EXERCISE - 1 [A]

1. **(a)**

$\Rightarrow 5t_5 8t$

$\Rightarrow 5(a+4d)=8(a+7d)$

$\Rightarrow 3a=-36d$

$\Rightarrow d=-\frac{1}{12}a$

Now, $T_{13}=a+12d$

$\Rightarrow a+12\left(-\frac{1}{12}a\right)=0$

2. **(c)**

$\Rightarrow T_7=a+6d$

$\Rightarrow a+6d+34$ ....(1)

$\Rightarrow T_{13}=a+12d$

$\Rightarrow a+12d=64$ ....(2)

Solving (1) and (2)

$\Rightarrow a=4$ and $d=5$

$\Rightarrow \therefore T_{18}=a+17d$

$\Rightarrow 4+17\times 5=89$

3. **(b)**

$\Rightarrow T_7=a+6d$

$\Rightarrow a+6d=40$

$\Rightarrow a=40-6d$

$\Rightarrow S_{13}=\frac{12}{2}[2a+12d]$

$\Rightarrow \frac{13}{2}[2(40-6d)+12d]$

$\Rightarrow \frac{13}{2}[80]=13\times 40=520$

4. **(a)**

$\Rightarrow S_{40}=\frac{40}{2}[2a+39d]$

$\Rightarrow 20[2(2)+39(4)]=3200$

5. **(c)**

Let the terms of A. P are $(a-d), a, (a+d)$

Now, $(a-d)+(a+d)=12$

$\Rightarrow 2a = 12$

$\Rightarrow a = 6$

$\Rightarrow \text{and}(a-d)a = 24$

$\Rightarrow (6-d) = 6 = 24$

$\Rightarrow d = 2$

$\Rightarrow \therefore \text{first tern } a-d = 6-2 = 4$

6. (c)

$\Rightarrow S_{2m} = \frac{2n}{2}\left[2(2)+(2n-1)(3)\right] = n\left[a+6n\right]$ ……….(1)

$\Rightarrow S_n = \frac{n}{2}\left[2(57)+(n-1)(2)\right] = n\left[56+n\right]$ ……….(2)

Solving (1) and (2)

$\Rightarrow 1+6n = 56+n$

$\Rightarrow n = 11$

7. (a)

$\Rightarrow S_{10} = 4S_5$

$\Rightarrow \frac{10}{2} = \left[2a+9d\right] = 4\times\frac{5}{2}\left[2a+4d\right]$

$\Rightarrow 2a = d$

$\Rightarrow \frac{a}{d} = \frac{1}{2}$

8. (a)

$\Rightarrow Sp = \frac{p}{2}\left[2a+(p-1)d\right] = x$ ………(1)

$\Rightarrow Sq = \frac{q}{2}\left[2a+(q-1)d\right] = y$ …..….(2)

$\Rightarrow Sr = \frac{r}{2}\left[2a+(q-1)d\right] = z$ ………(3)

Now substituting value from (1), (2), (3) in

$\Rightarrow \frac{x}{p} = (q-r)+\frac{y}{q}(r-p)+\frac{z}{r}(p-q)$

$\Rightarrow 2\left[2a-(p-1)d\right](q-r)+2\left[2a-(q-1)d\right](r-p)+2\left[2a-(r-1)d\right](p-q) = 0$

9. (a)

Odd two digit number will be 11, 13, 15, ………99 – total 45 numbers

$\Rightarrow S = \frac{45}{2}\left[2(11)+(45-1)2\right]$

$\Rightarrow \frac{45}{2}\left[22+88\right] = 2475$

10. (d)

$$\Rightarrow S=\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ldots\ldots+\frac{1}{\sqrt{2n}+\sqrt{2n+1}}$$

$$\Rightarrow \frac{\sqrt{3}-1}{2}+\frac{\sqrt{5}=\sqrt{3}}{2}+\frac{\sqrt{7}-\sqrt{5}}{2}+\ldots\ldots+\frac{\sqrt{2n+1}-\sqrt{2n}}{2}$$

$$\Rightarrow \frac{1}{2}\left(\sqrt{2n+1}-1\right)$$

11. (d)

$\Rightarrow a_1, a_2 \ldots\ldots\ldots a_{n+1}$ are in A. P.

Let $a_1 = a$ and common difference be d

Then, $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots\ldots+\frac{1}{a_na_{n+1}}$

$$\Rightarrow \frac{1}{a(a+d)}+\frac{1}{(a+d)(a+2d)}+\ldots\ldots+\frac{1}{(a+nd)(a+(n-1)d)}$$

$$\Rightarrow \frac{1}{d}\left[\frac{d}{a(a+d)}+\frac{d}{(a+d)(a+2d)}+\ldots\ldots+\frac{d}{(a+nd)(a+(n-1)d)}\right]$$

$$\Rightarrow \frac{1}{d}\left[\left(\frac{1}{d}-\frac{1}{a+d}\right)+\left(\frac{1}{a+d}-\frac{1}{a+2d}\right)+\ldots\ldots+\left(\frac{2}{a+(n-1)d}-\frac{1}{a+nd}\right)\right]$$

$$\Rightarrow \frac{1}{d}\left[\frac{1}{a}-\frac{1}{a+nd}\right]$$

$$\Rightarrow \frac{1}{d}\left[\frac{a+nd-a}{a(a+nd)}\right]$$

$$\Rightarrow \frac{n}{a_1a_{n+1}}$$

12. (d)

Let first be a and common difference be d.

$a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225$

$\Rightarrow 6a+23d=75$

Now, $S_{24}=\frac{24}{2}[2a+23d]=12[75]=900$

13. (a)

a, b, c are in A. P.

$$\Rightarrow \frac{a+c}{2}=b$$

$$\Rightarrow \frac{a+c}{2abc}=\frac{1}{ac}$$

$$\Rightarrow \frac{ab+cb}{2abc}=\frac{1}{ac}$$

$$\Rightarrow \frac{\frac{1}{ab}+\frac{1}{bc}}{2}=\frac{1}{ac}$$

$$\Rightarrow \therefore \frac{1}{bc},\frac{1}{ca},\frac{1}{ab} \text{ are in A. P.}$$

14. (b)

$$\Rightarrow \log 2 \log\left(2^n-1\right),\log\left(2^n+3\right) \text{are in A.P.}$$

$$\Rightarrow \therefore \log\left(2^n-1\right)=\frac{\log 2+\log 2^n+3}{2}$$

$$\Rightarrow 2\log\left(2^n-1\right)=\log\left(2\times\left(2^n+3\right)\right)$$

$$\Rightarrow \log\left(2^n-1\right)^2=\log\left(2^{n+1}+6\right)$$

$$\Rightarrow \left(2^n-1\right)^2=2^{n+1}+6$$

$$\Rightarrow 2^{2n}+1-2^{n+1}=2^{n+1}+6$$

$$\Rightarrow 2^{2n}-4.2^n-5=0$$

Let $^n=t$

$$\Rightarrow t^2-4t-5=0$$

$$(t-5)(t+1)=0$$

$\Rightarrow t=5$ or $t=-1$

$\Rightarrow 2^n=5$ or $2^n=-1$ (not possible)

$$\Rightarrow \log_2^5=n$$

15. (c)

$$x,|x+1|,|x-1|=\text{A.P}$$

For x < - 1

$$\Rightarrow x,-x-1,-x+1=\text{A.P}$$

$$\Rightarrow \therefore -x-1-x=-2x-1$$

$$\Rightarrow -x+1+x+1=2$$

From (1) and (2)

$$\Rightarrow -2x-1=2$$

$$\Rightarrow -2x=3x=\frac{-3}{2}$$

$$\Rightarrow \therefore S_{20}=\frac{20}{2}\left[2\left(\frac{-3}{2}\right)+(19)2\right]=350$$

16. (b)

$$\Rightarrow a=2n-1$$

$$\Rightarrow n=\frac{a+1}{2}$$

$\Rightarrow (1+3+5+.......p)+(1+3+5+......+q)=(1+3+5+.....+r)$

$\Rightarrow \left(\frac{p+1}{2}\right)^2+\left(\frac{q+1}{2}\right)^2=\left(\frac{r+1}{2}\right)^2$

$\Rightarrow (p+1)^2+(q+1)^2=(r+1)^2$

P > hence smallest pythagaion to put will be 6, 8, 10.
Therefore p = 7, q = 5, r = 9
Least value $p+q+r=21$

17. **(b)**
Let first term of G. P be A and common ratio be R.

$\Rightarrow T_p = AR^{p-1} = a$

$\Rightarrow T_q = AR^{-q-1} = b$

$\Rightarrow T_r = AR^{r-1} = c$

Now, $a^{q-r}.b^{r-p}.c^{p-q} = \left(AR^{(p-1)^{(q-r)}}\right).\left(AR^{(q-1)^{(r-1)}}\right).\left(AR^{(r-1)}\right)^{(p-q)}$

$\Rightarrow = A^0R^0 = 1$

18. **(c)**
Let the first term of G.P. be $\frac{a}{r^2}, \frac{1}{r}, a, ar, ar^2$

If third term is 4
$\Rightarrow a = 4$

$\therefore$ their product $=(a)^5=(4)^5$

19. **(d)**
$\Rightarrow x, 2x+2, 3x+3$ are in G. P.

then $(2x+2)^2 = x(3x+3)$

$\Rightarrow 4x^2+4+8x=3x^2+3x$

$\Rightarrow x^2+5x+4=0$

$\Rightarrow (x+1)(a+4)=0$

$\Rightarrow x=-4$ or $x=-1$

if $x=-1$ then term will be – 1, 0, 0 Not possible
if $x=-4$
Then term will be -4, -6, -8

$\Rightarrow a=-4$

$\Rightarrow r=\frac{-6}{-4}=\frac{3}{2}$

$\Rightarrow T_4 = ar^3 = -4\times\left(\frac{3^3}{2}\right) = -4\times\frac{27}{8} = -13.5$

20. **(b)**

$a = x$ let common ratio be r.

$\Rightarrow S_\infty = 5$

$\Rightarrow \frac{x}{1-r} = 5$

$\Rightarrow r = \frac{5-x}{5}$

Or $r \in (-1,1)$ for an infinite G. P.

$\Rightarrow -1 < \frac{5-x}{5} < 1$

$\Rightarrow 10 > x > 0$

21. (a)

A, b, c are in A.P.

$\Rightarrow \therefore \frac{a+c}{2} = b$ ....(1)

and $c - b = b - a$ ....(2)

and $b-a, c-b$a ar in G. P.

then $(c-b)^2 = a(b-a)$

form (2)

$\Rightarrow (b-a)^2 = a(b-a)$

$\Rightarrow b - a = a$ ....(3)

From (3)

$\Rightarrow b = 2a$

$\Rightarrow \frac{b}{a} = 2$

From (3)

$\Rightarrow b = 2a$

$\Rightarrow \frac{a+c}{2} = 2a$

$\Rightarrow \frac{c}{a} = 3$

$\Rightarrow \therefore a : b : c = 1 : 2 : 3$

22. (b)

Let $S = 3 + 33 + 333 + ...33.....33$

$\Rightarrow S = 3(1 + 11 + 111 + ... + 111.....111)$

$\Rightarrow 3\left(1 + (10+1) + (10^2 + 10 + 1) + ... + \left(10^n + 10^{a-1} + 10^{a-2} + ... + 10 + 1\right)\right)$

$\Rightarrow 3\left(n + 10(n-1) + 10^2(n-2) + ...10^n\right)$ .....(1)

Let $S^1 = n + 10(n-1) + 10^2(n-2) + ... + 10^n$ ......(2)

$\Rightarrow 10S^1 = 10n + 10^2(n-1) + ...210^n + 10^{n+1}$ ......(3)

(2) – (3)

$\Rightarrow -9S^1 = n - 10 - 10^2 - 10^3......10^{a+1}$

$\Rightarrow n - \left(10 + 10^2 + 10^3 + ...10^n + 10^{n+1}\right)$

$$\Rightarrow n - \frac{10\left(10^n - 1\right)}{10 - 1} = n - \frac{10^{n+1} - 10}{9}$$

$$\Rightarrow \frac{9n - 10^{n+1} + 10}{9}$$

$$\Rightarrow S^1 = \frac{10^{n+1} - 10 - 9n}{81}$$

$\Rightarrow \therefore$ From (1)

$$\Rightarrow S = \frac{10^{n+1} - 10 - 9n}{27}$$

23. (b)

1234, 2345, 3456 ……….

$d = 1111$

$T_n = 1234 + (n-1)1111$

$= 123 + 1111n$

24. (a)

$a = 2 + d$

$b = 2 + 2d$

$c = (2 + 2d)d$

$2(a + d)d.d = 160$

$\Rightarrow d.d(1 + d) = 4.4.5$

$\Rightarrow d = 4$

$a = 6, b = 10$

$c = 40$

$a + b + c = 56$

25. (a)

$T_6 = 8T_3$

$\Rightarrow ar^5 = 8ar^2$

$\Rightarrow \ell^3 = 8$

$\Rightarrow r = 2$

$T_7 + t_8 = 192$

$\Rightarrow ar^6 + ar^7 = 192$

$\Rightarrow a(64 + 128) = 192$

$\Rightarrow a = 1$ ……..(1)

$\Rightarrow a = 1$ …….(2)

$$T_5 + T_6 + ...T_{11} = \frac{2^4\left(2^7 - 1\right)}{2 - 1} = 2032$$

$T_6 + T_9 = 2^5 + 2^8 = 288$

26. **(c)**

$$\Rightarrow \sum_{n=1}^{\infty} \sin^{2n}\theta = \frac{1}{1-\sin^2\theta} \Rightarrow x = \frac{1}{\cos^2\theta}$$

$$\Rightarrow \sum_{n=1}^{\infty} \cos^{2n}\phi = \frac{1}{1-\cos^2\phi} \Rightarrow y = \frac{1}{\sin^2\phi}$$

$$\Rightarrow \sum_{n=1}^{\infty} \cos^{n}(\theta+\phi)\cos^{n}(\theta-\phi) = \frac{1}{1-\cos(\theta+\phi)\cos(\theta-\phi)}$$

$$\Rightarrow 2 = \frac{1}{1-\cos^2\theta+\sin^2\phi}$$

Now, $z = \dfrac{1}{1-\frac{1}{x}+\frac{1}{y}}$ or $z(xy-y+x) = xy$

$$\Rightarrow xyz - xy = yz - zx$$

27. **(b)**

Let $x = \sqrt{2}+1$

$\Rightarrow y = 1$

$\Rightarrow z = \sqrt{2}-1$

$$\Rightarrow \frac{x}{y} = \frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{1} = \frac{y}{z}$$

$\Rightarrow \therefore x, y, z$ are in G. P.

28. **(d)**

$$\Rightarrow (a)\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{b}$$

$$\Rightarrow \frac{b(b-c)+b(b-a)}{b(b-a)(b-c)} = (b-a)(b-c)$$

$$\Rightarrow b^2 + bc + b^2 - ab$$

$$\Rightarrow b^2 + ac - ab - bc$$

$$\Rightarrow b^2 = ac$$

a, b, c are in G.P.

but a, b, c are in H. P. so not correct

(b) as a, b, c, are in H. P.

$$\Rightarrow b = \frac{2ac}{a+c}$$

But $b = \dfrac{2ac}{a+c}$ is given so not correct

$$\Rightarrow (c)\frac{b+a}{b-a} + \frac{b+c}{b-c} = 1$$

$\Rightarrow (b+a)(b-c)+(b+c)(b-a)=(b-a)(b-c)$

$\Rightarrow b^2+bc+ab+ac+b^2-ab+bc-ac$

$\Rightarrow b^2-bc-ab+ac$

$\Rightarrow b^2+bc+ab=3ac$

No result

$\therefore$ Answer is none.

29. **(c)**

$\Rightarrow b=\dfrac{2ac}{a+c}$

Now, $\dfrac{b+a}{b-a}+\dfrac{b+c}{b-c}=\dfrac{\frac{b}{a}+1}{\frac{b}{a}-1}+\dfrac{\frac{b}{c}+1}{\frac{b}{a}-1}$

$\Rightarrow \dfrac{\frac{2c}{a+c}+1}{\frac{2c}{a+c}-1}+\dfrac{\frac{2a}{a+c}+1}{\frac{2a}{a+c}-1}=\dfrac{3c+a}{c-a}+\dfrac{3a+c}{c-a}=2$

30. **(a)**

$\Rightarrow y=\dfrac{2ab}{a+b}, x=\dfrac{2ay}{a+y}, z=\dfrac{2by}{b+y}$

or $y=\dfrac{2ab}{a+b}, x=\dfrac{4ab}{a+3b}, z=\dfrac{4ab}{3a+b}$

Now, $\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=\dfrac{a+b}{2ab}+\dfrac{a+3b}{4ab}+\dfrac{3a+b}{4ab}$

$\Rightarrow \dfrac{1}{a}+\dfrac{1}{b}=\dfrac{10}{9}$

31. **(c)**

$\Rightarrow a, b, c$ are in H. P.

$\Rightarrow \dfrac{1}{a}+\dfrac{1}{c}=\dfrac{2}{b}$ and $\dfrac{1}{c}=\dfrac{2}{b}-\dfrac{1}{a}$

Now, $\left(\dfrac{1}{b}+\dfrac{1}{c}-\dfrac{1}{a}\right)\left(\dfrac{1}{c}+\dfrac{1}{a}-\dfrac{1}{b}\right)=\left(\dfrac{1}{b}+\dfrac{2}{b}-\dfrac{1}{a}-\dfrac{1}{a}\right)\left(\dfrac{2}{b}-\dfrac{1}{b}\right)$

$\Rightarrow \left(\dfrac{3}{2}-\dfrac{2}{a}\right)\left(\dfrac{1}{b}\right)=\dfrac{3}{b^2}-\dfrac{2}{ab}$

32. **(c)**

$\Rightarrow \dfrac{a}{b}, \dfrac{b}{c}, \dfrac{c}{a}=\text{H.P.}$

$$\Rightarrow \frac{b}{c} = \frac{2\frac{a}{b}\frac{c}{a}}{\frac{a}{b}+\frac{c}{a}}$$

$$\Rightarrow \frac{b}{c} = \frac{2\frac{c}{b}}{\frac{a^2+c^2}{ab}}$$

$$\Rightarrow a^2b + b^2c = 2ac^2$$

33. (c)

$\Rightarrow a, b, c = \text{G.P.}$

$\Rightarrow b^2 = ac$

Now, $\frac{1}{\log_a^x} + \frac{1}{\log_b^x} = \log_x^a + \log_x^b = \log_x^{ab} = \log_x^{b^2}$

$\Rightarrow 2\log_x^b = 2\frac{1}{\log_b^x}$

$\Rightarrow \therefore \log_a^x, \log_b^x, \log_c^x = \text{H.P}$

34. (b)

Let $a = \frac{1}{\frac{1}{b}-d}$ and $c\frac{1}{\frac{1}{b}+d}$

$\Rightarrow a = \frac{b}{1-bd}$ and $c = \frac{b}{1+bd}$

Now, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = \frac{1}{4}$

$\Rightarrow \frac{1-bd}{b}+\frac{1}{b}+\frac{1+bd}{b} = \frac{1}{4}$ hence b = 12

Now, a + b + c = 37

$\Rightarrow \frac{12}{1-12d} + 12 + \frac{12}{1+12d} = 37$

$\Rightarrow \frac{24}{1-144d^2} = 25$

$\Rightarrow d = \frac{1}{60}$. Hence numbers are 15, 12, 10

35. (a)

$\Rightarrow d=\frac{19-3}{3+1}=\frac{16}{4}=4$

$\Rightarrow A_1=a+d=3+4=7$

$\Rightarrow A_2=a+2d=11$

$\Rightarrow A_3=a+3d=15$

36. (b)

A, b, c, d, e, f i.e. A. M. 's between 2 and 12

$\Rightarrow d=\frac{b-a}{n+1}=\frac{12-2}{6+1}=\frac{10}{7}$

$\Rightarrow S=\frac{8}{2}[2a+7d]=4[4+10]=56$

$\Rightarrow \therefore a+b+c+d+e+f=200-a-b$

$\Rightarrow 56-2-12=42$

37. (b)

$\Rightarrow r=\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$\Rightarrow n=2$

$\Rightarrow r=\left(\frac{64}{1}\right)^{\frac{1}{n+1}}=4$

$\Rightarrow G_1=ar=4$

$\Rightarrow G_2=ar^2=16$

38. (b)

$\Rightarrow G.M=3^{\frac{n+1}{2}}$

39. (a)

$\Rightarrow \frac{a+b}{2}=\frac{2ab}{a+b}$

$\Rightarrow a=b$

40. (a)

$\Rightarrow A_2+A_2=a+b, G_1G_2=ab$

$\Rightarrow \frac{1}{H_1}+\frac{1}{H_2}=\frac{a+b}{ab}$

$\Rightarrow \therefore \frac{H_1+H_2}{H_1H_2}=\frac{a+b}{ab}=\frac{A_1+A_2}{G_1G_2}$

$\Rightarrow \frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}$

41. **(d)**

Let the two number be a and b, then

$$\Rightarrow \frac{\frac{2ab}{a+b}}{\sqrt{ab}} = \frac{12}{13}$$

$$\Rightarrow \frac{2\sqrt{ab}}{a+b} = \frac{12}{13}$$

$$\Rightarrow \frac{(a+b)^2 - \left(2\sqrt{ab}\right)^2}{a+b} = \frac{5}{13}$$

$$\Rightarrow \frac{a-b}{a+b} = \frac{5}{13}$$

$$\Rightarrow 13a - 13b - 5a + 5b$$

$$\Rightarrow \frac{a}{b} = \frac{9}{4}$$

42. **(c)**

$$\Rightarrow \frac{a+b}{2} - \sqrt{ab} = 2$$

$$\Rightarrow \frac{a}{b} = \frac{4}{1} = a = 4b.$$

$$\Rightarrow \frac{4b+b}{2} - \sqrt{4b^2} = 2$$

$$\Rightarrow \frac{5}{2}b - 2b = 2$$

$$\Rightarrow b = 4 \text{ and } a = 16$$

43. **(c)**

$$\frac{\frac{a+b}{2}}{\frac{2ab}{a+b}} = \frac{m}{n}$$

$$\Rightarrow \frac{(a+b)^2}{4ab} = \frac{m}{n}$$

$$\Rightarrow \frac{(a+b)^2}{(a+b)^2 - 4ab} = \frac{m}{m-n}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{\sqrt{m}}{\sqrt{m-n}}$$

$$\Rightarrow \frac{a}{b} = \frac{\sqrt{m} + \sqrt{m-n}}{\sqrt{m} - \sqrt{m-n}}$$

44. **(c)**

$$x=\frac{\log 3}{\log 5}+\frac{\log 5}{\log 7}+\frac{\log 7}{\log 9}$$

$$\frac{x}{3}\geq\left(\frac{\log 3}{\log 5}.\frac{\log 5}{\log 7}.\frac{\log 7}{2og3}\right)$$

(By Am $\geq$am)

$$\Rightarrow\frac{x}{3}\geq\left(\frac{1}{2}\right)^{1/3}$$

$$\Rightarrow x\geq\frac{3}{3\sqrt{2}}$$

45. (a)

$A_1=G_1=H_1=A$

$A_{3n-1}=G_{2n-1}=H_{2n-1}=8$

So, $An=\frac{A+B}{2},Gn=\sqrt{AB},Hn=\frac{2AB}{A+B}$

$\Rightarrow \boxed{b^2=ac}$

46. (d)

If A. M. are inserted between two given number then product of rth A.M. from being and rth H.M. form and is equal to the product 0 there numbers.

Hence, $a_4\times h_7=2\times 3$ i.e. 6

47. (b)

$\Rightarrow a_1+a_{2n}=a_2+a_{2n-1}=a_3+a_{2n-2}=a+b$

$\Rightarrow g_1g_{2n}=g_2g_{2n-1}=g_3g_{2n-2}=.........=ab$

Hence, $\frac{a_1+b_{2n}}{g_1g_{2n}}+\frac{a_2+a_{2n-1}}{g_2g_{2n-1}}+\frac{a_3+a_{2n-2}}{g_3g_{2n-3}}+........=n\frac{a+b}{ab}$

But $\frac{2ab}{a+b}=h$, therefore $\frac{a_1+a_{2n}}{g_1g_{2n}}+\frac{a_2+a_{2n-1}}{g_2g_{2n-1}}+\frac{a_3+a_{2n-2}}{g_3g_{2n-3}}+.......=\frac{2n}{h}$

48. (b)

$\Rightarrow\frac{a+b}{2}=\frac{3}{2}$

$\Rightarrow a+b=3$

$\Rightarrow\frac{2ab}{a+b}=\frac{4}{3}$

$\Rightarrow 2ab=4$

$\Rightarrow ab=2$

$\Rightarrow\therefore x^2-3x+2=0$

49. (b)

$$\Rightarrow \frac{\pm\sqrt{\frac{c}{a}}}{\pm\sqrt{\frac{n}{1}}} = \pm\sqrt{\frac{cl}{an}}$$

50. (b)

$$\Rightarrow \frac{1}{xy-x^2}+\frac{1}{xy-y^2}=\frac{1}{x(y-x)}-\frac{1}{y(y-x)}$$

$$\frac{y-x}{(y-x)xy}=\frac{1}{xy}=\frac{1}{G^2}$$

51. (b)

$$\Rightarrow d=\frac{b-a}{n+1}=\frac{38-2}{n+1}=\frac{36}{n+1}$$

If n A. M. are inserted between 2 and 38 then total numbers of terms A. P. is n + 2

$$\Rightarrow S_{n+2}=\frac{n+2}{2}\left[2a+(n+2-1)d\right]$$

$$\Rightarrow \frac{n+2}{2}\left[2(2)+(n+1)\frac{36}{n+1}\right]=200$$

$$\Rightarrow \frac{n+2}{2}[4+36]=200$$

$$\Rightarrow n+2=10$$

$$\Rightarrow n=8$$

52. (b)

Let $S=1+2.2+3.2^2+4.2^3+.....+100.2^{99}$ ....(1)

$\Rightarrow 2S=2+2.2+3.2^2+.......+99.2^{99}+100.2^{100}$ ....(2)

(1) - (2)

$$\Rightarrow -1S=1+2+2^2+2^3+.......2^{99}-100.2^{100}$$

$$\Rightarrow 1\frac{(2^{100}-1)}{2-1}-100.2^{100}$$

$$\Rightarrow 2^{1000}-1-100.2^{100}$$

$$\Rightarrow S=99.2^{100}+1$$

53. (c)

$$\Rightarrow a^3+2^3+3^3+4^3+......15^3 \text{is}$$

$$\Rightarrow S=\left(\frac{n(n+1)}{2}\right)^2=\left(\frac{15(15+1)}{2}\right)^2=(120)^2=14400$$

54. (d)

$$\Rightarrow \left(1^2-t_1\right)+\left(2^2-t_2\right)+......+\left(n^2-t_n\right)=\frac{1}{3}n\left(n^2-1\right)$$

$$\Rightarrow 1^2+2^2+.........n^2-\left(t_1+t_2+........t_n\right)=\frac{1}{3}n\left(n^2-1\right)$$

$$\Rightarrow \frac{n\left(n+1\right)}{2}$$

$$\Rightarrow t_n=n$$

55. **(d)**

$$\Rightarrow \frac{1}{3\times 7}+\frac{1}{7\times 11}+\frac{1}{11\times 15}+.....$$

$$\Rightarrow \frac{1}{4}\left[\frac{1}{3}-\frac{1}{7}\right]+\frac{1}{4}\left[\frac{1}{7}-\frac{1}{11}\right]+\frac{1}{4}\left[\frac{1}{11}-\frac{1}{15}\right]$$

$$\Rightarrow=\frac{1}{4}\left[\frac{1}{3}-\frac{1}{\infty}\right]=\frac{1}{12}$$

56. **(b)**

$$\Rightarrow \frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+......$$

$$\Rightarrow 1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}........=1$$

57. **(a)**

$$\Rightarrow S=\frac{\left(1+2+3+.......+n\right)^2-\left(1^2+2^2+3^2+.......+n^2\right)}{2}$$

$$\Rightarrow \frac{n^2\left(n+1\right)^2}{8}-\frac{n\left(n+1\right)\left(2n+1\right)}{12}$$

$$\Rightarrow \frac{n\left(n+1\right)\left(n-1\right)\left(3n+1\right)}{24}$$

58. **(b)**

$$\Rightarrow \frac{1}{3}+\frac{1}{3^n}+\frac{1}{3^3}+.....\infty=\frac{1}{2}$$

Hence $y=\left(0.64\right)^{\log_{0.25}^{0.5}}$

$$\Rightarrow y=\left(0.64\right)^{\frac{1}{2}}=0.8$$

59. **(a)**

$$S=1.3^2+2.5^2 3.7^2+......$$

$$\Rightarrow T_n=n.\left(2n+1\right)^2$$

$$\Rightarrow T_n=4n^3+4n^2+n$$

$\Rightarrow S_n \sum 4n^3 + 4n^2 + n$

$\Rightarrow 4\left(n\frac{n+1}{2}\right)^2 + 4\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{n(n+1)}{2} \Rightarrow 188090$

60. **(b)**

Let $S = 1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + ...$ ....(1)

$\Rightarrow \frac{1}{2}s = \frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^2} + ...$ ....(2)

(1) – (2)

$\Rightarrow \frac{1}{2}s = 1 + \frac{2}{2} + \frac{2}{2^2} + \frac{2}{2^3} + ...$

$\Rightarrow 1 + 2\left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ...\right)$

$\Rightarrow 1 + 2\left(\frac{\frac{1}{2}}{1 - \frac{1}{2}}\right) = 1 + 2 = 3$

$\Rightarrow S = 6$

EXERCISE - 1 [B]

1. **(b)**

-4, -1, +2, +5 + ….

Is an A.P. with

First term a = -4

And common difference d = 3

Therefore

$T_n = a + (n-1)d$

$\Rightarrow T_{10} = -4 + (10-1)*3$

$\Rightarrow T_{10} = 23$

2. **(a)**

First term a = 2

Common difference d = 4

n = 40

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{40} = \frac{40}{2}\left[4 + (40-1)4\right]$

=1600

3. **(d)**

4, 9, 14, ….., 104

First term a = 4

Common difference d = 5

n^{th} tem is $T_n = 104$

$T_n = a + (n-1)d$

$\Rightarrow 104 = 4 + (n-1)5$

$\Rightarrow n = 21$

Therefore, middle term will be 11^{th} term

$T_{10} = 4 + (11-1)5$

= 54

4. **(b)**

$T_9 = 0$

$\Rightarrow a + (9-1)d = 0$

$\Rightarrow a = -8d$

Now,

$T_{29/}T_{19} = \frac{a + (29-1)d}{a + (19-1)d} = \frac{a + 28d}{a + 18d} \quad = \frac{8d + 28d}{-8d + 18d} = \frac{2}{1}$

$T_{29} : T_{19} = 2 : 1$

5. **(b)**
Number lying between 10 and 200 are the numbers which are multiple of 7
14, 21, 28, ……, 196
a = 14
d = 7
$T_n = 196$
$T_n = a + (n-1)d$
$\Rightarrow 196 = 14 + (n-1)7$
$\Rightarrow n = 27$
$S_n = \frac{n}{2}[2a + (n-1)d]$
$= \frac{27}{2}[2*14 + (27-1)7]$
= 2835

6. **(b)**
Let first term = a
Common difference = d
Then, A.P. be
$a, (a+d), (a+2d), (a+3d), ...$
$T_4 = a + 3d$
$\Rightarrow a + 3d + 3a$
$\Rightarrow a = \frac{3}{2}d$ ……(1)
$T_7 - 2(T_3) = 1$
$\Rightarrow a + 6d - 2(a + 2d) = 1$
$\Rightarrow 2d - a = 1$
Substituting value of a from (1)
$2d - \frac{3}{2}d = 1$
$\Rightarrow d = 2$

7. **(a)**
Let the term of A.P. is a
And common difference is d
So,
$T_p = a + (p-1)d = A$
$T_Q = a + (Q-1)d = B$
$T_r = a + (r-1)d = C$
Therefore,
$A(Q-r) + B(r-p) + C(p-Q)$

$= a(a+(p-1)d)(Q-r)+(a+(Q-1)d)(a+(r-1)d)(p-Q)$
$= 0$

8. **(b)**

$\frac{Sn}{Sn} = \left(\frac{n}{2}\right)(2a+(n-1)d)/\left(\frac{n}{2}\right)(2a'+(n-1)d')$

$\frac{Sn}{Sn'} = \frac{2a+(n-1)d}{2a+(n-1)d'}$

$\frac{2a+(n-1)d}{2a+(n-1)d'} = \frac{3n+8}{7n+15}$

Let, substituting n = 23

$\frac{2a+(23-1)d}{2a+(23-1)d'} = \frac{3*23+8}{7*23+15}$

$\frac{a+11d}{a+11d'} = \frac{77}{176}$

$T_{12/}T_{12'} = 7/16$

9. **(b)**

$S_n : n/2(2a+(n-1)d) = 2n^2+5n$

$S_1 : ½(2a) = 2+5 = 7$

$\Rightarrow a = 7$

$S_2 : (14+d) = 18$

$\Rightarrow d = 4$

$T_n = a+(n-1)d = 7+(n-1)4 = 4n+3$

10. **(b)**

Let the three terms of A.P. are a – d, a, a + d
Sum of first terns
$a-d+a+a+d = 3a = 51$

$\Rightarrow a = 17$

Product of first and third term
$(a-d)(a+d) = a^a - d^a$

$\Rightarrow 17^2 - d^2 = 273$

$\Rightarrow d^2 = 16$

$\Rightarrow d = 4$

So, third term
$a+d = 17+4 = 21$

11. **(b)**

Let the four terms of A.P. are
$a-3d, a-d, a+d, a+3d$

Then,

$a-3d+a-d+a+d+a+3d=20$

$\Rightarrow 4a=20$

$\Rightarrow 4=5$

$\frac{(a-3d)(a+3d)}{(a-d)(a+d)}=\frac{2}{3}$

$\Rightarrow \frac{3(a^2-9d^2)=2(a^a-d^a)}{(a-d)(a+d)}=\frac{2}{3}$

$\Rightarrow 3(a^2-9d^2)=2(a^a-d^a)$

$\Rightarrow a^2 25d^2$

$\Rightarrow a=5d$

$\Rightarrow d=1$

Smallest term

$a-3d=5-3=2$

12. **(a)**

Let the three numbers are

$a-d, a, a+d$

$(a-d)(a+d)=5a$

$\Rightarrow (a^2-d^2)=5a$ ...... (1)

$a+a+d=8(a-d)$

$\Rightarrow 6a=9d$

$\Rightarrow 2a=3d$ (2)

Solving (1) and (2), we get

$a=9, d=6$

So, the numbers are 3, 9, 15.

13. **(c)**

Sum of interior angles of an $n=\text{gon}=(n-2)\times 180^o$

Sum of n terms of A.P. $(a=120^0, d=5^o)=\frac{n}{2}\{2\times 120^o+(n-1)\times 5^o\}$.

Hence $\frac{n}{2}\{2\times 120^o+(n-1)\times 5^o\}=(n-2)\times 180^o$

$\Rightarrow n^2-25+144=0 \Rightarrow n=9$ or 16.

But for $n=16,$ greatest angle exceeds 180^o hence only 9 is correct.

14. **(b)**

Common difference of the two A.P.s are 4 & 5, hence common difference of A.P. formed by common terms will be 20. Also the first common term is 21. Now

$S=100(2\times 21\times 20)=402200$.

15. **(c)**

m^{th} term of first series $= 2m + 61$, m^{th} term of second series $= 7m - 4$. $7m - 4 = 2m + 61 \Rightarrow m = 13$.

16. **(c)**

$d_1 = 3 \,\&\, d_2 = 2 \Rightarrow d(\text{common terms}) = 6$

First common term = 5

Hence common term are 5, 11, 17,…

Now general term = 6n – 1.

60^{th} term of first A.P. = 179

50^{th} term of second A.P. = 101

Comparing $6n - 1$ with 101 gives n = 17

17. **(a)**

$a + e = b + d = 2c \Rightarrow a - 4b + 6c - 4d + 2 = 0$.

18. **(b)**

Given $11 + 11 + d + 11 + 2d + 11 + 3d = 56 \,\&$

$11 + (n-4)d + 11 + (n-3)d + 11 + (n-2)d + 11 + (n-1)d = 12$

$\Rightarrow d = 2 \,\&\, (2n-5)d = 34$ or $n = 11$.

19. **(c)**

$\frac{2n}{2}\{2 \times 2 + (2n-1) \times 3\} = \frac{n}{2}\{2 \times 57 + (n-1) \times 2\} \Rightarrow n = 11.$

20. **(c)**

$(a + 6d) - (a + d) = 20 \Rightarrow d = 4 \,\&\, a + 2d = 9 \Rightarrow a = 1$.

Now n^{th} term $= 4n - 3 = 2001 \Rightarrow n = 501.$

21. **(a)**

$(1 + 3 + 5 + \ldots p \text{ terms}) + (1 + 3 + 5 + \ldots q \text{ terms}) = (1 + 3 + 5 + \ldots r \text{ terms}) \Rightarrow p^2 + q^2 = r^2$

Now smallest pythagorian triplet will be 3, 4 ,5, hence least value of $p + q + r = 12.$

22. **(b)**

As a, x, y, z, b are in A.P. therefore $x + z = a + b \,\&\, y = \frac{a+b}{2}$

$\Rightarrow x + y + z = \frac{3}{2}(a + b)$. Hence $a + b = 10$

23. **(a)**

Let the number be $a - d, a, a + d$.

Now $a - d + a + a + d = 15 \Rightarrow a = 5$

As given $a - d + 1, a + 4, a + d + 19$ are in G.P. hence

$(a + 4)^2 = (a - d + 1)(a + d + 19) \Rightarrow 81 = 16 = (6 - d)(24 + d) \Rightarrow d = 3$.

Numbers are 2, 5, 8.

24. **(b)**

Let the first term is a

Common difference is d

Then,

$T_2 = a$

$T_3 = a + d$

$T_6 = a + 4d$

T_2, T_3 and T_6 are in G.P., Then

$(a+d)^2 = a(a+4d)$

$\Rightarrow a^2 + 2ad + d^2 = a^2 + 4ad$

$\Rightarrow d^2 = 2nd$

$\Rightarrow d = 2a$

Common ratio

$T_{3/}T_2 = (3a/a) = 3$

25. **(a)**

18, -12, 8, …….. - is in G.P.

Common ratio

$r = -\frac{12}{18} = -\frac{2}{3}$

$T_r = ar^a$

$\Rightarrow \frac{512}{729} = 18\left(-\frac{2}{3}\right)^n$

$\Rightarrow n - 1 = 8$

$\Rightarrow n = 9$

26. **(c)**

Let the first term of G.P. is a

And the common ratio is r

Then, the five consecutive terms of G.P. are

$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

$\Rightarrow a = 4$

Then,

$\frac{a}{r^2} * \frac{a}{r} * a * ar * ar^2 = a^5 = 4^5$

27. **(c)**

Let the first term of G.P. is a

And the common ratio is r

Then,

$T_3 = ar^2 = 15$ (1)

$T_7 = ar^6 = 135$ (2)

Solving (1) and (2), we get

$r^4 = 9$

$a = 5$

Therefore,

$T_5 = ar^4 = 5*9 = 45$

28. **(b)**

$1, x^2, 6 - x^2$ are in G.P. then

$$\frac{x^2}{1} = \frac{6 - x^2}{x^2}$$

$\Rightarrow x^4 = 6 - x^2$

$\Rightarrow x^4 + x^2 = 6 = 0$

29. **(a)**

$1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + ...$ are in G.P.

With common ratio $-\frac{1}{3}$

Then, the sum infinite G.P. is

$$S_n = \frac{a}{a - r} = \frac{1}{\left(1 + \frac{1}{3}\right)} = \frac{3}{4}$$

30. **(b)**

$1 + \frac{2}{x} + \frac{4}{x^3} + \frac{8}{x^3} + ...$

Sum of infinite term is finite when common ratio is less than 1

i.e. $\left|\frac{2}{x}\right| < 1$

$\Rightarrow |x|$

31. **(b)**

$96 + 48 + 24 + 12 + ... + \frac{3}{16}$

Then, the common ratio $\frac{48}{96} = \frac{1}{2}$

$T_n = ar^n$

$$\Rightarrow \frac{3}{16} = 96\left(\frac{1}{2}\right)^{n-1}$$

$\Rightarrow \frac{3}{2^{n-6}} = \frac{3}{16}$

$\Rightarrow n = 10$

32. **(c)**

$3 + 3a + 3a^2 + ... = \frac{45}{8}$ is a G.P.

$S_n = \frac{a}{1-r}$

$\Rightarrow \frac{3}{1-a} = \frac{45}{8}$

$\Rightarrow 24 = 45(1-a)$

$\Rightarrow 45a = 45 - 24 = \frac{21}{45} = \frac{7}{15}$

33. **(c)**

Let the number be a, ar, ar^2

Then,

$a + ar + ar^2 = 155$

$\Rightarrow a(1 + r + r^2) = 155$ (1)

And,

$ar^2 - a = 120$

$\Rightarrow a(r^2 - 1) = 120$ (2)

Solving (1) and (2), we get

$r = 5$ and $a = 5$

34. **(d)**

Let the numbers be a, ar, ar^2

Then there sum

$a + ar + ar^2 = 14$ (1)

And sum of their square

$a^2 + a^2r^2 + a^2r^4 = 84$ (2)

Squaring (1) and subtracting (2), we get

$(a + ar + ar^2)^2 - a^2 + a^2r^2 + a^2r^4 = 196 - 84$

$\Rightarrow 2ar(a + ar + ar^2) = 12$

$\Rightarrow ar = 4$

Subtracting this in (1) and solving, we get

$r = 2$ and $a = 2$

Therefore three numbers are 2, 4, 8

35. **(b)**

Let the four tern be a, ar, ar^2, ar^3

Then,

$a+ar^2=40$

$\Rightarrow a\left(1+r^2\right)=40$

$\Rightarrow \left(1+r^2\right)=\dfrac{40}{a}$ (1)

And

$ar+Ar^2=80$

$\Rightarrow ar\left(1+r^2\right)=80$

From (1)

$ar\left(\dfrac{40}{a}\right)=80$

$\Rightarrow r=2$ and $a=8$

36. **(b)**

a, b, c are in G.P.

Let the common ration is r

i.e. $\dfrac{b}{a}=\dfrac{c}{a}=r$

Then, for a^{-1}, b^{-1}, c^{-1}

$\dfrac{b^{-1}}{a^{-1}}=\dfrac{a}{b}=\dfrac{1}{r}$ and $\dfrac{c^{-1}}{b^{-1}}=\dfrac{b}{c}=\dfrac{1}{r}$

Therefore, a^{-1}, b^{-1}, c^{-1} are also in G.P.

37. **(a)**

Given $a\times ar\times ar^2=216$ & $a\times ar+ar\times ar^2+ar^2\times a=126.$

Or $(ar)^3=216$ & $a^2r\left(1+r+r^2\right)=126.$

$\Rightarrow 2r^2-5r+2=0.$ hence $r=\dfrac{1}{2}$ & $a=12.$

Now $a=12, b=6, c=3.$

38. **(a)**

$x=\log_{0.4}\left(\dfrac{1}{3}+\dfrac{1}{3^2}+\dfrac{1}{3^3}+\ldots\infty \text{ terms}\right)\Rightarrow c=\log_{0.4}\left(\dfrac{1}{2}\right)$

Hence $(0.16)^x=(0.16)^{-\log_{0.4}{}^{2}}=2^{-\log_{0.4}{}^{0.16}}$

Therefore $(0.16)^x=2^{-2}=\dfrac{1}{4}.$

39. **(c)**

$t_n=3\times 2^{n-1}.$ Now $12288=3\times 2^{12}$.

Hence $m=13.$

40. **(a)**

$$S_{10}=\frac{a\left(r^{10}-1\right)}{r-1} \,\&\, S_5=\frac{a\left(r^5-1\right)}{r-1}.\ \text{Now } \frac{S_{10}}{S_5}=244 \Rightarrow \frac{r^{10}-1}{r^5-1}=244 \text{ or } r=3.$$

41. **(a)**

Let the first term a and common ratio be b, then

$$x=ab^{p-1}, y=ab^{q-1}, z=ab^{r-1} \Rightarrow \frac{y}{x}=b^{q-p}, \frac{z}{y}=b^{r-q}, \frac{x}{z}=b^{p-r}$$

$$\text{Now } x^{q-r}y^{r-p}z^{p-q}=\left(\frac{y}{z}\right)^r\left(\frac{z}{y}\right)^p\left(\frac{x}{z}\right)^q=b^{r(q-p)+p(r-p)+q(p-r)}$$

$$\text{Or } x^{q-r}y^{r-p}z^{p-q}=b^0=1.$$

42. **(b)**

$$9^{1/3}.9^{1/9}.9^{1/27}...\infty \text{ terms}=9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...\infty \text{ terms}}$$

$$=9^{\frac{1/3}{9^{1-1/3}}}=9^{1/2}=3.$$

43. **(d)**

$$x=\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...\infty \text{ terms}=\frac{1}{2}$$

$$\text{Now } x^{\log_b a}=\left(\frac{1}{2}\right)^{\log_{\sqrt{5}} 0.2}=4.$$

44. **(c)**

$$\text{Given } a+ar+ar^2+...+ar^9=S_1 \,\&\, ar^{10}+ar^{11}+ar^{12}+...ar^{19}=S_2$$

$$\Rightarrow \frac{a\left(1-r^{10}\right)}{1-r}=S_1 \,\&\, \frac{ar^{10}\left(1-r^{10}\right)}{1-r}=S_2$$

$$\text{Or } \frac{S_2}{S_1}=r^{10}.$$

45. **(b)**

P, , r are in A.P.

$\Rightarrow Q.,-p=r-p$ (1)

$T_p=ar^{(p-1)}$

$T_Q=ar^{(Q-1)}$

$Tr=ar^{(r-1)}$

$$\frac{ar^{Q-1}}{ar^{p-1}}=r^{Q.-p}$$

And

$\frac{ar^{r-1}}{ar^{Q.-1}} = r^{r-Q.}$

From (1) we get

Common ratio is same

Then T_p, T_Q, T_r are in G.P.

46. (c)

$T_m = \frac{1}{a+(m-1)d} = n$

$\Rightarrow n(a+(m-1)d) = 1$ (1)

$T_n = \frac{1}{a+(n-1)d} = m$

$\Rightarrow m(a+(n-1)d) = 1$ (2)

From (1) and (2)

$n(a+(m-1)d) = m(a+(n-1)d)$

$na+(m-1)nd = ma+m(n-1)d$

$(n-m)a = (n-m)d$

$a = d$

$T_m = \frac{1}{a+(m-1)a} = n$

$\Rightarrow a = \frac{1}{mn}$

$T_r = \frac{1}{a+(r-1)d} = \frac{1}{a+(r-1)a} = \frac{mn}{r}$

47. (d)

First term is 1

N A.M's are inserted between the 1 and 51 then it become a A.P. of n+2 terms

Let the common difference is d

Then,

4^{th} A.M. will be the 5^{th} term of the A.P.

And 7^{th} A.M. will be the 8^{th} term of the A.P.

$T_5 = 1+(5-1)d = 1+4d$

$T_8 = 1+(8-1)d = 1+7d$

$\frac{1+4d}{1+7d} = \frac{3}{5}$

$\Rightarrow d = 2$

So, $T_{(n+2)} = 1+(n+2-1)d = 51$

$\Rightarrow (n+1)2 = 50$

$\Rightarrow n = 24$

48. **(b)**

x, y, z are in A.P.

a is the A.M. of x and y

$\Rightarrow a = \frac{x+y}{2}$ (1)

b is the A.M. of y and z

$\Rightarrow b = \frac{y+z}{2}$ (2)

Adding (1) and (2)

$\frac{a+b}{2} = y$

49. **(b)**

Let the common difference is d

Then,

$\frac{1}{3}, \frac{1}{3}+d, \frac{1}{4}+2d, \frac{1}{24}$ are in A.P.

$d = \frac{1}{24} = \frac{1}{3} - 2d$

$\Rightarrow d = \frac{-7}{72}$

$A_1 = \frac{1}{3} + \left(\frac{-7}{24}\right) = \frac{17}{72}$

$A_2 = \frac{1}{3} + 2\left(\frac{-7}{24}\right) = \frac{5}{36}$

50. **(c)**

H.M. between $\frac{a}{b}, \frac{b}{a}$ is

$$H = \frac{2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right)}{\left(\frac{a}{b}\right) + \left(\frac{b}{a}\right)} = \frac{2ab}{a^2b^2}$$

51. **(b)**

$\frac{2}{3}, a, b, c, d, \frac{2}{13}$ are in H.P

Then,

$\frac{3}{2}, \frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}, \frac{13}{2}$ are in A.P.

So, then Second H.M is the second A.M and it will be the 3^{rd} term of the A.P.

$T_6 = \frac{3}{2} + (6-1)d = \frac{13}{2}$

$\Rightarrow d = 1$

Therefore,

$\frac{1}{b} = \frac{3}{2} + (3-1)d$

$\Rightarrow b = \frac{2}{7}$

52. (b)

Let the one number be a the other number will be 4a

Then,

$AM + 2 = GM$

$\Rightarrow \frac{a+4a}{2} + 2 = \sqrt{a4a}$

$\Rightarrow \frac{5a}{2} + 2 = 2a$

$\Rightarrow a = 4$

53. (c)

Let the two numbers is a, b

Then,

$\frac{a+b}{2} = 34$ (1)

And,

$16^2 = ab$ (2)

Solving (1) and (2)

$a = 4, b = 64$

54. (c)

Let the two numbers is a, b

$\frac{a+b}{2} = A$

$ab = G^2$

$\frac{2ab}{a+b} = 4$

$\Rightarrow 8\left(\frac{a+b}{2}\right) = 2ab$

$\Rightarrow 4(A) = G^2$

$2A + G^2 = 27$

$\Rightarrow A = 4.5$ (1)

$\Rightarrow ab = 18$ (2)

Solving (1) and (2) we get

$a = 6$

$b = 3$

55. **(c)**

Let the two numbers is a, b

Then,

$\frac{a+b}{2} = x \quad ab = y$

$\frac{2ab}{a+b} = Z$

So, $z < y < x$

56. **(a)**

Let the two numbers is a, b

$AM = GM + 5$

$\frac{a+b}{2} = \sqrt{ab} + 5 \qquad (1)$

$GM = HM + 4$

$\sqrt{ab} = \frac{2ab}{a+b} + 4 \qquad (2)$

From (1), subtracting the value of $\sqrt{ab}$

$\frac{a+b}{2} = \frac{2ab}{a+b} + 4 \qquad (3)$

From (1)

$ab = \left(\frac{a+b}{2} - 5\right)^2 \qquad (4)$

Subtracting value of ab from (4) in (3) we get

$\frac{a+b}{2} - 5 = \frac{2}{a+b}\left(\frac{a+b}{2} - 5\right)^2 + 4$

Solving this we get

$a = 10$

$b = 40$

57. **(c)**

Let the sum is S

$S = 1 + 3x + 5x^2 + 7x^3 + ... \qquad (1)$

$xS = \quad x + 3x^2 + 5x^2 + ... \qquad (2)$

(1) – (2)

$(1-x)S = 1 + 2x + 2x^2 + 2x^3$

$(1-x)S = 1 + 2\left(x + x^2 + x^3 + ...\right)$

$(1-x)S = 1 + 2\left(\frac{x}{1-x}\right)$

$$S=\frac{1+x}{(1-x)^2}$$

58. **(d)**

$$S=1+2\left(1+\frac{1}{n}\right)+3\left(1-\frac{1}{n}\right)^2+..... \qquad (1)$$

$$\left(1-\frac{1}{n}\right)S=\left(1-\frac{1}{n}\right)+2\left(1-\frac{1}{n}\right)^2+3\left(1-\frac{1}{n}\right)^2+..... \qquad (2)$$

(1) – (2)

$$\frac{1}{n}S=1+\left(1-\frac{1}{n}\right)+\left(1-\frac{1}{n}\right)^2+\left(1-\frac{1}{n}\right)^3+......$$

$$\frac{1}{n}S=\frac{1}{1-\frac{1}{n}}=\frac{n}{n-1}$$

$$S=\frac{n^2}{n-1}$$

59. **(b)**

$$\sum_{k=1}^{n}k^3=\left(\frac{n(n+1)}{2}\right)^2=\left(\sum_{k=1}^{n}k\right)^2$$

60. **(b)**

$$S=(1+3+5+...20\text{ terms})+(2+4+8+...20\text{ terms})$$

$$\Rightarrow S=20^2+\frac{2\left(2^{20}-1\right)}{2-1}\text{ or }398+2^{21}$$

EXERCISE - 1 [C]

1. **(6534)**

All number divisible by 6 are 6, 12, 18, …., 198

Sum $= \frac{33(6+198)}{2} = 3366$

Now sum of all the even numbers less than $200 = \frac{99(2+198)}{2} = 9900$

Hence required Sum $= 9900 - 3366 = 6534.$

2. **(10)**

$S = \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...n$ terms = $1 - \frac{1}{2} + 1 - \frac{1}{4} + 1 - \frac{1}{8} + 1 - \frac{1}{16} + ...n$ terms

$\Rightarrow S = n - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...n \text{ terms}\right)$ Or $S = n - \frac{\frac{1}{2}\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} = n - 1 + 2^{-n}$.

As given $S = 9 + 2^{-10}$ hence n = 10.

3. **(64)**

$S = 1 + n + n^2 + ... + n^{127} = \frac{n^{128} - 1}{n - 1}$

$\Rightarrow S = (n^2+1)(n^4+1)(n^8+1)(n^{1+}+1)(n^{32}+1)(n^{64}+1)$

Hence $n^m + 1$ will divides s for $n = 2, 4, 8, 16, 32, 64.$

4. **(11)**

$S = \frac{1}{2} + \frac{1}{4} + ...\infty$ terms = 2 & $S_n = 1 + \frac{1}{2} + \frac{1}{4} + ...n$ terms = $2\left(1 - \frac{1}{2^n}\right)$

Now $2 - 2\left(1 - \frac{1}{2^n}\right) < \frac{1}{1000} \Rightarrow \frac{1}{2^n} < \frac{1}{2000}$ or $2^n > 2000.$

Hence ≥ 11.

5. **(900)**

In first rebound ball will travel $2 \times 100 \times \frac{4}{5}$, in second rebound ball will travel $2 \times 100 \times \left(\frac{4}{5}\right)^2$, in second rebound will travel $2 \times 100 \times \left(\frac{4}{5}\right)^3$, and so on infinitely.

Hence total distance travelled $= 100 + 200 \times \left(\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + ...\infty \text{ terms}\right) = 900$ mts.

6. **(2)**

As given $a+ar+ar^2+...+ar^{2n-1}=3\left(a+ar^2+ar^4+...+ar^{2n-2}\right)$

$$\Rightarrow \frac{a\left(1-r^{2n}\right)}{1-r}=3\frac{a\left(1-r^{2n}\right)}{1-r^2}\Rightarrow r=2.$$

7. **(16)**

$$(1+r)\left(1+r^2\right)\left(1+r^4\right)\left(1+r^8\right)=\frac{1-r^{16}}{1-r}\Rightarrow n=16.$$

8. **(6)**

Sum of n terms after first n terms $=S_{2n}-s_n=2S_n\Rightarrow S_{2n}\Rightarrow S_n=3S_n$

$$\Rightarrow \frac{2n}{2}\{2a+(2n-1)d\}=3\times\frac{n}{3}\{2a+(n-1)d\}$$

$$\Rightarrow a=(n+1)\frac{d}{2}.$$

Now $\frac{S_{3n}}{S_n}=\frac{\frac{3n}{2}\{2a+(3n-1)d\}}{\frac{n}{2}\{2a+(n-1)d\}}\Rightarrow\frac{S_n}{S_n}=\frac{3\{(n+1)d+(3n-1)d\}}{\{(n+1)d+(n-1)d\}}$

$$\Rightarrow\frac{S_{3n}}{S_n}=6.$$

9. **(1600)**

For integral roots discrmin ant must be a perfect square, hence $9+4a_i=k^2$.

The values of a_i for which it's a perfect square are $4,10,18,28,40.....,270$.

Now Let

$S_n=4+10+18+28+...+t_n$

$S_n=\quad 4+10+18+...+t_{n-1}+t_n$

$0=4+6+8+10+...n\text{ terms - }t_n$

$\Rightarrow t_n=\frac{n(n+3)}{2}$. Also 270 is 15th term.

Now $S_{15}=\frac{1}{2}\sum_{r=1}^{15}r^2+\frac{3}{2}\sum_{r=1}^{15}r$ or $S_{15}=\frac{15\times16\times31}{12}+\frac{3\times15\times16}{4}=1600$

10. **(8)**

$$\sum_{k=1}^{\infty}\frac{2^{k+2}}{3^k}=2^2\times\sum_{r=1}^{\infty}\left(\frac{2}{3}\right)^k\Rightarrow s=4\times\left(\frac{\frac{2}{3}}{1-\frac{2}{3}}\right)=8.$$

11. (14)

$\frac{a+b}{2}+\frac{2ab}{a+b}=25\,\&\,ab=144 \Rightarrow (a+b)^2-50(a+b)+576=0$. Hence $a+b=18$ or 32.

12. (2)

Let $b=ar\,\&\,c=ar^2, 2p=a+ar, 2q=ar+ar^2 \Rightarrow 2p=a(1+r), 2q=ar(1+r)$

$\frac{a}{p}+\frac{c}{q}=\frac{2}{1+r}+\frac{2ar^2}{ar(1+r)} \Rightarrow \frac{a}{p}+\frac{c}{q}=2.$

13. (188090)

$S=13^2+2.5^2+3.7^2+... \Rightarrow t_n=n(2n+1)^2$

$S_{20}\sum_{r=1}^{20}(4r^3+4r^2+r) \Rightarrow S_{20}=4\times\frac{20^2\times 21^2}{4}+4\times\frac{20\times 21\times 41}{6}+\frac{20\times 21}{2}$

Hence $S_{20}=188090.$

14. (1)

Given $t_n=\frac{1}{n(n+1)} \Rightarrow t_n=\frac{1}{n}-\frac{1}{n+1}$

Hence $S=\sum_{r=1}^{\infty}\left(\frac{1}{r}-\frac{1}{r+1}\right)=1$

15. (100)

$S_n=1+3+6+10+15+21+...+t_n$

$S_n=\quad 1+3+6+10+15+...+t_{n-1}+t_n$

$0=1+2+3+4+5+6+...n$ terms $-t_n$

$\Rightarrow t_n=\frac{n(n+1)}{2}$ Now $t_n=5050$ gives $n=100$.

PYQ : JEE Main

1. (d)
2. (c)
3. (a)
4. (b)
5. (d)
6. (b)
7. (c)
8. (b)
9. (b)
10. (a)
11. (d)
12. (c)
13. (b)
14. (b)
15. (d)
16. (b)
17. (b)
18. (b)
19. (d)
20. (b)
21. (d)
22. (b)
23. (d)

24. (b)

25. (b)

26. (d)

27. (c)

28. (a)

29. (c)

30. (d)

31. (b)

32. (c)

33. (c)

34. (c)

35. (c)

36. (b)

37. (c)

38. (a)

39. (d)

40. (b)

41. (c)

42. (c)

43. (b)

44. (7744)

45. (5143)

46. (3)

47. (832)

48. (2021)

49. (305)

50. (7)

51. (3)

52. (3)

53. (14)

54. (3)

55. (9)

56. (10)

57. (9)

58. (16)

59. (38)

60. (6993)

61. (53)

62. (50)

63. (16)

64. (702)

65. (2223)

66. (142)

67. (12)

68. (98)

69. (40)

70. (41651)

71. (27560)

72. (166)

73. (10620)

74. (120)

75. (286)

76. (1100)

77. (276)

EXERCISE - 2 [A]

1. **(c)**

$S=\log a+\log\frac{a^3}{b^2}+\ldots n \text{ terms}$

$\Rightarrow S=(1+2+3+..n \text{ terms})\log a-(1+2+3+\ldots n-1 \text{ terms})\log b$

$\Rightarrow S=\frac{n(n+1)}{2}\log a-\frac{n(n-1)}{2}\log b$

$\Rightarrow S=\frac{n^2}{2}\log\frac{a}{b}+\frac{n}{2}\log ab$.

2. **(c)**

First term = a, second term = b, last term = c.

Common difference, $d=b-a\Rightarrow c=a+(n-1)(b-a)$

Hence $n=\frac{b+c-2a}{b-a}$. Now sum of n terms will be

$S_n=\frac{a+c}{2}\left(\frac{b+c-2a}{b-a}\right)$.

3. **(b)**

$\ell=a+(n-1)d\Rightarrow d=\frac{\ell-a}{n-1}$. Also $S=\frac{n}{2}(a+\ell)\Rightarrow n=\frac{2S}{a+\ell}$.

Hence $d=\frac{\ell-a}{\frac{2S}{a+\ell}}$ i.e. $d=\frac{\ell^2-a^2}{2S-\ell-a}$

4. **(b)**

$2d=z-x$, also $2y=z+x\Rightarrow 4y^2=(z-x)^2+4zx$

Hence $4y^2=4d^2+4zx$ i.e. $d^2=y^2-zx$

5. **(b)**

$S_p=\frac{p}{2}\{2a+(p-1)d\}=0\Rightarrow d=-\frac{2a}{p-1}$

Now sum of next q term $=S_{p+q}-S_p$

$=\frac{p+q}{2}\{2a+(p+q-1)d\}$

$\Rightarrow$ Now Sum of next q term $=\frac{p+q}{2}\left\{2a-(p+1-1)\frac{2a}{p-1}\right\}=\frac{-(p+q)q}{p-1}$.

6. **(a)**

Total number of terms by the end of n^{th} group $=1+2+3+...+n=\frac{n(n+1)}{2}$

Sum of all the terms till n^{th} group $=\frac{\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2}+1\right)}{2}$

Sum of all the terms till $(n-1)^{th}$ group $=\frac{\frac{n(n-1)}{2}\left(\frac{n(n-1)}{2}+1\right)}{2}$

Sum of n^{th} group $=\frac{\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2}+1\right)}{2}-\frac{\frac{n(n-1)}{2}\left(\frac{n(n-1)}{2}+1\right)}{2}$

$\Rightarrow$ Sum of n^{th} group $=\frac{n(n^2+1)}{2}$

7. **(c)**

Let the numbers be $a-3d, a-d, a+d, a+3d$.

As given $a-3d+a-d+a+d+a+3d=48 \Rightarrow a=12$

Also $\frac{a^2-9d^2}{a^2-d^2}=\frac{27}{35} \Rightarrow \frac{144-9d^2}{144-d^2}=\frac{27}{35} \Rightarrow d=\pm 2$

Hence the numbers are 6, 10, 14, 18.

8. **(a)**

$$S=\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+...+10\text{ terms}$$

$$\Rightarrow S=\left(x^2+x^4+x^6+...+10\text{ terms}\right)+\left(\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^6}+...+10\text{ terms}\right)+20$$

$$\text{Or } S=\frac{x^2\left(x^{20}-1\right)}{x^2-1}+\frac{\frac{1}{x^2}\left(1-\frac{1}{x^{20}}\right)}{1-\frac{1}{x^2}}+20 \text{ i.e. } S=\left(\frac{x^{20}-1}{x^2-1}\right)\left(\frac{x^{22}+1}{x^{20}}\right)+20.$$

9. **(d)**

$$S=\sqrt{ax}\left(1+\sqrt{b}+b+b\sqrt{b}+...\infty\text{ terms}\right)+x\left(1+\sqrt{y}+y+y\sqrt{y}+...\infty\text{ terms}\right)$$

$$\Rightarrow S=\frac{\sqrt{ax}}{1-\sqrt{b}}+\frac{x}{1-\sqrt{y}}.$$

10. **(b)**

$a+ar+ar^2=S, a\times ar\times ar^2=P \;\&\; \frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}=R$.

$$\Rightarrow a\left(1+r+r^2\right)=S, a^3a^3=P \ \& \ \frac{a+r+r^2}{ar^2}=R$$

$$\Rightarrow P^2R^3\times\frac{\left(1+r+r^2\right)^3}{a^3r^6} \text{ or } P^2R^3=a^3\left(1+r+r^2\right)^3=S^3$$

11. (c)

$$S=1+r+r^2+\ldots\infty \text{ terms }=\frac{1}{1-r}\Rightarrow r=\frac{S-1}{S}.$$

Also $1+r^2+r^4+\ldots\infty$ terms $=\dfrac{1}{1-r^2}$

Hence $1+r^2+r^4+\ldots\infty$ terms $=\dfrac{S^2}{S^2-(S-1)^2}$ i.e. $\dfrac{S^2}{2S-1}$.

12. (b)

As given $x=a+ar+ar^2+\ldots\infty$ terms $=\dfrac{a}{1-r}$

and $y=a^2+a^2r^2+a^2r^4+\ldots\infty$ terms $\dfrac{a^2}{1-r^2}$

$$\Rightarrow r=\frac{x^2-y}{x^2+y}.$$

13. (b)

$$0.7+0.77+0.777+\ldots=\frac{7}{9}(0.9+0.99.+0.999+\ldots)$$

$$=\frac{7}{9}\left(10-\frac{1}{10}-\frac{1}{10^2}-\frac{1}{10^3}-\ldots\right)=\frac{7}{81}\left(89+\frac{1}{10^{10}}\right)$$

14. (a)

$$f(2n)=\sum_{r=1}^{2n}r^4\Rightarrow f(2n)=\sum_{r=1}^{n}(2r-1)^4+\sum_{r=1}^{n}(2r)^4$$

$$\Rightarrow f(2n)=\sum_{r=1}^{n}(2r-1)^4+16\times\sum_{r=1}^{n}r^4$$

$$\Rightarrow f(2n)=\sum_{r=1}^{n}(2r-1)^4+16f(n)$$

$$\Rightarrow f(2n)-16f(n)=\sum_{r=1}^{n}(2r-1)^4$$

15. (a)

For roots to be real $q^2\geq 4pr$. But as p, q, r are in A.P. hence $q=\dfrac{p+r}{2}$.

Thus $(p+r)^2 \geq 16pr$ or $p^2 - 14pr + r^2 \geq 0 \Rightarrow \left(\frac{p}{r} - 7\right)^2 \geq 48$

$\Rightarrow \left|\frac{p}{r} - 7\right| \geq 4\sqrt{3}$. Similarly $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$.

16. (b)

$\log_{5.2^x+1} 2, \log_{2^{1-x}+1} 4, 1$ are in H.P. hence $\log_2(5.2^x+1), \log_2(2^{1-x}+1)^{1/2}, \log_2 2$ are in A.P.

$\Rightarrow (5.2^x+1), (2^{1-x}+1)^{1/2}, 2$ are in G.P.

$\Rightarrow 2^{1-x} + 1 = (5.2^x+1) \times 2$ or $\frac{2}{2^x} + 1 = 10.2^x + 2$

$\Rightarrow 10(2^x)^2 + 2^x - 2 = 0$

$\Rightarrow 2^x = \frac{4}{5}$

Hence x is a negative real number.

17. (a)

$$\frac{S_{kx}}{S_x} = \frac{\frac{kx}{2}\{2a + (kx-1)d\}}{\frac{x}{2}\{2a + (x-1)d\}} \Rightarrow \frac{S_{kx}}{S_x} = \frac{k\{2a - d + kxd\}}{\{2a - d + xd\}}$$

Clearly if $2a = d$, then $\frac{S_{kx}}{S_x} = k^2$ i.e. independent of x.

18. (d)

p, q, r are in H.P. hence q is H.M. of p & r. Also $\sqrt{pr}$ is G.M. of p & r.

Now H.M. < G.M. $\Rightarrow q < \sqrt{pr}$ or $q^2 < pr$.

As discriminant is $4(q^2 - pr)$ hence roots must be imaginary.

19. (b)

$$S_n = \frac{a(r^n - 1)}{r-1}, S_{2n} = \frac{a(r^{2n} - 1)}{r-1} \& S_{3n} = \frac{a(r^{3n} - 1)}{r-1}$$

$$S_{2n} = (1 + r^n)S_n \& S_{3n} = (1 - r^n + r^{2n})S_n$$

$$S_{2n} - S_n = (r^n)S_n, S_{3n} - S_{2n} = r^n(r^n - 2)S_n, S_{3n} - S_n = r^n(r^n - 1)S_n$$

20. (a)

$$S = \frac{1}{2.4} + \frac{1.3}{2.4.6} + \frac{1.3.5}{2.4.6.8} + \frac{1.3.5.7}{2.4.6.8.10} + \ldots\infty \text{ terms}$$

$$S = \frac{1.(4-3)}{2.4} + \frac{1.3.(6-5)}{2.4.6} + \frac{1.3.5.(8-7)}{2.4.6.8} + \frac{1.3.5.7.(10-9)}{2.4.6.8.10} + \ldots\infty \text{ terms}$$

$$S=\frac{1}{2}-\frac{1.3}{2.4}+\frac{1.3}{2.4}-\frac{1.3.5}{2.4.6}+\frac{1.3.5}{2.4.6}-\frac{1.3.5.7}{2.4.6.8}+\frac{1.3.5.7}{2.4.6.8}-\frac{1.3.5.7.9}{2.4.6.8.10}+...\infty \text{ terms}$$

$$S=\frac{1}{2}.$$

21. **(c)**

$$S=1^2-\frac{2^2}{5}+\frac{3^2}{5^2}-\frac{4^2}{5^3}-\frac{5^2}{5^4}+....\infty \text{ terms}$$

$$\frac{1}{5}S=\quad \frac{1^2}{5}-\frac{2^2}{5^2}+\frac{3^2}{5^3}-\frac{4^2}{5^4}-...\infty \text{ terms}$$

$$\frac{6}{5}S=1-\frac{3}{5}+\frac{3}{5^2}-\frac{7}{5^3}+\frac{9}{5^4}-...\infty \text{ terms}$$

$$\frac{6}{5}S=1-\frac{3}{5}+\frac{3}{5^2}-\frac{7}{5^3}+\frac{9}{5^4}-...\infty \text{ terms}$$

$$\frac{6}{25}S=\quad \frac{3}{5}-\frac{3}{5^2}+\frac{7}{5^3}-\frac{9}{5^4}+...\infty \text{ terms}$$

$$\frac{36}{25}S=1-2\left(\frac{1}{5}-\frac{1}{5^2}+\frac{1}{5^3}-\frac{1}{5^4}+...\infty \text{ terms}\right)$$

Hence $S=\frac{25}{54}$.

22. **(b)**

According to Cauchy – Schwarz inequality

$$\left(x_1^{\ 2}+x_2^{\ 2}+x_3^{\ 2}+...+x_{n-1}^{\ 2}\right)\left(x_2^{\ 2}+x_3^{\ 2}+x_4^{\ 2}+...+x_3^{\ 2}\right)\ge\left(x_1x_2+x_2x_3+x_3x_4+...+x_{n-1}x_n\right)$$

Hence as given in question

$$\left(x_1^{\ 2}+x_2^{\ 2}+x_3^{\ 2}+...+x_{n-1}^{\ 2}\right)\left(x_2^{\ 2}+x_3^{\ 2}+x_4^{\ 2}+...+x_3^{\ 2}\right)=\left(x_1x_2+x_2x_3+x_3x_4+...+x_{n-1}x_n\right)$$

Now in Cauchy – Schwarz inequality equality occurs when

$\frac{x_1}{x_2}=\frac{x_2}{x_3}=\frac{x_3}{x_4}=...\frac{x_{n-1}}{x_n}$. Hence numbers are in G.P.

23. **(b)**

$$S_n=1+\frac{3}{2}+\frac{5}{3}+...\frac{2n-1}{n}\Rightarrow S_n=2-1+2-\frac{1}{2}+2-\frac{1}{3}+...+2-\frac{1}{n}$$

$\Rightarrow S_n=2n-\left(1+\frac{1}{2}+\frac{1}{3}+...\frac{1}{n}\right)$ or $S_n=2n-H_n$.

24. **(c)**

$$4x^2+9y^2+16z^2=6xy-12yz-8zx=0$$

$$\Rightarrow 8x^2+18y^2+32x^2-12xy-34yz-16zx=0$$

$$\Rightarrow(2x-3y)^2+(3y-4z)^2(3y-4z)^2+(4z-2x)^2=0$$

$\Rightarrow 2x=3y=4z$ or $\frac{x}{6}=\frac{y}{4}=\frac{z}{3}$.

Hence x, y, z are in H.P.

25. **(c)**

$$a_n=1+10+10^2+...+10^{n-1}\Rightarrow a_n=\frac{10^n-1}{9}$$

Now $a_{124}=\frac{10^{224}-1}{9}$. Observe that $271\times369=10^5-1$

Rewrite a_{124} as $\frac{\left(\left(10^5\right)^{124}-1\right)10^4+10^4-1}{9}$ or $\frac{27\ln}{9}+1111$

Now remainder when 1111 is divided by 27 is 27.

26. **(a)**

$a=\frac{a}{1-r},y=\frac{b}{1+r}\,\&\,z=\frac{c}{1-r^2}\Rightarrow\frac{xy}{z}=\frac{ab}{c}$.

27. **(c)**

$1+|\cos x|+|\cos x|^2+\ldots\infty$ terms $=\frac{1}{1-|\cos x|}\Rightarrow 8^{\frac{1}{1-|\cos x|}}=4^3$

Or $2^{\frac{3}{1-|\cos x|}}=2^6$ hence $\cos x=\pm\frac{1}{2}$

Now in $(-\pi,\pi)$, $x=-\frac{2\pi}{3},-\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3}$.

28. **(a)**

Let $b=ar,c=ar^2+d=ar^3$, then

$(a-c)^2+(b+c)^2+(b-d)^2-(a-d)^2=\left(a-ar^2\right)^2+\left(ar-ar^2\right)^2+\left(ar-ar^3\right)^2-\left(a-ar^3\right)^2$

$=a^2(1-r)^2(1+r)^2+a^2r^2(1-r)^2+a^2r^2(1-r)^2(1+r)^2-a^2\left(1+r+r^2\right)^2$

$=a^2(1-r)^2(1+r)^2\left(1+r^2\right)-a^2(1-r)^2\left(1+r^2\right)(1+r)^2=0$.

29. **(c)**

$S=1+(1+x)+\left(1+x+x^2\right)+\left(1+x+x^2x^3\right)+\ldots n$ terms

$\Rightarrow(1-x)S=(1-x)+\left(1-x^2\right)+\left(1-x^3\right)+\left(1-x^4\right)+\ldots n$ terms

$\Rightarrow(1-x)S=n-\left(x+x^2+x^3+\ldots n\text{ terms}\right)$

$\Rightarrow S=\frac{n}{1-x}-\frac{\left(x^n-1\right)}{(1-x)^2}$.

30. **(a)**

$2,\frac{7}{4},\frac{14}{9},\ldots$ is an H.P. hence $\frac{1}{2},\frac{4}{7},\frac{9}{14},\ldots.$ will be an A.P. with $d=\frac{1}{14}$

6^{th} term $=\frac{1}{2}+5\times\frac{1}{14}$ i.e. $\frac{6}{7}$.

Hence 6^{th} term of the given series will be $\frac{7}{6}$.

31. **(d)**

A.M. of n A.M.s between a & b is single A.M. between a & b, hence
Sum of n A.M.s between a & b, S = nA.

$\Rightarrow \frac{S}{A} = n$.

32. **(b)**

$2b = a + c, x^2 = ab, y^2 = bc \Rightarrow \frac{x^2 + y^2}{2} = b^2$.

33. **(b)**

Let the roots be $x_1 \& x_2$, then $x_1 + x_2 = 2A \& x_1 + x_2 = G^2$.

Required equation is $x^2 - 2Ax + G^2 = 0$.

34. **(c)**

$\frac{p+q}{2} = 2\sqrt{pq} \Rightarrow (p+q)^2 = 16pq$

$\Rightarrow \left(\frac{p}{q}\right)^2 - 14\left(\frac{p}{q}\right) + 1 = 0$. Hence $\frac{p}{q} = 7 + 2\sqrt{12}$. Now $7 + 2\sqrt{12} = \left(2+\sqrt{3}\right)^2 = \frac{2+\sqrt{3}}{2-\sqrt{3}}$.

35. **(a)**

G.M. of roots of $x^2 - 2ax + b^2 = 0$, $\sqrt{b^2} = |b|$

Now for $x^2 - 2bx + a^2 = 0, b = \frac{\text{sum of roots}}{2}$ i.e. AM.

36. **(b)**

$a + b + c = 3A, abc = G^3 \& \frac{3abc}{ab + bc + ca} = H \Rightarrow ab + bc + ca = \frac{3G^3}{H}$

Now required equation is $x^3 - 3Ax^2 + \left(\frac{3G^3}{H}\right)x - G^3 = 0$.

37. **(c)**

Given $t_n = (2n-1)(2n+1)(2n+3)$. Let $T_n = (2n-1)(2n+1)(2n+3)(2n+5)$

$T_{n-1} = (2n=3)(2n-1)(2n+1)(2n+3)$.

Now $T_n - T_{n-1} = 8t_n$

$S_n \sum_{r=1}^{n} t_r = \frac{1}{8} \times \sum_{r=1}^{n} (T_r - T_{r=1})$

Hence $S_n = \frac{T_n - T_0}{8}$ i.e.$S_n = \frac{(2n-1)(2n+1)(2n+3)(2n+5)+15}{8}$

38. **(a)**

$S = 1^2 + 2^2 + 3^3x^2 + 4^2x^2 + ...\infty$ terms

$$xS = \quad 1^2x + 2^2x^2 + 3^2c^2 + ...\infty \text{ terms}$$
$$(1-x)S = 1 + 3x + 5x^2 + 7x^3 + ...\infty \text{ terms}$$
$$x(1-x)S = \quad x + 3x^2 + 5x^2 + ...\infty \text{ terms}$$
$$(1-x)^2 S = 1 + 2(x + x^2 + x^3 + ...\infty \text{ terms})$$
$$\Rightarrow S = \frac{1}{(1-x)} + \frac{2x}{(1-x)^3}.$$

39. **(a)**

$$S = 1 - \frac{3}{2} + \frac{5}{4} + \frac{7}{8} + ...\infty \text{ terms}$$
$$\frac{1}{2}S = \frac{1}{2} - \frac{3}{4} + \frac{5}{8} ...\infty \text{ terms}$$
$$\frac{3}{2}S = 1 - 2\left(\frac{1}{2} - \frac{1}{2^2} + \frac{1}{2^3} - \frac{1}{2^4} + ...\infty \text{ terms}\right)$$
$$\Rightarrow S = \frac{2}{9}.$$

40. **(c)**

$$S = 1 + 3 + 7 + 15 + ...n \text{ terms}$$
$$\Rightarrow S = 2 - 1 + 4 - 1 + 8 - 1 + 16 - 1 + ...$$
$$\Rightarrow S = (2 + 4 + 8 + 16 + ...n \text{ terms}) - n$$
$$\Rightarrow S = 2^{n+1} - 2 - n.$$

41. **(d)**

a, b, c, d are in A.P. $\Rightarrow \frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd}$ will also be in A.P.

i.e. $\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}$ are in A.P.

$\Rightarrow bcd, acd, abd, abc$ are in H.P.

42. **(a)**

Roots are equal therefore Discrminant = 0.

$$(c-a)^2 = 4(b-c)(a-b) \Rightarrow a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ax = 0.$$
$$\Rightarrow (a + c + 2b)^2 = 0$$
$$\Rightarrow a + c = 2b.$$

43. **(b)**

Clearly the given expression is in form of sum of a G.P.

44. **(c)**

$$x = \frac{1}{1-a}, y = \frac{1}{a-b}, z = \frac{1}{1-c}$$

Now a, b, c are in A.P. hence $1-a, 1-b, 1-c$ will also be in A.P.

Hence $x=\frac{1}{1-a}, y=\frac{1}{1-b}, z\frac{1}{1-c}$ will be in H.P.

45. **(b)**

$\left(a^2+b^2\right)x^2-2ab\left(a+c\right)x+\left(b^2+x^2\right)=0 \Rightarrow \left(ax-b\right)^2+\left(bx-c\right)^2=0$

Hence $\frac{b}{a}=\frac{c}{}a=x$. Therefore a, b, c are in G.P.

EXERCISE - 2 [B]

1. **(a, c)**

$b = \dfrac{2ac}{a+c}$

Now, discriminate $= 4\left(b^2 - ac\right) < 0$ $\qquad \left(\begin{array}{l} \because b = 1 + m \text{ of } a, b, c \\ a + m \le \left(4m = \sqrt{ac}\right) \end{array}\right)$

Thus non-real roots

$\text{sum} = \dfrac{-2b}{a} = \text{real}$

$\boxed{\text{Ans}: a, c}$

2. **(b, d)**

$\left(a^2 + b^2 + c^2\right)p^2 - 2\left(ab + bc + cd\right)p + \left(b^2 + c^2 + a^2\right) \le 0$

For real p, discriminate ≤ 0 as expression is everywhere non-positive

But $a^2 + b^2 + c^2 > 0 \Rightarrow \text{expression} \ge 0$

Thus, $4\left(ab + bc + cd\right)^2 = \left(a^2 + b^2 + c^2\right)$

Solving, we get ad = bc

Also, $\dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c}$

$\boxed{\text{Ans}: b, d}$

3. **(c, d)**

$S_{even} = S_{odd} + 50d$

$S_{even} + S_{odd} = S_n = -1$

$1 + \left(1 + 50d\right) = -1$

Also, $\dfrac{100}{2}\left[2a + 99 \times \dfrac{3}{50}a\right] = -1$

$\Rightarrow 100a + \dfrac{99 \times 3 \times 100}{100} = -1$

$\Rightarrow 100a = -148$

Solving $t_{100} = \dfrac{74}{25}$

$\boxed{\text{Ans}: c, d}$

4. **(a, b, c)**

$a = AR^{p-1}$

$a = AR^{q-1}$

$c = AR^{r-1}$

$\log a = \log A + \left(p-1\right)\log R$

$\log b = \log A + \left(q-1\right)\log R$

$$\log c = \log A + (r-1)\log R$$

$$(q-r)\log a + (r-p)\log b + (p-q)\log c$$

$$= (\log A)(q-r+r-p+p-q)$$

$$+\left[(q-r)(p-1)+(r-p)(q-1)+(p-q)(r-1)\right]\log R$$

$$= 0$$

5. **(a, b, c)**

$$\alpha+\beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$

$$(\alpha-\beta)^2 = \left(\frac{-b}{a}\right)^2 - \frac{4c}{a} = \frac{+b^2-4ac}{a^2}$$

$$y+\delta = \frac{-q}{p}, y\delta = \frac{r}{p}$$

(a) $$\rightarrow \frac{b^2-4ac}{q^2-4pr} = \frac{a^2}{p^2} \times \frac{\left(\frac{b}{a}\right)^2 - 4\frac{c}{a}}{\left(\frac{q}{p}\right)^2 - 4\frac{q}{b}}$$

$$= \frac{a^2}{p^2} \times \frac{(\alpha+\beta)^2 - 4\alpha\beta}{(y+\delta)^2 - y\delta}$$

$$= \left(\frac{a^2}{p^2}\right)\left(\frac{\alpha-\beta}{y-\delta}\right)^2 = \frac{a^2}{p^2} \times \frac{\frac{b^2-4ac}{a^2}}{\frac{q^2-4pr}{p^2}}$$

$$= \frac{c^2}{r^2}$$

(b) $$\frac{\alpha-\beta}{\alpha\beta} = \left(\frac{\frac{b^2-4ac}{a^2}}{\frac{a}{c}}\right)^{\frac{1}{2}} = \left(\frac{b^2 4ac}{ac}\right)^{\frac{1}{2}}$$

Also, $\frac{1}{\beta}+\frac{1}{y} = \frac{1}{\alpha}+\frac{1}{\delta}$

$$\Rightarrow \frac{y-\delta}{ys} = \frac{\alpha-\beta}{\alpha\beta}$$

6. **(a, b, c)**

$f(0) = 1$

$f'(x) = 3ax^2 + 26x - 1$

f' exist everywhere

$f'(0)$ has two distinct roots.

So $(2b)^2 + 4(3a) > 0$

$4b^2 12a > 0$

$b^2 + 3a > 0$

Let the roots be $p, \frac{2pq}{p+q}$ and q.

Substituting them and eliminating p and q we get a, b, c

7. **(b, c, d)**

$\log\frac{a}{2b}, \log\frac{b}{1.5c}, \log\frac{3c}{a}$ is ab

So, $\left(\frac{b}{1.5c}\right)^2 = \frac{3c}{a} \times \frac{a}{2b}$

$\frac{b^2}{2.25c^2} = \frac{3c}{2b}$

$2b^3 = 6.7c^3$

$\left(\frac{b}{c}\right)^3 = 3.375$

$\Rightarrow \frac{b}{c} = \frac{3}{2}$

$a = 4.5k$

$a = 3k$

$c = 2k$

$\text{Thus}, 7 \to a, b, c, d$

8. **(a, b, c)**

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$E = \left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right)\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right)$

$= \left(\frac{1}{c} + D\right)\left(\frac{1}{c} - D\right)$ (D = common diff)

$= \frac{1}{c^2 - D^2}$

$= \frac{1}{c^2} - \left(\frac{1}{b} - \frac{1}{a}\right)^2$

$= \frac{1}{c^2} - \frac{1}{a^2} - \frac{1}{b^2} + \frac{2}{ab}$

Substituting $b = \frac{2ac}{ab}$

This simplifies to $\boxed{\text{Ans}: b, c, d}$

9. **(a, b, c)**

$A_1 = a + \frac{b-a}{3}$

$A_2 = \frac{a+2b}{3}$

$G_1 = a\sqrt[3]{\frac{ab}{a}} = \left(a^2 b\right)^{\frac{1}{3}}$

$G_2 = \left(b^2 a\right)^{\frac{1}{3}}$

$$\mu_1 = \frac{9ab}{3a+6b}, \mu_2 = \frac{9ab}{3b+6a}$$

Solving the choices we find $\boxed{\text{Ans}: a, b, c}$

10. **(a, b)**

$$S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{3}{q} \times \frac{1}{3} = \frac{1}{q}$$

Option $a \rightarrow (0.25)^{-1} = 4$

Option $b \rightarrow (0.008)^{\cos(\frac{1}{1})} = 8$

11. **(a, b, c, d)**

a, c

$1, 5, 25, \ldots\ldots$

12. **(a, b, c, d)**

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ is A.P.

So $\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ is A.P

So $\frac{a+b+c}{a} - 2, \frac{a+b+c}{b} - 2, \frac{a+b+c}{c} - 2$ is A.P.

$\Rightarrow \frac{b+c-a}{a}, \frac{a+c-b}{b}, \frac{a+b-c}{c}$ is A.P.

Thus $\frac{a}{b+c-a}, \frac{b}{a+c-b}, \frac{c}{a+b-c}$ is A.P.

Similarly rest of options.

$12 \rightarrow a, b, c$.

13. **(a, b)**

$$S = 1^2 + 2(2)^2 + 3^2 + 2(4)^2 \ldots$$

$$= (1^2 + 2^2 + 3^2 + 4^2 + \ldots)$$

$$4\left(1^2 + 2^2 + 3^2 + \ldots\left(\frac{n}{2}\right)^2\right)$$

N is even, we have

$$S = \frac{n(n+1)(2n+1)}{6} + \frac{4\left(\frac{n}{2}\right)\left(\frac{n}{2}+1\right)(n+1)}{6}$$

$$= \frac{n(n+1)^2}{2}$$

If n is odd, we have

$$S = \left(1^2 + 2^2 + ... + n^2\right) + 4\left(1^2 + 2^2 + ... + \left(\frac{n-1}{2}\right)^2\right)$$

$$= \frac{n(n+1)(2n+1)}{6} + \left(\frac{\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)(n-1)}{6}\right)$$

$$= \frac{1}{2}n^2$$

14. **(a, b, d)**

$t_a = 2, t_b = 31$

$d = \frac{31-2}{b-a} = \frac{29}{b-a} = \text{rational}$

So all terms rational

$\boxed{\text{Ans}: 14 \to a, b, d}$

15. **(a, b)**

$\underset{p}{\overset{\shortparallel}{a}}\ \underset{q}{\overset{\shortparallel}{ar}}\ \underset{r}{\overset{\shortparallel}{ar^2}}$

$p^2 + q^2 + 2pq\cos R = r^2$

$\Rightarrow a^2r^4 = a^2 + a^2r^2 + 2a^2r\cos R$

$\Rightarrow r^4 = 1 + r^2 + 2r\cos R$

$\Rightarrow \cos R = \frac{r^4 - r^2 - 1}{2r}$

$0 < \frac{r^4 - r^2 - 1}{2r} < 1$

16. **(c, d)**

$b^2 = \frac{2a^2c^2}{a^2 + c^2}$ & $b = \frac{a+c}{2}$

$\Rightarrow \left(a^2 + c^2 + 2ac\right)\left(a^2 + c^2\right) = 8a^2c^2$

or $\left(a^2 + c^2\right) + 2ac\left(a^2 + c^2\right) - 8(ac)^2 = 0$

$\Rightarrow \left(a^2 + c^2 - 2ac\right)\left(a^2 + c^2 + 4ac\right) = 0$

As $a \neq b \neq c$ hence $a^2 + c^2 - 2ac \neq 0$

Now $a^2 + 4ac + c^2 = 0$ gives $c = \left(-2 \pm \sqrt{3}\right)a$ & $2b = \left(-1 \pm \sqrt{3}\right)a$

17. **(a, b, c, d)**

$a_n = (\underbrace{111......11}_{n\text{ times}})$

If n is divisible by 3.

Then $a_n 3$

Thus not a prime

$\boxed{Ans:17 \rightarrow a,b,c}$

18. **(a, b, c, d)**

$ar^2 > 4(ar) - 3a \qquad (b_1 = a, b_2 = ar, b_3 = ar^3)$

$\Rightarrow r^2 < 4r - 3$

$r^2 - 4r + 3 > 0$

$r < 1 \,\&\, r > 3$

$\boxed{Ans:19 \rightarrow b,c}$

19. **(c, d)**

$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{2^n - 1}$

$< 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + (\text{missing})$

$\Rightarrow a_n < n$

By induction, for option $b, a(k) < \frac{k}{2}$

$a(k+1) = \frac{k}{2} + \frac{1}{2^k} + ... + \frac{1}{2^k - 1} < \frac{k}{2} + 1 < \frac{(k+1)}{2}$

Thus by induction, $a(n) < \frac{n}{2}$

Similarly $a(2n) > n$

20. **(a, b, c)**

Solving purely by principle of mathematical inductions.

(a) $\rightarrow \alpha(2 \times 1) = -\frac{1}{2} = \frac{1}{2}$

$\frac{1}{1+1} + \frac{1}{a+2} + ... + \frac{1}{2} \neq \frac{1}{2}$ wrong

(b) $\rightarrow$ for $n = 1, \quad \alpha(2n) < 1$

for $n = k, \alpha(2k) < 1$ (suppose)

$\alpha(2(k+1)) = \alpha(2k+2)$

21. **(c, d)**

22. **(a, b, c, d)**

23. **(b, c, d)**

24. **(a, b, c, d)**

25. **(a, b)**

26. **(a, d)**

27. **(b, d)**

28. **(a, b)**

29. **(b, c, d)**

30. **(a, b, c, d)**

31. **(b, d)**

32. **(a, c, d)**

33. **(b, c)**

34. **(a, c, d)**

35. **(a, d)**

36. **(b, c)**

37. **(a, b, c, d)**

38. **(a)**

39. **(a, b, c)**

40. **(b, c)**

41. **(a, b, c, d)**

42. **(a, b, c)**

43. **(a, b, c, d)**

44. **(a, b, c, d)**

45. **(a, b, c, d)**

Paragraph Type

Passage-I

1. **(a)**

$$a = 2r, d = 2r - 2$$

$$Ar = \frac{r}{2}\left(4r + (r-1)(2r-2)\right)$$

$$= \frac{r}{2}\left(4r + 2r^2 - 2r - 2r + 2\right)$$

$$= \frac{r}{2}\left(2r^2 + 2\right)$$

$$= r^3 + r$$

$$\sum_{r=1}^{n} Ar = \sum r^3 + \sum r$$

$$= \left| \left[\frac{r(r+1)^2}{2} \right] + \frac{r(r+1)}{2} \right| r = n$$

$$= \left| \frac{r(r+1)}{2} \left(\frac{r^2 + r + 12}{2} \right) \right| r = n$$

$$= \frac{n}{4}(n+1)(n^2 + n + 2)$$

$\boxed{\text{Ans : a}}$

2. **(c)**

$B_{10} = A_{12} - A_{11}$

$= 12^3 + 12 + 11^3 - 11$

$= 1728 + 12 - 1331 - 11$

$= 1740 - 1342$

$= 398$

3. **(b)**

$C_r = B_{r+1} - B_r$

$= A_{r+3} - A_{r+2} - A_{r+2} + A_{r+1}$

$= A_{r+3} + A_r - 2A_{r+2}$

$= (r+3)^3 + r^3 - 2(r+2)^3 + r + 3 + r - 2r - 4$

$= r^3 + 27 + 27r + 9r^2 + r^3 - 2r^3 - 16 - 24r - 12r^2 - 1$

$= -3r^2 + 3r + 8$

$\frac{d}{r} = -6r + 3.|\text{diff}| = 6$

Passage-II

4. **(d)**

$3, B, C, D$

Now, $\frac{2 \times 3 \times c}{c+3} = b$ or $\frac{c}{c+3} = \frac{b}{6}$

Also, $2c = b + d$

$c = \frac{b+d}{2} = \frac{b+(b+4)}{2} = b + 2$

So $\frac{b+2}{b+5} = \frac{b}{6}$

Or $6b + 12 = b^2 + 5b$

$b^2 - b - 12 = 0$

$c = 3, \quad b = 4, \; c = 6, \; d = 8$

$\boxed{\text{Ans : (d)}}$

5. **(c)**

$4, A_1, A_2, A_3, 6$

$\Rightarrow 4, 4.5, 5, 5.5, 6$

$5.5 = \frac{11}{2} = \frac{11}{4} = \frac{14+8}{4} = \frac{k+8}{4}$

$\boxed{\text{Ans}:(c)}$

6. **(b)**

$k = 14, b = 4, d = 8$

$kb^2 = 14.4^7 . kd^7 = 14.8^7$

$r = \left(\frac{14.8^7}{14.4^7}\right) = 2$

Passage-III

7. **(a)**

$$\frac{\frac{a}{3}+\frac{a}{3}+\frac{a}{3}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{4}+\frac{c}{4}+\frac{c}{4}+\frac{c}{4}}{4} \geq 10\sqrt{\left(\frac{a}{3}\right)^3\left(\frac{b}{3}\right)^3\left(\frac{c}{4}\right)^4}$$

$\boxed{\text{Ans}:(a)}$

8. **(d)**

$$\frac{a+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}}{10}$$

$$\geq \frac{10}{\frac{1}{a}+\frac{3}{b}+\frac{3}{b}+\frac{3}{b}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}}$$

or $\frac{1}{a}+\frac{9}{b}+\frac{36}{c} \geq \frac{100}{20}$ i.e. 5

9. **(b)**

$$\frac{3(a+x)+4(y+2)+5(z+4)}{12} \geq \sqrt[12]{(1+x)^3(y+2)^4(z+4)^5}$$

$$\Rightarrow \frac{5+3+8+20}{12} \geq \sqrt[12]{(1+x)^3(y+2)^4(z+4)^5}$$

3^{12} is max value.

$\boxed{\text{Ans}:(b)}$

Passage-IV

10. **(c)**

11. (d)

12. (b)

Passage-V

13. (c)

14. (d)

15. (a)

Passage-VI

16. (b)

17. (a)

18. (d)

Matrix-Match Type

1. **(A) → (P), (B) → (R), (C) → (Q), (D) → (P)**

$(A) \to (P)$

So $\frac{p}{64} = 2^{9-6} = 2^3 = 8$

(B) $\to (r)$

$$\frac{4^{x+\frac{1}{2}} + 4^{\frac{3}{2}} - x}{4} = \frac{1}{2}\left(\frac{x^{x+\frac{1}{2}} + 4^{\frac{3}{2}-x}}{x + \frac{1}{2} + \frac{3}{2} - x}\right)$$

This is greater than $\sqrt{4^{x+\frac{1}{2}} \times 4^{\frac{3}{2}-x}} = 4$

$\frac{1}{2}(4) = 2$

(C) $\to (q)$

$(x+2y-2)(2y+z-x)(z+x-y)$

$= (a+2a = 2d-a-2d)(2a+2d+a+2d-a)(a+2d+a-a-d)$

$= (2a)(2a+4d)(a+d)$

$= 2x \times 2z \times y$

$= 4xyz$

(D) $\to (p)$

$t_1 = 7(t_2 + \ldots + \ldots)$

$a = 7\left(\frac{a}{1-r} - a\right)$

$\Rightarrow 8a = \frac{7a}{1-r}$

$1 - r = \frac{7}{8}$

2. **(A) → (R), (B) → (P), (C) → (Q), (D) → (P)**

3. **(A) → (Q), (B) → (R), (C) → (Q), (D) → (R)**

4. **(A) → (T), (B) → (T), (C) → (R), (D) → (Q)**

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ is A.P.

So $\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ is A.P.

So $\frac{a+b+c}{a} - 2, \frac{a+b+c}{b} - 2, \frac{a+b+c}{c} - 2$ is A.P.

$\Rightarrow \frac{b+c-a}{a}, \frac{a+c-b}{b}, \frac{a+b-c}{b}, \frac{a+b+c}{c}$ is A.P.

Thus $\frac{a}{b+c-a}, \frac{b}{a+c-b}, \frac{c}{a+b-c}$ is A.P.

1. **(6)**

$$\frac{\frac{n}{2}\left[10+(n-1)4\right]}{\frac{n}{2}\left[14+(n-1)2\right]}=\frac{5}{4}$$

$$\frac{4n+6}{2n+12}=\frac{5}{4}$$

$$\frac{2n+3}{n+6}=\frac{5}{4}$$

$$\Rightarrow \boxed{n=6}$$

2. **(5)**

$$\frac{n}{2}\left[2\times 2+(n-1)3\right]=950$$

$$\frac{n}{2}\left[4+3n-3\right]=950$$

$$3n^2+n=1900$$

$$\boxed{n=25}$$

3. **(8)**

Let n be number of terms, then

$$S_{even}+S_{odd}=S_n$$

$$S_{even}=S_{odd}+\left(\frac{n}{2}\right)d$$

$$\Rightarrow 30=24+\frac{nd}{2}$$

$$\Rightarrow nd=12$$

also $a+(n-1)d-q=\frac{21}{2}$

4. **(7)**

$$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2a'+(n-1)d'\right]}=\frac{7n+1}{4n+27}$$

For $n=21$

$$\frac{2a+20d}{2a'+20d'}=\frac{147+1}{111}$$

$$\Rightarrow \frac{a+10d}{a'+20d'}=\frac{148}{111}$$

$$\Rightarrow \frac{t_{11}}{t_{11}}=\frac{148}{111}=\frac{4}{3}$$

$$4+3=\boxed{7}$$

5. **(6)**

$LCM=60$

So (div. by 15) + (div. by 20) – (div. by 60)

$= 20 + 15 - 5$

$= \boxed{30}$

6. **(3)**

$$\frac{\frac{n(n+1)}{2} - r}{n-1} = 5 \Rightarrow n^2 - 9n + 10 = 2r$$

Now, $2 \le n^2 - 9n + 10 \le 2n$

(i) $n^2 - 9n + 10 \ge 2$ $\qquad n^2 - 9n + 12 \ge 0$

$\Rightarrow n \le \frac{9-\sqrt{51}}{2}$ or $n \le \frac{9+\sqrt{51}}{2}$

(ii) $n^2 - 9n + 10 \le 2n$ $\qquad n^2 - 11n + 10 \le 0$

$\Rightarrow 1 \le n \le 5 + 10$

From (i) & (ii), $\frac{9\sqrt{51}}{2} \le n \le 10$

Hence n may be 8, 9 or 10

Hence sum of values of n = 27

7. **(4)**

D = common diff

$-2D + kD^2 + 8D^3 = -6D + 4D^2 - D^3$

$9D^3 + (k-4)D^2 + 4D = 0$

$9D^2 + (k-4)D + 4 = 0$

$(k-4)^2 - 4 \times 4 \times 9 \ge 0$

$(k-4)^2 - (12)^3 \ge 0$

$(k-16)(k+8) \ge 0$

$\boxed{k \ge 16}$

8. **(6)**

Numbers be a – d, a, a + d

$(a^2)^2 = (a-d)^2 (a+d)^2$

$\Rightarrow a^4 = (a^2 - d^2)^2$

$\qquad = a^4 + d^4 + 2a^2d^2$

$\Rightarrow d^4 = 2a^2a^2$

$\Rightarrow d^2 = 2a^2$

$$r = \frac{a^2}{(a-d)^2} = \left(\frac{a}{a-d}\right)^2 = \left(\frac{1}{1-\frac{d}{a}}\right)^2$$

$\frac{d}{a} = \pm\sqrt{2}$

So $r = \left(\frac{1}{a+\sqrt{2}}\right)^2$ or $\left(\frac{1}{1-\sqrt{2}}\right)^2$

$\qquad = (\sqrt{2}-1)^2$ or $(\sqrt{2}+1)^2$

$r_1 + r_2 = 2(2+1) = 6$

9. **(3)**

Let the number be $100ar^2 + 10at + a$

Now, $100ar^2 + 10at + a - 400 = 100(ar^2 - 4) + 10ar + a$

Given that $a + ar^2 - 4 = 2ar$

Also as each of a, ar, ar^2 is an integer lying between

1 & 9, hence $r = 2$ or 3

For $r = 2, a + ar^2 - 4 = 2ar \Rightarrow a = 4,$ but $ar^2 > 9$

For $r = 3, a + ar^2 - 4 = 2ar \Rightarrow a = 1$

Hence the number is 931 & sum of digits = 13

10. **(7)**

$$\frac{a(r^n - 1)}{r-1} > 1000$$

$$\frac{(3^n - 1)}{2} > 1000$$

$3^n > 2001$

$n = 7$

11. **(7)**

n includes all number div by 2, 3 or 5

We have

$$\sum \perp^n = \left(1 + \frac{1}{2} + \frac{1}{4} + ...\right) + \left(1 + \frac{1}{3} + \frac{1}{9} + ...\right)$$

$$+\left(1 + \frac{1}{5} + \frac{1}{25}....\right) - \left(1 + \frac{1}{6} + \frac{1}{x}....\right) - \left(1 + \frac{1}{15} + \frac{1}{225}\right)$$

$$-\left(1 = \frac{1}{10} + \frac{1}{100}.....\right) + 2\left(1 + \frac{1}{30} + \frac{1}{900} + ...\right)$$

12. **(4)**

$\frac{a}{r}, a, ar = GP$

$a^3 = 216 \qquad \Rightarrow a = 6$

$$\left(\frac{6}{r} \times 6\right) + (6 \times 6r) + (6)^2 = 156$$

$$\Rightarrow \frac{36}{r} + 36 + 36r = 156$$

$r = 3$

$sum = 18 + 6 + 2$

$= \boxed{26}$

13. **(6)**

$\frac{a}{r}, a, ar, ar^2.....$

$$\frac{a\left((r)^{24}-1\right)}{(r)-1}=1 \qquad \Rightarrow \frac{\left(\frac{1}{b}\right)\left(1-r^{24}\right)}{\left(r^{23}\right)(1-r)}$$

$$\Rightarrow ar^{23}=2$$

$$t_{24}=2$$

We have $\left(t_1\times t_{24}\right)=\left(\sqrt{2}\right)^2$

14. **(1)**

We have

$$\frac{p}{q}=1+\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)^2+3\left(\frac{1}{6}\right)^3.....$$

Now, $S=1+x+2x^2+3x^3....$

$Sx=x+x^2+2x^3+3x^4.....$

$S(1-x)=\left(x^2+x^3+3x^4\right)$

$$=\frac{x^2}{1-x}+\frac{(1-x)^2}{(1-x)}$$

$$S=\frac{x^2}{(1-x)^2}+\frac{1}{1-x}$$

$$\frac{p}{q}=\frac{\frac{1}{36}}{\frac{25}{36}}+\frac{\frac{1}{5}}{\frac{5}{6}}=\frac{1}{25}+\frac{6}{5}=\frac{31}{25}$$

$$\left|p^2-q^2\right|=4$$

15. **(1)**

$$=0.9\left(100-1+1000-1+....10^{101}-1\right)$$

$$=0.9\left(10^2\_....+10^{101}-100\times1\right)$$

$$=0.9\left(10^3+....10^{101}\right)$$

$$=0.9\frac{\left(10^3\right)\left(10^{99}-1\right)}{10-1}=100\left(10^{99}-1\right)$$

$$=10^{99}-100$$

16. **(3)**

$$t_1=b-2$$

$$t_3=b+6$$

$$r=\sqrt{\frac{b+6}{b-2}}$$

$$\frac{t_1+t_3}{2}=b+2$$

$$\frac{b+2}{b-2\sqrt{\frac{b+6}{b-2}}}=\frac{5}{3}\Rightarrow\frac{b+2}{\sqrt{(b-2)(b+6)}}=\left(\frac{5}{3}\right)^2$$

$$\Rightarrow \frac{(b+2)^2}{(b-2)(b+6)} = \left(\frac{5}{3}\right)^2 \Rightarrow \boxed{b=3}$$

17. **(9)**

After withdrawal of Acid = 729 – 3x

After II withdrawal of Acid $= 729 - 3x - \left(\frac{729-3x}{729}\right) \times 3x$

i.e. $\frac{(729-3x)^2}{729}$

After III withdrawal Acid $= \frac{(729-3x)^2}{729} - \left(\frac{729-3x}{729}\right)^2 \times 3x$

i.e. $\frac{(729-3x)^3}{729^2}$

After VI withdrawal Acid $= \frac{(729-3x)^6}{729^5} = 64$

Hence, $(729-3x)^6 = 2^6 3^{30}$ or $x = 81$

18. **(5)**

$$y = \frac{a+b}{2}$$

$$x = \frac{a+y}{2} = \frac{a}{2} + \frac{a}{4} + \frac{b}{4} = \frac{3a}{4} + \frac{b}{4} = \frac{3a+b}{4}$$

$$z = \frac{a+3b}{4}$$

$$xyz = 55 \Rightarrow 55 = \frac{(a+b)(3a+b)(a+3b)}{a+b}$$

Again, It $1+p, y = \frac{2ab}{a+b}$

$$x = \frac{2ay}{a+y} = \frac{2a \times \frac{2ab}{a+b}}{a + \frac{2ab}{a+b}} = \frac{4a^2b}{a^2+3ab}$$

$$z = \frac{4a^2b}{a^2+3ab}$$

$$xyz = \frac{4a^2b \times 4ab^2 \times 2ab}{ab(a+b)(b+3a)(a+3b)}$$

$$\frac{343}{55} = \frac{32a^3b^3}{ab(a+b)(3a+b)(a+3b)}$$

$$\therefore 55 \times \frac{343}{55} = a^3b^3$$

$$ab = 7$$

19. **(6)**

$$\frac{1}{t_1} = 2.5 = \frac{5}{2}$$

$\frac{1}{t_2}=\frac{23}{12}$

Smaller the t r_1, greater the term.

Common diff of $AP=\frac{23}{11}-\frac{30}{12}=\frac{-7}{12}$

So we have $\frac{30}{12},\frac{23}{12},\frac{16}{12},\frac{2}{12},\frac{-5}{12}$

so $\frac{2}{12}$

Reciprocal = $\frac{12}{2}=6$

20. **(1)**

$\sum_{r=0}^{88}\frac{1}{\cos r^0\cos(r+1)^0}$

$\frac{1}{\sin 1^0}\sum_{\ell=0}^{88}\frac{\sin 1^0}{\cos r^0.\cos(r+1)^0}$

$=\frac{1}{\sin 1^0}\sum_{\ell=0}^{88}\frac{\sin[(r+1)-r]}{\cos r^0.\cos(r+1)^0}$

$=\frac{1}{\sin 1^o}\left(\sum_{r=0}^{88}\frac{\sin(r+1)\cos r^0-\cos(r+1)\sin r}{\cos r.\cos(r+1)}\right)$

$=\frac{1}{\sin^0}\left(\sum_{\ell=0}^{88}\tan(r+1)^0-\tan r^0\right)$

$=\frac{1}{\sin^0}(\tan 89^0-\tan 0^0)$

$=\frac{\cot^0}{\sin 1^0}\qquad\Rightarrow\theta=1^0$

21. **(6)**

roots are

$-3x,-x,x,3x$

So, $10x^2=3m+2$

$9x^4=m^2$

$\Rightarrow\frac{100}{9}=\frac{9m^2+4+12m}{m^2}$

$\Rightarrow 100m^2=81m^2+108m+36$

$\Rightarrow 19m^2-108m-36=0$

$\Rightarrow 19m^2-114m+6m-36=0$

$\Rightarrow(19m+6)(m-6)=0$

$\Rightarrow m=6$

22. **(5)**

17, 21, 25 ……….. 417

16, 21, 26 ……….. 466

Common terms $\rightarrow$

21, 41, 61 ……………….

$T_n = 20n + 1 \le 417$
$\Rightarrow 20n \le 416$
$n \le 20.8$
$\boxed{n = 20} \Rightarrow K = 5$

23. **(8)**
$P2^x + \frac{4}{2^x} = 5$
$\Rightarrow pt^2 - 5t + 4 = 0$
Where $\left(t = 2^x > 0\right)$
So, $D \ge 0$
$\Rightarrow 25 - 16p \ge 0$
$p < \frac{25}{16}$
But, $p > 0$
So, $p = 0, p = 1$
at $p = 1$, 2 solution
at $p = 0$, 1 solution
No. of value of p is $Q = 1$
AP : $1, \frac{1}{x_1}, \frac{1}{x_2} ....\frac{1}{x_{20}}, \frac{1}{6}$
AP : $6, \frac{1}{x_1}, \frac{6}{x_2} ......\frac{6}{x_{20}}$
AP :

24. **(1)**
$a_1 = \frac{1}{2}$
$\left(n-1\right)a_{n-1} = \left(n+1\right)a_n$
$\Rightarrow n\left(n-1\right)a_{n-1} - n\left(n+1\right)a_n = 0$
$\Rightarrow \sum_{n=2}^{\infty} n\left(n-1\right)a_{n-1} - n\left(n+1\right)a_n = \sum 0$
$\Rightarrow 2.1.a, -n\left(n+1\right)a_n = 0$
$\Rightarrow a_n = \frac{1}{n\left(n+1\right)}$
$S_n = \sum_{r=1}^{n} \frac{1}{n\left(n+1\right)}$
$= \sum_{r=1}^{\infty} \frac{1}{r} - \frac{1}{r+1}$
$= 1 - 0 = 1$

25. **(4)**
(1), (2, 3, 4), (5, 6, 7, 8, 9) ……..
No. of terms in n^{th} jump is N_n
$N_1 = 1, N_2 = 3, N_3 = 5$
$N_n = \left(2n - 1\right)$

So, total no. of elements till $(n-1)$ groups

is $1+3+5+......(2n-3)=(n-1)^2$

total no. of elements till n groups

is $+3+5......2n-1=n^2$

sum of elements till $(n-1)$ groups is

$1+2+3+4..........(2-1)^2$

=

$$\frac{(n-1)^2\left((n-1)^2+1\right)}{2}$$

Sum of elements till n groups is

$1+2+3+.........n^2=\frac{n^2\left(n^2+1\right)}{2}$

Sum of elements in 12 groups is

Ans. $-4\ \frac{12^2\left(11^2+1\right)}{2}-\frac{11^2\left(11^2+1\right)}{2}=6238$

26. **(1)**

t^{th} **M**ean $=(r+1)^{th}$ term

$T_{r+1}=x+r.\frac{(2y-x)}{n+1}$

$=\frac{(n+1-r)x+2xy}{n+1}$

$T_{r+1}=2x+r\frac{(y-23)}{n+1}$

$\Rightarrow\frac{(n+1-\ell)2x+2y}{n+1}$

$\Rightarrow(n+1-r)x+2xy=(n+1-r)2x+xy$

$\Rightarrow(n+1-\ell)x=xy$

$\Rightarrow\frac{y}{x}=\frac{x+1}{r}-1$

$\Rightarrow\frac{n+1}{r}-\frac{y}{z}=1$

Ans – 1

27. **(4)**

$\left(\frac{1}{5}\right)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+........\infty\right)}$

$=\left(\sqrt{5}\right)^{-2\log_{\sqrt{5}}\left(\frac{1/4}{1-1/2}\right)}$

$=\left(\sqrt{5}\right)^{\log_{\sqrt{5}}(1/2)^{-2}}$

$=4$

28. **(5)**

$$\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)........\infty$$

$$=\frac{\left(1-\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)}{\left(1-\frac{1}{3}\right)}.........\infty$$

$$=\frac{\left(1-\frac{1}{3^2}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)}{2/3}..........\infty$$

29. **(9)**

$$x=\frac{a+b}{2}$$

$$y=\sqrt{ab}$$

$$z=\frac{2ab}{a+b}$$

$$y^2=xz$$

$$z=\frac{x}{9}(\text{given})$$

$$\Rightarrow y^2=\frac{x^2}{9}$$

$$\Rightarrow \frac{x^2}{y^2}=9$$

30. **(0)**

$$\frac{a^n+b^n}{a^{n-1}+b^{n-1}}=\frac{2ab}{a+b}$$

$$\Rightarrow a^{n+1}+b^{n+1}+ab\left(b^{n-1}+a^{n-1}\right)$$

$$=2ab\left(a^{n-1}+b^{n-1}\right)$$

$$\Rightarrow a^{n+1}+b^{n+1}=ab\left(a^{n-1}+b^{n-1}\right)$$

$$\Rightarrow a^n(a-b)=b^n(a-b)$$

$$\Rightarrow a^n=b^n$$

$$\Rightarrow n=0$$

31. **(8)**

$$S=\frac{1}{a^5}+\frac{1}{a^4}+\frac{3}{a^3}+a^8+a^{10}$$

By $AM\geq GM$

$$\frac{1}{a^5}+\frac{1}{a^4}+\frac{1}{a^3}+\frac{1}{a^3}+\frac{1}{a^3+8^{10}}$$

$$\Rightarrow \boxed{S\geq 1}$$

32. **(1)**

33. **(5)**

34. **(3)**

35. **(5)**

36. **(6)**

37. **(6)**

38. **(9)**

39. **(3)**

40. **(3)**

PYQ : JEE Advanced

1. (a, d)
2. (b)
3. (b, c)
4. (3)
5. (0)
6. (3 or 9)
7. (8)
8. (5)
9. (4)
10. (9)
11. (6)
12. (3748)
13. (157)
14. (1)
15. (8)
16. (18900)