SHM ~~MEDICAL~~ Oscillations (AKMP)

(Level II)

(I) $A = 4\,cm$ and $T = 4\,sec$

Eq. of SHM

$$x = A\cos\frac{2\pi t}{T}$$

when $x = \frac{A}{2}$

$$\cos\frac{2\pi t}{T} = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\frac{2\pi t}{T} = \frac{\pi}{3}$$

$$t = \frac{T}{6} = \frac{2}{3}\,sec.$$

(II) $A = 4\,cm.$ and $T = 1.2\,sec.$

Particle starts from $x = +4\,cm.$

Eq. of SHM

$$x = A\cos\frac{2\pi t}{T}$$

when $x = 2\,cm. = \frac{A}{2}$

$$\cos\frac{2\pi t}{T} = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\frac{2\pi t}{T} = \frac{\pi}{3}$$

$$t = \frac{T}{6} = 0.2\,sec.$$

Total time taken by particle to go from $x = 2\,cm.$ to $x = 4\,cm.$ and come back

$$\Delta t = 2t = 0.4\,sec$$

(III) $y = 3\sin\omega t + 4\cos\omega t$

let $y_1 = 3\sin\omega t$

and $y_2 = 4\cos\omega t$

phase diff $\varphi = \frac{\pi}{2}$

$$A_{net} = \sqrt{3^2 + 4^2} = 5$$

(IV) $y = 4\cos^2\frac{t}{2}\sin 1000t$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos^2\theta = \frac{\cos 2\theta + 1}{2}$$

$$y = 4 \times \frac{1}{2}(1 + \cos t)\sin 1000t$$

$$y = 2(1 + \cos t)\sin 1000t$$

$$y = 2\sin 1000t + 2\cos t\sin 1000t$$

This expression can be obtained by superposition of three SHM.

(V) $y = 10\sin 20\pi t$

Comparing with

$$y = A\sin\omega t$$

$$\omega = 20\pi$$

$$f = \frac{\omega}{2\pi} = 10\,Hz.$$

(VI) $y_1 = A\sin\omega t$

$$y_2 = \frac{A}{2}\sin\omega t + \frac{A}{2}\cos\omega t$$

Net amplitude of y_2

$$A_{net} = \sqrt{A_1^2 + A_2^2} = \frac{A}{2}\sqrt{2}$$

$$y_2 = \frac{A}{\sqrt{2}}$$

Ratio of amplitude of y_1 & y_2

$$\boxed{\frac{y_1}{y_2} = \sqrt{2}}$$

(7) General eq. of SHM

$$y = a \sin\left(\frac{2\pi t}{T} + \alpha\right)$$

where $a \rightarrow$ amplitude
$T \rightarrow$ time period
& $\alpha \rightarrow$ phase diff.

(8) $T = 0.1$ sec and $A = 2 \times 10^{-3}$ m.

Max. velocity

$$V_{max} = A\omega$$

$$V_{max} = A\left(\frac{2\pi}{T}\right)$$

$$V_{max} = 2 \times 10^{-3} \times \frac{2\pi}{0.1}$$

$$V_{max} = \frac{\pi}{25} \text{ m/s}$$

(9) $A = 4$ cm. and $V_{max} = 10$ cm/s

let eq. of SHM

$$X = A \sin \omega t$$

$$V = \frac{dx}{dt} = A\omega \cos \omega t$$

$$V = V_0 \cos \omega t$$

when $V = 5 \text{ cm/s} = \frac{V_0}{2}$

$$\cos \omega t = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\frac{2\pi t}{T} = \frac{\pi}{3}$$

$$\boxed{t = \frac{T}{6} \text{ sec.}}$$

$$X = A \sin \omega t$$

$$X = A \sin \frac{2\pi}{T} \times \frac{T}{6} =$$

$$X = A \sin \frac{\pi}{3} = \frac{A\sqrt{3}}{2} = 2\sqrt{3} \text{ cm}$$

Alternate

Velocity as a function of displacement

$$V = \omega\sqrt{A^2 - X^2} \quad \text{--- (1)}$$

$$V_{max} = A\omega = 10 \text{ cm/s}$$

$$\omega = \frac{10}{4} = 2.5$$

replacing in eq (1)

$$V = 2.5\sqrt{4^2 - x^2} = 5$$

$$\sqrt{4^2 - x^2} = 2$$

$$4^2 - x^2 = 2^2$$

$$x^2 = 4^2 - 2^2 = 12$$

$$\boxed{X = 2\sqrt{3} \text{ cm}}$$

(10) $V = \omega\sqrt{A^2 - X^2}$

$$V^2 = \omega^2(A^2 - x^2)$$

$$(10)^2 = \omega^2(A^2 - 4^2) \quad \text{--- (I)}$$

$$(8)^2 = \omega^2(A^2 - 5^2) \quad \text{--- (II)}$$

Eq (I) − (II)

$$36 = \omega^2(5^2 - 4^2)$$

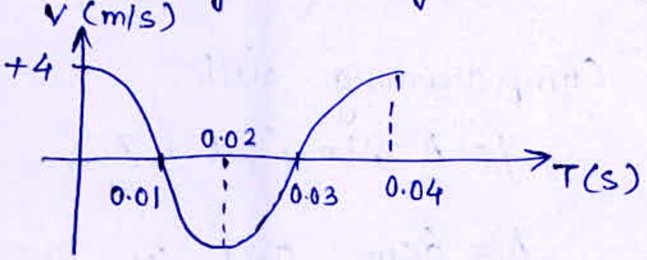
$$\omega^2 = \frac{36}{9} = 4$$

$$\omega = 2$$

Time period

$$T = \frac{2\pi}{\omega} = \pi$$

(11) Velocity time graph



Time period $T = 0.04$ sec

frequency $f = \frac{1}{T} = 25$

(12) Potential Energy

$$U = \frac{1}{2}kx^2$$

when $x = \frac{A}{2}$

$$U = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = 2.5\text{ J.}$$

$$= \frac{1}{8}kA^2 = 2.5\text{ J.}$$

Total Energy of SHM

$$E = \frac{1}{2}kA^2 = 10\text{ J.}$$

(13) Total Energy of SHM

$$E = \frac{1}{2}ka^2$$

Hence it depends on k & a.

(14) For two springs

$$k_1 = 1500\text{ N/m.}$$

$$k_2 = 3000\text{ N/m}$$

$$F = k_1x_1 = k_2x_2$$

Potential Energy

$$U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{F}{k}\right)^2$$

$$U = \frac{1}{2k}F^2$$

Ratio of potential Energy,

$$\frac{U_1}{U_2} = \frac{k_2}{k_1} = 2:1$$

(15) when displacement $= x$

P.E. $E_1 = \frac{1}{2}kx^2$ —— (I)

when displacement $= y$

$E_2 = \frac{1}{2}ky^2$ —— (II)

when displacement $= x+y$

$E = \frac{1}{2}k(x+y)^2$ —— (III)

$$E = \frac{1}{2}k(x^2+y^2+2xy)$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}ky^2 + \frac{1}{2}k(2xy)$$

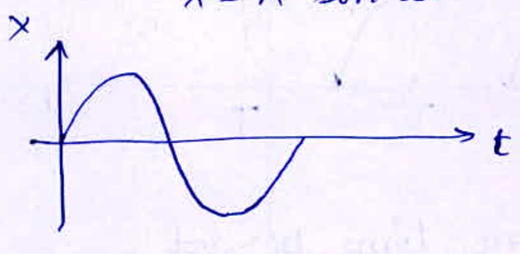
$$\boxed{E = E_1 + E_2 + 2\sqrt{E_1E_2}}$$

Using eq. (III)

$$\boxed{\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}}$$

(16) Eq. of SHM

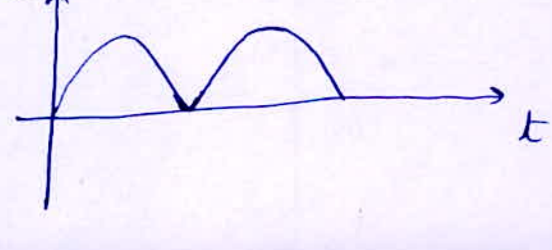
$$x = A\sin\omega t$$



Potential Energy

$$U = \frac{1}{2}kx^2$$

$$U = \frac{1}{2}kA^2\sin^2\omega t$$



(17) $A = 4\,cm.$

Total Energy $E = \frac{1}{2} k A^2$

when $K.E = P.E. = \frac{E}{2}$

$$\frac{1}{2} k x^2 = \frac{1}{2}\left(\frac{1}{2} k A^2\right)$$

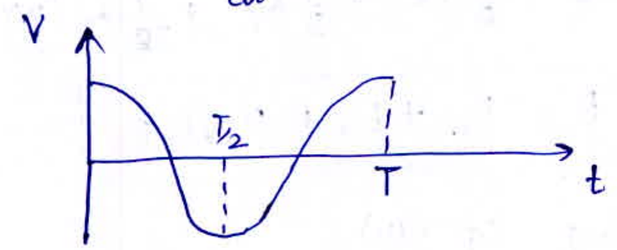
$$x^2 = \frac{1}{2} A^2$$

$$x = \frac{A}{\sqrt{2}} = 2\sqrt{2}\ cm.$$

(18) let eq. of SHM

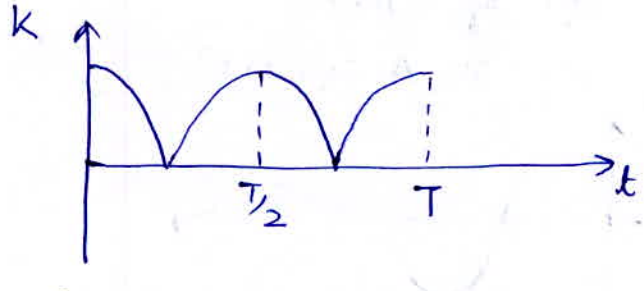
$$x = A \sin \omega t$$

$$V = \frac{dx}{dt} = A\omega \cos \omega t$$



$$K.E. = \frac{1}{2} m v^2$$

$$K.E = \frac{1}{2} m A^2 \omega^2 \cos^2 \omega t$$



In one time period

No. of cycles completed = 2

frequency = 2f

(19) $y = 6 \sin\left(100 t + \frac{\pi}{4}\right)$.

Comparing with

$$y = A \sin\left(\omega t + \frac{\pi}{4}\right)$$

$A = 6\,cm.$ and $\omega = 100$

$$V_{max} = A\omega$$

$$= (6 \times 10^{-2}) \times 100$$

$$V_{max} = 6\,m/s$$

$$K_{max} = \frac{1}{2} m (V_{max})^2$$

$$= \frac{1}{2} \times 1 (6)^2 = 18\,J.$$

(20) At mean position, K.E. is max.

$$(K.E.)_{max} = \frac{1}{2} k A^2 = 16\,J.$$

$$A = 25\,cm.$$

$$K = \frac{16 \times 2}{(0.25)^2} = 512\ N/m.$$

Time period $T = 2\pi \sqrt{\frac{m}{K}}$

$$T = 2\pi \sqrt{\frac{5.12}{512}} = \frac{\pi}{5}\ sec.$$

(22) If a tunnel is dug through center of earth

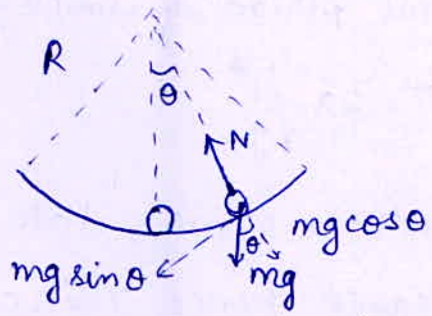
Time period of oscillation

$$T = 2\pi \sqrt{\frac{R}{g}} = 84.6\ min$$

where $R \rightarrow$ radius of earth

Time taken to go from one end to other $t = \frac{T}{2} = 42.3$

(21)



If sphere is displaced slightly from mean position

Restoring force

$$F = mg \sin\theta \quad \text{—— (1)}$$

where $\theta = \dfrac{x}{R-r}$

If 'θ' is very small

$\sin\theta \to \theta$

Replacing in eq (1)

$$F = mg\left(\frac{x}{R-r}\right)$$

$F \propto x$

Force const. $k = \dfrac{mg}{R-r}$

Time period $T = 2\pi\sqrt{\dfrac{m}{k}}$

$$\boxed{T = 2\pi\sqrt{\frac{R-r}{g}}}$$

(23) Time period of SHM of a liquid in U-tube

$$T = 2\pi\sqrt{\frac{L}{2g}} \quad \text{—— (1)}$$

where L = total length of liquid coloumn

Mass of liquid $= m$

density " " $= d$

Volume $= \dfrac{m}{d}$

Cross-section area $= a$

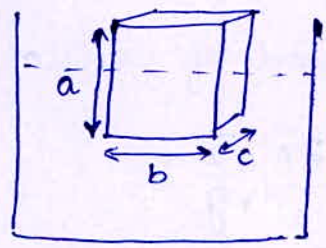
length of liquid coloumn

$$L = \frac{V}{a} = \frac{m}{ad}$$

Replacing in eq (1)

$$T = 2\pi\sqrt{\frac{m}{2adg}}$$

(24) A block of dimensions a, b, c is floating in water.



Let Length of block inside water $= L$

At equilibrium

$$F_B = mg$$

$$1 \times (L \times b \times c) \times g = d(a \times b \times c) \times g$$

$$\boxed{L = ad}$$

where d = relative density

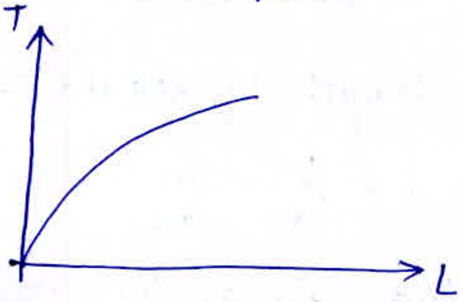
Time period of oscillation

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{ad}{g}}$$

(25) Time period of simple pendulam

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T \propto \sqrt{L}$$



(26)

Time period of oscillation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where $L \rightarrow$ distance of C.O.M. from point of suspension

Initially C.O.M. is at center of sphere.

As water flows out C.O.M. goes down. After some time it again rises up and when all liquid flows out C.O.M. again comes at center.

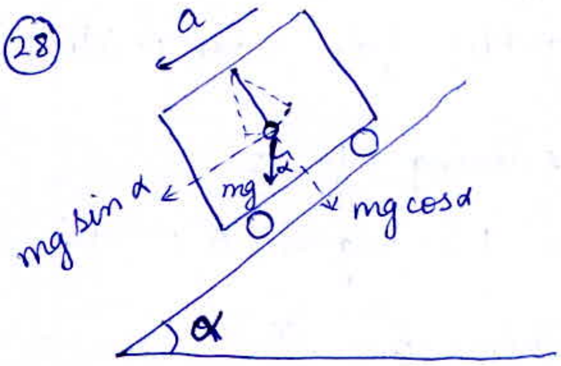
Hence initially time period increases then decreases.

(27) Time period of simple pendulam

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Inside a mine, Value of g decreases hence T increases.

(28)



acc. of vehicle

$$a = g\sin\alpha$$

$$g_{eff} = g\cos\alpha$$

$$T = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

(29)
$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T \propto \sqrt{L}$$

$$\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta L}{L}\right)$$

If length is increased by 300%

$$\Delta L = 3L$$

New length $L' = 4L$

$$\frac{T'}{T} = \sqrt{\frac{L'}{L}} = 2$$

Increase in time period

$$\Delta T = T' - T = T$$

$$\% \text{ increase} = \frac{\Delta T}{T} \times 100 = 100\%$$

(30) $T = 2\pi \sqrt{\frac{L}{g}}$

$T \propto \sqrt{L}$

$\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta L}{L}\right)$

If length is decreased by 2%

$\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{2}{100}\right) = \frac{1}{100}$

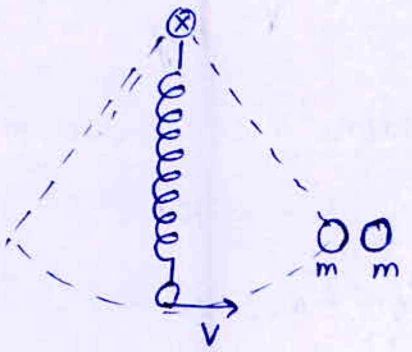
$\Delta T = \frac{1}{100}$ sec.

For one day

$\Delta T = \frac{24 \times 60 \times 60}{100}$

$\Delta T = 24 \times 36 = 864$ sec

(31)

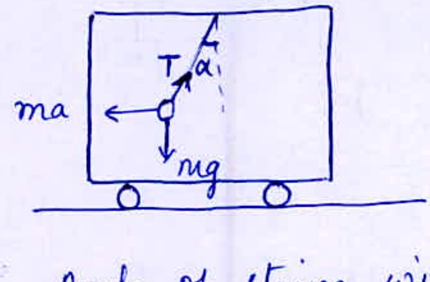


At extreme position

Velocity $v = 0$

Hence momentum transferred is also zero.

(32)



Angle of string with vertical

$\tan\alpha = \frac{a}{g} = \frac{49 \times 10^{-2}}{9.8} = 0.05$

$\alpha \approx 3°$

(33)

The two springs are in series combination

$K_{eq} = \frac{k_1 k_2}{k_1 + k_2}$

frequency of oscillation

$f = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{m}}$

$f = \frac{1}{2\pi}\sqrt{\frac{K_1 K_2}{m(k_1 + k_2)}}$

(34) When a mass 'm' is attached to a spring

At equilibrium

$mg = kx$

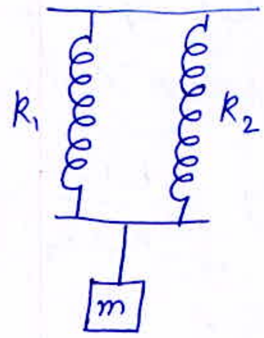
$\frac{m}{k} = \frac{x}{g}$

Time period of SHM

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x}{g}}$

$T = 2\pi\sqrt{\frac{0.2}{9.8}} = \frac{2\pi}{7}$ sec.

(35)



The two springs are in parallel combination

$$K_{eq} = K_1 + K_2$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{K_1+K_2}}$$

(36) $T = 2\pi\sqrt{\frac{m}{K}} = 2\,sec. \text{ —— (I)}$

If mass is increased by 2 kg

$$T' = 2\pi\sqrt{\frac{m+2}{K}} = 3\,sec \text{ —— (II)}$$

Eq $\frac{(II)}{(I)}$ $\quad \frac{T'}{T} = \sqrt{\frac{m+2}{m}} = \frac{3}{2}$

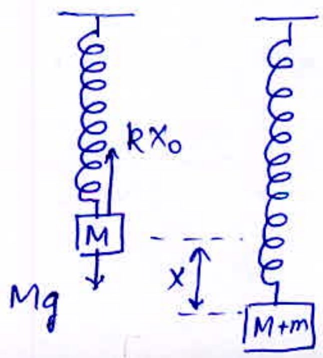
$$\frac{m+2}{m} = \frac{9}{4}$$

$$4(m+2) = 9m.$$

$$5m = 8$$

$$\boxed{m = 1.6\,kg}$$

(37)



Initially

$$kx_0 = Mg \text{ —— (I)}$$

After mass 'm' is added

$$k(x+x_0) = (M+m)g \text{ —— (II)}$$

Eq (II) − (I)

$$kx = mg$$

$$k = \frac{mg}{x} \text{ —— (III)}$$

Time period of oscillation

$$T = 2\pi\sqrt{\frac{M+m}{k}}$$

$$T = 2\pi\sqrt{\frac{(M+m)x}{mg}}$$

(38) Workdone = increase in P.E.

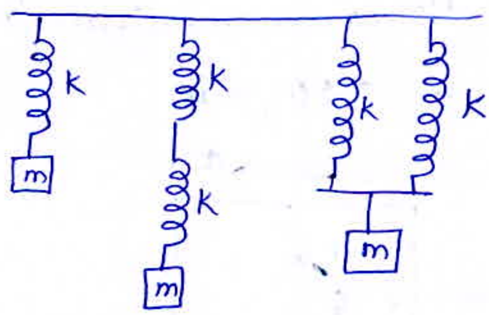
$$W = \frac{1}{2}kx^2$$

If $K_A > K_B$

and $x_A = x_B$

then $\boxed{W_A > W_B}$

(39)



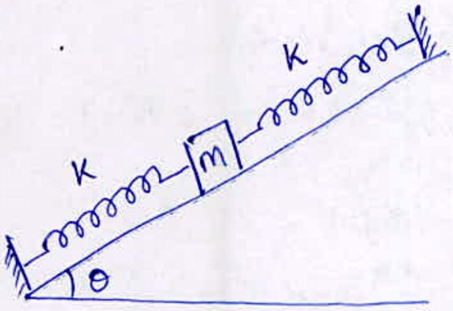
$$T_1 = 2\pi\sqrt{\frac{m}{k}}$$

$$T_2 = 2\pi\sqrt{\frac{2m}{k}}$$

$$T_3 = 2\pi\sqrt{\frac{m}{2k}}$$

$$T_1 : T_2 : T_3 = 1 : \sqrt{2} : \frac{1}{\sqrt{2}}$$

(40)

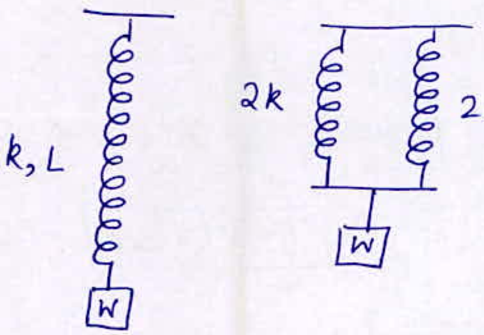


The two springs are in parallel Combination because extension in one spring = compression in other

$$K_{eq} = k_1 + k_2 = 2K$$

$$T = 2\pi\sqrt{\frac{M}{2K}}$$

(41)



For a spring

$$k \times L = \text{Const.}$$

If spring is cut into two equal parts

$$L_1 = L_2 = \frac{L}{2}$$

$$k_1 = k_2 = 2k$$

Now springs are connected in parallel combination

$$K_{eq} = k_1 + k_2 = 4k$$

Force balance

$$W = kx = 4k \cdot x_1 \Rightarrow \boxed{x_1 = \frac{x}{4}}$$

(42)

Springs are in parallel comb.

$$K_{eq} = k_1 + k_2$$

frequency $f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$

(43) Initially time period

$$T = 2\pi\sqrt{\frac{M}{k}} \quad \text{——— (I)}$$

If mass is increased by 'm'

$$\frac{5}{4}T = 2\pi\sqrt{\frac{M+m}{k}} \quad \text{——— (II)}$$

$$\frac{(II)}{(I)} \quad \frac{5}{4} = \sqrt{\frac{M+m}{M}}$$

$$\frac{M+m}{M} = \frac{25}{16}$$

$$1 + \frac{m}{M} = \frac{25}{16}$$

$$\boxed{\frac{m}{M} = \frac{9}{16}}$$

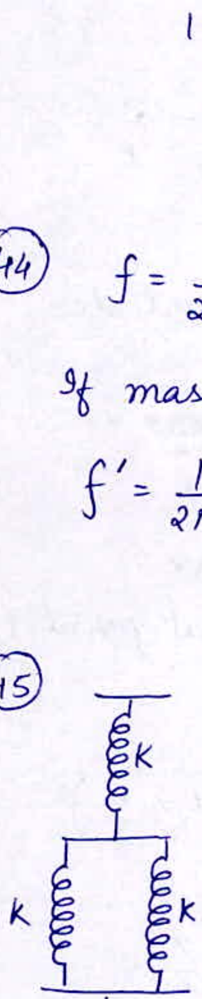
(44) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 0.5\ Hz$

If mass m is reduced to $\frac{m}{4}$

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m/4}} = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$$

$$f' = 2f = 1\ Hz.$$

(45)



Lower two springs are in parallel comb.

$$K_{eq} = 2K$$

For system

$$\frac{1}{K_{eq}} = \frac{1}{K} + \frac{1}{2K} \Rightarrow \boxed{K_{eq} = \frac{2K}{3}}$$

Time period of oscillation

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}}$$

where $K_{eq} = \frac{2K}{3}$

$$\boxed{T = 2\pi\sqrt{\frac{3m}{2K}}}$$

(46) $f = 60\,Hz$ and $A = 0.01\,m$.

$\omega = 2\pi f = 120\pi$

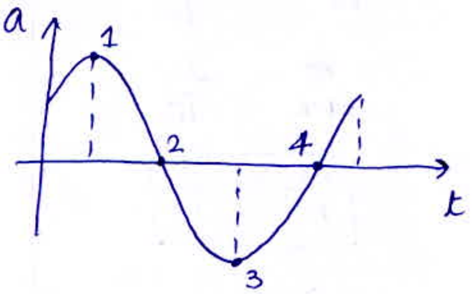
max. acceleration

$$a_{max} = A\omega^2$$

$$a_{max} = 0.01\,(120\pi)^2$$

$$\boxed{a_{max} = 144\pi^2}$$

(47)



Acceleration of a particle

$$a = -\omega^2 x$$

when $x = -X_{max}$

$$a = +a_{max}$$

Hence particle is at point 1.

(48) $m = 0.1\,kg$

$$x = (10\,cm)\cos\left(10t + \frac{\pi}{2}\right)$$

Comparing with

$$x = A\cos\left(\omega t + \frac{\pi}{2}\right)$$

$A = 10\,cm$.

$\omega = 10$

Max. acceleration

$$a_{max} = A\omega^2 = 0.1(10)^2 = 10\,m/s^2$$

(50) $m = 10\,gm$, $A = 0.5\,m$

and $T = \frac{\pi}{5}$ sec.

$$F_{max} = m \times a_{max}$$

$$= m(A\omega^2) = mA\left(\frac{2\pi}{T}\right)^2$$

$$= 0.01 \times 0.5\,(10)^2$$

$$= 0.5\,N.$$

(51) acc $a = -\omega^2 x$

when $x = 3\,cm$

$$a = \cdot 3\omega^2 = 12\,cm/s^2$$

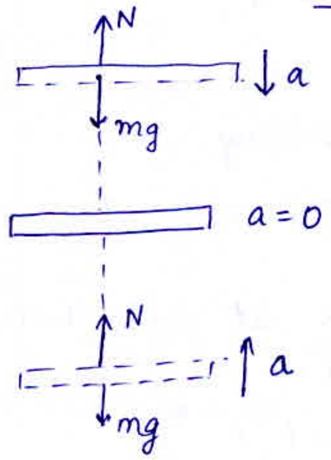
$$\omega^2 = 4$$

$$\omega = 2$$

Time period $T = \frac{2\pi}{\omega} = \pi$ sec

$$\boxed{T = 3.14\text{ sec}}$$

(52)



At mean position $a = 0$

As surface moves up, Normal reaction decreases.

At highest point

$$N = mg - ma = 0$$

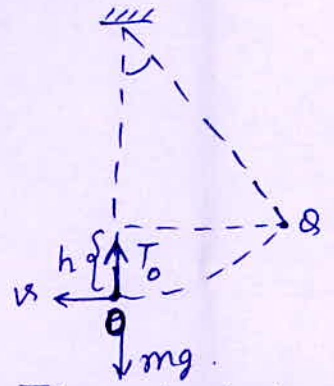
$$a = g$$

$$A\omega^2 = 10 \Rightarrow \omega^2 = 1000$$

$\cancel{\omega = 10 \text{ rad/sec}}$

$$f = \frac{\omega}{2\pi} = \frac{10\sqrt{10}}{2\pi} \approx 5 \text{ Hz}.$$

53



Time period $T = 2$ sec.

$$T = 2\pi\sqrt{\frac{L}{g}} = 2 \text{ sec}.$$

$$L = \frac{g}{\pi^2} \quad \text{—— (I)}$$

Energy Cons.

$$\frac{1}{2}mv^2 = mgh$$

$$v^2 = 2gh \quad \text{—— (II)}$$

At mean position

$$T_0 - mg = \frac{mv^2}{L}$$

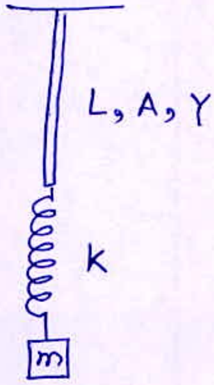
Tension $T_0 = mg + \dfrac{mv^2}{L}$

$$T_0 = mg + \frac{m(2gh)}{g/\pi^2}$$

$$T_0 = mg + 2mh\pi^2$$

$$\boxed{T_0 = m(g + 2h\pi^2)}$$

54



A metal elastic wire can be considered as a spring

force const. $K' = \cancel{\frac{L}{AY}}\ \frac{AY}{L}$

Wire & spring are in series Combination

$$K_{eq} = \frac{K_1 K_2}{K_1 + K_2} = \frac{KAY}{L\left(\frac{AY}{L} + K\right)}$$

$$K_{eq} = \frac{KAY}{AY + KL}$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m(AY+KL)}{KAY}}$$

55

$$a_m = \frac{a_0}{a\omega^2 - b\omega + c}$$

For single resonant frequency

$$b^2 - 4ac < 0$$

$$b^2 < 4ac.$$