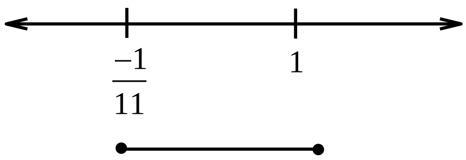
$$25y^2+1+10y-36y^2\geq 0$$

$$11y^2-10y-1\leq 0$$

$$11y^2-11y+y-1\leq 0$$

$$11y(y-1)+(y-1)\leq 0$$

$$(11y+1)(y-1)\leq 0$$



$$y_{\max m}=1$$

51. **(D)**

$$y=\frac{x^2+34x-71}{x^2+2x-7}$$

$$yx^2+2xy-7y=x^2+34x-71$$

$$(y-1)x^2+2x(y-17)+(71-7y)=0$$

For real x,

$D\geq 0$

$$4(y-17)^2-4(y-1)(71-7y)\geq 0$$

$$y^2-34y+289-(-7y^2+78y-71)\geq 0$$

$$y^2-34y+289+7y^2-78y+71\geq 0$$

$$8y^2-112y+360\geq 0$$

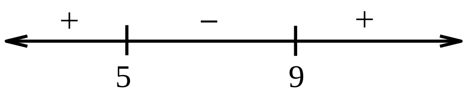
$$2y^2-289+90\geq 0$$

$$y^2-14y+45\geq 0$$

$$y^2-5y-9y+45\geq 0$$

$$y(y-5)-9(y-5)\geq 0$$

$$(y-5)(y-9)\geq 0$$



$$y\in(-\infty+5)\cup[9,\infty)$$

52. **(B)**

$$y_{min} = \frac{-D}{4a} = -\frac{\left(12^2 - 4.40\right)}{4}$$

$$y_{min} = -\frac{4 \times \not{4}\left(3^2 - 10\right)}{\not{4}}$$

$$= -4 \times -1 = 4$$

53. **(C)**

$$y = \frac{x^2 - 2x + 1}{x + 1}$$

$$yx + y = x^2 - 2x + 1$$

$$x^2 - x(2 + y) + (1 - y) = 0$$

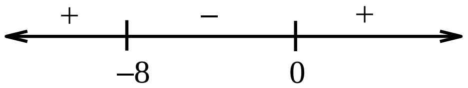
For real $x, D \geq 0$

$$(2 + y)^2 - 4(1 - y) \geq 0$$

$$\not{4} + y^2 + 4y - \not{4} + 4y \geq 0$$

$$y^2 + 8y \geq 0$$

$$y(y + 8) \geq 0$$



$$y \in (-\infty, -8] \cup [0, \infty)$$

54. **(D)**

$$\Rightarrow 4x^3 + 20x^2 - 23x + 6 = 0$$

Let's take roots are α, α, β

$$\Rightarrow 2\alpha + \beta = \frac{-20}{4} = -5 \qquad \text{……..(1)}$$

$$\Rightarrow \alpha^2 + \alpha\beta + \alpha\beta = \frac{-23}{4}$$

$$\Rightarrow \alpha^2\beta = \frac{-6}{4}$$

$$\Rightarrow \alpha^2(-5 - 2\alpha) = -\frac{6}{4}$$

$$\Rightarrow 5\alpha^2 + 2\alpha^3 - \frac{6}{4} = 0$$

$$\Rightarrow \alpha^3 + 20\alpha^3 - 6 = 0$$

Which gives $\alpha = \frac{1}{2}$

So $2\left(\frac{1}{2}\right) + \beta = -5 \Rightarrow \beta = -6$

So roots are $-6, \frac{1}{2}, \frac{1}{2}$

55. **(D)**

$\Rightarrow 2x^3 - 5x^2 + 3x - 1 = 0$

$\Rightarrow \alpha\beta\gamma = \frac{1}{2}$

$\Rightarrow \alpha + \beta + \gamma = \frac{5}{2}$

$\Rightarrow \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\beta\alpha} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{\left(\frac{5}{2}\right)}{\left(\frac{1}{2}\right)} = 5$

56. **(A)**

$\Rightarrow x^3 - 4x^2 + x + 6 = 0$

Roots are $2\alpha, 3\alpha, \beta$

So, $2\alpha, 3\alpha, \beta = 4$

$\Rightarrow 5\alpha + \beta = 4$ ……….(1)

$\Rightarrow (2\alpha)(3\alpha)(\beta) = -6$

$\Rightarrow \alpha^2\beta = -1$ ………..(2)

By (1) & (2)

$\Rightarrow \alpha^2(4 - 5\alpha) = -1$

$\Rightarrow 5\alpha^3 - 4\alpha^2 - 1 = 0$

By this we get $\alpha = 1$

So roots are $2, 3, -1$

57. **(B)**

$\Rightarrow x^3 + 3x + 2 = 0$

$\Rightarrow a + b + c = 0$

$\Rightarrow ab + bc + ca = 3$

$\Rightarrow abc = -2$

$\Rightarrow a^3 + b^3 + c^3 - 3abx = (a + b + c)(a^2 + b^2 + c^2 + ab + bc + ca) = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3(-2) = -6$

58. **(A)**

$\Rightarrow x^3 + 1 = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = 0$

$\Rightarrow \alpha\beta\gamma = 1$

$\Rightarrow (\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 0$

$\Rightarrow (\alpha^2 + \beta^2 + \gamma^2)^2 = \alpha^4 + \beta^4 + \gamma^4 + 2(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2)$

$\Rightarrow 0 = \alpha^4 + \beta^4 + \gamma^4 + 2\left[(\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2(\alpha\beta.\beta\gamma + \beta\gamma.\gamma\alpha + \gamma\alpha.\alpha\beta)\right]$

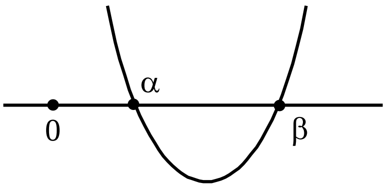
$= \alpha^4 + \beta^4 + \gamma^4 + 2\left[0 - 2(\beta + \gamma + \alpha)\right]$ $\quad (\because \alpha\beta\gamma = 1)$

$\Rightarrow 0 = \alpha^4 + \beta^4 + \gamma^4 + 2\left[0 - 2(0)\right]$

$\Rightarrow \alpha^4 + \beta^4 + \gamma^4 = 0$

59. **(A)**

Given $D \geq 0$



$1 - 4.2.K \geq 0$

$8K \leq 1$

$K \leq 1/8$

$f(0) > 0 \therefore K > 0$

$\frac{+1}{2.2} > 0 \qquad \therefore \frac{1}{a} > 0$

$\therefore$ roots are the there fore $K > 0$

$K \in \left(0, \frac{1}{8}\right]$

60. **(A)**

Product < 0

$a^2 - 4a < 0$

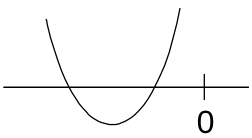
$a \in (0, 4)$

61. **(B)**

$D \geq 0$

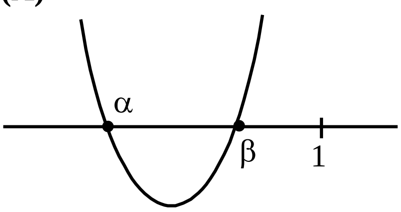
& $-\frac{b}{2a} < 0$

& $f(0) > 0$



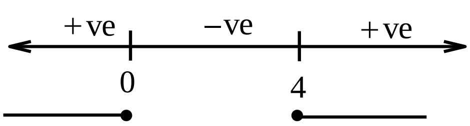
Take intersection of all there inequalities.

62. **(A)**



(i) $D \geq 0 \quad \therefore a^2 - 4a \geq 0$

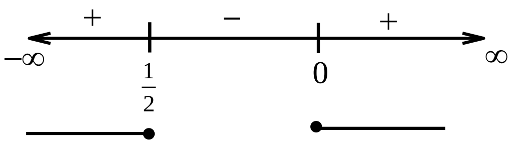
$a(a - 4) \geq 0$



$a \in (-\infty, 0] \cup [4, \infty)$ .... (i)

(ii) $a(a+a+1)>0$

$a(2a+1)>0$



$a \in \left(-\infty, \frac{-1}{2}\right) \cup (0,\infty)$ .... (ii)

$a \in \left(-\infty, \frac{1}{2}\right) \cup [4,\infty)$ (i) & (ii)

EXERCISE - 1 [B]

1. **(C)**

$|\alpha - \beta| > 0$

$(\alpha+\beta)^2 - 4\alpha\beta > 0$

$(P+2)^2 - 4x2P > 0$

$P^2 + 4 + 4P - 8P > 0$

$(P-2)^2 > 0$

$\therefore P \in I - \{2\}$

2. **(A)**

Sum = 4 + 3 = 7

Product = 3 × 2 = 6

Roots = 6, 1

3. **(A)**

Given $|\alpha - \beta| = 3$

$(\alpha-\beta)^2 = 9$

$(\alpha+\beta)^2 - 4\alpha\beta = 9$

$\left(\frac{a-4}{a-2}\right)^2 + \frac{8}{(a-2)} = 9$

$(a-4)^2 + 8(a-2) = 9(a-2)^2$

$a^2 - \cancel{8a} + \cancel{16} + \cancel{8a} + \cancel{16} = 9a^2 + 36 - 36a$

$8a^2 - 36a + 36 = 0$

$2a^2 - 9a + 9 = 0$

$2a^2 - 3a - 6a + 9 = 0$

$a(2a-3) - 3(2a-3) = 0$

$(a-3)(2a-3) = 0$

$a = \frac{3}{2}, 3$

4. **(D)**

Given $\alpha + \beta = -p$

$\alpha\beta = q$

$r + \delta = -p$

$r\delta = r$

$(\alpha - r)(\alpha - \delta) = \alpha^2 - \alpha\delta - \alpha r + r\delta$

$= \alpha^2 - \alpha(\delta + r) + r\delta$

$= \alpha^2 + p\alpha + r$

$= q + r$

(Given $\alpha^2 + p\alpha - q = 0$)

$\alpha^2 + p\alpha = q$

5. **(C)**

Given $\alpha + \beta = \frac{35}{2}$

$\alpha\beta = 1$

$\left[(2\alpha - 35)(2\beta - 35)\right]^3 = \left[4\alpha\beta - 70(\alpha + \beta) + 35^2\right]^3$

$= \left[4 - 70 \times \frac{35}{2} + 35^2\right]^3$

$= \left(4 - 35^2 + 35^2\right)^3 = 64$

6. **(B)**

If $ax^2 + bx + c = 0$ is dx identify the

$a = b = c = 0$

$\therefore \quad \lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - \lambda - 2\lambda + 2 = 0$

$\lambda(\lambda - 1) - 2(\lambda - 1) = 0$, $(\lambda - 1)(\lambda - 2) = 0$

$\lambda = 1, 2$

$\lambda^2 - 2\lambda - 3\lambda + 6 = 0$

$\lambda(\lambda - 2) - 3(\lambda - 2) = 0 \qquad \lambda = 2, 3$

And $\lambda^2 = 4$

$\lambda = \pm 2$

$\therefore \quad \lambda = 2$ only common value

7. **(B)**

$\alpha + \beta = -b/a$

$\alpha\beta = c/a$

$$\frac{\beta}{a\left(\alpha+\frac{b}{a}\right)}+\frac{\alpha}{a\left(\beta+\frac{b}{a}\right)}=\frac{\beta}{a\left(\cancel{\alpha}-\cancel{\alpha}-\beta\right)}+\frac{\alpha}{a\left(\cancel{\beta}-\alpha-\cancel{\beta}\right)}$$

$$=\frac{-\beta}{a\beta}-\frac{\alpha}{a\alpha}$$

$$=-\left(\frac{1}{a}+\frac{1}{a}\right)=\frac{-2}{a}$$

8. (A)

$$\alpha+\beta=\frac{-b}{a}$$

$$\alpha\beta=c/a$$

Given $\alpha+\beta=\alpha^2+\beta^2$

$$\alpha+\beta=(\alpha+\beta)^2-2\alpha\beta$$

$$\frac{-b}{a}=\frac{b^2}{a^2}-2.\frac{c}{a}$$

$$\frac{-b}{\cancel{a}}=\frac{b^2-2ac}{a^{\cancel{2}}}$$

$$b^2-2ac=-ab$$

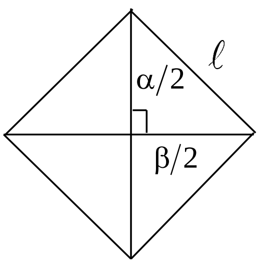
$$b^2+ab=2ac$$

9. (A)

$$\ell=\sqrt{\frac{\alpha^2}{4}+\frac{\beta^2}{4}}$$

$$=\sqrt{\frac{\alpha^2+\beta^2}{2}}$$

$$=\frac{\sqrt{(\alpha+\beta)^2-2\alpha\beta}}{2}$$



10. (D)

$$\alpha+\beta=z$$

$$\alpha^3+\beta^2=98$$

$$(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=98$$

$$8-3.\alpha\beta.z=98$$

$$-90=6.\alpha\beta$$

$$\alpha\beta=\frac{-30}{2}=-15$$

Revered equation $x^2-2x-15=0$

11. **(C)**

$|\alpha-\beta|=1$

$(\alpha+\beta)^2-4\alpha\beta=1$

$p^2-4q=1 \Rightarrow p^2=1+4q$

$p^2+4q^2=1+4q+4q^2$

$=(1+2q)^2$

12. **(D)**

Product of roots = 1

13. **(C)**

Product of roots < 0

14. **(C)**

Other root $=\frac{3-5i}{2}$

$\frac{c}{2}=$ Product

15. **(A)**

$|\alpha-\beta|=\alpha\beta$

$\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\alpha\beta$

$\alpha+\beta=\frac{a+1}{2}$

$\alpha\beta=\frac{a-1}{2}$

16. **(A)**

$\alpha+\alpha^2=1$

$\alpha\cdot\alpha^2=\alpha^3=-k$

17. **(B)**

Sum of roots = 0

18. **(C)**

$\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$\alpha+\beta=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$\frac{-b}{a}=\frac{\frac{b^2}{a^2}-\frac{2.c}{a}}{\frac{c^2}{a^2}}$

$$\frac{-b}{a}=\frac{b^2-2ac}{c^2}$$
$$-bc^2=b^2a-2a^2c$$
$$b^2a+bc^2=2a^2c$$
$$\frac{b^2 a}{a^2 c}+\frac{bc^2}{a^2 c}=2$$
$$\frac{b^2}{ac}+\frac{bc}{a^2}=2$$

19. **(A)**

$$\alpha-\beta=\frac{\sqrt{D_1}}{a}=\frac{\sqrt{D_2}}{P}$$
$$\frac{D_1}{D_2}=\frac{a^2}{P^2}$$

20. **(A)**

$$\frac{ax-ab+bx-ab}{(x-a)(x-b)}=1$$
$$(a+b)x-2ab=x^2-(a+b)x+ab$$
$$x^2-2(a+b)x+3ab=0 \qquad \text{.......... (i)}$$

Given, sum of roots $=0$

$$2(a+b)=0$$
$$\therefore a+b=0$$

21. **(C)**

Given $\alpha_1-\beta_1=\alpha_2-\beta_2$

$$(\alpha_1-\beta_1)^2=(\alpha_2-\beta_2)^2$$
$$(\alpha_1+\beta_1)^2-4\alpha_1\beta_1=(\alpha_2+\beta_2)^2-4\alpha_2\beta_2$$
$$p^2-4q=q^2-4p$$
$$p^2-q^2+4(p-q)=0$$
$$(p-q)(p+q+4)=0$$
$$\therefore p+q=-4 \quad \because (p\neq a)$$

22. **(B)**

Let roots be $\alpha, 2\alpha$

Sum = $3\alpha=\dfrac{1-3a}{a^2-5a+3} \Rightarrow \alpha=\dfrac{1-3a}{3(a^2-5a+3)}$

Product = $\alpha\cdot 2\alpha=2\alpha^2=\dfrac{2}{a^2-5a+3}$

Solve.

23. **(D)**

$|\alpha-\beta|=1$

$(\alpha+\beta)^2-4\alpha\beta=1$

24. **(A)**

$|\alpha-\beta|<\sqrt{5}$

$(\alpha+\beta)^2-4\alpha\beta<5$

25. **(C)**

$D<0$

$4a^2-4(10-3a)<0$

$a^2+3a-10>0$

$a\in(-5,2)$

26. **(B)**

Given $\mathrm{D}=4(\mathrm{bc}+\mathrm{ad})^2-4(\mathrm{a}^2+\mathrm{b}^2)(\mathrm{c}^2+\mathrm{d}^2)=0$

$\therefore \mathrm{b}^2\cancel{\mathrm{c}^2}+\cancel{\mathrm{a}^2}\mathrm{d}^2+2\mathrm{abcd}-\mathrm{a}^2\mathrm{c}^2-\cancel{\mathrm{a}^2}\mathrm{d}^2-\mathrm{b}^2\cancel{\mathrm{c}^2}-\mathrm{b}^2\mathrm{d}^2=0$

$(\mathrm{ac}-\mathrm{bd})^2$

$\therefore \mathrm{ac}=\mathrm{bd}$

27. **(B)**

Given $\mathrm{D}=\mathrm{b}^2-4\mathrm{ac}<0$

If $\mathrm{b}=0,\mathrm{a}>0,\mathrm{c}>0$

Then $\mathrm{D}<0$

28. **(A)**

$D=0$

29. **(C)**

$D<0$

30. **(C)**

$D=0$

31. **(B)**

$\mathrm{D}<0$

$\mathrm{b}^2-4\mathrm{ac}<0$

$\mathrm{b}^2<4\mathrm{ac}$

32. **(C)**

$\mathrm{f}(1)<0$

$\mathrm{ax}^2+\mathrm{bx}+\mathrm{c}<0\ \ \forall\ \mathrm{x}\in\mathrm{R}$ $\qquad$ $\mathrm{c}<0$

33. **(B)**

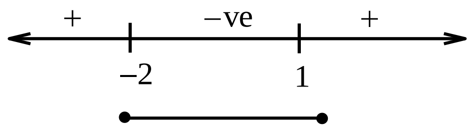
(i) $\left(\lambda^2+\lambda-2\right)x^2+\left(\lambda+2\right)x-1<0 \qquad \forall x \in R$

$\lambda^2+\lambda-2<0$

$\lambda^2+2\lambda-\lambda-2<0$

$\lambda\left(\lambda+2\right)-\left(\lambda+2\right)<0$

$\left(\lambda-1\right)\left(\lambda+2\right)<0$



$\lambda \in \left(-2,1\right)$ .... (i)

(ii) $\left(\lambda+2\right)^2+4\left(\lambda^2+\lambda-2\right)<0$

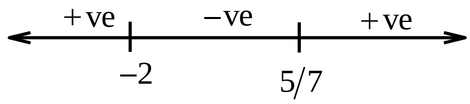
$\lambda^2+4\lambda+4+4\lambda^2+4\lambda-8<0$

$5\lambda^2+8\lambda-4<0$

$5\lambda^2+10\lambda-2\lambda-4<0$

$5\lambda\left(\lambda+2\right)-2\left(\lambda+2\right)<0$

$\left(5\lambda-2\right)\left(\lambda+2\right)<0$



$-2<\lambda<5/2$

From (i) and (ii)

$\lambda \in \left(-2,1\right)$

34. **(B)**

Product of roots <0 then roots are real and opposite in sign

35. **(C)**

$D \geq 0$

36. **(D)**

α, β, γ are roots of $x^3-19x-1=0$

$\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ are roots of $x^3+19x^2-1=0$

(Put $x=\frac{1}{x}$)

37. **(C)**

$y=x^2+2$

$\therefore x=\sqrt{y-2}$

Replace x by $\sqrt{x-2}$ in the given equation we get revered equation whose roots are

$\alpha^2+2 \, \& \, \beta^2+2$

$2(x-2)-3\sqrt{x-2}-6=0$

$2x-4-6=3\sqrt{x-2}$

$2x-10=3\sqrt{x-2}$

$4x^2+100-40x=9(x-2)$

$4x^2-40x+100=9x-18$

$4x^2-49x+118=0$

38. **(A)**

$y=\dfrac{x-1}{x+1} \qquad \therefore yx+y=x-1$

$y+1=x(1-y)$

$x=\dfrac{1+y}{1-y}$

Replace x by $\dfrac{1+x}{1-x}$ we get revered equation

$\left(\dfrac{1+x}{1-x}\right)^2-2\dfrac{(1+x)}{(1-x)}+3=0$

$(1+x)^2-2(1+x)(1-x)+3(1-x)^2=0$

$x^2+1+2x-2\left(1-x^2\right)+3\left(1+x^2-2x\right)=0$

$x^2+1+2x-2+2x^2+3+3x^2-6x=0$

$6x^2-4x+2=0$

$3x^2-2x+1=0$

39. **(B)**

$x=-\dfrac{1}{x}$

40. **(B)**

$3\alpha+2, 3\beta+2$

41. **(A)**

Replace x by $1/x$ we get

$cx^2+bx+a=0$

42. **(B)**

Given α, β be the roots of

$ax^2+bx+c=0$

$ax^2+bx+c=0$ .... (i)

$a\beta^2+b\beta+c=0$ .... (ii)

From (i) & (ii) $2\alpha, 2\beta$ satisfy equation

$ax^2+2bx+4c=0$

43. **(A)**

$\Rightarrow x^3+3x^2+2x+4=0$

$\Rightarrow \alpha\beta\gamma=-3$

$\Rightarrow \sum\alpha\beta=2$

$\Rightarrow \alpha\beta\gamma=-4$

$\Rightarrow x^3-(2\alpha+2\beta+2\gamma)x^2+\left(\sum\alpha 2\beta\right)x-(2\alpha.2\beta.2\gamma)=0$

$\Rightarrow x^3+6x^2+8x+32=0$

44. **(A)**

$\alpha+\beta,\beta+\gamma,\gamma+\alpha$

$\alpha+\beta+\gamma=p$

Hence roots of the equations are $p-\alpha,p-\beta,p-\gamma$

$x=p-\alpha\Rightarrow\alpha=p-x$

Equation is

$(p-x)^3-p(p-x)^2+q(p-x)-r=0$

$\Rightarrow x^3-3px^2+3p^2x-p^3+p^3-2xp^2+px^2-qp+qx+r=0$

$\Rightarrow x^3-2px^2+2(p^2+q)x+r-pq=0$

45. **(A)**

$x=\dfrac{1+\alpha}{1-\alpha}$

$\Rightarrow \alpha=\dfrac{x-1}{x+1}$

$\left(\dfrac{x-1}{x+1}\right)^3-\left(\dfrac{x-1}{x+1}\right)-1=0$

$\Rightarrow (x-1)^3-(x-1)(x+1)^2-(x+1)^3=0$

$\Rightarrow -6x^2-2-x^3-x^2+x+1=0$

$\Rightarrow x^3+7x^2-x+1=0$

$\Rightarrow x^3+7x^2-x+1=0$

46. **(B)**

$(a_1b_2-a_2b_1)(b_1c_2-bc_1)=(c_1a_2-c_2a_1)^2$

47. **(B)**

$x^3-2x^2+2x-1=0$

$x^3-x^2-x^2+x+x-1=0$

$x^2(x-1)-x(x-1)+(x-1)=0$

$(x-1)(x^2-x+1)=0$ .... (i)

From (i) two common roots are imaginary

$$\frac{a}{1}=\frac{b}{-1}=\frac{a}{1}$$
$$a=-b$$
$$a+b=0$$

48. **(C)**

Use condition for one common root

49. **(D)**

$$4+2a+b=0$$
$$4+2c+d=0$$

Subtract $2(a-)c+b-d=0$

$$b-d=2(c-a)$$

50. **(A)**

Let α be common root

$$a\alpha^2+b\alpha+c=0$$
$$c\alpha^2+b\alpha+a=0$$

$$\frac{\alpha^2}{ab-bc}=\frac{\alpha}{c^2-a^2}=\frac{1}{ab-bc}$$
$$\alpha=\frac{(a-c)b}{(c-a)(c+a)}=\frac{-b}{a+c}$$
$$\alpha=\frac{(c-a)(c+a)}{-b(c-a)}=\frac{c+a}{-b}$$
$$\therefore\frac{-b}{a+c}=\frac{(c+a)}{-b}$$
$$(a+c)^2=b^2$$
$$(a+c)^2-b^2=0$$
$$(a+b+c)(a-b+c)=0$$

51. **(C)**

Given $\frac{a}{2}=\frac{b}{-3}=\frac{c}{4}$

$$\therefore 6a=-4b=3c$$

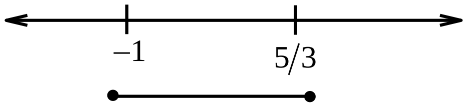
52. **(A)**

$$yx^2+yx+y=x^2+3x+1$$
$$(y-1)x^2+(y-3)x+y-1=0$$

For real x, $D\geq 0$

$$(y-3)^2-4(y-1)(y-1)\geq 0$$
$$(y-3)^2-(2y-2)^2\geq 0$$
$$(y-3+2y-2)(y-3-2y+2)\geq 0$$

$$(3y-5)(-y-1)\geq 0$$
$$(3y-5)(y+1)\leq 0$$



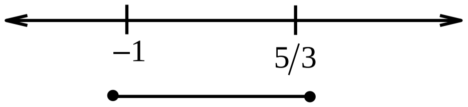
$$y\in[-1,5/3]$$

53. **(A)**

$$yx^2+yx+y=x^2+3x+1$$
$$(y-1)x^2+(y-3)x+y-1=0$$

For real x, $D\geq 0$

$$(y-3)^2-4(y-1)(y-1)\geq 0$$
$$(y-3)^2-(2y-2)^2\geq 0$$
$$(y-3+2y-2)(y-3-2y+2)\geq 0$$
$$(3y-5)(-y-1)\geq 0$$
$$(3y-5)(y+1)\leq 0$$



$$y\in[-1,5/3]$$

54. **(A)**

$$Kx^2+Kx+K=x^2-x+1$$
$$(K-1)x^2+(K+1)x+(K-1)=0$$
$$D\geq 0$$

55. **(B)**

$$y_{min}=\frac{-D}{4a}$$
$$=-\frac{(36+4.8.3)}{4\times -8}$$
$$=\frac{-\cancel{12}^{3}(3+8)}{-\cancel{4}\times 8}$$
$$=\frac{3\times 11}{8}=33/8$$

56. **(A)**

$$y=\frac{1}{4\left(x^2+2.\frac{1}{4}x+1\right)}$$

$$= \frac{1}{4\left(x^2 + 2.\frac{1}{4}x + \frac{1}{16} - \frac{1}{16}\right) + 1}$$

$$y = \frac{1}{4\left(x + \frac{1}{4}\right)^2 + \frac{3}{4}}$$

$$y_{maximum} = \frac{4}{3}$$

57. **(C)**

$f(x) = (x+2)^2 - 3$

$f(-4) = 1$

$f(x) \geq 1$

When $x \leq -4$

58. **(D)**

$$y = \frac{x^2 - x + c}{x^2 + x + 2c}$$

$yx^2 + xy + 2cy = x^2 - x + c$

$(y-1)x^2 + (y+1)x + (2y-1)c = 0$

For real x, $(y+1)^2 - 4(y-1)(2y-1)c \geq 0$

$y^2 + 2y + 1 - 4c(2y^2 - 3y + 1) \geq 0$

$(1-8c)y^2 + 2y(1+6c) + (1-4c) \geq 0$ ________(ii)

(ii) Is valid for all y if

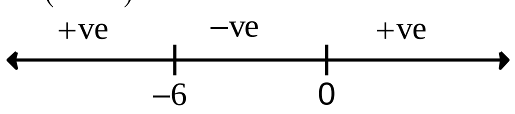
$D \leq 0$

$4(1+6C)^2 - 4(1-8C)(1-4C) \leq 0$

$1 + 36C^2 + 12C - 1 + 12C - 32C^2 \leq 0$

$4C^2 + 24C \leq 0$

$4C(C+6) \leq 0$



$C \in [-6, 0]$

59. **(A)**

$\alpha^2 + \beta^2 = (a-2)^2 + 2(a+1)$

$= a^2 - 2a + 6 = (a-1)^2 + 5$ is least when $a = 1$

60. **(B)**

Roots are $\sqrt{3} - \sqrt{2}, \sqrt{3} + \sqrt{2}, -\sqrt{3} - \sqrt{2}, -\sqrt{3} + \sqrt{2}$.

61. **(A)**

$\Rightarrow x^3 + ax + b = 0$

$\Rightarrow \alpha+\beta+\gamma=0$

$\Rightarrow \alpha\beta+\beta\gamma+\gamma\alpha=a$

$\Rightarrow \alpha\beta\gamma=-b$

$\Rightarrow \alpha^3+\beta^3+\gamma^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)+\gamma^3$

$=(-\gamma)\left((\alpha+\beta)^2-2\alpha\beta-\alpha\beta\right)+\gamma^3$

$=(-\gamma)(\gamma^2-3\alpha\beta)+\gamma^3$

$=+3\alpha\beta\gamma=-3b$

$\Rightarrow \alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$\Rightarrow \frac{\alpha^3+\beta^3+\gamma^3}{\alpha^2+\beta^2+\gamma^2}=\frac{-3b}{-2a}=\frac{3b}{2a}$

62. **(B)**

$\Rightarrow 4x^3-12x^2+11x+k=0$

Sum of roots $(a-d+a+a+d)=\frac{12}{4}=3$

$\Rightarrow a=1$

As a is root of equation so it will satisfy if.

$\Rightarrow 4(a)^3-12a^2+11a+k=0$

$\Rightarrow 4-12+11+k=0$

$\Rightarrow k-3$

63. **(B)**

$\Rightarrow x^3-11x^2+37x-35=0$

If root is $3+\sqrt{2}$, second root has to be $3-\sqrt{2}$

Let's take third root α

So, $(3+\sqrt{2})+(3-\sqrt{2})+\alpha=11$

$\Rightarrow \alpha=5$

64. **(A)**

α,β,γ are roots of $x^3-5x^2+5x-3=0$

$\alpha+\beta+\gamma=0$

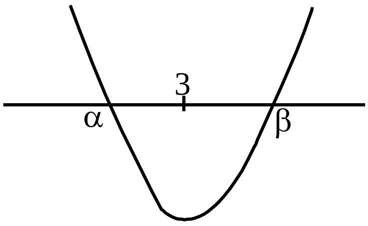
65. **(A)**

$D>0$

$(1-2k)^2-4(k^2-k-2)>0$

$4k^2-4k+1-4k^2+4k+8>0+\ 9>0$

$f(3)<0$



$9+3(1-2k)+(k^2-k-2)<0$

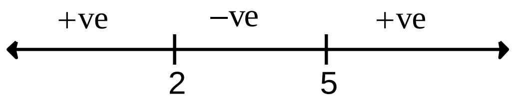
$9+3-6k+k^2-k-2<0$

$k^2-7k+10<0$

$k^2-2k-5k+10<0$

$k(k-2)-5(k-2)<0$

$(k-2)(k-5)<0$



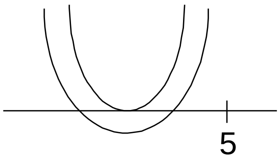
$k \in (2,5)$

66. **(C)**

$D \geq 0$

& $\frac{-b}{2a} < 5$

& $f(5) > 0$

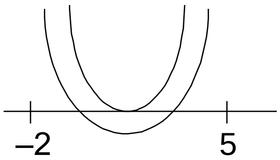


67. **(B)**

$D \geq 0$

$-2 < \frac{-b}{2a} < 4$

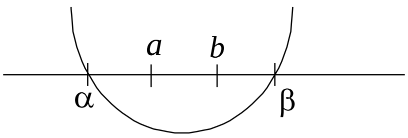
$f(-2) > 0,\ f(4) > 0$



68. **(D)**

$f(a) < 0$

$f(b) < 0$



$\alpha < a$ & $\beta < b$

1. **(1)**

$mx^2 - 2x + (2m-1) = 0$

Product = 3

$\frac{2m-1}{m} = 3$

$m = 1$

$m + 2 = 1$

2. **(3)**

$(k-2)x^2 - (k-4)x = 0$

$(k-2)x^2 - (k-2-2)x - 2 = 0$

$(k-2)x^2 - (k-2)x + 2x - 2 = 0$

$[(k-2)x + 2][x-1] = 0$

$\therefore\ x = \frac{-2}{k-2}$ or $x = 1$

Difference = 3, i.e.

$1 + \frac{2}{k-2} = 3$ or $\frac{-2}{k-2} - 1 = 3$

$k = 3k - 6 \qquad 4k - 8 = -2$

$k = 3 \qquad k = \frac{8}{4}$

The integral value of k will be 3

3. **(1)**

$(a^2 - 1)x^2 + (a-1)x + a^2 - 4a + 3 = 0$

For an quadratic to be an identity.

All coefficients should be 0.

$\therefore\ a^2 - 1 = 0, \quad a - 1 = 0 \quad \& \quad a^2 - 4a + 3 = 0$

$(a-1)(a+1) = 0, \quad a = 1 \quad \& \quad (a-1)(a-3) = 0$

$\therefore\ a = -1, 1, \quad a = 1 \quad \& \quad a = 1, 3$

The only common value which satisfies all 3 is $a = 1$.

4. **(1)**

$x - 3 = \sqrt{5}$

Squaring on both side

$x^2 - 6x + 9 = 5$

$x^2 - 6x = -4 \qquad ...(1)$

Square on both side

$x^4 - 12x^3 + 36x^2 = 16$

Adding 8 × (1) on both side.

$x^4 - 12x^3 + 36x^2 + 8x^2 - 48x = 16 - 32$

$x^4 - 12x^3 + 44x^2 - 48x = -16$

Add 17 on both side

$x^4 - 12x^3 + 44x^2 - 48x + 17 = 1$

5. **(4)**

Since both roots are common.

$\frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{5}{10} = \frac{2}{q}$

$q = 4$

6. **(15)**

Let common root $b \in \alpha$.

$\therefore\ \alpha^2 + a\alpha + 12 = 0 \qquad ...(1)$

$\alpha^2 + bd + 15 = 0 \qquad ...(2)$

$\alpha^2 + (a+b)\alpha + 36 = 0 \qquad ...(3)$

Let's consider (1) + (2) – (3)

$\therefore\ \alpha^2 - 9 = 0$

$\alpha = 3 \quad \because$ given α is positive

Put $\alpha = 3$ in (3)

$\therefore$ It $(a+b)3 + 36 = 0$

$a + b + 15 = 0$

$\therefore\ a + b + 30 = 15$

7. **(7)**

Given: $y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$x^2y + 3xy + 4y = x^2 - 3x + 4$

$x^2(y-1) + (3y+3)x + 4y - 4 = 0$

Since, x is real.

$b^2 - 4ac > 0$

$(3y+3)^2 - 4 \times (y-1) \times (4y-4) > 0$

$9y^2 + 18y + 9 - 16(y^2 - 2y + 1) > 0$

$-7y^2 + 50y - 7 \geq 0$

$7y^2 - 50y + 7 \leq 0$

$7y^2 - 49y - y + 7 \leq 0$

$7y(y-7) - 1(y-7) \leq 0$

$(7y-1)(y-7) \leq 0$

For y between $\left[\frac{1}{7}, 7\right]$, the above inequality holds.

$\therefore$ maximum value of $y = 7$.

8. **(6)**

Given roots of $x^2+2(k-3)x+9=0$

Lie between $(-6,1)$.

$\therefore$ at $x=-6$, quadratic > 0. $(\therefore\ a>0)$

i.e. $(36)+2(k-3)(-6)+9\geq 0$

$12(k-3)\leq 45$

$k-3\leq\frac{15}{4}$

$k\leq\frac{27}{4}$

Also at $x=1$, quadratic ≥ 0

$1+2(k-3)+9\geq 0$

$2(k-3)\geq -10$

$k-3\geq -5$

$k\geq -2$

$\therefore\ -2<k<\frac{27}{4}$

$\therefore$ Min value of $k=-2$.

9. **(7)**

10. **(0)**

$3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$

$3\left(x^2+\frac{1}{x^2}+2-2\right)-2\left(x+\frac{1}{x}\right)+5=0$

$3\left(\left(x+\frac{1}{x}\right)^2\right)-2\left(x+\frac{1}{x}\right)-1=0$

Let $x+\frac{1}{x}=t$

$\therefore\ 3t^2-2t-1=0$

$3t^2-3t+t-1=0$

$(3t+1)(t-1)=0$

$t=1,\quad t=-\frac{1}{3}$

$\therefore\ x+\frac{1}{x}=1$ or $x+\frac{1}{x}=-\frac{1}{3}$

$x^2-x+1=0$ or

$D=\sqrt{1-4}$ or $D=\sqrt{1-36}$

$D=\sqrt{-3}\qquad D=\sqrt{-35}$

For both the cases no real root is possible as $D<0$.

$\therefore$ No. of real roots = 0.

11. **(4)**

$S=\left\{x:x\in R\ \&\ \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10\right\}$

We have a set *S*, such that for real x

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10$

& we need to find n (*S*) = ?

$\sqrt{3}+\sqrt{2}=\left(\sqrt{3}-\sqrt{2}\right)^{-1}$ or $\sqrt{3}-\sqrt{2}=\dfrac{1}{\sqrt{3}+\sqrt{2}}$

$\therefore$ Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=t$

We will set $t+\dfrac{1}{t}=10$

$t^2-10t+1=0$

$t=\dfrac{10\pm\sqrt{96}}{2}$

$t=5+\sqrt{24}$ or $t=5-\sqrt{24}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\sqrt{3}+\sqrt{2}\right)^2$ or $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=\left(\sqrt{3}-\sqrt{2}\right)^2$

From here | or $x^2-4=-2$ $\left(\because \dfrac{1}{\sqrt{3}-\sqrt{2}}\right)=\sqrt{3}+2$

$x^2-4=2$ | $\therefore\ x^2=2$

$x=\pm\sqrt{6}$ | $x=\pm\sqrt{2}$

$\therefore$ x satisfying the conditions are $\sqrt{6},\sqrt{2},-\sqrt{2},-\sqrt{6}$.

$\therefore\ n(S)=4$

12. **(58)**

$ax^2-2bx+15=0$ has repeated root

i.e. $\alpha=r$

$\therefore\ 2\alpha=\dfrac{2b}{a}$ or $\dfrac{b}{a}=\alpha$

& $\alpha^2=\dfrac{15}{a}\Rightarrow\dfrac{b^2}{a^2}=\dfrac{15}{a}\Rightarrow\alpha=\dfrac{15}{b}$

Now, equation $x^2-2bx+21=0$ has roots α & β

$\alpha+\beta=26$ $\alpha\beta=21$.

Since α is a root.

$\therefore\ \left(\dfrac{15}{b}\right)^2-2b\times\dfrac{15}{b}+21=0$

$\left(\dfrac{15}{b}\right)^2=9$

$b^2=25$

$\therefore\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\alpha^2+\beta^2=4b^2-42=58$

13. **(24)**

$3x^2+\lambda x-1=0 \Rightarrow \alpha+\beta=-\frac{\lambda}{3},\ \alpha\beta=-\frac{1}{3}\ \&\ \frac{1}{\alpha^2}+\frac{1}{\beta^2}=15$

$\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}=15$

$(\alpha+\beta)^2-2\alpha\beta=\frac{5}{3}$

$(\alpha+\beta)^2=\frac{5}{3},-\frac{2}{3}$

$\lambda^2=9$

$\lambda=\pm 3$

Now, $6(\alpha^3+\beta^3)^2=6(\alpha+\beta)^2\left((\alpha+\beta)^2-3\alpha\beta\right)^2$

$=6\cdot 1\cdot(1+1)^2$

$=24$.

JEE Main : PYQ

1. **(C)**

$x^2+5^{\frac{1}{2}}=-(20)^{\frac{1}{4}}x \Rightarrow \left(x^2+\sqrt{5}\right)^2=\sqrt{20}x^2$

$\Rightarrow x^4+5+2\sqrt{5}x^2=2\sqrt{5}x^2 \Rightarrow x^4=-5 \Rightarrow x^8=25$

So, $\alpha^8+\beta^8=50$

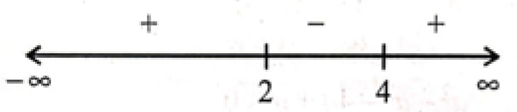
2. **(B)**

$x^2-2(3k-1)x+8k^2-7>0, \forall x\in R$

$\Rightarrow D<0 \Rightarrow 4(3k-1)^2-4\cdot 1\cdot\left(8k^2-7\right)<0$

$\Rightarrow 9k^2-6k+1-8k^2+7<0 \Rightarrow k^2-6k+8<0$

$\Rightarrow (k-2)(k-4)<0$



$\Rightarrow k\in(2,4)$; So, $k=3$

3. **(C)**

As, $\left(\alpha^2+\sqrt{3}\right)=-(3)^{1/4}\cdot\alpha$

$\Rightarrow \left(\alpha^4+2\sqrt{3}\alpha^2+3\right)=\sqrt{3}\alpha^2$ (On squaring)

$\therefore \left(\alpha^4+3\right)=(-)\sqrt{3}\alpha^2$

$\Rightarrow \alpha^8+6\alpha^2+9=3\alpha^4$ (Again squaring)

$\therefore \alpha^8+3\alpha^4+9=0$

$\Rightarrow \alpha^8=-9-3\alpha^4$ (Multiple by α^4)

So, $\alpha^{12}=-9\alpha^4-2\alpha^8$

$\therefore\ \alpha^{12} = -9\alpha^4 - 3\left(-9 - 3\alpha^4\right)$

$\Rightarrow\ \alpha^{12} = -\cancel{9\alpha^4} + 27 + \cancel{9\alpha^4}$

Hence, $\alpha^{12} = (27)^2$

$\Rightarrow\ \left(\alpha^{12}\right)^8 = (27)^8$

$\Rightarrow\ \alpha^{96} = (3)^{24}$

Similarly, $\beta^{96} = (3)^{24}$

$\therefore\ \alpha^{96}\left(\alpha^{12} - 1\right) + \beta^{96}\left(\beta^{12} - 1\right) = (3)^{24} \times 52$

4. **(D)**

Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \ldots}}}$

$\Rightarrow\ x = 3 + \cfrac{1}{4 + \cfrac{1}{x}}\ \Rightarrow\ (x-3)(4x+1) = x$

$\Rightarrow\ 4x^2 - 11x - 3 = x \Rightarrow\ 4x^2 - 12x - 3 = 0$

$4\left(x - \frac{3}{2}\right)^2 = 12\ \Rightarrow\ x = \sqrt{3} + \frac{3}{2}\left(\sqrt{3} + \frac{3}{2}\text{ rejected}\right)$

5. **(A)**

$\because\ \alpha + \beta = 64, \alpha\beta = 256$

$\frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}} = \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{\left(2^8\right)^{5/8}} = \frac{64}{32} = 2$

6. **(B)**

Let α and β be the roots of the given quadratic equation,

$4x^2 + 2x - 1 = 0 \qquad \ldots(\text{i})$

Then, $\alpha + \beta = -\frac{1}{2}\ \Rightarrow\ -1 = 2\alpha + 2\beta$ and $4\alpha^2 + 2\alpha - 1 = 0$ [$\because \alpha$ is root of eq. (i)]

$\Rightarrow\ 4\alpha^2 + 2\alpha + 2\alpha + 2\beta = 0\ \Rightarrow\ \beta = -2\alpha(\alpha + 1)$

7. **(D)**

Let α and β be the roots of the quadratic equation

$7x^2 - 3x - 2 = 0$

$\therefore\ \alpha + \beta = \frac{3}{7},\ \alpha\beta = \frac{-2}{7}$

Now, $\frac{\alpha}{1 - \alpha^2} + \frac{\beta}{1 - \beta^2}$

$= \frac{\alpha - \alpha\beta(\alpha + \beta) + \beta}{1 - \left(\alpha^2 + \beta^2\right) + (\alpha\beta)^2}$

$$=\frac{(\alpha+\beta)-\alpha\beta(\alpha+\beta)}{1-(\alpha+\beta)^2+2\alpha\beta+(\alpha\beta)^2}$$

$$=\frac{\frac{3}{7}+\frac{2}{7}\times\frac{3}{7}}{1-\frac{9}{49}+2\times\frac{-2}{7}+\frac{4}{49}}=\frac{27}{16}$$

8. **(A)**

$ax^2-2bx+5=0$

If α and β are roots of equation, then sum of roots

$2a=\frac{2b}{a}\Rightarrow\alpha=\frac{b}{a}$ and product of roots $=\alpha^2=\frac{5}{a}\Rightarrow\frac{b^2}{a^2}=\frac{5}{a}$

$\Rightarrow b^2=5a\quad(\alpha\neq 0)\qquad ...(i)$

For $x^2-2bx-10=0$

$\alpha+\beta=2b\qquad ...(ii)$

and $\alpha\beta=-10\qquad ...(iii)$

$\alpha=\frac{b}{a}$ is also root of $x^2-2bx-10=0$

$\Rightarrow b^2-2ab^2-10a^2=0$

By equation (i) $\Rightarrow 5a-10a^2-10a^2=0$

$\Rightarrow 20a^2=5a$

$\Rightarrow a=\frac{1}{4}$ and $b^2=\frac{5}{4}$

$\alpha^2=20$ and $\beta^2=5$

Now, $\alpha^2+\beta^2=5+20=25$

9. **(D)**

$\alpha\cdot\beta=2$ and $\alpha+\beta=-p$ also $\frac{1}{\alpha}+\frac{1}{\beta}=-q$

$\Rightarrow p=2q$

Now, $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$$=\left[\alpha\beta+\frac{1}{\alpha\beta}-\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right]\left[\alpha\beta+\frac{1}{\alpha\beta}+2\right]$$

$$=\frac{9}{2}\left[\frac{5}{2}-\frac{\alpha^2+\beta^2}{2}\right]=\frac{9}{4}\left[5-\left(p^2-4\right)\right]$$

$$=\frac{9}{4}\left(9-p^2\right)\quad\left[\because\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta\right]$$

10. **(C)**

The given quadratic equation is

$(\lambda^2+1)x^2-4\lambda x+2=0$

$\because$ One root is in the interval (0, 1)

$\therefore\ f(0)f(1)\le 0$.

$\Rightarrow\ 2\left(\lambda^2+1-4\lambda+2\right)\le 0$

$\Rightarrow\ 2\left(\lambda^2-4\lambda+3\right)\le 0$

$(\lambda-1)(\lambda-3)\le 0\ \Rightarrow\ \lambda\in[1,3]$

But at $\lambda=1$, both roots are 1 so $\lambda\ne 1$

$\therefore\ \lambda\in(1,3]$

11. **(C)**

Since, α and β are the roots of the equation $5x^2+6x-2=0$

Then, $5\alpha^2+6\alpha-2=0, 5\beta^2+6\beta-2=0$

$5\alpha^2+6\alpha=2$

$$5S_6+6S_5=5\left(\alpha^6+\beta^6\right)+6\left(\alpha^5+\beta^5\right)$$

$$=\left(5\alpha^4+6\alpha^5\right)+\left(5\beta^6+6\beta^5\right)$$

$$=\alpha^4\left(5\alpha^2+6\alpha\right)+\beta^4\left(5\beta^2+6\beta\right)$$

$$=2\left(\alpha^4+\beta^4\right)=2S_4$$

12. **(D)**

Since, $2-\sqrt{3}$ is a root of the quadratic equation $x^2+px+q=0$

$\therefore\ 2+\sqrt{3}$ is the other root

$$\Rightarrow\ x^2+px+q=\left[x-\left(2-\sqrt{3}\right)\left[x-\left(2+\sqrt{3}\right)\right]\right.$$

$$=x^2-\left(2+\sqrt{3}\right)x-\left(2-\sqrt{3}\right)x+\left(2^2-\left(\sqrt{3}\right)^2\right)=x^2-4x+1$$

Now, by comparing $p=-4,\ q=1$

$\Rightarrow\ p^2-4q-12=16-4-12=0$

13. **(C)**

Sum of roots $=\dfrac{3}{m^2+1}$

$\because$ sum of roots is greatest, $\therefore\ m=0$

Hence equation becomes $x^2-3x+1=0$

Now, $\alpha+\beta=3,\ \alpha\beta=1\Rightarrow\ |-\alpha-\beta|=\sqrt{5}$

$$\left|\alpha^3-\beta^3\right|=\left|(\alpha-\beta)\left(\alpha^2+\beta^2+\alpha\beta\right)\right|=\sqrt{5}(9-1)=8\sqrt{5}$$

14. **(B)**

Let roots of the quadratic equation are α,β.

Given, $\lambda=\dfrac{\alpha}{\beta}$ and $\lambda+\dfrac{1}{\lambda}=1\Rightarrow\ \dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=1$

$$\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=1 \qquad ...(i)$$

The quadratic equation is, $3mh2x^2+m(m-4)x+2=0$

$\therefore\ \alpha+\beta=\frac{m(4-m)}{3m^2}=\frac{4-m}{3m}$ and $\alpha\beta=\frac{2}{3m^2}$

Put these values in eq. (i),

$$\frac{\left(\frac{4-m}{3m}\right)^2}{\frac{2}{3m^2}}=3 \Rightarrow (m-4)^2=18 \Rightarrow m=4\pm\sqrt{18}$$

Therefore, least value is $4-\sqrt{18}=4-3\sqrt{2}$

15. **(D)**

Let α and β be the roots of the equation,

$81x^2+kx+256=0$

Given, $(\alpha)^{\frac{1}{3}}=\beta \Rightarrow \alpha=\beta^3$

$\because$ Product of the roots $=\frac{256}{81}$

$\therefore\ (\alpha)(\beta)=\frac{256}{81}$

$\Rightarrow\ \beta^4=\left(\frac{4}{3}\right)^4 \Rightarrow \beta=\frac{4}{3} \Rightarrow \alpha=\frac{64}{27}$

$\because$ Sum of the roots $=-\frac{k}{81}$

$\therefore\ \alpha+\beta=-\frac{k}{81} \Rightarrow \frac{4}{3}+\frac{64}{27}=-\frac{k}{81}$

$\Rightarrow\ k=-300$

16. **(D)**

Consider the quadratic equation

$(c-5)x^2-2cx+(c-4)=0$

Now, $f(0).f(3)>0$ and $f(0).f(2)<0$

$\Rightarrow\ (c-4)(4c-49)>0$ and $(c-4)(c-24)<0$

$\Rightarrow\ c\in(-\infty,4)\cup\left(\frac{49}{4},\infty\right)$ and $c\in(4,24)$

$\Rightarrow\ c\in\left(\frac{49}{4},24\right)$

Integral values in the interval $\left(\frac{49}{4},24\right)$ are 13, 14, …., 23.

$\therefore\ S=\{13,14,...,23\}$

17. **(D)**

The given quadratic equation is

$x^2+(3-\lambda)x+2=\lambda$

Sum of roots $=\alpha+\beta=\lambda-3$

Product of roots $=\alpha\beta=2-\lambda$

$\alpha+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(\lambda-3)^2-2(2-\lambda)$

$=\lambda^2-4\lambda+5$

$=(\lambda-2)^2+1$

For least $(\alpha^2+\beta^2)\lambda=2$.

18. (A)

The roots of $6x^2-11x+\alpha=0$ are rational numbers.

$\therefore$ Discriminant D must be perfect square number.

$D=(-11)^2-4\cdot6\cdot\alpha$

$=121-24\alpha$ must be a perfect square

Hence, possible values for α are $\alpha=3,4,5$.

$\therefore$ 3 positive integral values are possible.

19. (B)

Given quadratic equation is : $x^2-mx+4=0$

Both the roots are real and distinct.

So, discriminant $B^2-4AC>0$.

$\therefore\ m^2-4\cdot1\cdot4>0$

$\therefore\ (m-4)(m+4)>0$

$\therefore\ m\in(-\infty,-4)\cup(4,\infty)$ ...(i)

Since, both roots lies in [1, 5]

$\therefore\ -\frac{-m}{2}\in(1,5)$

$\Rightarrow\ m\in(2,10)$ ...(ii)

And $1\cdot(1-m+4)>0\ \Rightarrow\ m<5$

$\therefore\ m\in(-\infty,5)$ ...(iii)

And $1\cdot(25-5m+4)>0\ \Rightarrow\ m<\frac{29}{5}$

$\therefore\ m\in\left(-\infty,\frac{29}{5}\right)$ ...(iv)

From (i), (ii), (iii) and (iv), $m\in(4,5)$

20. (B)

$\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$

$\frac{x+p+x+q}{(x+p)(x+q)}=\frac{1}{r}$

$(2x+p+q)r=x^2+px+qx+pq$

$x^2+(p+q-2r)x+pq-pr-qr=0$

Let α and β be the roots.

$\therefore\ \alpha+\beta=-(p+q-2r)$ ...(i)

& $\alpha\beta=pq-pr-qr$ ...(ii)

$\because\ \alpha=-\beta$ (given)

$\therefore$ in eq. (i), we get

$\Rightarrow\ -(p+q-2r)=0$ ...(iii)

Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(-(p+q-2r))^2-2(pq-pr-qr)$ ...(from (i) and (ii))

$=p^2+q^2+4r^2+2pq-4pr-4qr-2pq+2pr+2qr$

$=p^2+q^2+4r^2-2pr-2qr$

$=p^2+q^2+2r(2r-p-q)$ ...(from (iii))

$=p^2+q^2$

21. **(B)**

Let, the roots of the equation, $x^2+(2-\lambda)x+(10-\lambda)=0$ are α and β.

Also roots of the given equation are

$$\frac{\lambda-2\pm\sqrt{4-4\lambda+\lambda^2-40+4\lambda}}{2}=\frac{\lambda-2\pm\sqrt{\lambda^2-36}}{2}$$

The magnitude of the difference of the roots is $\left|\sqrt{\lambda^2-36}\right|$

So, $\alpha^3+\beta^3=\dfrac{(\lambda-2)^3}{4}+\dfrac{3(\lambda-2)(\lambda^2-36)}{4}$

$=\dfrac{(\lambda-2)(4\lambda^2-4\lambda-104)}{4}=(\lambda-2)(\lambda^2-l-26)=f(\lambda)$

As $f(\lambda)$ attains its minimum value at $\lambda=4$.

Therefore, the magnitude of the difference of the roots is $\left|i\sqrt{20}\right|=2\sqrt{5}$

22. **(D)**

We have

$f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2,\ (x\in R)$

$\Rightarrow\ f(x)=(x+b)^2+2c^2-b^2$ and $g(x)=-(x+c)^2+b^2+c^2$

Now, $f_{\min}=2c^2-b^2$ and $g_{\max}=b^2+c^2$

Given: $\min f(x)>\max g(x)$

$\Rightarrow\ 2c^2-b^2>b^2+c^2$

$\Rightarrow\ c^2>2b^2$

$\Rightarrow\ |c|>|b|\sqrt{2}$

$\Rightarrow\ \dfrac{|c|}{|b|}>\sqrt{2}\ \Rightarrow\ \left|\dfrac{c}{b}\right|<\sqrt{2}$

$\Rightarrow\ \left|\dfrac{c}{b}\right|\in(\sqrt{2},\infty)$

23. **(D)**

If a and -1 are the roots of the polynomial, then we get

$f(x)=x^2+(1-a)x-a$.

$\therefore\ f(1)=2-2a$ and $f(2)=6-3a$

As, $f(1)++f(2)=0 \Rightarrow 2-2a+6-3a=0 \Rightarrow a=\frac{8}{5}$

Therefore, the other root is $\frac{8}{5}$

24. **(C)**

$x^2+bx-1=0$ ...(1) and

$x^2+x+b=0$ ...(2)

Subtracting this to get common root as $x=\frac{b+1}{b-1}$

Substituting the common root in equation (2)

$\left(\frac{b+1}{b-1}\right)^2+\left(\frac{b+1}{b-1}\right)+b=0$

$(b+1)^2+(b+1)(b-1)+b(b-1)^2=0$

$b^2+2b+1+b^2-1+b\left(b^2+1-2b\right)=0$

$b^3+3b=0$

$b=0,\ \pm\sqrt{3}i$

0 is not possible.

$|b|=\sqrt{3}$

25. **(B)**

$(a-1)\left(x^4+x^2+1\right)+(a+1)\left(x^2+x+1\right)^2=0$

$\Rightarrow\ (a-1)\left(x^2+x+1\right)\left(x^2-x+1\right)+(a+1)\left(x^2+x+1\right)^2=0$

$\Rightarrow\ \left(x^2+x+1\right)\left[(a-1)\left(x^2-x+1\right)+(a+1)\left(x^2+x+1\right)\right]=0$

$\Rightarrow\ \left(x^2+x+1\right)\left(ax^2+x+a\right)=0$

For roots to be distinct and real, $a\neq 0$ and $1-4a^2>0$

$\Rightarrow\ a\neq 0$ and $a^2<\frac{1}{4} \Rightarrow a\in\left(-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right)$

26. **(B)**

$\alpha=2+3i;\ \beta=2-3i,\ \gamma=?$

$\alpha\beta\gamma=\frac{13}{2}$ $\left[\because \text{product of roots}=-\frac{d}{a}\right]$

$\Rightarrow\ (4+9)\gamma=\frac{13}{2} \Rightarrow \gamma=\frac{1}{2}$

27. **(A)**

Let $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ be the roots of $ax^2+bx+1=0$

$$\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}=\left(\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}}\right)=-\frac{b}{a}$$

$$\frac{1}{\sqrt{\alpha}\sqrt{\beta}}=\frac{1}{a} \Rightarrow a=\sqrt{\alpha\beta}$$

$$b=-\left(\sqrt{\alpha}+\sqrt{\beta}\right)$$

$$x\left(x+b^3\right)+\left(a^3-3abx\right)=0$$

$$\Rightarrow \quad x^2+\left(b^3-3ab\right)x+a^3=0$$

Putting values of a and b, we get

$$x^2+\left[\left(-\sqrt{a}+\sqrt{\beta}\right)^3+3\left(\sqrt{\alpha\beta}\right)\left(\sqrt{\alpha}+\sqrt{\beta}\right)\right]+\left(\alpha\beta\right)^{3/2}=0$$

$$\Rightarrow \quad x^2-\left[\alpha^{3/2}+\beta^{3/2}+3\sqrt{\alpha\beta}\left(\sqrt{\alpha}+\sqrt{\beta}\right)-x^2--3\sqrt{\alpha\beta}\left(\sqrt{\alpha}+\sqrt{\beta}\right)\right]x+\left(\alpha\beta\right)^{3/2}=0$$

$$\Rightarrow \quad x^2-\left(\alpha^{3/2}+\beta^{3/2}\right)x+\alpha^{3/2}\beta^{3/2}=0$$

Roots of this equation are $\alpha^{3/2}, \beta^{3/2}$

28. **(B)**

Given $\alpha^3+\beta^3=-p$ and $\alpha\beta=q$

Let $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ be the roots of required quadratic equation

So, $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{-p}{q}$ and $\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=q$

Hence, required quadratic equation is

$$x^2-\left(\frac{-p}{q}\right)x+q=0$$

$$\Rightarrow \quad x^2+\frac{p}{q}x+q=0 \quad \Rightarrow \quad qx^2+px+q^2=0$$

29. **(C)**

Given quadratic equation is

$$x^2+px+\frac{3p}{4}=0$$

So, $\alpha+\beta=-p, \alpha\beta=\frac{3p}{4}$

Now, given $|\alpha-\beta|=\sqrt{10}$

$$\Rightarrow \quad \alpha-\beta=\pm\sqrt{10}$$

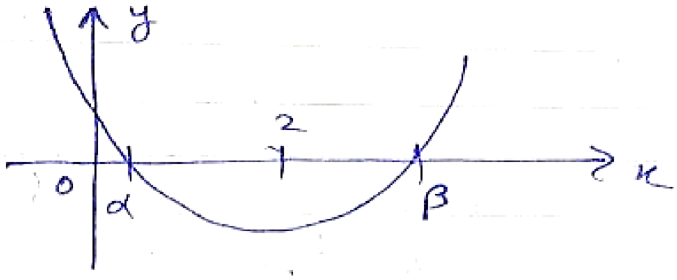
$$\Rightarrow \quad \left(\alpha-\beta^2\right)=10 \Rightarrow \alpha^2+\beta^2-2\alpha\beta=10$$

$$\Rightarrow \quad \left(\alpha+\beta\right)^2-4\alpha\beta=10$$

$$\Rightarrow \quad p^2-4\times\frac{3p}{4}=10 \Rightarrow p^2-3p-10=0$$

$\Rightarrow \quad p = -2, 5 \Rightarrow p \in \{-2, 5\}$

30. **(C)**



Quadrate equation is: $x^2 - (a + 1)x + a^2 + a - 8 = 0$
One root exceeds 2 and other is lesser than
$\Rightarrow$ 2 lies between the roots.
A rough diagram it as shown below.
Where α and β are its roots.
$\Rightarrow$ For 2 to lie.
between roots.
The only condition is
$f(2) < 0$
$\Rightarrow 2^2 - (a + 1)(2) + a^2 + a - 8 < 0$
$\Rightarrow 4 - 2a - 2 + a^2 + a - 8 < 0$
$\Rightarrow a^2 - a - 6 < 0$
$\Rightarrow a^2 - 3a + 2a - 6 < 0$
$\Rightarrow a(a - 3) + 2(a - 3) < 0$
$\Rightarrow (a + 2)(a - 3) < 0$
$\Rightarrow$ using sign reheme method.
$a \in (-2, 3)$
i.e. $-2 < a < 3$

31. **(324)**

$x^2 - x - 1 = 0$ roots = α, β

$\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^{n+1} = \alpha^n + \alpha^{n-1}$

$\beta^2 - \beta - 1 = 0 \Rightarrow \beta^{n+1} = \beta^n + \beta^{n-1}$

$+$

$P_{n+1} = P_n + P_{n-1}$

$29 = P_n + 11$

$P_n = 18$

$P_n^2 = 324$

32. **(66)**

$$\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$$

$x \in R - \{1, 2\}$

$\Rightarrow k(2x - 4 - x + 1) = 2(x^2 - 3x + 2)$

$\Rightarrow k(x - 3) = 2(x^3 - 3x + 2)$

For $x \neq 3,\ k = 1\left(x - 3 + \frac{2}{x-3} + 3\right)$

$x - 3 + \frac{2}{x-3} \geq 2\sqrt{2}, \forall\, x > 3\ \&\ x - 3 + \frac{2}{x-3} \leq -2\sqrt{2}, \forall\, x < -3$

$\Rightarrow \left(x - 3 + \frac{2}{x-3} + 3\right) \in \left(-\infty, 6 - 4\sqrt{2}\right] \cup \left[6 + 4\sqrt{2}, \infty\right)$

For no real roots $k \in \left(6 - 4\sqrt{2}, 6 + 4\sqrt{2}\right) - \{0\}$

Integral $k \in \{1, 2, ..., 11\}$

Sum of $k = 66$

33. (18)

Since α is root of both the quadratic equations,

$3\alpha^2 - 10\alpha + 27\lambda = 0 \quad ...(i)$

$\alpha^2 - \alpha + 2\lambda = 0 \quad ...(ii)$

On solving equations (i) and (ii)

$-7\alpha + 21\lambda = 0 \Rightarrow \alpha = 3\lambda$

Put $\alpha = 3\lambda$ in equation (i), we get

$9\lambda^2 - 3\lambda + 2\lambda = 0$

$9\lambda^2 = \lambda \Rightarrow \lambda = \frac{1}{9}$ as $\lambda \neq 0$

Now $\alpha = 3\lambda \Rightarrow \lambda = \frac{1}{3}$

Now sum of roots

$\alpha + \beta = 1 \Rightarrow \beta = \frac{2}{3}$

$\alpha + \gamma = \frac{10}{3} \Rightarrow \gamma = 3$

$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3} \times 3}{\frac{1}{9}} = 18$

34. (2)

Let $e^x = t, (t > 0)$

$t^4 - t^3 - 4t^2 - t + 1 = 0$

$\Rightarrow t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0$

$\Rightarrow \left(t^2 + \frac{1}{t^2}\right) - \left(t^3 + t\right) - 4 = 0$

$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 6 = 0$

Let $t + \frac{1}{t} = u \quad (u > 2)$

$\Rightarrow u^2 - u - 6 = 0$

$\Rightarrow (u - 3)(u + 2) = 0$

$\Rightarrow\ u=3,-1$ (rejected)

$\Rightarrow\ u=3$

Since, $u=t+\dfrac{1}{t}$

$\Rightarrow\ t+\dfrac{1}{t}=3\ \Rightarrow\ t^2-3t+1=0$

$\Rightarrow\ t=\dfrac{3\pm\sqrt{5}}{2}=e^x$

$\Rightarrow\ x=\ln\dfrac{3+\sqrt{5}}{2},\ \ln\dfrac{3-\sqrt{5}}{2}$

So, the number of real rots is 2.

35. **(1)**

$x^2+5\sqrt{2}x+10=0\ \&\ P_n=\alpha^n-\beta^n$ (Given)

Now $\dfrac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}=\dfrac{P_{17}\left(P_{20}+5\sqrt{2}P_{19}\right)}{P_{18}\left(P_{19}+5\sqrt{2}P_{18}\right)}$

$$\frac{P_{17}\left(\alpha^{20}-\beta^{20}+5\sqrt{2}\left(\alpha^{19}-\beta^{19}\right)\right)}{P_{18}\left(\alpha^{19}-\beta^{19}+5\sqrt{2}\left(\alpha^{18}-\beta^{18}\right)\right)}$$

$$\frac{P_{17}\left(\alpha^{19}\left(\alpha+5\sqrt{2}\right)-\beta^{19}\left(\beta+5\sqrt{2}\right)\right)}{P_{18}\left(\alpha^{18}\left(\alpha+5\sqrt{2}\right)-\beta^{18}\left(\beta+5\sqrt{2}\right)\right)}$$

Since, $\alpha+5\sqrt{2}=-\dfrac{10}{\alpha}$ ...(1)

& $\beta+5\sqrt{2}=-\dfrac{10}{\beta}$ ...(2)

Now put there values in above expression

$=-\dfrac{10P_{17}P_{18}}{-10P_{18}P_{17}}=1$