EXERCISE - 1 [A]

1. **(A)**

$\Rightarrow \tan\frac{2}{3}\theta = \sqrt{3}$

$\Rightarrow \frac{2\theta}{3} = n\pi + \frac{\pi}{3}$

$\Rightarrow 2\theta = 3n\pi + \pi$

$\Rightarrow \theta = 3n\frac{\pi}{2} + \frac{\pi}{2}, n \in I$

2. **(C)**

$\Rightarrow \sec\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$

3. **(C)**

$\Rightarrow \cos\left(\frac{-\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right) = 0$

$\Rightarrow \frac{\theta}{2} = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\pi$

4. **(B)**

$\Rightarrow \cos^2\theta = 1$

$\Rightarrow \cos\theta = \pm 1$

$\Rightarrow \cos\theta = 1$ or $\cos\theta = -1$

$\Rightarrow \theta = 2n\pi$ $\qquad \theta = (2n+1)\pi$

$\Rightarrow \therefore \theta = n\pi$

5. **(C)**

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = \frac{\pi}{3}, \frac{4\pi}{3}$

$\Rightarrow \operatorname{cosec}\theta = -\frac{2}{\sqrt{3}} \Rightarrow \sin\theta = \frac{-\sqrt{3}}{2}$

$\Rightarrow \theta = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$

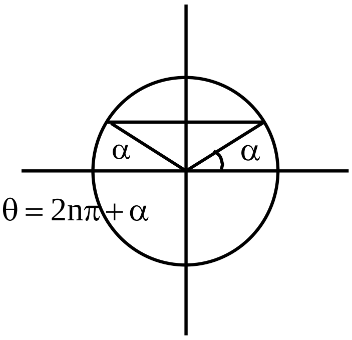
$\Rightarrow \therefore \theta = \frac{4\pi}{3}$

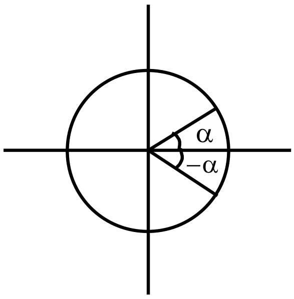
$\Rightarrow \theta = 2n\pi + \frac{4\pi}{3}$

6. **(A)**

$\Rightarrow \sin\theta = \sin\alpha$ $\qquad$ $\cos\theta = \cos\alpha$

$\Rightarrow \theta = n\pi + (-1)^n \alpha$ $\qquad$ $\theta = 2n\pi \pm \alpha$





7. **(A)**

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = n\pi + \frac{\pi}{3}$

$\Rightarrow \frac{\pi}{3}$ and $\frac{4\pi}{3}$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{3}$

$\Rightarrow \sec\theta = 2$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

8. **(A)**

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}$

P.S. $\frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{6}$

P.S. $= \frac{\pi}{6}, \frac{11\pi}{6}$

Common value $=\frac{\pi}{6}$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{6}$

9. **(B)**

$\Rightarrow \sin\theta = \sqrt{3}\cos\theta$ ….$-\pi < \theta < 0$

$\Rightarrow \tan\theta = \sqrt{3}$ …..$\cos\theta \neq 0$

$\Rightarrow \theta = n\pi + \frac{\pi}{3}$ …..$\theta \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = -\pi + \frac{\pi}{3}$ …..for $n = -1, \theta \neq \frac{\pi}{2}$

$= -\frac{2\pi}{3}$

$= -\frac{4\pi}{6}$

10. **(B)**

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \theta = n\pi + \frac{5\pi}{6}$

P.S. $= \frac{5\pi}{6}, \frac{11\pi}{6}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{5\pi}{6}$

$\Rightarrow$ P.S. $= \frac{5\pi}{6}, -\frac{5\pi}{6}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}$

P.S. $= \frac{5\pi}{6}, \frac{11\pi}{6}$

Common solution $= \frac{5\pi}{6}$

11. **(A)**

$\Rightarrow 2\sin x + \tan x = 0$

$\Rightarrow \cos x \neq (2n+1)\pi$

$\Rightarrow 2\sin x + \frac{\sin x}{\cos x} = 0$

$\Rightarrow \sin x\left(\frac{2\cos x + 1}{\cos x}\right) = 0$

$\Rightarrow \sin x = 0$ or $2\cos x + 1 = 0$ …….$\cos x \neq 0$

$$\Rightarrow x = n\pi \qquad x = 2n\pi \pm 2\frac{\pi}{3}$$

$$\Rightarrow \therefore x = (3n \pm 1)\left(\frac{2\pi}{3}\right) \text{and } n\pi$$

12. **(B)**

$$\Rightarrow (2\cos x - 1)(3 + 2\cos x) = 0$$

$$\Rightarrow 0 \le x \le 2\pi$$

$$\Rightarrow \cos x = \frac{1}{2} \quad \text{or} \quad \cos = -\frac{3}{2} \qquad \text{....(not possible)}$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3} \text{in} \{0, 2\pi\}$$

$$\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$$

13. **(B)**

$$\Rightarrow (1 + \tan\theta)(1 + \tan\varphi) = 2$$

$$\Rightarrow (1 + \tan\theta\tan\varphi + \tan\theta + \tan\varphi) = 2$$

$$\Rightarrow \tan\theta + \tan\varphi = 1 - \tan\theta\tan\varphi$$

$$\Rightarrow \frac{\tan\theta + \tan\varphi}{1 - \tan\theta\tan\varphi} = 1 = \tan\frac{\pi}{4}$$

$$\Rightarrow \tan(\theta + \varphi) = \tan\frac{\pi}{4}$$

$$\Rightarrow \theta + \varphi = 45^{o}$$

14. **(A)**

$$\cos\theta + \cos 2\theta = 2$$

$$\cos\theta + 2\cos^2\theta - 1 = 2$$

$$2\cos^2\theta + \cos\theta - 3 = 0$$

$$\cos\theta = \frac{-3}{2}, 1$$

$$\cos\theta = 1 \Rightarrow \theta = 2n\pi, n \in I$$

15. **(A)**

$$\Rightarrow \sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$$

$$\Rightarrow 1 - \cos^2 - 2\cos\theta + \frac{1}{4} = 0$$

$$\Rightarrow \frac{5}{4} - \cos^2\theta - 2\cos\theta = 0$$

$$\Rightarrow \cos^2\theta + 2\cos\theta - \frac{5}{4} = 0$$

$$\Rightarrow (\cos\theta + 1)^2 - \frac{9}{4} = 0$$

$$\Rightarrow \left(\cos\theta + 1 - \frac{3}{2}\right)\left(\cos\theta + 1 + \frac{3}{2}\right) = 0$$

$$\Rightarrow \left(\cos\theta - \frac{1}{2}\right)\left(\cos\theta + \frac{5}{2}\right) = 0$$

$$\Rightarrow \cos\theta = \frac{1}{2}; \quad \cos\theta = \frac{5}{2} \qquad \text{(not possible)}$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$$

16. **(C)**

$$\Rightarrow \tan^2\theta + 2\sqrt{3}\tan\theta = 1$$

$$\Rightarrow \tan^2\theta + 2\sqrt{3}\tan\theta - 1 = 0$$

$$\Rightarrow \tan\theta = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = \frac{-2\sqrt{3} \pm 2\times 2}{2}$$

$$\Rightarrow \tan\theta = -\sqrt{3} \pm 2$$

$$\Rightarrow \tan\theta = 2 - \sqrt{3} \qquad \text{or} \qquad \tan\theta = -2 - \sqrt{3}$$

$$\Rightarrow \tan 15^o = 2 - \sqrt{3} \qquad \tan 75^o = 2 + \sqrt{3}$$

$$\tan - 75^o = -2 - \sqrt{3}$$

$$\Rightarrow \tan\theta = \tan 15^o = \tan 2 - \sqrt{3}$$

$$= \tan\frac{\pi}{12}$$

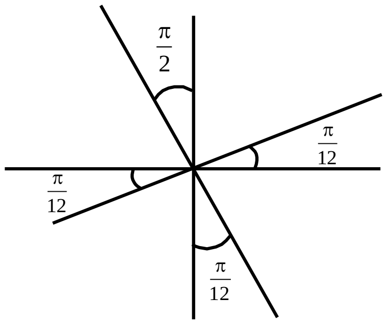
$$\theta = n\pi + \frac{\pi}{12}$$

$$\Rightarrow \tan\theta = \tan\left(-2 - \sqrt{3}\right)$$

$$= \tan - 75$$

$$= \tan - \frac{5\pi}{12}$$

$$\Rightarrow \theta = n\pi - \frac{5\pi}{12}$$



$$\Rightarrow \theta = n\pi + \frac{7\pi}{12}$$

$$= (2n+1)\frac{\pi}{2} + \frac{\pi}{12}$$

$$\Rightarrow \therefore\ \theta = (6n+1)\frac{\pi}{12}$$

17. **(B)**

$$\Rightarrow 25\cos^2\theta + 5\cos\theta - 12 = 0$$

$\Rightarrow \alpha$ is the root then

$$\Rightarrow \cos\alpha = \frac{-5 \pm \sqrt{25+1200}}{50}$$

$= -\frac{4}{5}, \frac{3}{5}$

$\Rightarrow \cos\alpha = -\frac{4}{5}$ ……..II quadrant

$\Rightarrow \sin\alpha = \frac{3}{5}$

$\Rightarrow \sin 2\alpha = 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) = -\frac{24}{25}$

18. **(A)**

$\Rightarrow \cos x + \sec x = 2$

We know arithmetic mean > Geometric mean

$\Rightarrow \frac{a + \frac{1}{a}}{2} \geq \sqrt{a \times \frac{1}{a}}$

$\Rightarrow a + \frac{1}{a} \geq 2$ (not possible so only equality holds)

Now for $a = \cos x = 1$

$\Rightarrow n = 2n\pi$

19. **(B)**

$\Rightarrow \left(\sin^2\theta\right)\sec\theta + \sqrt{3}\tan\theta = 0$

$\Rightarrow \sec\theta \neq \infty$; $\cos\theta \neq 0$

$\Rightarrow \theta \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) + \sqrt{3}\tan\theta = 0$

$\Rightarrow \tan\theta\left(\sin\theta + \sqrt{3}\right) = 0$

$\Rightarrow \tan\theta = 0$

$\Rightarrow \sin\theta \neq -\sqrt{3}$

$\Rightarrow \theta = n\pi$

20. **(C)**

$\Rightarrow 3\left(\sec^2\theta + \tan^2\theta\right) = 5$

$\Rightarrow 3\left(1 + \tan^2\theta + \tan^2\theta\right) = 5$

$\Rightarrow 3 + 6\tan^2\theta = 5$

$\Rightarrow 6\tan^2\theta = 2$

$\Rightarrow \tan^2\theta = \frac{1}{3} = \tan^2\frac{\pi}{6}$

$\Rightarrow \theta = n \pm \frac{\pi}{6}$

21. **(A)**

$\Rightarrow 2\cot^2\theta = \text{cosec}^2\theta$

$= 1 + \cot^2\theta$

$\Rightarrow \cot^2\theta = 1$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$

22. **(A)**

$2\cos^2 x+3\sin x-3=0$

$2\left(1-\sin^2 x\right)+3\sin x-3=0$

$2-2\sin^2 x+3\sin x-3=0$

$2\sin^2 x-3\sin x+1=0$

$\sin x=1$ or $\sin x=\frac{1}{2}$

FQ $0\le x\le 180^o$

$x=90^o,\ 30^o,\ 150^o$

23. **(B)**

$2\sin^2\theta=3\cos\theta$

$2\left(1-\cos^2\theta\right)=3\cos\theta$

$2-2\cos^2\theta=3\cos\theta$

$2\cos^2\theta+3\cos\theta-2=0$

$2\cos^2\theta+4\cos\theta-\cos\theta-2=0$

$\left(\cos\theta+2\right)\left(2\cos\theta-1\right)=0$

$\cos\theta\ne-2$

$\cos\theta=\frac{1}{2}$

$\theta=\frac{\pi}{3},\frac{5\pi}{3}$

24. **(A)**

$\Rightarrow \sin 7\theta+\sin\theta=\sin 4\theta$

$\Rightarrow 2\sin 4\theta\cos 3\theta=\sin 4\theta$

$\Rightarrow \sin 4\theta\left(2\cos 3\theta-1\right)=0$

$\Rightarrow \sin 4\theta=0$ or $2\cos 3\theta-1=0$

$\Rightarrow 4\theta=n\pi$ $\cos 3\theta=\frac{1}{2}$

$\Rightarrow \theta=\frac{n\pi}{4}$ $3\theta=2m\pi\pm\frac{\pi}{3}$

$\theta=\frac{2m\pi}{3}\pm\frac{\pi}{9}$

between $0<\theta<\frac{\pi}{2}$

$\Rightarrow \theta=\frac{\pi}{4}$ $\theta=\frac{\pi}{9}$

$\Rightarrow \theta\in\left(0,\frac{\pi}{2}\right)$

25. **(B)**

$\cot\theta=\sin 2\theta$

$\frac{\cos\theta}{\sin\theta}=2\sin\theta\cos\theta$

$\cos\theta=2\sin^2\theta\cos\theta$

$\cos\theta\left(2\sin^2\theta-1\right)=0$

$-\cos\theta\cdot\cos 2\theta=0$

$\cos\theta=0$ or $\cos 2\theta=0$

$\theta=90^o$ or $\theta=45^o$

26. **(C)**

$\Rightarrow 2\tan^2 a=\sec^2\theta$

$=1+\tan^2\theta$

$\Rightarrow \tan^2\theta=1$

$\Rightarrow \theta=n\pi\pm\frac{\pi}{4}$

27. **(C)**

Snx + cosx = 1

$\frac{1}{\sqrt{2}}\text{Snx}+\frac{1}{\sqrt{2}}\cos x=\frac{1}{\sqrt{2}}$

$\text{Sn}\left(x+\frac{\pi}{4}\right)=\text{Sn}\,\frac{\pi}{4}$

$x+\frac{\pi}{4}=n\pi+\left(-1\right)^n\frac{\pi}{4}$

$x=n\pi+\left(-1\right)^n\frac{\pi}{4}$

$-\frac{\pi}{4}$

28. **(B)**

$\Rightarrow \cos p\theta=\cos q\theta$

$\Rightarrow p\theta=2n\pi\pm q\theta$

$\Rightarrow p\theta\pm q\theta=2n\pi$

$\Rightarrow \theta\left(p\pm q\right)=2n\pi$

$\Rightarrow \theta=\frac{2n\pi}{p\pm q}$

29. **(A)**

$\Rightarrow \tan 5\theta=\cot 2\theta$

$\Rightarrow \tan 5\theta=\tan\left(\frac{\pi}{2}-2\theta\right)$

$\Rightarrow 5\theta=n\pi+\frac{\pi}{2}-2\theta$

$\Rightarrow 7\theta=\left(2n+1\right)\frac{\pi}{2}$

$\Rightarrow \theta=\left(2n+1\right)\frac{\pi}{14}$

$\Rightarrow \theta=\frac{n\pi}{7}+\frac{\pi}{14}$

$\Rightarrow \tan 5\theta\neq\infty$

$\Rightarrow 5\theta\neq\left(2n+1\right)\frac{\pi}{2}$

$\Rightarrow \theta \neq (2n+1)\frac{\pi}{14}$

Hence $(2n+1) \neq 7, 21,$ etc.

30. **(B)**

$\Rightarrow \tan\theta + \cot\theta = 2$

$\Rightarrow 2\cos ec 2\theta = 2$

$\Rightarrow \sin 2\theta = 1$

$\Rightarrow 2\theta = 2n\pi + \frac{\pi}{2}$

$\Rightarrow \theta = n\pi + \frac{\pi}{4}$

31. **(C)**

$\Rightarrow \cot\theta + \tan\theta = 2\cos ec\theta$

$\Rightarrow 2\cos ec\theta 2\theta = 2\cos ec\theta$

$\Rightarrow \sin\theta = \sin 2\theta$

$\Rightarrow 2\sin\theta\cos\theta = \sin\theta$

$\Rightarrow \sin\theta(2\cos\theta - 1) = 0$

$\Rightarrow \sin\theta \neq 0 \because \theta \neq m\pi$

$\Rightarrow \therefore \cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow \theta = 2\pi\pi \pm \frac{\pi}{3}$

$\Rightarrow \tan\theta \neq 0$

$\Rightarrow \theta = n\pi$

$\Rightarrow \cot\theta \neq \infty$

$\Rightarrow \theta \neq (2n+1)\frac{\pi}{2}$

32. **(B)**

$\sin\theta - \cos\theta = 0 \Rightarrow \sin\theta = \cos\theta \Rightarrow \tan\theta = 1$

If $\theta \in \left(0, \frac{\pi}{2}\right]$ then $\theta = \frac{\pi}{4}$

33. **(B)**

$x \in \left(0, \frac{\pi}{2}\right)$ and $\sin x \cdot \cos x = \frac{1}{4}$

$\Rightarrow \frac{\sin 2x}{2} = \frac{1}{4} \Rightarrow \sin 2x = \frac{1}{2}$

$\therefore 2x = n\pi + (-1)^n \frac{\pi}{6}$

$x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}$

$n = 0,\ x = \frac{\pi}{12},\ n = 1,\ x = \frac{\pi}{2} + \frac{-\pi}{12} = \frac{5\pi}{12}$

34. **(C)**

$\Rightarrow 1 + \cot\theta = \cos ec\theta$

$\Rightarrow \dfrac{\cos\theta+\sin\theta}{\sin\theta}=\dfrac{1}{\sin\theta}$

$\Rightarrow \sin\theta+\cos\theta=1$

$\Rightarrow \dfrac{1}{\sqrt{2}}\sin\theta+\dfrac{1}{\sqrt{2}}\cos\theta=\dfrac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta+\dfrac{\pi}{4}\right)=\sin\dfrac{\pi}{4}$ ...... $\sin\theta\neq 0;\theta\neq n\pi$

$\Rightarrow \theta+\dfrac{\pi}{4}=n\pi+(-1)^n\dfrac{\pi}{4}$

$\Rightarrow \theta=n\pi+(-1)^n\dfrac{\pi}{4}-\dfrac{\pi}{4}$

$\Rightarrow n=0 \qquad \theta=0$

$\Rightarrow n=1 \qquad \pi-\dfrac{\pi}{4}-\dfrac{\pi}{2}=\dfrac{\pi}{2}$

$\Rightarrow n=2 \qquad 2\pi$

$\Rightarrow n=3 \qquad 3\pi-\dfrac{\pi}{2}=\dfrac{5\pi}{2}$

$\Rightarrow \theta=2n\pi+\dfrac{\pi}{2}$

35. **(C)**

$\Rightarrow \sin^2\theta+\sin\theta-2=0$

$\Rightarrow (\sin\theta+2)(\sin\theta-1)=0$

Not possible $\qquad \theta=n\pi+(-1)^n\dfrac{\pi}{2}$

$\Rightarrow \theta=2n\pi+\dfrac{\pi}{2}$

36. **(A)**

$\Rightarrow 2\sin^2\theta=4+3\cos\theta$

$\Rightarrow 2-2\cos^2\theta=4+3\cos\theta$

$\Rightarrow 2\cos^2\theta+3.\cos\theta+2=0$

$\Rightarrow 0=b^2-4ac=9-16=-7$

No real roots.

37. **(D)**

$\Rightarrow 3\cos x+4\sin x=6$

$\Rightarrow -5\leq 3\cos x+4\sin x\leq 5$

For real roots it will never equal 6.

38. **(D)**

$\Rightarrow \cos^2\theta+\sin\theta+1=0$

$\Rightarrow 1-\sin^2\theta+\sin\theta+1=0$

$\Rightarrow (1-\sin\theta)(1+\sin\theta)+(1+\sin\theta)=0$

$\Rightarrow (1+\sin\theta)(1-\sin\theta+1)=0$

$\Rightarrow (1+\sin\theta)(2-\sin\theta)=0$

$\Rightarrow \sin\theta=-1$ or $\sin\theta=2$ (not possible)

$\Rightarrow \theta = \frac{3\pi}{2}$ (principle solution)

39. **(C)**

$\Rightarrow \sin 5x + \sin 3x + \sin x = 0$ ….. $0 \le x \le \frac{\pi}{2}$

$\Rightarrow \sin 5x + \sin x + \sin 3x = 0$

$\Rightarrow 2\sin 2x \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0$

$\Rightarrow 3x = m\pi$

$\Rightarrow x = \frac{m\pi}{3}$

P.S. $= 0, \frac{\pi}{3}, \frac{2\pi}{3}$

$\Rightarrow \cos 2x = -\frac{1}{2}$

$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = x\pi \pm \frac{\pi}{3}$

$\Rightarrow x = \frac{\pi}{3}, -\frac{\pi}{3}$

Common value between 0 and $\frac{\pi}{2}$ is $\frac{\pi}{3}$

40. **(A)**

$\Rightarrow \sin 2\theta = \cos 3\theta$

$\Rightarrow \cos 3\theta = \sin 2\theta$

$\Rightarrow \cos 3\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow 3\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right)$ or $3\theta = 2n\pi - \left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow 5\theta = 2n\pi + \frac{\pi}{2}$ $\theta = 2n\pi - \frac{\pi}{2}$

$\Rightarrow \theta = \frac{(4n+1)}{10}\pi$

$\Rightarrow \theta = \frac{\pi}{10}, \frac{5\pi}{10}, \frac{9\pi}{10}$ etc.........

$\Rightarrow \therefore$ acute angle $= \frac{\pi}{10}$

$\Rightarrow \theta = 18^o$

$\Rightarrow \sin 18^o = \frac{\sqrt{5}-1}{4}$

41. **(C)**

$\Rightarrow \sqrt{3}(\cot\theta + \tan\theta) = 4$

As, $\cot\theta + \tan\theta = 2\operatorname{cosec} 2\theta$

$\Rightarrow \tan\theta \neq \infty \neq (2n+1)\frac{\pi}{2}$

$\Rightarrow \cot\theta \neq \infty \neq n\pi$

$\Rightarrow \therefore 2\sqrt{3}\,\text{cosec}\,2\theta = 4$

$\Rightarrow \text{cosec}\,2\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow 2\theta = n\pi + (-1)^n \frac{\pi}{3}$

$\Rightarrow \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{6}$

42. **(D)**

$\Rightarrow \sin x + \frac{1}{\sin x} = \frac{7}{2\sqrt{3}}$

$\Rightarrow 2\sqrt{3}\sin^2 x + 2\sqrt{3} = 7\sin x$

$\Rightarrow 2\sqrt{3}\sin^2 x - 7x + 2\sqrt{3} = 0$

$\Rightarrow \sin x = \frac{7 \pm \sqrt{49 - 4\left(2\sqrt{3}\right)^2}}{4\sqrt{3}}$

$= \frac{7 \pm \sqrt{49-48}}{4\sqrt{3}}$

$= \frac{7 \pm 1}{4\sqrt{3}} = \frac{2}{\sqrt{3}}, \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin = \frac{2}{\sqrt{3}}$ ……..(not possible)

$\Rightarrow \therefore \sin x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = 60^o$

43. **(C)**

$\cos 3x \cdot \cos 7x = \cos^2 2x$

$2\cos 7x + \cos 3x = 2\cos^2 2x$

$\cos 10x + \cos 4x = 2\cos^2 2x$

$\cos 10x + \cos 4x = 1 + \cos 4x$

$\cos 10x = 1 \Rightarrow x = \frac{x\pi}{5}, n \in I$

44. **(D)**

$6\sin^2 x + \sin x - 1 = 0$ …(1)

$6\sin^2 x + 3\sin x - 2\sin x - 1 = 0$

$(2\sin x + 1)(3\sin x - 1) = 0$

$\sin x = \frac{-1}{2}, \sin x = \frac{1}{3}$

Then sum of roots of
Equation 1 in $x \in [0, 2\pi]$
is 4π

45. **(C)**

$2\sin^2 x + 5\sin x + 2 = 0$
$2\sin^2 x + 4\sin x + \sin x + 2 = 0$
$(\sin x + 2)(2\sin x + 1) = 0$

$\sin x = -2 \quad \text{or} \quad \sin x = \frac{-1}{2}$

Then $x = x\pi + (-1)^n \left(\frac{-\pi}{6}\right), n \in I$

46. **(D)**

$\tan^2\theta = 1 - \sec 2\theta$

$\tan^2\theta = 1 - \left(\frac{1 + \tan^2\theta}{1 - \tan^2\theta}\right)$

$\tan^2\theta = \frac{1 - \tan^2\theta - 1 - \tan^2\theta}{1 - \tan^2\theta}$

$\tan^2\theta(1 - \tan^2\theta) = -2\tan^2\theta$

$\tan^2\theta - \tan^4\theta = -2\tan^2\theta$
$\tan^4\theta = 3\tan^2\theta$
$\tan^2\theta(\tan^2\theta - 3) = 0$

$\tan^2\theta = 0$

$\tan^2\theta = (\sqrt{3})^2$

$\tan\theta = 0 \qquad \tan^2\theta = \tan^2\frac{\pi}{3}$

$\theta = n\pi \qquad \theta = n\pi \pm \frac{\pi}{3}$

Then $\theta = \frac{n\pi}{3}, n \in I$

47. **(D)**

$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta \cdot \tan 2\theta = \sqrt{3}$
$\tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta \cdot \tan 2\theta)$

$\Rightarrow \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta \cdot \tan 2\theta} = \sqrt{3}$

$\Rightarrow \tan(3\theta) = \sqrt{3}$

$3\theta = n\pi + \frac{\pi}{3}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{9} = (3n+1)\frac{\pi}{9}$

48. **(D)**

$5\cos 2\theta + (1 + \cos\theta) + 1 = 0$

$5\left(2\cos^2\theta-1\right)+1+\cos\theta+1=0$

$10\cos^2\theta+\cos\theta-3=0$

$\cos\theta=\frac{1}{2},-\frac{3}{5}$

$\theta=\frac{\pi}{3},\quad \pi-\cos\left(\frac{3}{5}\right)$

49. **(C)**

$1+\tan^2 x=\sqrt{2}\left(1-\tan^2 x\right)$

$\frac{1}{\sqrt{2}}=\cos^2 x$

$2x=2n\pi\pm\frac{\pi}{4}$

$x=n\pi\pm\frac{\pi}{8}$

50. **(C)**

$\Rightarrow a\sin x+b\cos x=c$

Will have solution when $|c|<\sqrt{a^2+b^2}$

51. **(D)**

$\Rightarrow \cos y\cos\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-y\right)\cos x+\sin y\cos\left(\frac{\pi}{2}-x\right)+\cos x\sin\left(\frac{\pi}{2}-y\right)=0$

$\Rightarrow \cos y\sin x-\sin y\cos x+\sin y\sin x+\cos x\cos y=0$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\left(x-y\right)+\frac{1}{\sqrt{2}}\cos\left(x-y\right)=0$

$\Rightarrow \sin\left(x-y+\frac{\pi}{4}\right)=0$

$\Rightarrow x-y+\frac{\pi}{4}=n\pi$

$\Rightarrow x=n\pi-\frac{\pi}{4}+y$

52. **(C)**

$\left(\sin x+\cos x\right)^{(1+\sin 2x)}=2,\ x\in[-\pi,\pi]$

$\sin x+\cos x\le\sqrt{2}\ \Rightarrow\ \sin x+\cos x=\sqrt{2}$ and $1+\sin 2x=2$

$\therefore\ \sin 2x=1$

Also, $\sin x+\cos x=\sqrt{2}\ \Rightarrow\ \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\sqrt{2}$

$\therefore\ \sin\left(x+\frac{\pi}{4}\right)=1$

$\therefore\ x=\frac{\pi}{4}$

53. **(B)**

$\cos^5 x=1+\left(1-\cos^2 x\right)^2$

$\cos^5 x = 1 + 1 + \cos^4 x - 2\cos^2 x$
Let $\cos x = t$
$t^5 - t^4 + 2t^2 - 2 = 0$
$(t-1)(t^4 + 2t + 2) = 0$
$t = 1$ or $t^4 + 2t + 2 = 0$
$\cos x = 1$
$x = 2n\pi, n \in I$

54. **(D)**

55. **(B)**
y = Snx – cos x
$y \in \left[-\sqrt{2}, \sqrt{2}\right]$

56. **(D)**
$\sin\left(e^x\right) = 2^x + \frac{1}{2^x}$
As we know that $2^x + \frac{1}{2^x} > 2 \ \forall \ x \in (0, \infty)$ and
$\therefore$ L.H.S $\neq$ R.H.S
Hence no solutions.

57. **(A)**
$\cos x \cos 6x = -1$
Case – 1: $\cos x = 1,$ $\cos 6x = -1$
$x = 2n\pi,$ then $6x = 12n\pi$
cos 6x = 1
Not Possible
Case – 2 : cos x = -1, & cos 6x = 1
$x = (2n+1)\pi$
Then $6x = 6(2n+1)\pi$
$\cos 6x = 1$
$x = (2n+1)\pi$

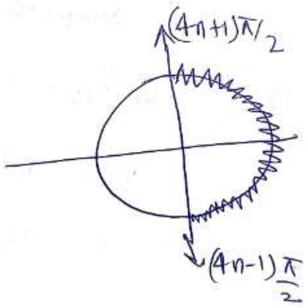
58. **(A)**
$\text{Snx} + \text{Sny} = 2$
$x = \frac{\pi}{2}, \ y = \frac{\pi}{2}$
$n + y = \pi$

59. **(C)**
$\sin x \leq 1$
$\therefore \ x \in R$

60. **(B)**

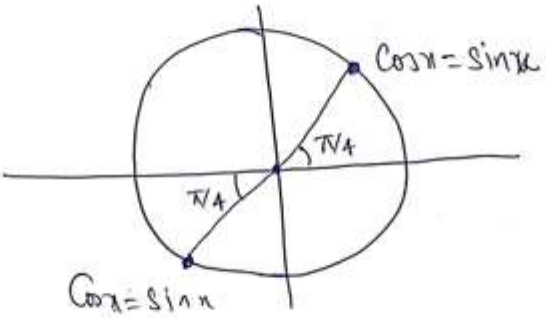
$0 \le \cos x \le 1$

$\therefore \bigcup_{n \in J}\left[(4n-1)\frac{\pi}{2},(4n+1)\frac{\pi}{2}\right]$



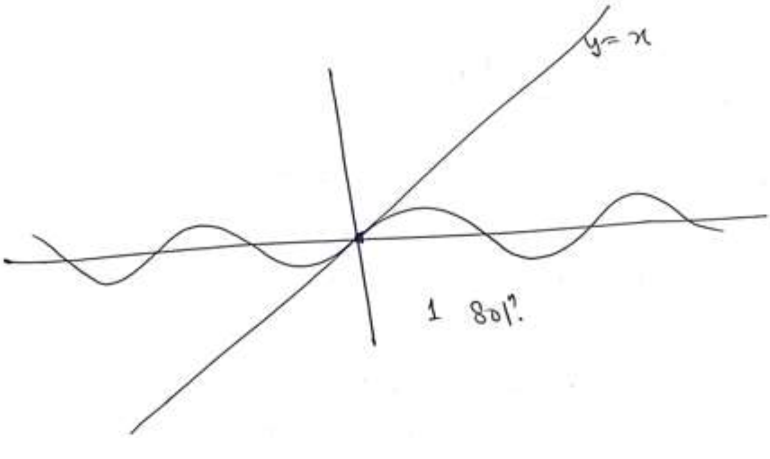
61. **(C)**

$\sin x = \cos x$



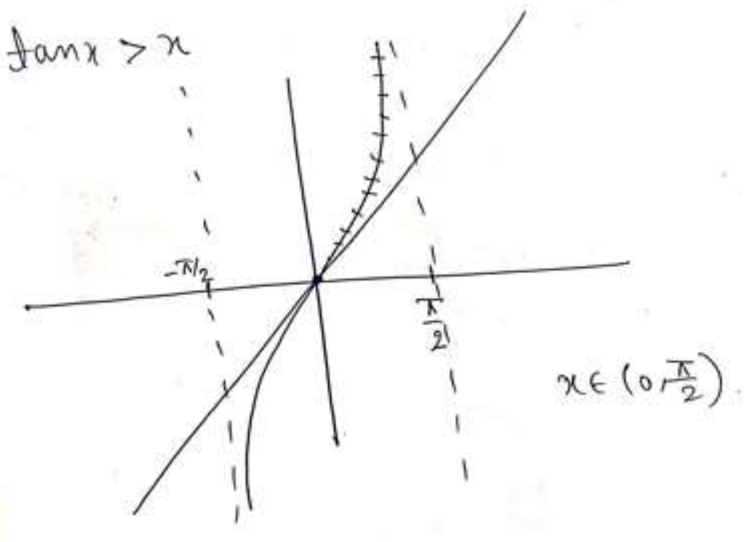
62. **(B)**

$\sin x = x$



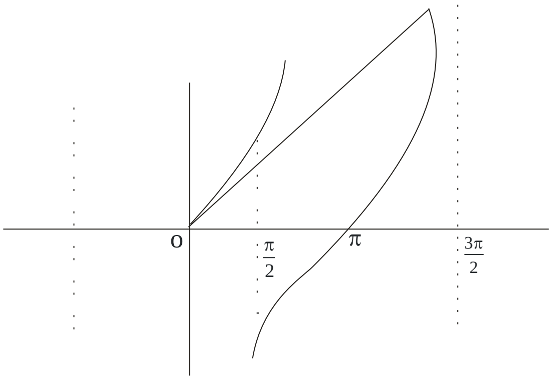
63. **(C)**

$\tan x > x$



64. **(C)**

tan x = x



Soln. lies $\left(\pi, \frac{3\pi}{2}\right)$

65. **(D)**

$x^2 - 4x + 5 = \sin y$

$\underbrace{(x-2)^2 + 1}_{\geq 1} = \underbrace{\sin y}_{\leq 1}$

$\therefore \sin y = 1 \quad \& \quad x - 2 = 0$

$y = \frac{\pi}{2} \qquad x = 2$

EXERCISE - 1 [B]

1. **(D)**

$\frac{1}{2}(\sin 8\theta + \sin 2\theta) = \frac{1}{2}(\sin 16\theta + \sin 2\theta)$

$\therefore \sin 8\theta = \sin 16\theta$

$\sin 16\theta - \sin 8\theta = 0$

$2\sin(4\theta)\cos(12\theta) = 0$

$4\theta = n\pi$

$\theta = \frac{n\pi}{4}, \qquad 12\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{24}$

$0, \frac{\pi}{4}, \frac{\pi}{2}, \quad \frac{\pi}{24}, \frac{3\pi}{24}, \frac{5\pi}{24}, \frac{7\pi}{12}, \frac{9\pi}{24}, \frac{11\pi}{12}$

= 9

2. **(B)**

$\lambda = \frac{\sin 4x \cos 4x}{2}$

$= \frac{\sin 8x}{4}$

$\therefore \frac{-1}{4} \leq \lambda \leq \frac{1}{4}$

3. **(D)**

$3\tan^2 x \ge 4\sin^2 x$

$3\sin^2 x \ge 4\sin^2 x\cos^2 x$ [where $x \ne \frac{\lambda}{2}$]

$\sin^2 x\left(3-4\cos^2 x\right) \ge 0$

$\sin x = 0$

$x = nz$

$3-4\cos^2 x \ge 0$

$4\cos^2 x - 3 \le 0$

$\left(\cos x = \frac{\sqrt{3}}{2}\right)\left(\cos x + \frac{\sqrt{3}}{2}\right) \le 0$

$-\frac{\sqrt{3}}{2} \le \cos x \le \frac{\sqrt{3}}{2}$

Where $\cos x \ne 0$

$x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right] - \left\{\frac{z}{2}\right\} \cup \{0, z\}$

Hence (D)

4. **(B)**

$\frac{\cos 3x}{4} = \frac{1}{4}, \quad \cos 3x = 1$

$3x = 2n\pi$

$x = 2n\frac{\pi}{3}$

$\left\{0 + \frac{2\pi}{3} + \frac{4\pi}{3} + \frac{6\pi}{3} + \ldots\ldots + \frac{18\pi}{3}\right\}$

$= \frac{10}{2}\left(0 + \frac{18\pi}{3}\right) = 30\pi$

5. **(B)**

$\sin^3 x + \sin x\cos x + \cos^3 x = 1$

$\left(\sin x + \cos x\right)\left(1 - \sin x\cos x\right) + \left(\sin x\cos x - 1\right) = 0$

$\left(\sin x + \cos x\right) = 1$

Or

$\left(\sin x\cos x\right) = 1$

$\sin x\cos x \ne 1 \qquad \therefore \sin x + \cos x = 1$

$2\sin\frac{\pi}{2}\cos\frac{\pi}{2} = 2\sin^2\frac{\pi}{2}$

$\sin\frac{\pi}{2} = 0$ or $\tan\frac{\pi}{2} = 1$

$x = 2n\pi \qquad$ or $\qquad \frac{x}{2} = n\pi + \frac{\pi}{4}$

$x = 2n\pi + \frac{\pi}{2}$

6. **(D)**

$7\cos^2 x + \sin x \cos x - 3 = 0$

$x = (2n+1)\frac{\pi}{2}$ is not a solution

Divided by $\cos^2 x$

$7 + \tan x - 3(1 + \tan^2 x) = 0$

$3\tan^2 x - \tan x - 4 = 0$

$\tan = -1$ or $\frac{4}{3}$

$x = n\pi + \frac{3\pi}{4}$ or $k\pi + \tan^{-1}\left(\frac{4}{3}\right)$

7. **(C)**

$4\sin^2 x + 4\sin x + a^2 - 3 = 0$

$(2\sin x + 1)^2 + a^2 - 4 = 0$

$(2\sin x + 1)^2 = 4 - a^2$

$-2 \le 2\sin x \le 2$

$-1 \le 2\sin x + 1 \le 3$

$0 \le (2\sin x + 1)^2 \le 9$

$0 \le (4 - a^2) \le 9$

$-9 \le a^2 - 4 \le 0$

$-5 \le a^2 \le 4$

$a^2 \le 4$

$-2 \le a \le 2$

8. **(A)**

$3\tan(\theta - 15^o) = \tan(\theta + 15^o)$

$\frac{\tan(\theta + 15^o)}{\tan(\theta - 15^o)} = \frac{3}{1}$

$\frac{\sin 2\theta}{\sin 30^o} = \frac{4}{2} = 2$

$\sin 2\theta = 1$

$2\theta = 2n\pi + \frac{\pi}{2}$

$\theta = n\pi + \frac{\pi}{4}$

9. **(B)**

$\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 4$

$\left(\frac{1 + \tan\theta}{1 - \tan\theta}\right) + \left(\frac{1 - \tan\theta}{1 + \tan\theta}\right) = 4$

$2 + 2 + a^2\theta = 4(1 - \tan^2\theta)$

$6 + a^2\theta = 2$

$\tan^2\theta = \frac{1}{3}$, $\theta = n\pi \pm \frac{\pi}{6}$

10. **(A)**

let t = tan θ

$$t + \frac{t+(-1)}{1-(t)(-1)} = 2$$

$$\frac{t(1+t)+t-1}{1+t} = 2$$

$$t^2 + \cancel{2t} - 1 = \cancel{2t} + 2$$

$t^2 = 3$, $\tan^2\theta = \tan^2\frac{\pi}{3}$

$$\theta = n\pi \pm \frac{\pi}{3}$$

11. **(D)**

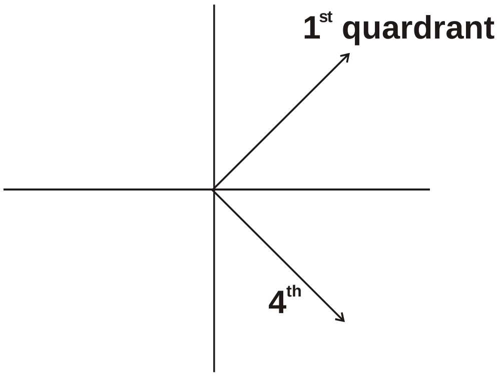
$$2(\sec^2 x - 1) - 5\sec x = 1$$

$$2\sec^2 x - 5\sec x - 3 = 0$$

$$\sec x = \frac{5 \pm \sqrt{25+24}}{4}$$

$$= \frac{12}{4} \text{ or } \frac{-1}{2}$$

$$= 3 \text{ or } \frac{-1}{2}$$



seen = 3, $[0, 6\pi]$: 6 soln.

$\left[6\pi, 6\pi + \frac{3\pi}{2}\right]$: 1 soln.

$\left[0, \frac{15\pi}{2}\right]$: 7 soln.

$n_{max} = 15$

12. **(D)**

6 $\tan^2$x – 2$\cos^2$ x = $\cos^2$x

6 $\tan^2$x – (1+cos 2x) = cos 2x

$$6\left(\frac{1-\cos 2x}{1+\cos 2x}\right) = 2\cos^2 x + 1$$

$$6 - 6\cos^2 x = (1+\cos 2x)(2\cos^2 x + 1)$$

$$6 - 6\cos^2 x = 2\cos^2 2x + 2\cos^2 x + 1 + \cos^2 x$$

$$2\cos^2 2x + 9\cos^2 x - 5 = 0$$

$$\cos^2 x = \frac{1}{2} \text{ or } -5$$

13. **(B)**

Snx – 3Sn2x + Sn3x

= cos x – 3 cos2x + cos3x

2Sn2x cos x – 3 sn 2x

= 2cos 2x cosx – 3 cos2x

Sn 2x (2 cos x – 3) – cos 2x (2cos x – 3) = 0
(Sn 2x – cos 2x) (2 cos x – 3) = 0

$\tan x = 1 \text{ or } \cos x = \frac{3}{2}$

$2x = n\pi + \frac{\pi}{4}$

$x = \frac{n\pi}{2} + \frac{\pi}{8}$

14. **(C)**

Snx + cosx = 1

$\frac{1}{\sqrt{2}}\text{Snx} + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$

$\text{Sn}\left(x + \frac{\pi}{4}\right) = \text{Sn}\frac{\pi}{4}$

$x + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$

$x = n\pi + (-1)^n \frac{\pi}{4}$

$-\frac{\pi}{4}$

15. **(C)**

$6\text{Sn}\,\theta + t\cos\theta = 9$

Let t = $\tan\frac{\theta}{2}$

$6\left(\frac{2t}{1+t^2}\right) + 7\left(\frac{1-t^2}{1+t^2}\right) = 9$

$12t + 7 - 7t^2 = 9 + 9t^2$

$8t^2 - 6t + 1 = 0$

$t = \frac{6 \pm \sqrt{36-32}}{16} = \frac{6 \pm 2}{16} = \frac{1}{2} \text{ or } \frac{1}{4}$

$\tan\frac{\theta}{2} = \frac{1}{2}, \qquad \tan\frac{\theta}{2} = \frac{1}{4}$

$\tan\theta = \frac{2\left(\frac{1}{2}\right)}{1-\frac{1}{4}} = \frac{4}{3}, \qquad \tan\theta = \frac{2\left(\frac{1}{4}\right)}{1-\frac{1}{16}}$

$= \frac{1/2}{15/16} = \frac{8}{15}$

16. **(A)**

$\tan x + \sec x = 2\cos x \quad x \in [0, 2\pi)$

$\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

$\sin x + 1 = 2\cos^2 x \qquad 1 + \sin x = 2\left(1 - \sin^2 x\right)$

$(1+\sin x) = 2(1+\sin x)(1-\sin x)$

$\therefore\ 1 = 2(1-\sin x)$ OR $1+\sin x = 0$

$1-\sin x = \frac{1}{2}$ $\sin x = -1$

$\sin x = \frac{1}{2}$ $\cos x = 0$ [Reject]

$\therefore\ \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}.$ 2 Solutions.

17. **(C)**

$\mathrm{K}\cos x = 3\mathrm{Snx} = \mathrm{K}+1$

$|\mathrm{K}\cos x - 3\mathrm{Snx}| \le \sqrt{k^2+9}$

$\cancel{k^2} + 2k + 1 \le \cancel{k^2} + 9$

$2k \le 9-1$

$k \le 4$

18. **(C)**

$\frac{\mathrm{Sn}\,3\theta}{2\cos 2\theta + 1} = \frac{1}{2}, \quad \frac{\mathrm{Sn}\,3\theta\ \mathrm{Sn}\theta}{2\mathrm{Sn}\theta\cos 2\theta + \mathrm{Sn}\theta} = \frac{1}{2}$

$\theta \ne n\pi$

$\frac{\mathrm{Sn}3\theta\ \mathrm{Sn}\theta}{\mathrm{Sn}3\theta - \mathrm{Sn}\theta + \mathrm{Sn}\theta} = \frac{1}{2}$

$\mathrm{Sn}\theta = \frac{1}{2}$

$\theta = n\pi + (-1)^n \frac{\pi}{6}$

19. **(C)**

$a^2 - 4a + 6 = (a-2)^2 + 2 \ge 2$

So, $\min\limits_{a \leftarrow n} \{1, a^2 - 4a + 6\}$

$\mathrm{Sn}\,x + \mathrm{asx} = 1$

$\mathrm{Sn}\left(x + \frac{\pi}{4} = \frac{1}{\sqrt{2}}\right)$

$x + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$

$x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

20. **(B)**

$1 + \mathrm{Snx}^4 = \cos^2 3x$

$\mathrm{Sn}^2 3x + \mathrm{Sn}^4 x = 0$

$\mathrm{Sn}3x = 0\ \&\ \mathrm{Snx} = 0$

$3x = n\pi\ \ \&\ x = n\pi$

$\therefore x = n\pi$

$x = 2\pi$ greatest

21. **(A)**

$$\text{Snx} + \cos x = \sqrt{y + \frac{1}{y}} \geq \sqrt{2}$$

$\therefore \quad \text{Sn}\,x + \cos x = \sqrt{2}$

$x = \frac{\pi}{4}$, & y = 1

22. **(C)**

$\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta \; \tan 4\theta \; \tan 7\theta$

Refer to (Q. 4) soln.

23. **(C)**

$\cos\theta + \cos 7\theta + \cos 3\theta + \cos 5\theta = 0$

$2\cos 4\theta \cos 3\theta + 2\cos 4\theta \cos\theta = 0$

$2\cos 4\theta \left(\cos 2\theta \; \cos\theta\right) = 0$

$\frac{1}{8}\frac{\text{Sns}\,\theta}{\text{Sn}\,\theta} = 0\,, \quad \theta \neq n\pi$

$\text{S}\theta = n\pi$

$\theta = n\frac{\pi}{8}, n \neq 8k$

24. **(B)**

$\left(2\,\text{Sn}\,2x \cos x + 3\text{Sn}\,2x\right) = \left(2\cos x \cos^2 x\right) + 3\cos 2x$

$\text{Sn}\,2x\left(2\cos x + 3\right) - \cos 2x\left(2\cos x + 3\right) = 0$

$\tan 2x = 1$ or 2 cos x+ 3 = 0

$2x = n\pi + \frac{\pi}{4}$

$x = \frac{n\pi}{2} + \frac{\pi}{8}$

$\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9}{8}, \frac{13\pi}{8}$

25. **(D)**

$4\,\text{Sn}\theta \cos\theta - 2\cos\theta - 2\sqrt{3}\text{Sn}\,\theta + \sqrt{3} = 0$

$2\cos\theta\left(2\text{Sn}\,\theta - 1\right) - \sqrt{3}\left(2\,\text{Sn}\,\theta - 1\right) = 0$

$\left(2\cos\theta - \sqrt{3}\right)\left(2\,\text{Sn}\theta - 1\right) = 0$

$\text{Sn}\,\theta = \frac{1}{2}, \quad \cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6,}\frac{5\pi}{6}, \frac{11\pi}{6}$

26. **(A)**

$\left(\text{Sn}\,x + \cos x\right)\left(1 - \text{Sn}\,x \cos x0\right) + 3\sin x \cos x = 1$

$\text{Sn}\,x + \cos x = t$

$\text{Sn}\,x \cos x = \frac{t^2 - 1}{2}$

$$t\left(1-\left(\frac{t^2-1}{2}\right)\right)+3\left(\frac{t^2-1}{2}\right)=1$$

$$t\left(\frac{3-t^2}{2}\right)+\frac{3t^2}{2}-\frac{5}{2}=0$$

$$\frac{t}{2}-\frac{t^3}{2}+\frac{3t^2}{2}-\frac{5}{2}=0$$

$$t^3-3t^2-3t+5=0$$

1	1	-3	-3	5
	↓	1	-2	-5
	1	-2	-5	0

$$t^2-2t-5=0$$

$$t=1\pm\sqrt{6}$$

But $\quad t\leftarrow\left[-\sqrt{2},\sqrt{2}\right]$

So, $\quad t=1$

Sn x + cos x = 1

$$x=2n\pi \quad \text{or} \quad 2n+\frac{\pi}{2}$$

27. **(D)**

$$4Sn^2x-8Snx+3\geq 0$$

$$(2Snx-1)(2Snx-3)\leq 0$$

$$\frac{1}{2}\leq Snx\leq\frac{3}{2}$$

$$Snx\geq\frac{1}{2}$$

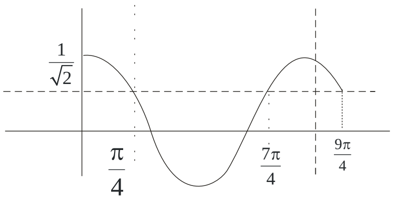
$$x\leftarrow\left[\frac{\pi}{6},\frac{5}{6}\right]$$

28. **(C)**

$$\cos x-Sn\,x\geq 1 \qquad x\leftarrow[0,2\pi]$$

$$\cos\left(x+\frac{\pi}{4}\right)\geq\frac{1}{\sqrt{2}} \qquad \theta\leftarrow\left[\frac{\pi}{4},\frac{9\pi}{4}\right]$$

$$\cos\theta\geq\frac{1}{\sqrt{2}}$$



$$\frac{7\pi}{4}\leq\theta\leq\frac{9\pi}{4}$$

$$\frac{7\pi}{4}\leq x+\frac{\pi}{4}\leq\frac{9\pi}{4}$$

$$\frac{3\pi}{2} \le x \le 2\pi$$

x = 0 also satisfied

$$x \leftarrow \left[\frac{3\pi}{2}, 2\pi\right] \cup \{0\}$$

29. **(B)**

$$\frac{2^{Sn\theta} + 2^{-\cos\theta}}{2} \ \ge \ \sqrt{2^{(Sn\theta - \cos\theta)}}$$

$$\ge \sqrt{2^{-\sqrt{2}}}$$

$$\ge 2^{-1/\sqrt{2}}$$

So $2^{Sn\theta} + 2^{-\cos\theta} \ \ge \ 2^{1-\frac{1}{\sqrt{2}}}$

When

$$Sn\theta = -\cos\theta = -\frac{-1}{\sqrt{2}}$$

i.e. $\theta = \dfrac{2n\pi + 7\pi}{4}$

30. **(A)**

$$\left(\sqrt{3}-1\right)Sn\,\theta + \left(\sqrt{3}+1\right)\cos\theta = 2$$

$$\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)Sn\,\theta + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\cos\theta = \frac{1}{\sqrt{2}}$$

$$Sn\,15^{o}\,Sn\,\theta + \cos 15^{o}\,\cos\theta = \cos 45^{o}$$

$$\cos\left(\theta - 15^{o}\right) = \cos 45^{o}$$

$$\theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4}$$

$$\theta = 2n\pi K \pm \frac{\pi}{4} + \frac{\pi}{12}$$

31. **(C)**

$$4Sn\theta\ Sn\,2\theta Sn\,4\theta = Sn\,3\theta$$

$$\Rightarrow 2\left(\cos\theta - \cos 3\theta\right)Sn\,4\theta - Sn\,3\theta$$

$$\Rightarrow \left(Sn\,5\theta + Sn\,3\theta - \left(Sn\,7\theta + Sn\,\theta\right)\right) = Sn\,3\theta$$

$$Sn\,\theta + Sn\,7\theta - Sn\,5\theta = 0$$

$$Sn\,\theta + 2Sn\,\theta\cos 6\theta = 0$$

$Sn\theta = 0$ or $\cos 6\theta = \dfrac{-1}{2}$

$\theta = n\pi$ $\qquad 6\theta = 2n\pi \pm \dfrac{2\pi}{3}$

$$\theta = \frac{n\pi}{3} \pm \frac{\pi}{9}$$

$$\theta = \left(3n \pm 1\right)\frac{\pi}{9}$$

32. **(C)**

$$8\cos x\cos 2x\cos 4x = \frac{\text{Sn}\,6x}{\text{Snx}}$$

$n \neq n\pi$

$$\frac{\text{Sn}\,8x}{\text{Sn}\,x} = \frac{\text{Sn}\,6x}{\text{Sn}\,x}$$

$\text{Sn}\,8x = \text{Sn}\,6x$

$2\text{Sn}\,x\ \cos 7x = 0$

$x = n\pi$ or $7x = (2n+1)\frac{\pi}{2}$

$x = (2n+1)\frac{\pi}{14}$

$x = \frac{n\pi}{7} + \frac{\pi}{14}$

33. **(B)**

$\text{Sn}\,3\alpha = 4\text{Sn}\,\alpha\left(\text{Sn}^2x - \text{Sn}^2\alpha\right)$

$3\text{Sn}\alpha - 4\text{Sn}^3\alpha = 4\text{Sn}\alpha\ \ \text{Sn}^2x - 4\text{Sn}^3\alpha$

$3\text{Sn}\alpha = 4\text{Sn}\alpha \quad \text{Sn}^2x$

$\text{Sn}^2x = \frac{3}{4}$

$x = n \pm \frac{\pi}{3}$

34. **(B)**

$\tan(\cot x) = \cot(\tan x)$

$= \tan\left(\frac{\pi}{2} - \tan x\right)$

$\cot x = \frac{\pi}{2} - \tan x + n\pi$

$\frac{2}{\text{Sn}\,2x} = n\pi + \frac{\pi}{2} = (2n+1)\frac{\pi}{2}$

$\text{Sn}\,2x = \frac{4}{(2n+1)\pi}$

35. **(C)**

$12\cos^3 x - 7\cos^2 x + 4\cos x - 9 = 0$

1	12	-7	4	-9
	↓	12	5	0
	12	5	9-5	

$(\cos x - 1)\left(12\cos^2 x + 5\cos x + 9\right) = 0$

cos x = 1

$x = (2n\pi)$

Infinite Soln.

36. **(A)**

$\tan 3\theta + \tan\theta = 2\tan 2\theta$

$\dfrac{\text{Sn } 2\theta}{\cos 3\theta \cos\theta} = \dfrac{2\,\text{Sn } 2\theta}{\cos 2\theta}$

$\therefore \quad \cos 2\theta = 2\cos 3\theta\cos\theta$

$\cos 2\theta = \cos 4\theta + \cos 2\theta$

$\therefore \quad \cos 4\theta = 0$

$4\theta = (2n+1)\dfrac{\pi}{2}$

$\theta \neq (2n+1)\dfrac{\pi}{2}$

$(2n+1)\dfrac{\pi}{6}$

$(2n+1)\dfrac{\pi}{4}$

Either

$\text{Sn } 2\theta = 0$

$2 = n\pi$

$\theta = \dfrac{n\pi}{2}$

But $\theta \neq \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ etc.

$\theta = m\pi$

$\theta = (2n+1)\dfrac{\pi}{8}$

37. **(D)**

$\tan\left(\dfrac{p\pi}{4}\right) = \tan\left(\dfrac{\pi}{2} - \dfrac{2}{4}\right)$

$\dfrac{p\pi}{4} = n\pi + \dfrac{\pi}{2} - \dfrac{9\pi}{4}$

$(p+q) = 4n+2$

$= 2(2n+1)$

38. **(C)**

$\tan(\pi\cos n) = \cot(\pi\,\text{Sn } x)$

$= \tan\left(\dfrac{\pi}{2} - \pi\,\text{Snx}\right)$

$\pi\cos x = \dfrac{\pi}{2} - \pi\text{Sn } x + n\pi$

$\sqrt{2} \leq \text{Snx} + \cos x = n + \dfrac{1}{2} \leq \sqrt{2}$

$\sqrt{2}\cos\left(\dfrac{\pi}{4} - x\right) = n + \dfrac{1}{2}, \quad n = 0, -1$

$= \pm\dfrac{1}{2}$

$$\cos\left(\frac{\pi}{4}-x\right)=\pm\frac{1}{2\sqrt{2}}$$

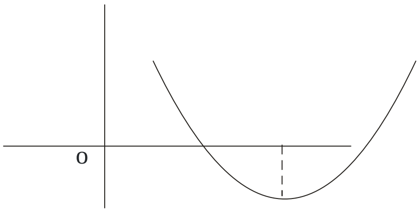
39. **(C)**

$\cos^4 x + a\cos^2 x + 1 = 0$

$D \ge 0$ $\quad a^2 - 4 \ge 0$

$|a| > 2$

Product of roots = 1



So both roots cannot lie in [0, 1]

Hence one root > 1 & one root lie $\in$ (0, 1)

So $\quad f(1) < 0$

$1 + a + 1 \le 0$

$a \le -2$

$\therefore \quad a \leftarrow (-0, -2]$

40. **(C)**

$\tan^4 x - 2\tan^2 x - 2 + a^2 = 0$

$\tan^4 x + 2ta^2x + 1$

$= 3 - a^2$

$\left(ta^2x - 1\right)^2 = 3 - a^2 \ge 0$

$|a| \le \sqrt{3}$

41. **(A)**

$x^2 + 4 + 3Sn\left(ax+b\right) - 2x = 0$

$\left(x^2 - 2x + 1\right) + 3\left(1 + Sn\left(ax+b\right)\right) = 0$

$\left(x-1\right)^2 + 3\left(1 + Sn\left(ax+b\right)\right) = 0$

$\left(x-1\right) = 0$ & $Sn\left(ax+b\right) = -1$

$x = 1$ & $Sn\left[a+b\right] = -1$

$a + b = \frac{3\pi}{2}, \frac{7\pi}{2}$ etc.

$\therefore a + b = \frac{7\pi}{2}$

42. **(B)**

$3Sn\,x + 4\cos ax = 7$

Sn x = 1 & cos ax = 1

$x \quad = 2n\pi + \frac{\pi}{2}, ax = 2n\pi$

$= \left(4n+1\right)\frac{\pi}{2} \qquad \therefore \frac{a\pi}{2}\left(4n+1\right) = 2m\pi$

$a = \frac{4m}{4n+1}$

$a = \frac{4m}{4n+1}$ $\quad\quad m(4n+1)K$

$a = \frac{4n(4n+DK)}{4n+1}$

A = 4mk

43. **(A)**

$|\text{Sn}\,x + \cos x| = |\text{Sn}\,x| + |\cos x|$

$\therefore \quad \text{Sn}\,x \, \cos x \geq 0$

I & III Quadrant

44. **(B)**

$\frac{\text{Sin}^3\theta - \cos^3\theta}{\text{Sn}\,\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{1+\cot^2\theta}} \quad\quad \theta \neq \frac{\pi}{4}$

$-2\tan\theta\,\cot\theta = -1$

$1 + \text{Sn}\,\theta\,\cos\theta - \cos\theta|\text{Sn}\,\theta| - 2 = -1$

$\text{Sn}\,\theta\cos\theta = \cos\theta|\text{Sn}\,\theta|$

$|\text{Sn}\,\theta| = \text{Sn}\,\theta$

$\theta \leftarrow \left(\frac{\pi}{2}, \pi\right)$

45. **(B)**

$\frac{2^{\text{Sn}\,x} + 2^{\text{Cos}\,x}}{2} \geq \sqrt{2^{(\text{Sn}\,x + \cos x)}}$

$\geq \sqrt{2^{-\sqrt{2}}}$

$\geq 2^{-1/2}$

$\therefore 2^{\text{Snx}} + 2^{\cos x} \geq 2^{1-\frac{1}{2}}$

Equal if Snx = cosx = $\frac{-1}{2}$

i.e $x = \frac{5\pi}{4}$

$x = 2n\pi + \frac{5\pi}{4}$

$x = (2n+1)\pi + \frac{\pi}{4}$

46. **(C)**

$\text{Sn}\left(\pi\left(x^2 + x\right)\right) = \text{Sn}\ \pi x^2$

$\pi\left(x^2 + x\right) = n\pi + (-1)^n\left(\pi x^2\right)$

$\pi x^2 + \pi x = 2n\pi + \pi x^2$ or $\pi x^2 + \pi x = (2n+1)\pi - \pi x^2$

x = 2n

but $x \neq I$

$2x^2 + x - (2n+1) = 0$

$$x=\frac{-1\pm\sqrt{1+8(2n+1)}}{4}$$
$$x=\frac{\sqrt{1+8(2n+1)}-1}{4}$$
$$x=\frac{\sqrt{odd}-1}{}4$$

47. **(C)**

$\cos x=1 \qquad x=2n\pi$

$\cos 2\lambda x=1 \qquad 2\lambda x=2m\pi$

$4\lambda n\pi=2m\pi$

$\lambda=\frac{m}{2n}$ will have

Infinite soln. if λ is rational

If λ is Inatiael the

$\lambda=0$ is the only solution

$\therefore$ λ is inativel

48. **(D)**

$$2^{(1-2Sn^2x)}-3\left(2^{-2Sn^2x}\right)+1=0$$
$$2\left(2^{-\sin^2 x}\right)^2-3\left(2^{-Sn^2x}\right)+1=0$$
$$2t^2-3t+1=0$$

t = 1, $\frac{1}{2}$

$2^{Sn^2x}=1$ or 1

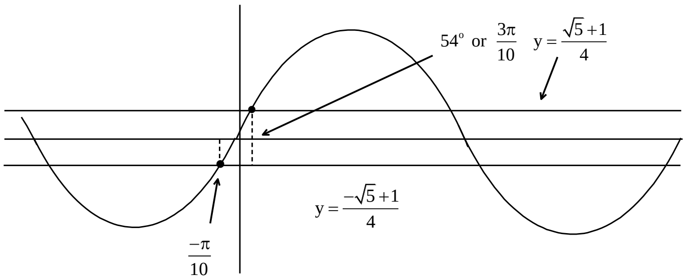
$x=n\pi,\quad n\pi\pm\frac{\pi}{2}$

49. **(C)**

$$\sqrt{3}\cos\theta-3Sn\theta=4Sn2\theta\cos3\theta$$
$$=2(Sn5\theta)-2Sn\theta$$
$$\sqrt{3}\cos\theta-Sn\theta=2Sn5\theta$$
$$2\sin\left(\frac{\pi}{3}-\theta\right)=2Sn5\theta$$
$$Sn\left(\frac{\pi}{3}-\theta\right)=m5\theta$$
$$\theta=2Sn\left(3\theta-\frac{\pi}{6}\right)\cos\left(2\theta+\frac{\pi}{6}\right)$$
$$3\theta-\frac{\pi}{6}=n\pi \quad \& \quad 2\theta+\frac{\pi}{6}=(2n+1)\frac{\pi}{2}$$
$$3\theta=n\pi+\frac{\pi}{6}, \qquad \theta=n\pi+\frac{\pi}{3}$$
$$\theta=\frac{n\pi}{3}+\frac{\pi}{18}$$

50. **(A)**

$\frac{-\sqrt{5}+1}{4} < \text{Sn}\,x < \frac{\sqrt{5}+1}{4}$

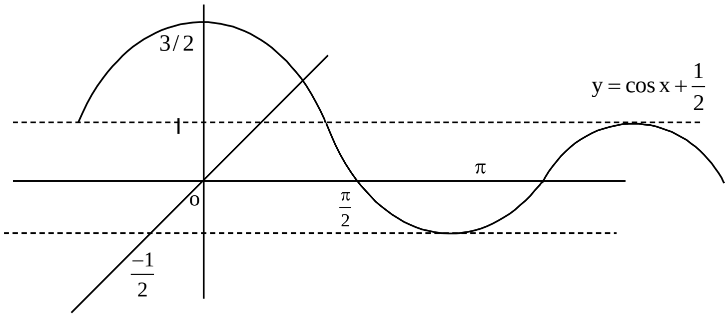


$x \leftarrow \left(\frac{-\pi}{10}, \frac{3\pi}{10}\right)$

51. **(A)**

$\cos x - x + \frac{1}{2} = 0$

$x = \cos x + \frac{1}{2}$ $\quad\quad$ y = x



Soln. in $\left(0, \frac{\pi}{2}\right)$

52. **(B)**

$a\,\text{Sn}\,x + 1 - 2\text{Sn}^2 x = 2a - 7$

$2\text{Sn}^2 x - a\,\text{Sn}\,x + 2a - 7 - 1 = 0$

$2\text{Sn}^2 x - a\,\text{Sn}\,x + 2(a-4) = 0$

At least one root $\in [-1,1]$

$D = a^2 - 16(a-4) \geq 0$

$a^2 - 16a + 6 \geq 0$

$(a-8)^2 \geq 0$

Snx= $\frac{a \pm (a-8)}{4}$

$= \frac{8}{4}$ or $\frac{2a-8}{4}$

= 2 or $\frac{a-4}{2}$

$-a \leq \frac{a-4}{2} \leq 1$

$-2 \leq a - 4 \leq 2$

$2 \leq a \leq 6$

53. **(D)**

$\text{Sn}\,x+\cos x=y^2-y+a$

$y^2-y=\left(y=\frac{1}{2}\right)^2+\frac{3}{4}\ge\frac{3}{4}$

$\therefore\quad y^2-y+a\ge\frac{3}{4}+a$

$\sin x+\cos x\le\sqrt{2}$

$\therefore$ if $\frac{3}{4}+a>\sqrt{2}$ then no soln.

$a>\sqrt{2}-\frac{3}{4}$

$a>1.414-0.75$

$a\in\left(\sqrt{3},\infty\right)$

54. **(A)**

$4\text{Sn}^2x+\tan^2x+\text{cosec}^2x+\cot^2x-6=0$

$\left(2\text{Sn}x-\text{cosec}\right)^2+\left(\tan x-\cot x\right)^2=0$

$2\text{Sn}\,x-\frac{1}{\text{Sn}\,x}=0$ & $\tan-\cot x=0$

$\text{Sn}^2x=\frac{1}{2}$ & $\tan^2x=1$

$x=n\pi k\pm\frac{\pi}{4}$

55. **(C)**

$\tan\left(\pi\cos\theta\right)=\cot\left(\pi\sin\theta\right)$

$\tan\left(\pi\cos\theta\right)=\tan\left(\frac{\pi}{2}-\pi\sin\theta\right)$

$\pi\cos\theta=\frac{\pi}{2}-\pi\sin\theta$

$\pi\left(\sin\theta+\cos\theta\right)=\frac{\pi}{2}$

$\sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta\right]=\frac{1}{2}$

$\sqrt{2}\left[\sin\left(\theta+\frac{\pi}{4}\right)\right]=\frac{1}{2}$

$\sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}$

56. **(B)**

$\Rightarrow\tan2\theta\tan\theta=1$

$\Rightarrow\tan2\theta\tan\theta-1=0$

$\Rightarrow\frac{\sin2\theta\sin\theta}{\cos2\theta\cos\theta}-1=0$

$\Rightarrow\frac{\sin2\theta\sin\theta-\cos2\theta\cos\theta}{\cos2\theta\cos\theta}=0$

$\Rightarrow \dfrac{\cos 3\theta}{\cos\theta\cos 2\theta} = 0$

$\Rightarrow 3\theta = (2n+1)\dfrac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\dfrac{\pi}{6}$

$\Rightarrow \theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$\Rightarrow \theta = n\pi \pm \dfrac{\pi}{6}$

$\Rightarrow \cos\theta \neq 0$

$\Rightarrow \theta \neq (2n+1)\dfrac{\pi}{2}$

$\Rightarrow \cos 2\theta \neq 0$

$\Rightarrow \theta \neq (2n+1)\dfrac{\pi}{4}$

$\Rightarrow \dfrac{\pi}{2}$ and $\dfrac{9\pi}{6}$ are ruled out.

57. **(A)**

$\Rightarrow \sin\left(\dfrac{\pi}{4}\cos\theta\right) = \cos\left(\dfrac{\pi}{4}\tan\theta\right)$

$\Rightarrow \cos\left(\dfrac{\pi}{2} - \dfrac{\pi}{4}\cot\theta\right) = \cos\left(\dfrac{\pi}{4}\tan\theta\right)$

$\Rightarrow \therefore \dfrac{\pi}{2} - \dfrac{\pi}{4}\cot\theta = 2n\pi \pm \dfrac{\pi}{4}\tan\theta$

$\Rightarrow \dfrac{\pi}{2} - 2n\pi = \dfrac{\pi}{4}\cot\theta \pm \dfrac{\pi}{4}\tan\theta$

$\Rightarrow -2n\pi$ can be taken as $2n\pi$ ($\because$ n can be negative integer)

$\Rightarrow \therefore 2n\pi + \dfrac{\pi}{2} = \dfrac{\pi}{4}(\cot\theta \pm \tan\theta)$

$\Rightarrow 8n + 2 = \cot\theta \pm \tan\theta$

$\Rightarrow 2 = \cot\theta \pm \tan\theta$

$\Rightarrow 2 = \dfrac{1}{\tan\theta} \pm \tan\theta$

$\Rightarrow 2 = \dfrac{1}{\tan\theta} + \tan\theta$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = n\pi + \dfrac{\pi}{4}$

$\Rightarrow 2 = \dfrac{1}{\tan\theta} - \tan\theta$

$\Rightarrow 2\tan\theta = 1 - \tan^2\theta$

$\Rightarrow \tan^2\theta + 2\tan\theta - 1 = 0$

$\Rightarrow \tan\theta = \dfrac{-2 \pm \sqrt{8}}{2}$

$\Rightarrow \tan\theta = -1 \pm \sqrt{2}$

58. **(B)**

$\tan\theta = \cot 2\theta$

$\tan\theta = \tan\left(\frac{\pi}{2} - 2\theta\right)$

$\theta = n\pi + \frac{\pi}{2} - 2\theta$

$3\theta = n\pi + \frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{6}$, where

$n \in I, n \neq 3m+1,\ m \in I$.

$= \frac{\pi}{2}, \frac{3\pi}{2} \ldots\ldots\ldots$ $\theta = (2n+1)\frac{\pi}{4}$

$\frac{\pi}{4}, \frac{3\pi}{4} \ldots\ldots\ldots$

59. **(D)**

$\tan\theta + \tan 2\theta + \tan\theta \tan 2\theta = 1$

$\tan\theta + \tan 2\theta = 1 - \tan\theta \tan 2\theta$

$\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta \tan 2\theta} = 1$

$\tan(\theta + 2\theta) = 1$

$\tan 3\theta = 1$

$3\theta = n\pi + \frac{\pi}{4}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{12}, n \in I$

Where $\frac{n\pi}{3} + \frac{\pi}{12} \neq \left\{\frac{(2n+1)\pi}{4}(2k+1)\frac{\pi}{2}\right\}$

$m, k \in I$

60. **(A)**

For $x \in \left(0, \frac{\pi}{2}\right)$

$\cos^2 x = 1 - \sin 2x$

$1 - \sin^2 x = 1 - 2\sin x \cos x$

$\sin x(\sin x - 2\cos x)$

$\sin x = 0$ $\tan x = 2$

$x = \tan^{-1} 2$

61. **(D)**

$|\sin x|^2 + |\sin x| + b = 0$

$t^2 + t + b = 0$ $\in [0,1]$

The root is negative, other roots must it lie is [0,1] for 2 values of x.

$f(0) \leq 0$ $\Rightarrow$ $b \leq 0$

$f(1) > 0$ $b > -2$

$(-2, 0]$

62. **(B)**

$a\cos x + s\sin x = 13$

For no real solution

$\sqrt{a^2+25} < 13$

$a^2 + 25 < 169$

$a^2 - 144 < 0$

$(a-12)(a+12) < 0$

$-12 < a < 12$

$a \in (-12, 12)$

63. **(D)**

$|\sin x| < \frac{1}{2}$

$\frac{-1}{2} < \sin x < \frac{1}{2}$

Hence $x \in$

$\left(\frac{-\lambda}{6}, \frac{\lambda}{6}\right) \cup \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$

Dig.

$\left(2n\pi - \frac{\pi}{6}, 2n\pi + \frac{\pi}{6}\right) \cup \left(2n\pi + \frac{5\pi}{6}, 2n\pi + \frac{7\pi}{6}\right)$

Where $n \in I$

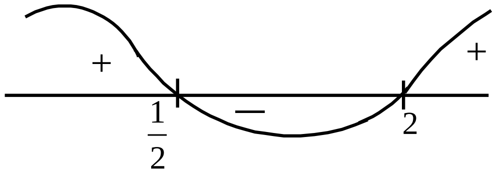
64. **(D)**

$2\sin^2\theta - 5\sin\theta + 2 > 0$

$2\sin^2\theta - 4\sin\theta - \sin\theta + 2 > 0$

$2\sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) > 0$

$(\sin\theta - 2)(2\sin\theta - 1) > 0$



$\sin\theta < \frac{1}{2}$ or $\sin\theta > 2$ (not possible)

$\theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

65. **(B)**

$|\cos x| = \sin x$

$\Rightarrow |\cos x|$ is always positive

$\therefore$ x must lie in 1st and 2nd quadrant,

So $\sin x$ is positive

and $|\cos x| = \sin x$ at $x = \frac{\pi}{4}, \frac{3\pi}{4}$

$\Rightarrow 0 \le x \le 4\pi$ is 2 periods

So total solution is 4.

1. **(2)**

$\because \cos x = \sqrt{1-\sin 2x}$

$\Rightarrow \cos x = |\sin x - \cos x|$

There are two cases arise

Case I $\sin x \le \cos x$

$\Rightarrow \quad \cos x = \cos x - \sin x$

$\Rightarrow \sin x = 0$

Where, $x \in \left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

$\Rightarrow x = 2\pi$, neglecting $x = \pi$

Case II $\sin x > \cos x$

$\Rightarrow \quad \tan x = 2$

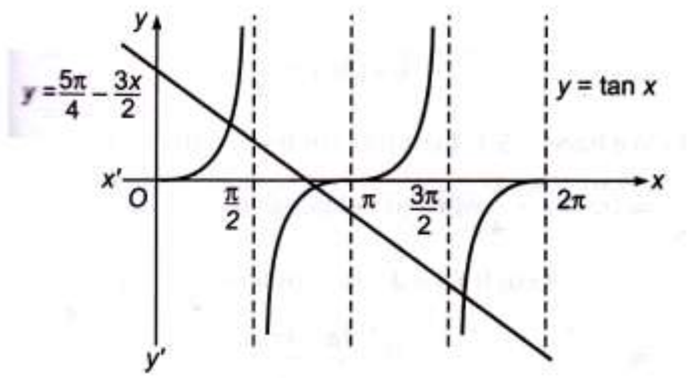
Where, $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\because \tan x = 2$

$\Rightarrow x = \tan^{-1}(2)$

Thus, the given equation has two solutions.

2. **(3)**



$\Rightarrow \tan x = \frac{5\pi}{4} - \frac{3x}{2}$

Let $\frac{5\pi}{4} - \frac{3x}{2} = y$

And $y = \tan x$

$\therefore$ Graph of $y = \frac{5\pi}{4} - \frac{3x}{2}$ and $y = \tan x$ meet exactly three times in $[0, 2\pi]$. Thus, the number of solutions of given equation is 3.

3. **(6)**

$3\sin^2 x - 6\sin x - \sin x + 2 = 0$

$\Rightarrow 3\sin x(\sin x - 2) - 1(\sin x - 2) = 0$

$\Rightarrow (3\sin x - 1)(\sin x - 2) = 0$

$\Rightarrow \sin x = \frac{1}{3} \text{ or } 2$

$\Rightarrow \sin x = \frac{1}{3}$ $\quad (\because \sin x \neq 2)$

Let $\sin^{-1}\frac{1}{3} = \alpha, 0 < \alpha < \frac{\pi}{2}$

Then, $\alpha, \pi-\alpha, 2\pi+\alpha, 3\pi-\alpha, 4\pi+\alpha, 5\pi-\alpha$ are the solutions in $[0, 5\pi]$

$\therefore$ Required number of solutions $= 6$

4. **(0)**

Since, $1+\sin x \sin^2\frac{x}{2} = 0$

$\therefore 1+\sin x\left(\frac{1-\cos x}{2}\right) = 0$

$\Rightarrow 2+\sin x - \sin x \cos x = 0$

$\Rightarrow \sin 2x - 2\sin x = 4$

Which is not possible for any x in $[-\pi, \pi]$

5. **(4)**

Now, $1+|\cos x|+\cos^2 x+|\cos^{-1} x|+.....\infty = \frac{1}{1-|\cos x|}$

$\therefore \quad 8^{\frac{1}{1-|\cos x|}} = 4^3$

$\Rightarrow \frac{3}{2^{1-|\cos x|}} = 2^6 \Rightarrow 1 = 2-2|\cos x|$

$\Rightarrow |\cos x| = \frac{1}{2}$

$\Rightarrow \cos x = \pm\frac{1}{2}$

$\Rightarrow x = \frac{\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, -\frac{2\pi}{3}$

$\therefore$ Number of solutions $= 2$

6. **(0)**

Given, $e^{\sin x} - e^{-\sin x} - 4 = 0$

$\Rightarrow e^{2\sin x} - 4e^{\sin x} - 1 = 0$

$\Rightarrow e^{\sin x} = \frac{4 \pm \sqrt{16+4}}{2} = 2+\sqrt{5}$

$\Rightarrow \sin x = \log\left(2+\sqrt{5}\right)$ [$\because \log\left(2-\sqrt{5}\right)$ is not defined]

Since, $2+\sqrt{5} > e \Rightarrow \log\left(2+\sqrt{5}\right) > 1$

$\Rightarrow \sin x > 1$, which is not possible.

Hence, no solution exist.

7. **(0)**

$2\sin x = 5x^2 + 2x + 3$

$\Rightarrow 2\sin x = 4x^2 + (x+1)^2 + 2$

But $2\sin x \leq 2$

And $4x^2 + (x+1)^2 + 2 > 2$, so it has no solution

8. **(2)**

We have, $(5+4\cos\theta)(2\cos\theta+1)=0$

$\Rightarrow \cos\theta=-\dfrac{5}{4}$ which is not possible.

$\therefore 2\cos\theta+1=0 \Rightarrow \cos\theta=-\dfrac{1}{2}$

$\Rightarrow \theta=\dfrac{2\pi}{3},\dfrac{4\pi}{3}$

$\therefore$ Solution set is $\left\{\dfrac{2\pi}{3},\dfrac{4\pi}{3}\right\}\in[0,2\pi]$

9. **(5)**

We have,

$\cos 3x+\cos 2x=\sin\dfrac{3x}{2}+\sin\dfrac{x}{2}$

$\Rightarrow 2\cos\dfrac{5x}{2}\cos\dfrac{x}{2}=2\sin x\cos\dfrac{x}{2}$

Either $\cos\dfrac{x}{2}=0 \Rightarrow \dfrac{x}{2}=(2n+1)\dfrac{\pi}{2}$

$\Rightarrow x=(2n+1)\pi$ or $\cos\dfrac{5x}{2}=\sin x$

$\Rightarrow \cos\dfrac{5x}{2}=\cos\left(\dfrac{\pi}{2}-x\right)$

$\Rightarrow \dfrac{5x}{2}=2n\pi\pm\left(\dfrac{\pi}{2}-x\right)$

Taking the positive sign $\dfrac{7x}{2}=2n\pi+\dfrac{\pi}{2} \Rightarrow x=\dfrac{4n\pi}{7}+\dfrac{\pi}{7}$

Taking negative sign

$\dfrac{3x}{2}=2n\pi-\dfrac{\pi}{2} \Rightarrow x=\dfrac{4n\pi}{3}-\dfrac{\pi}{3}$

For $0\le x\le 2\pi$

$x=\dfrac{\pi}{7},\dfrac{5\pi}{7},\dfrac{9\pi}{7},\dfrac{13\pi}{7},\pi$

Thus, number of solutions $=5$

10. **(0)**

$\because \sin^5 x-\cos^5 x=\dfrac{\sin x-\cos x}{\sin x\cos x}$

$\Rightarrow \sin x\cos x\left[\dfrac{\sin^5 x-\cos^5 x}{\sin x-\cos x}\right]=1$

$\Rightarrow \dfrac{1}{2}\sin 2x\left[\sin^4 x+\sin^3 x\cos x+\sin^2 x\cos^2 x+\sin x\cos^2 x+\cos^4 x\right]=1$

$\Rightarrow \sin 2x\left[\left(\sin^2 x+\cos^2 x\right)^2-2\sin^2 x\cos^2 x+\sin x\cos x\left(\sin^2 x+\cos^2 x\right)+\sin^2 x\cos^2 x\right]=2$

$\Rightarrow \sin 2x\left[1-\sin^2 x\cos^2 x+\sin x\cos x\right]=2$

$\Rightarrow \sin^3 2x-2\sin^2 2x-4\sin 2x+8=0$

$\Rightarrow (\sin 2x-2)^2(2\sin 2x+2)=0$

$\Rightarrow \sin 2x = \pm 2$, which is not possible for any x.

11. **(16)**

$\sqrt{\sin^2 x - \sin x + \frac{1}{2}} = \sqrt{\left(\sin x - \frac{1}{2}\right)^2 + \frac{1}{4}} \geq \frac{1}{2}, \forall x$ and $\sec^2 y \geq t, \forall y$, so $2^{\cos^2 y} \geq 2$.

Hence, the above inequality holds only for those values of x and y for which $\sin x = \frac{1}{2}$ and $\sec^2 y = 1$.

Hence, $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ and $y = 0, \pi, 2\pi, 3\pi$.

Hence, required number of ordered pairs are 16.

12. **(0)**

Since, $2\cos^2 \frac{x}{2} \sin^2 x < 2$

But $x^2 + \frac{1}{x^2} \geq 2$

Thus, the equation has no solution

13. **(2)**

Given, $\sin^4 x + \cos^4 x = \sin x . \cos x$

$\Rightarrow \left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x . \cos^2 x = \sin x . \cos x$

$\Rightarrow 1 - \frac{\sin^2 2x}{2} = \frac{\sin 2x}{2}$

$\Rightarrow \sin^2 2x + \sin 2x - 2 = 0$

$\Rightarrow \left(\sin 2x + 2\right)\left(\sin 2x - 1\right) = 0$

$\Rightarrow \sin 2x = 1 \qquad \left(\because \sin 2x \geq -1\right)$

$\therefore 2x = \left(4n+1\right)\frac{\pi}{2}$

$\Rightarrow x - \left(4n+1\right)\frac{\pi}{4}$

$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

Hence, two solutions exist.

14. **(0.33)**

$\frac{1 + \tan\theta}{1 - \tan\theta} = 3\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$

On simplification, we get

$3\tan^4\theta - 6\tan^2\theta + 8\tan\theta - 1 = 0$

$\therefore$ Product of roots

$= \tan\alpha . \tan\beta . \tan\gamma . \tan\delta = -\frac{1}{3}$

15. **(1)**

$\because x^3 + x^2 + 4x + 2\sin x = 0$

$\Rightarrow x^3 + \left(x+2\right)^2 + 2\sin x = 4$

$x = 0$, satisfies this equation

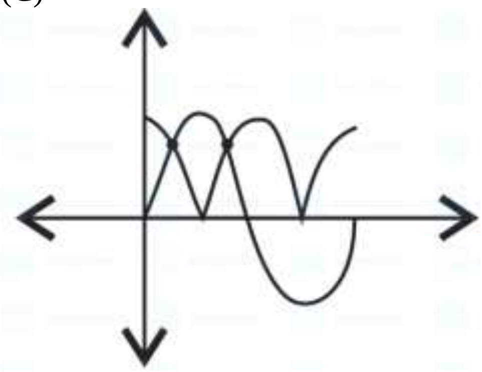
Now, in $0 < x \le \pi, x^3 + (x+2)^2 + 2\sin x > 4$

And in $\pi < x \le 2\pi, x^3 + (x+2)^2 + 2\sin x > 27 + 25 - 2 = 50$

Hence, $x = 0$ is the only solution.

JEE Main : PYQ

1. (C)



2 solutions in $(0, 2\pi)$

So, 8 solutions in $[-4\pi, 4\pi]$

2. (C)

$8^{2\sin^2\theta} + 8^{2-2\sin^2\theta} = 16$

$y + \frac{64}{y} = 16$

$\Rightarrow y = 8$

$\Rightarrow \sin^2\theta = \frac{1}{2}$

$n(S) + \sum_{\theta \in S} \frac{1}{\cos(\pi/4 + 2\theta)\sin(\pi/4 + 2\theta)}$

$= 4 + (-2) \times 4 = -4$

3. (3)

$2\sin^2\theta - \cos 2\theta = 0$

$2\sin^2\theta - (1 - 2\sin^2\theta) = 0$

$\Rightarrow \sin^2\theta = \left(\frac{1}{2}\right)^2$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\therefore \sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

So, the common solution is

$\theta=\frac{7\pi}{6},\frac{11\pi}{6}$

Sum $=\frac{l\pi+11\pi}{6}=3\pi=k\pi$

$K=3$

4. (16)

$7\cos^2\theta-3\sin^2\theta-2\cos^2 2\theta=2$

$4\cos^2\theta+3\cos 2\theta-2\cos^2 2\theta=2$

$2(1+\cos 2\theta)+3\cos 2\theta-2\cos^2 2\theta=2$

$2\cos^2 2\theta-5\cos 2\theta=0$

$\cos 2\theta(2\cos 2\theta-5)=0$

$\cos 2\theta=0$

$2\theta=(2n+1)\frac{\pi}{2}$

$\theta=(2n+1)\frac{\pi}{4}$

$S=\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$

For all four values of θ

$x^2-2\left(\tan^2\theta+\cot^2\theta\right)x+6\sin^2\theta=0$

$\Rightarrow x^2-4x+3=0$

Sum of roots of all four equations $=4\times4=16$

5. (B)

$\sin\theta\tan\theta+\tan\theta=\sin 2\theta$

$\tan\theta(\sin\theta+1)=\frac{2\tan\theta}{1+\tan^2\theta}$

$\tan\theta=0\Rightarrow\theta=-\pi,0,\pi$

$(\sin\theta+1)=2.\cos^2\theta=2(1+\sin\theta)(1-\sin\theta)$

$\sin\theta=-1$ which is not possible

$\sin\theta=\frac{1}{2}$ $\qquad\theta=\frac{\pi}{6},\frac{5\pi}{6}$

$n(s)=5$

$T=\cos 0+\cos 2\theta+\cos 2\pi+\cos\frac{\pi}{3}+\cos\frac{5\pi}{3}$

$T=4$

$T+n(s)=9$

6. (4)

$x\in\left(\frac{\pi}{4},\frac{7\pi}{4}\right)$

$14\operatorname{cosec}^2x-2\sin^2x=21-4\cos^2x$

$=21-4\left(1-\sin^2x\right)$

$=17+4\sin^2x$

$14\operatorname{cosec}^2 x - 6\sin^2 x = 17$

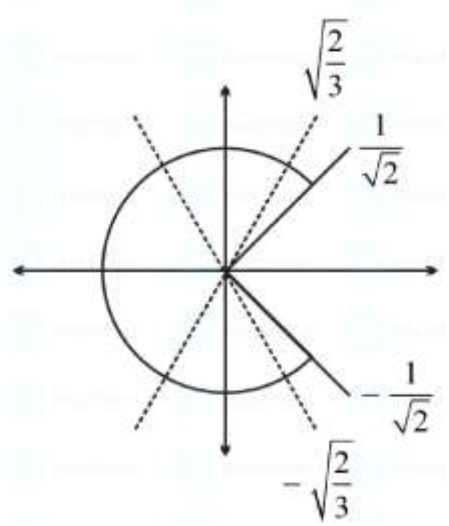
Let $\sin^2 x = p$

$\frac{14}{p} - 6p = 17 \Rightarrow 14 - 6p^2 = 17p$

$6p^2 + 17p - 14 = 0$

$p = -3.5, \frac{2}{3} \Rightarrow \sin^2 x = \frac{2}{3}$

$\Rightarrow \sin x = \pm\sqrt{\frac{2}{3}}$



$\therefore$ Total 4 solutions

7. (32)

$3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$

$3\cos^2 2\theta + 6\cos 2\theta - 5(1 + \cos 2\theta) + 5 = 0$

$3\cos^2 2\theta + \cos 2\theta = 0$

$\cos 2\theta = 0 \text{ OR } \cos 2\theta = -1/3$

$\theta \in [-4\pi, 4\pi]$

$2\theta = (2n+1).\frac{\pi}{2}$

$\therefore \theta = \pm\pi/4. \pm 3\pi/4.......\pm 15\pi/4$

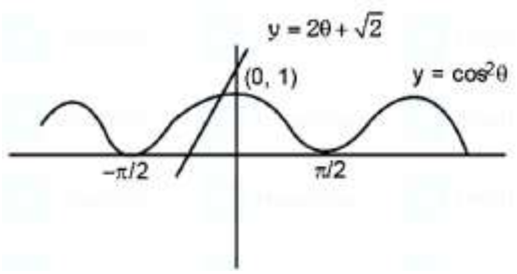
Similarly $\cos 2\theta = -1/3$ gives 16 solution

8. (1)

$2\theta - \cos^2\theta + \sqrt{2} = 0$

$\Rightarrow \cos^2\theta = 2\theta + \sqrt{2}$

$y = 2\theta + \sqrt{2}$



Both graphs intersect at one point.

9. (4)

$\sin^2 x+\sin x-1=0$

$\sin x=\dfrac{-1+\sqrt{5}}{2}=+\text{ve}$

Only 4 roots

10. (A)

$2\cos x\left(4\sin\left(\dfrac{\pi}{4}+x\right)\sin\left(\dfrac{\pi}{4}-x\right)-1\right)=1$

$\Rightarrow 2\cos x\left(2\cos(2x)=2\cos\left(\dfrac{\pi}{2}\right)-1\right)=1$

$\Rightarrow 2\cos x\left(4\cos^2 x-3\right)=1$

$\Rightarrow \cos 3x=\dfrac{1}{2}$

$\Rightarrow 3x=\dfrac{\pi}{3},\dfrac{5\pi}{3},\dfrac{7\pi}{3}\Rightarrow x=\dfrac{\pi}{9},\dfrac{5\pi}{9},\dfrac{7\pi}{9}$

Number of solutions $=n=3$

Sum of solutions $=S=\dfrac{13\pi}{9}$

11. (B)

$32^{\tan^2 x}+32^{\sec^2 x}=81$

$\Rightarrow 32^{\tan^2 x}+32^{1+\tan^2 x}=81$

$\Rightarrow 33\times 32^{\tan^2 x}=81$

$\Rightarrow 32^{\tan^2 x}=\dfrac{27}{11}$

$\Rightarrow \tan^2 x=\ln_{32}\left(\dfrac{27}{11}\right)$

$\Rightarrow \tan x=\sqrt{\ln_{32}\left(\dfrac{27}{11}\right)}\in(0,1)$

$\Rightarrow$ One solution in $\left[0,\dfrac{\pi}{4}\right]$

12. (56)

Given, $\sin^4\theta+\cos^4\theta-\sin\theta\cos\theta=0$

$\theta\in[0,4\pi]$

$\Rightarrow 1-2\sin^2\theta\cos^2\theta-\sin\theta\cos\theta=0$

$\Rightarrow 2-\sin^2 2\theta-\sin 2\theta=0$

$\Rightarrow \sin^2 2\theta+\sin 2\theta-2=0$

$\Rightarrow (\sin 2\theta+2)(\sin 2\theta-1)=0$

$\Rightarrow \sin 2\theta=1, 2\theta[0,8\pi]$

$\theta=\dfrac{\pi}{4},\dfrac{5\pi}{4},\dfrac{9\pi}{4},\dfrac{13\pi}{4}$

Sum of solutions $S=\dfrac{28\pi}{4}$

Then, $\frac{8S}{\pi}=\frac{8\pi}{\pi}\times\frac{28\pi}{4}=56$

13. (A)

We have, $\frac{\cos x}{1+\sin x}=|\tan 2x|$

$$\Rightarrow \frac{\cos^2\left(\frac{x}{2}\right)-\sin^2\left(\frac{x}{2}\right)}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}=|\tan 2x|$$

$$\Rightarrow \frac{\left[\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right]\left[\cos\frac{x}{2}+\sin\frac{x}{2}\right]}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}=|\tan 2x|$$

$$\Rightarrow \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}=|\tan 2x|$$

$$\Rightarrow \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=|\tan 2x|$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)=\tan^2 2x$$

$$\Rightarrow 2x=n\pi\pm\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\Rightarrow 2x=n\pi+\frac{\pi}{4}-\frac{x}{2}$$

Or $2x=n\pi-\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$$\Rightarrow \frac{5x}{2}=\left(n+\frac{1}{4}\right)\pi$$

Or $\frac{3x}{2}=\left(n-\frac{1}{4}\right)\pi$

$$\Rightarrow \frac{-\pi}{2}<\frac{2}{5}\left(n+\frac{1}{4}\right)\pi<\frac{\pi}{2}$$

Or $\frac{-\pi}{2}<\frac{2}{3}\left(n-\frac{1}{4}\right)\pi<\frac{\pi}{2}$

$$\Rightarrow -\frac{5}{4}<n+\frac{1}{4}<\frac{5}{4}$$

Or $\frac{-3}{4}<n-\frac{1}{4}<\frac{3}{4}$

$$\Rightarrow \frac{-6}{4}<n<1$$

Or $\frac{-1}{2}<n<1$

$\Rightarrow n=-1,0$

Or $n=0$

When $n=-1, x=\left(\frac{-3}{10}\right)\pi$

Or when $n=0, x=-\frac{\pi}{6}$

$n=0, x=\left(\frac{1}{10}\right)\pi$

$\therefore$ Required sum

$=\left(\frac{-3}{10}\right)\pi+\left(\frac{1}{10}\right)\pi+\left(\frac{-1}{6}\right)\pi=\left(\frac{-11}{30}\right)\pi$

14. (C)

$\sin^7 x+\cos^7 x=1 \quad \ldots\ldots(i)$

As,

$\sin^2 x+\cos^2 x=1 \quad \ldots\ldots(ii)$

Now, $\sin^7 x\le\sin^2 x$ and $\cos^7 x\le\cos^2 x$.

But according to question, Eqs (i) and (ii), it is only possible, when

$\sin^7 x=\sin^2 x$ and $\cos^7 x=\cos^2 x$

So, $\sin^2 x+\cos^2 x=1$

When $\sin x=0$ and $\cos x=1$

Or $\sin x=1$ and $\cos x=0$

$\Rightarrow x=0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$

$\therefore$ 5 solutions.

15. (A)

Let $\alpha=\max\left(8^{2\sin 3x}.4^{4\cos 3x}\right)$

$=\max\left(2^{6\sin 3x}.2^{8\cos 3x}\right)$

$=\max\left(2^{6\sin x+8\cos 3x}\right)$

and $\beta=\max\left(8^{2\sin 3x}.4^{4\cos 3x}\right)$

$=\max\left(2^{6\sin 3x}.2^{8\cos 3x}\right)$

$=\max\left(2^{6\sin 3x}+8\cos 3x\right)$

Now, determine the range of

$6\sin 3x+8\cos 3x$

($\because$ Range of $a\sin x+b\cos x$

$=\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$

$=\left[-\sqrt{6^2+8^2}, \sqrt{6^2+8^2}\right]$

$=[-10,10]$

So, $\alpha=2^{10}$ and $\beta=2^{-10}$

Now, $\alpha^{1/5}=\left(2^{10}\right)^{1/5}=4$

$\beta^{1/5}=\left(2^{-10}\right)^{1/5}=\frac{1}{4}$

Quadratic equation with roots 4 and $\frac{1}{4}$ is

$$x^2-\left(4+\frac{1}{4}\right)x+4\times\frac{1}{4}=0$$

$$\Rightarrow x^2-\frac{17}{4}x+1=0$$

Multiplying both sides by 8,

$$8x^2-34x+8=0$$

On comparing, $8x^2+bx+c$, we get

$b=-34$ and $c=8$

So, $c-b=8-(-34)=42$

16. (1)

Given,

$$|\cot x|=\cot x+\frac{1}{\sin x} \quad \text{.......(i)}$$

If $\cot x>0$, then $|\cot x|=\cot x$

From Eq. (i), $\cot x=\cot x+\frac{1}{\sin x}$

$$\Rightarrow \frac{1}{\sin x}=0 \quad \text{(not possible)}$$

If $\cot x<0$, then $|\cot x|=-\cot x$

From Eq.(ii), $-\cot x=\cot x+\frac{1}{\sin x}$

$$\Rightarrow 2\cot x+\frac{1}{\sin x}=0\Rightarrow 2\cos x=-1$$

$$\Rightarrow x=\frac{2\pi}{3}\text{ or }\frac{4\pi}{3}$$

Here, $x=\frac{4\pi}{3}$ rejected because $\frac{4\pi}{3}\in$ third quadrant and in third quadrant cot x is positive.

Since, we considered $\cot x<0$. $\therefore\ x=2\pi/3$ is the only one solution.

17. (A)

Given, $x+2\tan x=\frac{\pi}{2}$

$$\Rightarrow 2\tan x=\frac{\pi}{2}-x$$

$$\Rightarrow \tan x=\frac{\pi}{4}-\frac{x}{2}$$

$$\Rightarrow \tan x=\left(-\frac{1}{2}\right)x+\frac{\pi}{4} \quad \text{.......(i)}$$

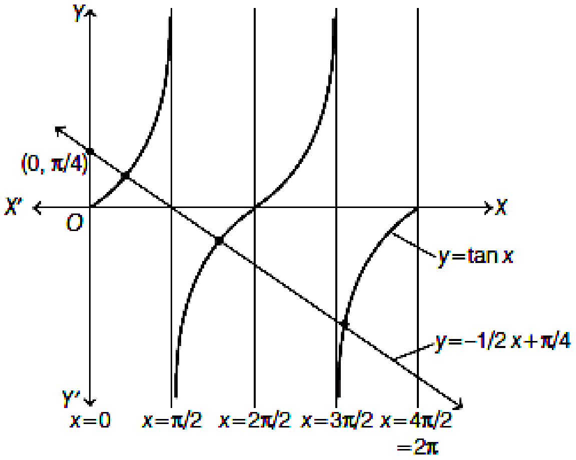
Approach In this type of problem solving, graphical approach is best because we have to find only number of solutions, not the solution (i.e. not the value (s) of x).

Concept: To find the number of solution(s) for Eq. (i), first of all, let

$$y=\tan x \quad \text{....(ii)}$$

and $y=\left(\frac{-1}{2}\right)x+\frac{\pi}{4}$ ....(iii)

and then draw the graph of Equation (ii) and (iii)
Now, total number of solution(s) = Total number of point(s) of intersection of the graph(ii) and (iii).



From above, we see that the red line i.e., $y=-\frac{1}{2}x-\frac{\pi}{4}$ intersects the black curve i.e., $y=\tan x$ at three distinct points in $[0,2\pi]$.
$\therefore$ Total number of solutions $=3$

18. (B)
Given, $81^{\text{in}^2 x}+81^{\cos^2 x}=30$
$\Rightarrow 81^{\sin^2 x}+81^{(1-\sin^2 x)}=30$
$\Rightarrow 81^{\sin^2 x}+\frac{81}{81^{\sin^2 x}}=30$
Let $81^{\sin^2 x}=y$
$\therefore \quad y+\frac{81}{y}=30$
$\Rightarrow y^2-30y+81=0$
$\Rightarrow (y-27)(y-3)=0$
$\Rightarrow y=3 \text{ or } y=27$
$81^{\sin^2 x}=3 \text{ or } 81^{\sin^2 x}=27$
$3^{4\sin^2 x}=3 \text{ or } 3^{4\sin^2 x}=3^3$
$\Rightarrow 4\sin^2 x=1 \text{ or } 4\sin^2 x=3$
$\Rightarrow \sin^2 x=1/4 \text{ or } \sin^2 x=3/4$
$\Rightarrow \sin^2 x=\sin^2(\pi/6)$
Or $\sin^2 x=\sin^2(\pi/3)$
$\Rightarrow x=n\pi\pm\pi/6 \text{ or } x=n\pi\pm\pi/3$
From $[0,\pi]$
$x=\pi/6, 5\pi/6 \text{ or } =\pi/3, 2\pi/3$
Hence, the total number of solutions =4

19. (1)
Given, $\sqrt{3}\cos^2 x=(\sqrt{3}-1)\cos x+1$,
$x\in[0,\pi/2]$

Let $\cos x = t$, then

$\sqrt{3}t^2 = \left(\sqrt{3}-1\right)t+1$

$\Rightarrow \sqrt{3}t^2 - \sqrt{3}t + t - 1 = 0$

$\Rightarrow \left(\sqrt{3}t^2 - \sqrt{3}t\right) + (t-1) = 0$

$= \sqrt{3}t(t-1) + 1(t-1) = 0$

$\Rightarrow (t-1)\left(\sqrt{3}t+1\right) = 0$

This gives $t = 1$ and $t = \frac{-1}{\sqrt{3}}$

Put, $t = \cos x$, then

$\cos x = 1$ and $\cos x = \frac{-1}{\sqrt{3}}$

$\cos x = -1/\sqrt{3}$ is rejected as $x \in [0, \pi/2]$

$\therefore \cos x = 1$

Since, $x \in \left[0, \frac{\pi}{2}\right]$, then $\cos x = \cos 0$

This gives $x = 0$ is only solution. Therefore, number of solution when $x \in [0, \pi/2]$.

20. (11)

Given, $3\sin x + 4\cos = k+1$ ......(i)

Multiply and divide LHS of Eq. (i) by $\sqrt{3^2+4^2} = 5$

i.e. $5\left(\frac{3}{5}\sin x + \frac{4}{5}\cos x\right) = k+1$

$\Rightarrow 5(\cos\alpha \sin x + \sin\alpha \cos x) = k+1$

[Let $\cos\alpha = 3/5$ then $\sin\alpha = \sqrt{1-(3/5)^2} = \frac{4}{5}$]

$5\sin(x+\alpha) = k+1$

[Use $\sin(a+b) = \sin a \cos b + \cos a \sin b$]

$\Rightarrow \sin(x+\alpha) = \frac{k+1}{5}$

Let, $x + \alpha = \theta$

Then, $\sin\theta = \frac{k+1}{5}$

$\because -1 \le \sin\theta \le 1$

$\Rightarrow -1 \le \frac{k+1}{5} \le 1$

$\Rightarrow -5 \le k+1 \le 5$

$\Rightarrow -6 \le k \le 4$

$\therefore$ Possible integral values of k are $-6, -5, -4, -3, -2, -1, 0, 1, 2, 3$ and 4

i.e. Total 11 integral values of k are possible for which Eq. (i) has solution.

21. (B)

The expression, $\cos^4\theta + \sin^4\theta$

$= \left(\cos^2\theta + \sin^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta$

$=1-\frac{1}{2}\sin^2(2\theta)$

$\because \sin^2(2\theta)\in[0,1]$

$\Rightarrow -\frac{1}{2}\sin^2(2\theta)\in\left[-\frac{1}{2},0\right]$

$\Rightarrow 1-\frac{1}{2}\sin^2(2\theta)\in\left[\frac{1}{2},1\right]$

Now, as $\cos^4\theta+\sin^4\theta+\lambda=0$

$\Rightarrow \lambda=-\left(\cos^4\theta+\sin^4\theta\right)$

$\Rightarrow \lambda\in[-1,-1/2]$ for real solution of the given equation $\cos^4\theta+\sin^4\theta+\lambda=0$ for θ.

Hence, option (B) is correct.

22. (8)

Given equation,

$\log_{1/2}|\sin x|=2-\log_{1/2}|\cos x|$

$\Rightarrow -\log_2|\sin x|=2+\log_2|\cos x|$

$\Rightarrow \log_2|\sin x|+\log_2|\cos x|+\log_2 4=0$

$\Rightarrow \log_2\left(4|\sin x||\cos x|\right)=0$

$\Rightarrow 4|\sin x||\cos x|=1$

$\Rightarrow \sin 2x=\pm\frac{1}{2}$

$\because x\in[0,2\pi]\Rightarrow 2x\in[0,4\pi]$

$\therefore$ For $2x\in[0,2\pi]\sin 2x=\pm\frac{1}{2}$ has four solutions and $2x\in[2\pi,4\pi],\sin 2x=\pm\frac{1}{2}$ has four more solutions.

$\therefore$ Total number of solutions are 8. Hence, answer is 8

23. (A)

We have $\theta\in[-2\pi,2\pi]$

And $2\cos^2\theta+3\sin\theta=0$

$\Rightarrow 2\left(1-\sin^2\theta\right)+3\sin\theta=0$

$\Rightarrow 2-2\sin^2\theta+3\sin\theta=0$

$\Rightarrow 2\sin^2\theta-3\sin\theta-2=0$

$\Rightarrow 2\sin^2\theta-4\sin\theta+\sin\theta=0$

$\Rightarrow 2\sin\theta(\sin\theta-2)+1(\sin\theta-2)=0$

$\Rightarrow (\sin\theta-2)(2\sin\theta+1)=0$

$\Rightarrow \sin\theta=\frac{-1}{2}$ $\quad [\because (\sin\theta-2)\neq 0]$

$\Rightarrow \theta=2\pi-\frac{\pi}{6},-\pi+\frac{\pi}{6},-\frac{\pi}{6},\pi+\frac{\pi}{6}$ $\quad [\because \theta\in[-2\pi,2\pi]]$

Now, sum of all solutions

$$=2\pi-\frac{\pi}{6}-\pi+\frac{\pi}{6}-\frac{\pi}{6}+\pi+\frac{\pi}{6}=2\pi$$

24. (B)

Given equation is $1+\sin^4 x=\cos^2(3x)$

Since, range of $(1+\sin^4 x)=[1,2]$

And range of $\cos^2(3x)=[0,1]$

So, the given equation holds if

$1+\sin^4 x=1=\cos^2(3x)$

$\Rightarrow \sin^4 x=0 \text{ and } \cos^2 3x=1$

Since, $x\in\left[-\frac{5\pi}{2},\frac{5\pi}{2}\right]$

$\therefore x=-2\pi,-\pi,0,\pi,2\pi$

Thus, there are five different values of x is possible

25. (D)

The given trigonometric equation is

$\cos 2x+\alpha\sin x=2\alpha-7$

$\Rightarrow 1-2\sin^2 x+\alpha\sin x=2\alpha-7$

$[\because \cos 2x=1-2\sin^2 x]$

$\Rightarrow 2\sin^2 x-\alpha\sin x+2\alpha-8=0$

$\Rightarrow 2(\sin^2 x-4)-\alpha(\sin x-2)=0$

$\Rightarrow 2(\sin^2 x-2)(\sin x+2)-\alpha(\sin x-2)=0$

$\Rightarrow (\sin x-2)(2\sin x+4-\alpha)=0$

$\Rightarrow 2\sin x+4-\alpha=0 \qquad [\because \sin x+2\neq 0]$

$\Rightarrow \sin=\frac{\alpha-4}{2} \qquad \text{.......(i)}$

Now, as we know $-1\le\sin x\le 1$

$\therefore \quad -1\le\frac{\alpha-4}{2}\le 1 \qquad$ [From Eq. (i)]

$\Rightarrow -2\le\alpha-4\le 2$

$\Rightarrow 2\le\alpha\le 6\Rightarrow\alpha\in[2,6]$

26. (A)

We have, $\sin x-\sin 2x+\sin 3x=0$

$\Rightarrow (\sin x+\sin 3x)-\sin 2x=0$

$\Rightarrow 2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)-\sin 2x=0$

$\left[\because \sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$

$\Rightarrow 2\sin 2x\cos x-\sin 2x=0 \quad [\because \cos(-\theta)=\cos\theta]$

$\Rightarrow \sin 2x(2\cos x-1)=0$

$\Rightarrow \sin 2x=0 \text{ or } 2\cos x-1=0$

$\Rightarrow 2x=0,\pi,\text{......or} \cos x=\frac{1}{2}$

$\Rightarrow x = 0, \frac{\pi}{2} ..... \text{or } x = \frac{\pi}{3}$

In the interval $\left[0, \frac{\pi}{2}\right)$ only two values satisfy, namely $x = 0$ and $x = \frac{\pi}{3}$

27. (C)

Given, $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow \left(1 - \cos^2 2\theta\right) + \cos^4 2\theta = \frac{3}{4}$

$\left(\because \sin^2 x = 1 - \cos^2 x\right)$

$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0$

$\Rightarrow \left(2\cos^2 2\theta - 1\right)^2 = 0$

$\Rightarrow 2\cos^2 2\theta - 1 = 0$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} \Rightarrow \cos 2\theta = \pm\frac{1}{\sqrt{2}}$

If $\theta \in \left(0, \frac{\pi}{2}\right)$, then $2\theta \in (0, \pi)$

$\therefore \cos 2\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow 2\theta = \frac{\pi}{4}, \frac{3\pi}{4},$

$$\left[\begin{aligned} \because \cos\left(\frac{3\pi}{4}\right) &= \cos\left(\pi - \frac{\pi}{4}\right) \\ &= -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}} \end{aligned}\right]$$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8}$

Sum of value of $\theta = \frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}$

28. (D)

Given expression $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$

$= 3\cos\theta + 5\left(\sin\theta\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos\theta\right)$

$= 3\cos\theta + 5\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right)$

$= 3\cos\theta - \frac{5}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$

$= \frac{1}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$

$\because$ The maximum value of $a\cos\theta + b\sin\theta$ is $\sqrt{a^2 + b^2}$

So, maximum value of $\frac{1}{2}\cos\theta+\frac{5\sqrt{3}}{2}\sin\theta$ is

$$=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{5\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{75}{4}}$$

$$=\sqrt{\frac{76}{4}}=\sqrt{19}$$

29. (D)

Given, $5\left(\tan^2 x-\cos^2 x\right)=2\cos 2x+9$

$$\Rightarrow 5\left(\frac{1-\cos 2x}{1+\cos 2x}-\frac{1+\cos 2x}{2}\right)=2\cos 2x+9$$

Put $\cos 2x=y$, we have

$$5\left(\frac{1-y}{1+y}-\frac{1+y}{2}\right)=2y+9$$

$$\Rightarrow 5\left(2-2y-1-y^2-2y\right)=2(1+y)(2y+9)$$

$$\Rightarrow 5\left(1-4y-y^2\right)=2\left(2y+9+2y^2+9y\right)$$

$$\Rightarrow 5-20y-5y^2=22y+18+4y^2$$

$$\Rightarrow 9y^2+42y+13=0$$

$$\Rightarrow 9y^2+3y+39y+13=0$$

$$\Rightarrow 3y(3y+1)+13(3y+1)=0$$

$$\Rightarrow (3y+1)(3y+13)=0$$

$$\Rightarrow y=-\frac{1}{3},-\frac{13}{3}$$

$$\therefore \cos 2x=-\frac{1}{3},-\frac{13}{3}$$

$$\Rightarrow \cos 2x=-\frac{1}{3} \qquad \left[\because \cos 2x\neq-\frac{13}{3}\right]$$

Now, $\cos 4x=2\cos^2 2x-1=2\left(-\frac{1}{3}\right)^2-1$

$$=\frac{2}{9}-1=-\frac{7}{9}$$

30. (C)

Given equation is

$$\cos x+\cos 2x+\cos 3x+\cos 4x=0$$

$$\Rightarrow (\cos x+\cos 3x)+(\cos 2x+\cos 4x)=0$$

$$\Rightarrow 2\cos 2x\cos x+2\cos 3x\cos x=0$$

$$\Rightarrow 2\cos x\left(\cos 2x+\cos 3x\right)=0$$

$$\Rightarrow 2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right)=0$$

$$\Rightarrow \cos x.\cos\frac{5x}{2}.\cos\frac{x}{2}=0$$

$$\Rightarrow \cos x = 0 \text{ or } \cos\frac{5x}{2} = 0 \text{ or } \cos\frac{x}{2} = 0$$

Now, $\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$[\because 0 \le x < 2\pi]$

$$\cos\frac{5\pi}{2} = 0 \Rightarrow \frac{5x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2} \ldots\ldots,$$

$$\Rightarrow x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5} \qquad [\because 0 \le x < 2\pi]$$

And $\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots$

$$\Rightarrow x = \pi \qquad [\because 0 \le x < 2\pi]$$

Hence, $x = \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

31. (D)

$$A = \sin^2 x + \cos^4 x$$

$$\Rightarrow A = 1 - \cos^2 x + \cos^4 x$$

$$= \cos^4 x - \cos^2 x + \frac{1}{4} + \frac{3}{4}$$

$$= \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4} \qquad \ldots\ldots(i)$$

Where, $0 \le \left(\cos^2 x - \frac{1}{2}\right)^2 \le \frac{1}{4}$ .....(ii)

$$\therefore \frac{3}{4} \le A \le 1$$

32. (A)

$$\sin\theta + \sin 4\theta + \sin 7\theta = 0$$

$$\Rightarrow \sin 4\theta + (\sin\theta + \sin 7\theta) = 0$$

$$\Rightarrow \sin 4\theta + 2\sin 4\theta.\cos 3\theta = 0$$

$$\Rightarrow \sin 4\theta\{1 + 2\cos 3\theta\} = 0$$

$$\Rightarrow \sin 4\theta = 0, \cos 3\theta = -\frac{1}{2}$$

As, $0 < \theta < \pi$

$\therefore 0 < 4\theta < 4\pi$

$\therefore 4\theta = \pi, 2\pi, 3\pi$

$$\cos 3\theta = -\frac{1}{2}$$

$0 < 3\theta < 3\pi$

$$\Rightarrow 3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$$

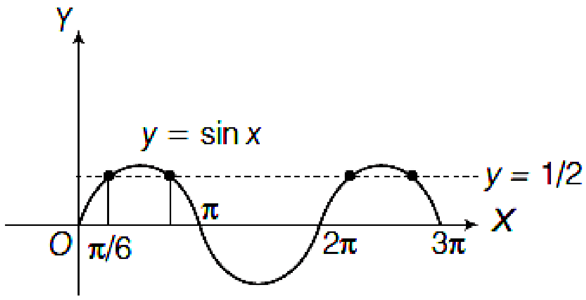
$$\Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$$

33. (D)

Given equation is $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x + 3) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ $\quad\quad [\because \sin x \neq -3]$



It is clear from figure that the curve intersect the line at four points in the given interval.
Hence, number of solution are 4.

34. (B)

Given, $\cos x + \sin x = \frac{1}{2}$

$\therefore \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1}{2}$

Let $\tan\frac{x}{2} = t$

$\Rightarrow \frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} = \frac{1}{2}$

$\Rightarrow 2(1-t^2+2t) = 1+t^2$

$\Rightarrow 3t^2 - 4t - 1 = 0$

$\Rightarrow t = \frac{2 \pm \sqrt{7}}{3}$

As $0 < x < \pi \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{2}$

So, $\tan\frac{x}{2}$ is positive.

$\therefore t = \tan\frac{x}{2} = \frac{2+\sqrt{7}}{3}$

Now, $\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} = \frac{2t}{1-t^2}$

$\Rightarrow \tan x = \frac{2\left(\frac{2+\sqrt{7}}{3}\right)}{1-\left(\frac{2+\sqrt{7}}{3}\right)^2}$

$\Rightarrow \tan x = \frac{-3(2+\sqrt{7})}{1+2\sqrt{7}} \times \frac{1-2\sqrt{7}}{1-2\sqrt{7}}$

$\Rightarrow \tan x = -\left(\frac{4+\sqrt{7}}{3}\right)$

35. (D)

Given that, $f(x) = \sin x - \sqrt{3}\cos x + 1$

$\because -2 \le \sin x - \sqrt{3}\cos x \le 2$

$\left[\because \sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}\right]$

$\Rightarrow -1 \le \sin x - \sqrt{3}\cos x + 1 \le 3$

$\therefore$ Range of $f(x) = [-1, 3]$

36. (B)

Since, α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$

$\therefore\ 25\cos^2\theta + 5\cos\theta - 12 = 0$

$\Rightarrow (5\cos\alpha - 3)(5\cos\alpha + 4) = 0$

$\Rightarrow \cos\alpha = -\frac{4}{5}$ and $\frac{3}{5}$

But $\frac{\pi}{2} < \alpha < \pi$ i.e. in second quadrant

$\therefore \cos\alpha - \frac{4}{5}$

$\Rightarrow \sin\alpha = \frac{3}{5}$

Now, $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2 \times \frac{3}{5} \times \left(-\frac{4}{5}\right) = -\frac{24}{25}$

37. (C)

Since, $|c| > \sqrt{a^2+b^2}$

$\Rightarrow c < -\sqrt{a^2+b^2}$

And $c > \sqrt{a^2+b^2}$

But $-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$ ….(i)

And $a\sin x + b\cos x = c$ …..(ii)

From equation (i) and (ii), we see that no solution exists.

EXERCISE - 2 [A]

1. (AD)

cos x = tan x

$\Rightarrow \cos^{-2} x = \sin x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\sin x = \frac{-1 \pm \sqrt{5}}{2}$

$$= \frac{\sqrt{5}-1}{2} \simeq 0.62$$

So, $x \in \left(30^0, 45^0\right)$ or $\left(135^0, 150^0\right)$

2. (AB)

$\sin^3 A + \cos^2 B = 2$

$\sin^2 A \le 1$ & $\cos^2 B \le 1$

So, $\sin A = \pm 1$, $\cos B = \pm 1$

$A = (2n+1)\frac{\pi}{2}$, $B = n\pi$

3. (AB)

$2\sin^2 x + 5\sin x \cos x + \cos^2 x + 1 = 0$

Multiple & dividing by $\cos^2 x$

$\Rightarrow 2\tan^2 x + 5\tan x + 1 + 1 + \tan^2 x = 0$

$\Rightarrow 3\tan^2 x + 5\tan x + 2 = 0$

$\Rightarrow (3\tan x + 2)(\tan x + 1) = 0$

tan = –1 or $-2/3$

4. (BD)

sin x. cos 3x – cos x . sin 3x > 0

$\Rightarrow \sin(-2x) > 0$

$\Rightarrow \sin 2x < 0$

$2x \in (\pi, 2\pi) \cup (3\pi, 4\pi)$

$x \in \left(\frac{\pi}{2}, \pi\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

5. (A, C, D)

$|x| + |y| = 10$

(A) sin (x + y) = 0

$\Rightarrow x \in y = n\pi$; n = 0, 1, 2, 3, –1, –2, –3

7 line 2 points = 14

(B) sin2x = sin2y

$\Rightarrow 2\sin(x-y)\cos(x+y) = 0$

$\Rightarrow x - y = n\pi$ or $x + y = (2m+1)\pi/2$

n = 0, 1, 2, 3, –1, –2, –3 14 points

m = 0, $\pm 1, \pm 2$ m = 3 12 points

= 26

(C) sin2x . sin2y = 0

$2x = n\pi, 2y = n\pi$

$n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6$ 26 points

$m = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6$ 26 points

Hence 52

(D) $|\sin x| = |\sin y|$

$\Rightarrow x = n\pi \pm y_2$

$\Rightarrow x \pm y = n\pi$

$n = 0, \pm 1, \pm 2, \pm 3$ 28 points

6. (AB)

$\Rightarrow \cos(\theta-\alpha)=a$ $\Rightarrow \sin(\theta-\alpha)=\sqrt{1-a^2}$

$\Rightarrow \sin(\theta-\beta)=b$ $\Rightarrow \sin(\theta-\beta)=\sqrt{1-b^2}$

Now, $\sin(\alpha-\beta)=\sin((\theta-\beta)-(\theta-\alpha))$

$=\sin(\theta-\beta)\cos(\theta-\alpha)-\cos(\theta-\beta)\sin(\theta-\alpha)$

$=ab-\sqrt{1-b^2}\sqrt{1-\alpha^2}$

$=ab-\sqrt{1-a^2-b^2+a^2b^2}$ …(A)

And $\cos(\alpha-\beta)=\cos((\theta-\beta)-(\theta-\alpha))$

$=\cos(\theta-\beta)\cos(\theta-\alpha)-\sin(\theta-\beta)\sin(\theta-\alpha)$

$=\sqrt{1-b^2}\times a+b\times\sqrt{1-a^2}$

$=a\sqrt{1-b^2}+b\sqrt{1-a^2}$ … (B)

7. (ABCD)

$\Rightarrow \sin 5\theta = a\sin^5\theta + b\sin^3\theta + c\sin\theta + d$

$\Rightarrow \sin 5\theta = \sin(3\theta+2\theta)$

$\Rightarrow \sin 3\theta\cos 2\theta + \cos 3\theta \sin 2\theta$

$=(3\sin\theta+4\sin^3\theta)(\cos^2\theta-\sin^2\theta)+(4\cos^3\theta-3\cos\theta)(2\sin\theta\cos\theta)$

$=(3\sin\theta+4\sin^3\theta)(\cos^2\theta-\sin^2\theta)+(4\cos^3\theta-3\cos\theta)(2\sin\theta\cos\theta)$

$=(3\sin\theta+4\sin^3\theta)(1-2\sin^2\theta)+2(4\cos^2\theta-3)\sin\theta(1-\sin^2\theta)$

Let $\sin\theta=t$

Then, $\sin 5\theta = 16t^5-20t^3+5t$

Therefore a = 16

$\Rightarrow$b = –20

$\Rightarrow$c = 5

$\Rightarrow$d = 0

$\Rightarrow$ a + b + c + d = 10 – 20 + 5 + 0 = 1 … (A)

$\Rightarrow$ a + b + c = 16 – 20 + 5 =1 … (B)

$\Rightarrow$ 5a + 4b = 80 – 80 = 0 … (C)

$\Rightarrow$b + 4c = –20 + 20 = 0 … (D)

8. (BC)

$\Rightarrow x+y=\frac{\pi}{4}$

$\Rightarrow$ tan x + tan y = 1

$\Rightarrow$ tan x = 1 – tan y

$\Rightarrow \tan\left(\frac{\pi}{4}-y\right)=1-\tan y$

$\Rightarrow \frac{1-\tan y}{1+\tan y}=1-\tan y$

$\Rightarrow (1-\tan y)\left(\frac{1}{1+\tan y}-1\right)=0$

$\Rightarrow (1-\tan y)(1-\tan y)=0$

$\Rightarrow$ 1 – tan y = 0 or tan y = 0

$\Rightarrow \tan y = 1 \qquad \tan x = 1$

$\Rightarrow y = n\pi + \frac{\pi}{4} \qquad x = n\pi + \frac{\pi}{4}$

But $x + y = \frac{\pi}{4} \qquad x + y = \frac{\pi}{4}$

$\Rightarrow \therefore x = -n\pi \qquad \therefore y = -n\pi$

(C) (B)

9. (BD)

$$\Rightarrow \frac{4\sin^2 x\cos^2 x + 4\sin^4 x - 4\sin^2 x\cos^2 x}{4 - 4\sin^2 x\cos^2 x - 4\sin^2 x} = \frac{1}{9}$$

$$\Rightarrow \frac{4\sin^4 x}{4\left(1-\sin^2 x\right) - \sin^2 x\cos^2 x} = \frac{1}{9}$$

$$\Rightarrow \frac{\sin^4 x}{\cos^2 x - \sin^2 x\cos^2 x} = \frac{1}{9}$$

$$\Rightarrow \frac{\sin^4 x}{\cos^2 x\left(1-\sin^2 x\right)} = \frac{1}{9}$$

$$\Rightarrow \frac{\sin^4 x}{\cos^4} = \frac{1}{9}$$

$$\Rightarrow \tan^4 x = \frac{1}{9}$$

$$\Rightarrow \tan^2 x = \pm\frac{1}{3}$$

$\Rightarrow \tan^2 x = \frac{1}{3}$ or $\tan^2 x = -\frac{1}{3}$

$\Rightarrow \tan x = \pm\frac{1}{\sqrt{3}}$ or not possible

$$\Rightarrow \tan x = \pm\frac{1}{\sqrt{3}}$$

$\Rightarrow \tan x = \frac{1}{\sqrt{3}}$ or $\tan x = -\frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

(B) (D)

10. (AB)

$\Rightarrow 4\sin^4 x + \cos^4 x = 1$

$\Rightarrow 4\sin^4 x = 1 - \cos^4 x$

$\Rightarrow 4\sin^4 x = \left(1-\cos^2 x\right)\left(1+\cos^2 x\right)$

$\Rightarrow 4\sin^4 x - \sin^2 x\left(1+\cos^2 x\right) = 0$

$\Rightarrow \sin^2 x - \left(4\sin^2 x - 1 - \cos^2 x\right) = 0$

$\Rightarrow \sin^2 x\left(4\sin^2 x - 1 - \cos^2 x\right) = 0$

$\Rightarrow \sin^2 x\left(4\sin^2 x - 1 - 1 + \sin^2 x\right) = 0$

$\Rightarrow \sin^2 x\left(5\sin^2 x - 2\right) = 0$

$\Rightarrow \sin^2 x = 0$ or $5\sin^2 x = 2$

$\Rightarrow x = n\pi$ $\qquad \sin^2 x = \frac{2}{5}$

(A) $\qquad \sin x = \pm\sqrt{\frac{2}{5}}$

$x = n\pi \pm \sin^{-1}\sqrt{\frac{2}{5}}$

(B)

11. (AC)

$\Rightarrow \tan^2\theta + \cos 2\theta = 1$

$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} + \cos^2\theta - \sin^2\theta = 1$

$\Rightarrow \sin^2\theta + \cos^4\theta - \sin^2\theta\cos^2\theta = \cos^2\theta$

$\Rightarrow \sin^2\theta + \left(1 - \cos^2\theta\right) = \cos^2\theta - \cos^4\theta$

$\Rightarrow \sin^4\theta = \cos^2\theta\left(1 - \cos^2\theta\right)$

$\Rightarrow \sin^4\theta - \sin^2\theta\cos^2\theta = 0$

$\Rightarrow \sin^2\theta\left(\sin^2\theta - \cos^2\theta\right) = 0$

$\Rightarrow \sin^2\theta = 0$ $\qquad \sin^2\theta - \cos^2\theta = 0$

$\Rightarrow \theta = n\pi$ $\qquad \tan^2\theta = 0$

$\theta = n\pi \pm \frac{\pi}{4}$

12. (BD)

$\Rightarrow \sin\theta + \sqrt{3}\cos\theta = 6x - x^2 - 11$ $\qquad \theta \in [0, 4\pi]$

$\Rightarrow \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta = \frac{6x - x^2 - 11}{2}$

$\Rightarrow \sin\theta\cos\frac{\pi}{3} + \cos\theta\sin\frac{\pi}{3} = \frac{6x - x^2 - 11}{2}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{3}\right) = \frac{6x - x^2 - 11}{2}$

L.H.S. $= \sin\left(\theta + \frac{\pi}{3}\right)$

$\Rightarrow -1 \le \sin\left(\theta + \frac{\pi}{3}\right) \le 1$

R.H.S. $= \frac{6x - x^2 - 11}{2}$

$\Rightarrow 6x - x^2 - 11$ max value will ne $\frac{-D}{4a}$

$\Rightarrow \frac{-D}{4a} = \frac{-(36 - 44)}{4 \times (-1)} = -2$ $\qquad$ at $\frac{-b}{2a} = \frac{-6}{-2}$

So max value of $\frac{6x - x^2 - 11}{2}$ is –1

Therefore, L.H.S. and R.H.S. are equal at –1 for x = 3

$\Rightarrow \sin\left(\theta + \frac{\pi}{3}\right) = -1$

$\Rightarrow \theta = (2n+1)\pi - \frac{\pi}{3}$

$\Rightarrow \theta = \pi - \frac{\pi}{3}$ and $\theta = 3\pi - \frac{\pi}{3}$

$\Rightarrow \theta = \frac{2\pi}{3}$ and $\Rightarrow \frac{8\pi}{3}$

For x = 3

$\Rightarrow \theta = \frac{2\pi}{3}$ and $\frac{8\pi}{3}$

13. (ABD)

$\Rightarrow \cos\left(x + \frac{\pi}{3}\right) + \cos x = a$

$\Rightarrow \cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3} + \cos x = a$

$\Rightarrow \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x + \cos x = a$

$\Rightarrow \frac{3}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = a$

$\Rightarrow \therefore |a| \le \sqrt{\left(\frac{3}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}$

Integral solution of $|a| \le \sqrt{3}$ is 0, –1, 1 (A)

Sum of integral values of a is = 0 + 1 + (–1) = 0 (B)

For a = 1

$\Rightarrow \frac{3}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = 1$

$\Rightarrow \sqrt{3}\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right) = 1$

$\Rightarrow \cos\left(\frac{\pi}{6} + x\right) = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{\pi}{6} + x = 2n\pi \pm \cos^{-1}\frac{1}{\sqrt{3}}$

$\Rightarrow x = 2nm \pm \cos^{-1}\frac{1}{\sqrt{3}} + \frac{\pi}{6}$

No solution in $[0, 2\pi]$ (D)

14. (AC)

$\Rightarrow x + y = \frac{2\pi}{3}$ $\frac{\sin x}{\sin y} = 2$

$\Rightarrow y = \frac{2\pi}{3} - x$ sin x = 2 sin y

$\Rightarrow \therefore \sin x = 2\sin\left(\frac{2\pi}{3} - x\right)$

$= 2\left(\sin\frac{2\pi}{3}\cos x - \cos\frac{2\pi}{3}\sin x\right)$

$= 2\left(\cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x\right)$

$= 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right)$

$= \sqrt{3}\cos x + \sin x$

$\Rightarrow \sqrt{3}\cos x = 0$

$\Rightarrow$ cos x = 0

$\Rightarrow x = (2n+1)\frac{\pi}{2}$

$\therefore$ 2 solutions in $[0, 2\pi]$

And 4 solutions in $[0, 4\pi]$

Now, $x + y = \frac{2\pi}{3}$

$\Rightarrow y = \frac{2\pi}{3} - x = \frac{2\pi}{3} - (2n+1)\frac{\pi}{2}$

$= \frac{2\pi}{3} - n\pi - \frac{\pi}{2}$

$= \frac{\pi}{3} - n\pi$

$= m\pi + \frac{\pi}{3}$

$\therefore$ 2 solutions in $[0, 2\pi]$

And 4 solutions in $[0, 4\pi]$

15. (BD)

$\Rightarrow |\cos x| = \cos x - 2\sin x$

Case – I

$\Rightarrow \cos x \geq 0$ $\Rightarrow$ cos x = cos x – 2 sin x

$\Rightarrow$ 2 sin x = 0 $\Rightarrow$ sin x = 0

$\Rightarrow x = n\pi$

For $n \rightarrow$ even for $n \rightarrow$ odd

$\Rightarrow$ cos x = 1 cos x = –1

But cos $\geq$ x

$\Rightarrow \therefore x = 2n\pi$ (B)

Case – II

$\Rightarrow$cos x < 0 $\Rightarrow$ – cos x = cos x – 2 sin x

$\Rightarrow$–2cos x = –2 sin x $\Rightarrow$ cos x = sin x

$\Rightarrow$ tan x = 1 $\Rightarrow x = n\pi + \frac{\pi}{4}$

$\Rightarrow$ n $\rightarrow$ even n $\rightarrow$ odd

$\Rightarrow \cos x = \frac{1}{\sqrt{2}}$ $\cos x = -\frac{1}{\sqrt{2}}$

As coc x < 0

$\Rightarrow \therefore x = (2n+1)\pi + \frac{\pi}{4}$ (D)

16. (AC)

$\Rightarrow \cos\left(\pi\sqrt{x-4}\right)\cos\left(\pi\sqrt{x}\right)=1$

Case – 1

$\Rightarrow \cos\left(\pi\sqrt{x-4}\right)=\cos\left(\pi\sqrt{x}\right)=1$

$\Rightarrow \cos\pi\sqrt{x-4}=1$ and $\cos\pi\sqrt{x}=1$

$\Rightarrow \pi\sqrt{x-r}=2n\pi$ $\quad \pi\sqrt{x}=2m\pi$

$\Rightarrow \sqrt{x-4}=2n$ $\quad \sqrt{x}=2m$

$\Rightarrow \therefore x=4n^2+4=4m^2$

$\Rightarrow n^2-m^2=1$

$\Rightarrow n=\pm1$ and m = 0 as $x=\pm1$ and n = 0

$\Rightarrow \therefore$ x = 4

Case – 2

$\Rightarrow \cos\pi\sqrt{x-4}=\cos\pi\sqrt{x}=-1$

$\Rightarrow \cos\pi\sqrt{x-4}=-1$ and $\cos\pi\sqrt{x}=-1$

$\Rightarrow \pi\sqrt{x-4}\left(2n+1\right)\pi$ $\quad \pi\sqrt{x}=\left(2m+1\right)\pi$

$\Rightarrow \sqrt{x-4}=\left(2n+1\right)$ $\quad \sqrt{x}=\left(2m+1\right)\pi$

$\Rightarrow x=\left(2n+1\right)^2+4$ $\quad x = (2m + 1)^2$

$\Rightarrow \therefore (2n + 1)^2 + 4 = (2m + 1)^2$

$\Rightarrow (2m + 1)^2 – (2n + 1)^2 = 4$

$\Rightarrow (2m +1 + 2n + 1) (2m + 1 – 2n – 1)= 4$

$\Rightarrow 2(m + n + 1) 2 (m – n) = 4$

$\Rightarrow (m + n + 1) (m – n) = 1$

$\Rightarrow m + n + 1 = 1$ $\quad m + n + 1 = –1$

$m – n = 1$ $\quad m – n = –1$

Hence, 1 solution

17. (BD)

y = 2 sin x $\quad y\in\left[-2,2\right]$

$y = 5x^2 + 2x + 3,$ $\quad y\in\left[-\frac{\left(40-60\right)}{20},\infty\right)$

$y\in\left[\frac{56}{20},\infty\right)$

Hence no solution

18. (BC)

$x^3 + x^2 + 4x + 2 \sin x = 0$

x = 0 is a solution

sin x < 0, $\quad x>\pi$

When for, $x>\pi$, $x^3 + x^2 + 4x > 1$

Hence, no solution in $x\in\left(\pi,2\pi\right)$

Hence, x =0 is the only solution

19. (AD)

$\tan^2\left(x+y\right)+\cot^2\left(x+y\right)\ge 2$

$1-2x+x^2=2-\left(1+x\right)^2\le 2$

Equations hold at x = –1 and $x+y=\frac{n\pi}{2}+\frac{\pi}{4}$

$$y=\frac{n\pi}{2}+\frac{n\pi}{4}+1$$

20. (ACD)

$|\cos x|^{\sin^2 x-\frac{3}{2}\sin x+\frac{1}{2}}=1$

$\Rightarrow$ either $|\cos x|=1 \Rightarrow x=n\pi$

or, $\sin^2 x-\frac{3}{2}\sin x+\frac{1}{2}=0$

$\Rightarrow$ sin x = 1, $\sin x=\frac{1}{2}$

But at sin x = 1, cos x = 0 (not possible)

So, $\sin x = 1/2$

$x=n\pi+(-1)^n \pi/6$

21. (ACD)

$\cos^2(\pi x)-\sin^2(xy)=1/2$

$\Rightarrow 1+\cos\pi x-1+\cos 2\pi y=1$

$\Rightarrow 2\cos(\pi(x+y)).\cos(\pi(x-y))=1$

$\Rightarrow \cos\pi(x+4)=1$

$\Rightarrow \pi(x+y)=2n\pi$

$\Rightarrow$x + y = 2n

$x-y=1/3$

$x=n+\frac{1}{6}, y=n-\frac{1}{6}$

$\left(\frac{7}{6},\frac{5}{6}\right),\left(\frac{-5}{6},\frac{-7}{6}\right)$

$\left(\frac{13}{6},\frac{11}{6}\right)$

22. (ABC)

$\sqrt{\cos 2x}+\sqrt{1+\sin 2x}=2\sqrt{\cos x+\sin x}$

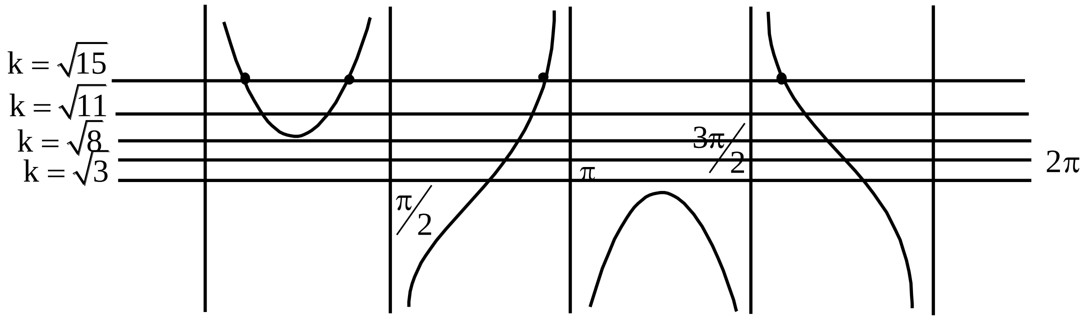
$\Rightarrow \sqrt{\cos^2 x-\sin^2 x}+\sqrt{(\cos x+\sin x)^2}-2\sqrt{\cos x+\sin x}=0$

$\Rightarrow \cos x+\sin x=0$ $\quad\quad$ $\sqrt{\cos x-\sin x}+\sqrt{\cos x+\sin x}=2$

$\Rightarrow$ tan = –1 $\quad\quad$ $\Rightarrow$ cos x = 1

$\Rightarrow x=n\pi-\frac{\pi}{4}; n\in I$ $\quad\quad$ $\Rightarrow x=2\pi; n\in I$

23. (BCD)



4 solution $k=\sqrt{15},\sqrt{11}$

3 solution $k=\sqrt{8}$

2 solution $k=\sqrt{3}$

24. (ABC)

2 (sin x + sin y) – 2 cos (x – y) = 3

$\Rightarrow 4\sin\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)-4\cos^2\left(\frac{x-y}{2}\right)+2=3$

$\Rightarrow 4\cos^2\left(\frac{x-y}{2}\right)-4\sin\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)+1=0$

$\Rightarrow \because D=16\sin\left(\frac{x+y}{2}\right)^2-16$

For $D\geq 0,\sin\left(\frac{x+y}{2}\right)=\pm 1$

For smallest positive x & y

$\sin\left(\frac{x+y}{2}\right)=1\Rightarrow \frac{x+y}{2}=\frac{\pi}{2}$

$\cos\left(\frac{x-y}{2}\right)=1/2$

$\frac{x-y}{2}=\frac{\pi}{3}$ or $\frac{y-x}{2}=\frac{\pi}{3}$

$\left(x=\frac{5\pi}{6},y=\frac{\pi}{6}\right)$ or $\left(x=\frac{\pi}{6},y=\frac{5\pi}{6}\right)$

2 solutions

PASSAGE – I

25. (B)

Roots are x = 1, x = cos x, x = sin x

$x_1^L+x_L^2+x_3^2=2$

26. (C)

For two roots equal

Either cos = 1, or $\sin\theta=1$ or $\sin\theta=\cos\theta$

So, $\theta=0,2\pi,\theta=\frac{\pi}{4},\theta=\frac{\pi}{2},\theta=\frac{5\pi}{4}$

5 values

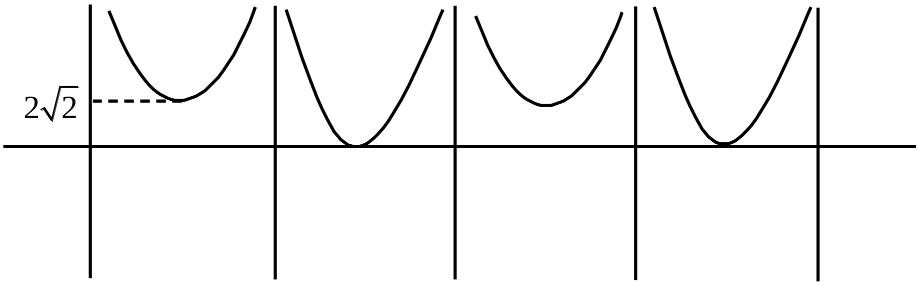
27. (A)

$\left|\max\left(\sin\theta-1\right)\right|=2$

$\left|\max\left(\cos\theta - 1\right)\right| = 2$

$\left|\max\left(\sin\theta - \cos\theta\right)\right| = \sqrt{2}$

PASSAGE – 2



28. (A)
For 8 solution, $a > 2\sqrt{2}$

29. (C)
For 6 solution, $a = 2\sqrt{2}$

30. (D)
For 2 solution, a = 0

Passage – 3

31. (A)
$\sin x.\cos 2y = \left(a^2 - 1\right)^2 + 1$ …(1)
$\cos x.\sin 2y = a + 1$ … (2)
For any value of x & y
$-1 \le \sin x.\cos 2y \le 1$
For equation (1) $a^2 = 1$ is the only value
$\Rightarrow a = \pm 1$
Out of these 2 only a = –1 satisfy equation (2)
So only one value of a

32. (B)
If a = –1
Sin x . cos 2y = 1
$x = \pi/2, y = 0, \pi, 2\pi$
$x = 3\pi/2, y = \pi/2, 3\pi/2$
cos x . sin 2y = 0
(1) $x = \pi/2, 3x/2, y \in R$
(2) $y = 0, \pi/2, \pi, 3\pi/2, x \in R$
$\therefore$ total 2 solutions

33. (D)
From above y has 5 solutions for a = –1
$y \in \left\{0, \pi/2, \pi, 3\pi/2, 2\pi\right\}$

PASSAGE – 4

34. (A)

$x(\cos y+\sin y)^3 = 27$

$x(\cos y-\sin y)^3 = 1$

Taking power 2/3 on both the sides and adding

$x^{2/3}(2)=9+1$

$x^{2/3}=5$

$x=\pm 5\sqrt{5}$

35. (D)

Dividing (1)/(2) form above

$\frac{\cos y+\sin y}{\cos y-\sin y}=3$

cos y + sin y = 3 cos y – 3 sin y

4 sin y = 2 cos y

$\tan y = 1/2$

Total 6 solutions in $(0, 6\pi]$

36. (B)

As $\tan y = 1/2$

$\cos = \frac{2}{\sqrt{5}}$

$\therefore \sin^2 y + 2\cos^2 y = 1+\cos^2 y$

$=1+\frac{4}{5}$

$= 9/5$

PASSAGE – 5

37. (B)

ABCD is a quadrilateral

$\sin^2 A + \sin^2 B + \sin^2 C + \sin^2 D = (x+1)^2 + 4$

for equality to hold true.

$A = B = C = D = 90^0$ & $(x+1)^2 = 0 \Rightarrow x = -1$

Then ABCD must be a rectangle

38. (C)

$\tan\theta = x = -1$ (from above question)

$\theta = n\pi - z/4$

39. (D)

$\tan^4 x - 10\tan^2 x + 9 = 0$

$(\tan^2 x - 9)(\tan^2 x - 1) = 0$

$\tan x = \pm 3, \pm 1$

Total 8 solutions in $[0, 2\pi]$

40. (C)

$D > 0$

$(-10)^2 - 4\times1\times a > 0$

$a = 25$

$a \in (-\infty, 25)$

Also roots should be positive

$\therefore$ product of roots > 0, $\frac{a}{1} > 0$

$\therefore a \in (0, 25)$

PASSAGE – 7

41. (C)

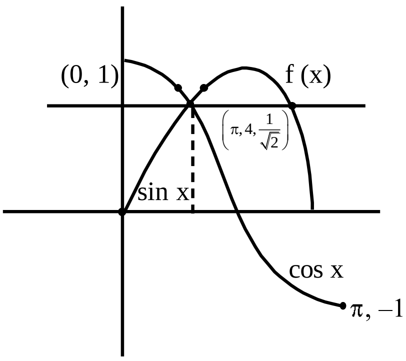
$f(x) = \max\{\sin x, \cos x\} = 4/5$

$\frac{4}{5} = 0.8$

$\frac{1}{\sqrt{2}} = 0.75$

$\because \frac{4}{5} > \frac{1}{\sqrt{2}}$, $y = \frac{4}{5}$ cuts f (x) at 3 points

$\therefore$ 3 solutions



42. (B)

Reasoning Type

43. (A)

Statement $(\sin x + \cos x)^{1+\sin^2 x}$

$= (\sin x + \cos x)^{(\sin x + \cos x)^2}$

$\because$ max. value of $\sin x + \cos x = \sqrt{2}$

Occurs at $x = \pi/4$

$\left(\sqrt{2}\right)^{\left(\sqrt{2}\right)^2} = 2$

44. (A)

45. (D)

Statement 1 is false

Statement 2 is true

46. (A)

$\sqrt{1-\sin 2x} = \sin x$

$|\cos x - \sin x| = \sin x$

When $x \in \left[0, \pi/4\right]$ cos x > sin x

$\therefore$ cos x – sin x= sin x

$\Rightarrow \tan x = 1/2$

$\Rightarrow$ one solution

Statement 2 correct explanation

47. (D)

$$\frac{\tan 4x - \tan 2x}{1 + \tan 4x \tan 2x} = 1$$

$\Rightarrow$ tan (4x – 2x) = 1

tan 2x = 1

In this case tan 4x is always not designed

So no solution

Matrix Match

48. **A - R, B - S, C - P, D - Q**

$\cos^2_{2x} + \cos^2 x = 1$

$\cos^2_{2x} = \sin^2 x$

$\cos^2_{2x} + \left(\cos\left(\pi/2 - x\right)\right)^2$

$2x = n\pi \pm \left(\pi/2 - x\right)$

$3x = n\pi + \pi/2$

$x = \frac{n\pi}{3} + \pi/6$ $\quad\quad$ $x = x\pi - \pi/2$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Hence $x = \left\{n\pi \pm \frac{\pi}{6}\right\} \cup \left\{2n\pi \pm \frac{\pi}{2}\right\}$ $\quad$ (R)

(B) $\cos x + \sqrt{3} \sin x = \sqrt{3}$

$\Rightarrow \cos\left(x - \frac{\pi}{5}\right) = \cos \pi/6$

$\Rightarrow x - \frac{\pi}{3} = 2n\pi \pm \pi/6$

$\Rightarrow x = 2n\pi + \frac{\pi}{2}, 2n\pi + \pi/6; n \in I$ $\quad$ (S)

(C) $1 + \sqrt{3} \tan^2 x = \left(1 + \sqrt{3}\right) \tan x$

$\Rightarrow \sqrt{3} \tan^2 x - \left(1 + \sqrt{3}\right) \tan x + 1 = 0$

$\Rightarrow \left(\sqrt{3} \tan x - 1\right)\left(\tan x - 1\right) = 0$

$\tan x = \frac{1}{\sqrt{3}}, \tan x = 1$

$x = \left\{n\pi + \frac{\pi}{4}\right\}, \left\{n\pi + \frac{\pi}{6}\right\}; n \in z$ $\quad$ (P)

(D) tan 3x – tan2x – tan x = 0

$\Rightarrow$ tan 3x = tan2x + tan x

$\Rightarrow \frac{\tan 2x + \tan x}{1 - \tan 2x . \tan x} = \tan 2x$ of tan x

$\Rightarrow$ either tan 2x = – tan x

Or tan 2x . tan x= 0

$\Rightarrow x = n\pi$

Or $\quad 2x = n\pi - x$

$\Rightarrow x = \frac{n\pi}{3}$

Hence $n \in \left(\frac{n\pi}{3}\right)$ (Q)

49. **A - R, B - Q, C - R, D - S**

(A) – (R)

$\cos^7 x + \sin^2 x = 1$

$\Rightarrow \cos^7 x = \cos^2 x$

$\Rightarrow \cos^2 x (1 - \cos^5 x) = 0$

$\Rightarrow \cos x = 0, 1$

Total 3 solution in $(-\pi, \pi)$

(B) – (Q)

$\sqrt{3}\,\mathrm{cosec}\,20^0 - \sec 20^0$

$\Rightarrow \frac{\sqrt{3}}{\sin 20^0} - \frac{1}{\cos 20^0}$

$\Rightarrow 4 \times \frac{\frac{\sqrt{3}}{2}\cos 20^0 - \frac{1}{2}\sin 20^0}{2\sin 20^0 . \cos 20^0}$

$\Rightarrow 4\frac{\sin\left(60^0 - 20^0\right)}{\sin 40^0}$

$\Rightarrow 4$

(C) – (R)

$4\cos 36^0 - 4\cos 72^0 + 4\sin 18^0 \cos 36^0$

$= 4\cos 36^0 - 4\cos 72^0 + 2[\sin 54^0 - \sin 8^0]$

$= 6\cos 36^0 - 6\cos 72^0$

$= 6\left(\frac{\sqrt{5}+1}{4}\right) - 6\left(\frac{\sqrt{5}-1}{4}\right)$

$= 3$

(D) – (S)

cosec x = 1 + cot x

cosec x + cot x = 1 $\{\text{As } \mathrm{cosec}^2 x - \cot^2 x = 1\}$

2cosec x = 2, cosec x = 1

$\therefore$ 2 solution in $[-2\pi, 2\pi]$

50. **A - P, B - P, C – Q**

(A) – (P)

If $\cos\theta + \cos\phi = 2$

$\Rightarrow \cos\theta = 1$ & $\cos\phi = 1$

$\Rightarrow \sin\theta = 0$ & $\sin\phi = 0$

So. No value of θ & ϕ will statisfy. $\sin\theta + \sin\phi = 1/2$

So, no solution

(B) – (P)

$\sin^2\alpha + \sin\left(\pi/3 - \alpha\right)\sin\left(\pi/3 + \alpha\right) = \sec\alpha$

$\cancel{\sin^2}\alpha\left(\sin^2 \pi/3\right) - \cancel{\sin^2}\alpha = \sec\alpha$

$\frac{3}{4} = \sec\alpha$

No solution

(C) – (Q)

$\tan\theta = 3\tan\phi$

$\tan^2(\theta - \phi)$

$= \left[\frac{\tan\theta - \tan\phi}{1 + \tan\theta + \tan\phi}\right]^2$

$= \left[\frac{2\tan\phi}{1 + 0(\tan\phi)^2}\right]^2$

Let $y = \frac{2x}{1+3x^2}$, take $x = \tan\phi$

$y + 3x^2y = 2x$

$(3y)x^2 - 2x + y = 0$

As x is real $D \geq 0$

$(-2)^2 - 4 \times 3y.y \geq 0$

$4 - 4.3y^2 \geq 0$

$3y^2 \leq 1$

$y^2 \leq 1/3$

$\therefore \left[\frac{2\tan\phi}{1 + 3(\tan\phi)^2}\right]^2 \leq 1/3$

EXERCISE - 2 [B]

1. **(0)**

LHS $\leq 2 \Delta$ RHS ≥ 2

$\therefore$ Equality appears when LHS = RHS = 2

$\therefore$ for RHS = 2 x = $\pm$ 1

But @ x = $\pm$ 1 LHS $\neq 2$

$\Rightarrow$ simultaneously LHS & RHS can't be $\Rightarrow$ No solution

2. **(4)**

$\frac{\sin^2 x + \cos^2 x}{\sin x \ \cos x} = \frac{2}{\sin x}$

$\Rightarrow \cos x = \frac{1}{2}$

$\therefore$ In $[-2\pi, 2\pi]$ No. of solution = 4

3. **(0)**

Let $\sin x + \cos x = t$

$\Rightarrow \sin 2x = t^2 - 1$

$3t-2(t)\left(1-\frac{(t^2-1)}{2}\right)=8$

$t - 2 + t^3 - t = 8$

$t^3 = 8$

$\Rightarrow \sin x+\cos x=2$

No solution

4. **(2)**

$\sin^4 x+\cos^4 x=\sin x\cos x$

$\Rightarrow 1-2\sin^2 x\ \cos^2 x=\frac{\sin^2 x}{2}$

$\Rightarrow 1-\frac{\sin^2 2x}{2}=\frac{\sin 2x}{2}$

$\sin^2 2x+\sin 2x-2=0$

Sin 2x = - 2 or sin 2x = - 1

Discard in $[0,2\pi]$ possible @ 2 values of x

5. **(1)**

$1-\cos^2\theta+3\cos\theta=3$

$\cos^2\theta-3\cos\theta+2=0$

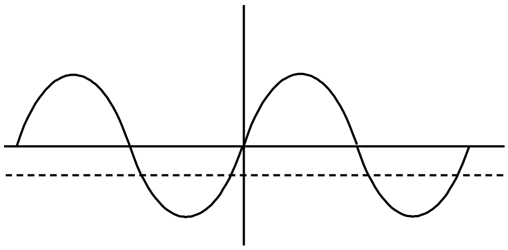
$\cos\theta=2,\qquad \cos\theta=1$

$\theta=0$ 1 solution

6. **(4)**

$\sin^2 x-\sin x-1=0$

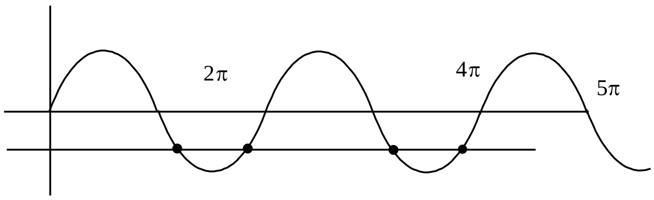
$\sin=\frac{1\pm\sqrt{5}}{2}$ discard $\frac{1+\sqrt{5}}{2}$



4 intersection point $\Rightarrow$ 4 soln.

7. **(4)**

$\sin\theta=1+\sqrt{2}$ or $1-\sqrt{2}$



Discard

$\sin\theta=1-\sqrt{2}$

Least n = 4

& $\text{Me}\alpha\,n=5$

Ans. 4

8. **(0)**

cos x + sin x = 2

$\Rightarrow \cos x = \sin x = 1$

$\Rightarrow \phi$

9. **(0)**

Me write equation

$2\sin\left(2e^{x}\right) = 2^{x} + 2^{-x}$

Now $\quad LHS \le 2$ & $RHS \le 2$

$\therefore$ in solution to exist

LHS = RHS – 2

$\therefore$ for RHS = 2 x = 0

But @ x = 0 LHS $\ne$ 2

$\therefore$ no soln.

10. **(6)**

cos x sin y = 1

$\Rightarrow$ either cos x = -1 & sin y = - 1

$x = \pi, 3\pi$ & $y = \frac{3\pi}{2}$

$\left\langle \pi, \frac{3}{2} \right\rangle, \left\langle 3\pi, \frac{3\pi}{2} \right\rangle$

Or cos x = 1 sin y = 1

$x = 0, 2\pi,\qquad y = \frac{\pi}{2}$ & $\frac{5\pi}{2}$

$\therefore$ Total ordered pair 6

11. **(6)**

$2\sin\theta = \left(r^{2} - 1\right)^{2} + 2$

Now LHS ≤ 2, RHS ≥ 2

$\therefore$ for soln. LHS = RHS = 2

$\therefore \quad r = \pm 1 \quad \& \quad \sin\theta = 1$

$\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

$\therefore$ ordered pair 6

12. **(0)**

$\sin x \cos x\left(\sin^{2} x + \sin x \cos x + \cos x\right) = 1$

$\Rightarrow \frac{\sin 2x}{2}\left(1 + \frac{\sin 2x}{2}\right) = 1$

Let sin 2x = y

$2y + y^{2} = 4$

$y^{2} + 2y - 4 = 0$

$\Rightarrow \sin 2x = \cancel{-1+\sqrt{5}}, \quad \cancel{-1-\sqrt{5}}$

discard discard

13. **(4)**

$\sin^{4} x - \sin x\left(1 - \sin^{2} x\right) + 2\sin^{2} x + \sin x = 0$

$\sin^4 x + \sin^3 x + 2\sin^2 x = 0$

$\Rightarrow \sin^2 x = 0 \quad \text{or} \quad \sin^2 x - \sin x 12 = 0$

discard

$x = 0, \pi, 2\pi, 3\pi$

14. **(5)**

$(1-\tan\theta)\left(1+\frac{2\tan\theta}{1+\tan^2\theta}\right) = 1 - 1\tan\theta$

$(1-\tan\theta)\frac{(1+\tan\theta)^2}{1+\tan^2\theta} = 1+\tan\theta$

$\Rightarrow \tan\theta = -1$ or $\frac{1-\tan^2\theta}{1+\tan^2\theta} = 1$

2 solution

In $[0, 2\pi]$

$1-\tan^2\theta = 1+\tan^2\theta$

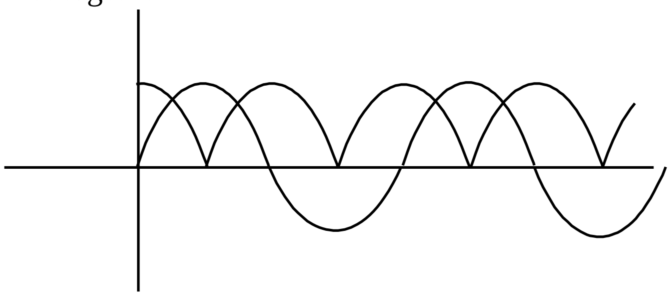
$\tan\theta = 0$

3 soln.

Total 5 solution

15. **(4)**

Plot together



4 soln

16. **(3)**

$\cos x = |\cos x - \sin x|$

$\cos x \geq \sin x \Rightarrow \cos x = \cos x - \sin x$

Soln. here is $0, 2\pi$

$\cos x < \sin x \Rightarrow \cos x = -\cos x + \sin x$

$\text{nt}\left(\frac{\pi}{4}, \frac{5}{4}\right)$ tan x = 2

Only 1 soln. here also

A < 3

17. **(8)**

That is only possible when

$\log_{|\cos n|}|\sin x| = 1$

$\Rightarrow |\sin x| = |\cos x|$

tan x = y

ijn $(-2\pi, 2\pi)$

18. **(2)**

Case I : $\cot x \geq 0$

$\cot x = \cot x + \frac{1}{\sin x}$

$\Rightarrow \sin x = \infty$

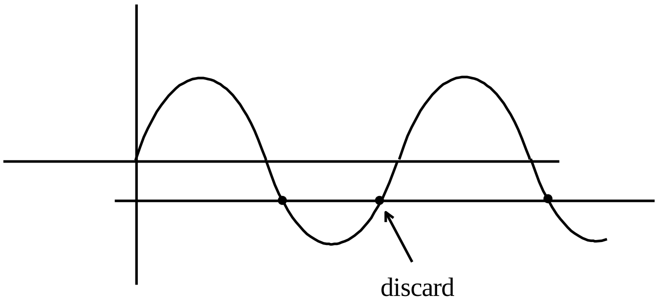
$\Rightarrow$ no soln

Case II: $\cot x < 0$

$\Rightarrow -\cot x = \cot x + \frac{1}{\sin x}$

$\frac{-2\cos x}{\sin x} = \frac{1}{\sin x}$

$\Rightarrow \cos x = \frac{-1}{2}$



$\Rightarrow 2$ soln.

19. **(1)**

$\sum \cos x = 5$ only possible when

$\cos x = \cos 2x = \cos 3x = \cos^4 x = \cos^5 x = 1$

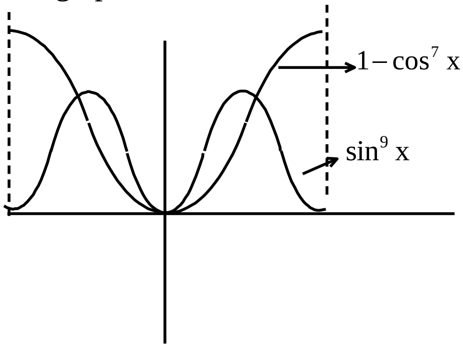
Simultaneously possible of x = 0

$\therefore$ 1 soln.

20. **(3)**

$\sin^4 x = 1 - \cos^7 x$

Rough plot $\sin^9 x$ & $1 - \cos^7 x$



$\therefore$ We see 3 soln. 5

21. **(1)**

$\text{LHS} \le 1$ & $\text{RHS} = \left(x - \sqrt{3}\right)^2 + 1$

$\Rightarrow \text{RHS} \ge 1$

$\therefore$ for soln. to exist LHS= RHS = 1

$\Rightarrow x = \sqrt{3}$ only hence 1 soultion

22. **(6)**

$\sin x + \sin y = \sin(x + y)$

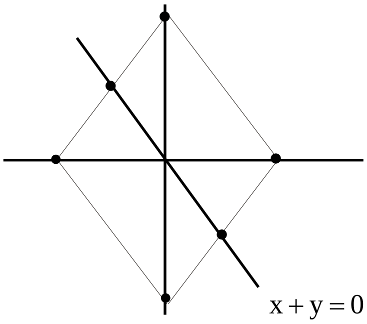
$$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right)$$

$$\Rightarrow \sin\left(\frac{x+y}{2}\right) \quad \text{or} \quad \cos\left(\frac{x-y}{2}\right)=\cos\left(\frac{x+y}{2}\right)$$

Or $x=2m\pi$ or $y=2k\pi$

$x+y=2n\pi$

Here any x + y = 0, x = 0 y = 0 will intersect $|x|+|y|=1$

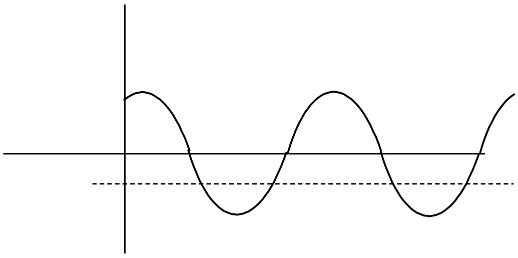


6 soln.

23. **(4)**

$\sin x\ \cos=\frac{3}{4}$ & $\cos x\ \ \cos y=\frac{1}{4}$

$\Rightarrow \cos(x-y)=1$ & $\cos(x+y)=\frac{-1}{2}$



(i) $x=y \Rightarrow \cos^2 x=\frac{-1}{2}$

4 soln.

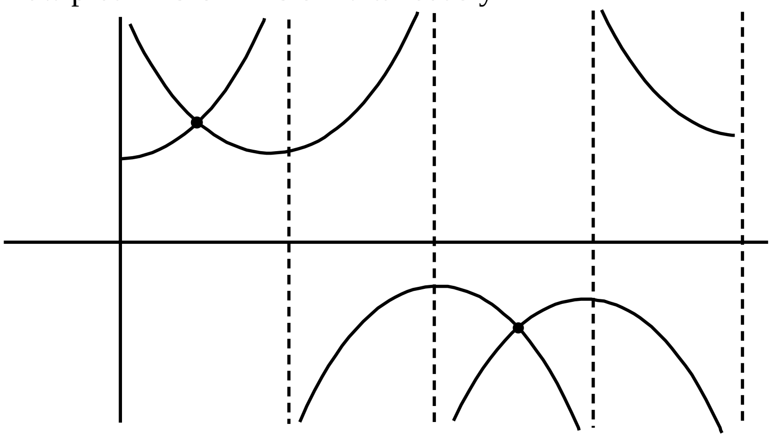
(ii) $x=y+2\pi$ & $x=y-2\pi$ not possible as $0<x,y<2\pi$

A 4 soln.

24. **(0)**

$\sin^5 x+\frac{1}{\sin x}=\frac{1}{\cos x}+\cos^5 x$

Now plot LHS & RHS simultaneously



There are 2 intersection points but here $\sin x = \cos x$

$\Rightarrow$ Overall No soln. A 0

25. **(1)**

$a_1 + a_2 \sin x + a_3 \cos x + a_4 \sin 2x + a_5 \cos 2x = 0$

Since this is an identify in x

Put x = 0, $x = \frac{\pi}{2}, x = \frac{-\pi}{2}, x = \pi, x = -\pi$

& soln. to get

$a_1 = a_2 = a_3 = a_5 = 0$

$\therefore$ one possibility $\langle 0,0,0,0,0 \rangle$

26. **(5)**

$(\sin\theta + 2)(\sin\theta + 3)(\sin\theta + 4) = 6$

$\sin\theta = -1$ then only LHS = 6

Otherwise LHS > 6

= $\sin\theta = -1$ $\therefore [0, 4\pi]$

Possible at $\frac{3\pi}{2}, \frac{7\pi}{2}$

Sum = $5\pi \Rightarrow k = 5$

27. **(2)**

$1 - \cos^2 x + a\cos x + a^2 > 1 + \cos x$

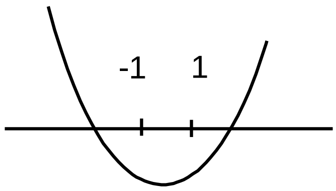
$\cos^2 x + (1-a)\cos x - a^2 < 0$

Put cos x = t

$f(t) = t^2 + (1-a)t - a^2 < 0 \, \forall t [-1, 1]$

$f(-1) < 0$

$f(1) < 0$



(i) $f(-1) \le 0 \Rightarrow a \in [-\infty, 0] \cup [1, \infty]$

(ii) $f(1) \le 0 \Rightarrow a \in (-\infty, -1) \cup [3, \infty]$

$a \in [-\infty, 1] \cup [3, \infty]$

28. **(4)**

Simplify

cos y sin x – sin y cos x + sin y sin x + cos x cos y = 0

$\sin(x+y) + \cos(x-y) = 0$

$\tan(x-y) = -1$ $-2\pi < x - y < 2\pi$

$x - y$ has 4 solutions in $(-2\pi, 2\pi)$

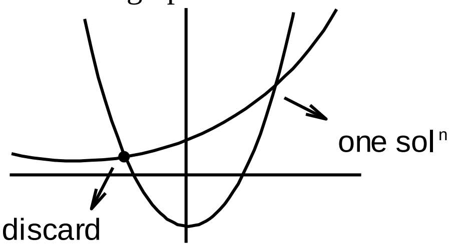
29. **(1)**

Put $\tan^2\theta = t$

$\left(1-t^2\right)-2^t=0$

$2^t=-1+t^2$

Plot both graph when t > 0



We know t = 3 satisfies $\tan^2\theta=3$

$\therefore$ 2 soln.

30. **(2)**

Use principal sol.

P sin x = $\frac{\pi}{2}-p\cos$

$P\left(\sin x+\cos x\right)=\frac{\pi}{2}$

Now least P = $\frac{\pi}{2\sqrt{2}}$ when is > 1

$\therefore$ least +in internal P = 2

31. **(2)**

sin x + cos x = 1

$\Rightarrow \sin x=0$ & $\cos x=1$ cos x = 0 7 sin x = 4

$x=2n\pi$ $x=\left(4n+1\right)\frac{\pi}{2}$

x = 0 $\frac{\pi}{2}$

2 soln.

32. **(0)**

Let $\sin x+\cos x=1\Rightarrow \sin 2x=t^2-1$

$\Rightarrow t=2\left(\frac{t^2-1}{2}+1\right)$

$\Rightarrow t^2=t^2-1+2\Rightarrow 2=1$

No soln.

33. **(1)**

$RHS\geq 1$ & $LHS\geq 1$

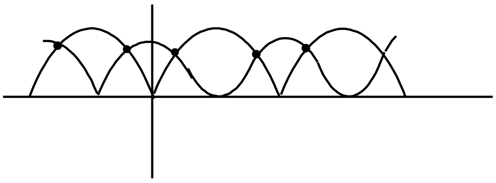
Only possible @ x = 0 & x = 1

Now pur x = 0 satisfies

Put x = 1 doesn't satisfy

$\therefore$ one soln. x = 0

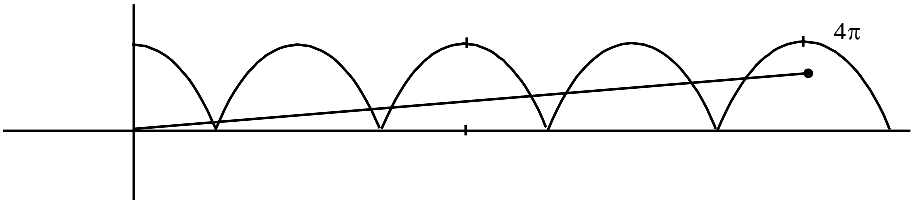
34. **(6)**



6 soln.

35. **(8)**

$|\cos x| = \frac{x}{30} \rightarrow$ this passes through $\left(4\pi, \frac{4\pi}{30}\right)$



36. **(8)**

$3\left(2\cos^2 x - 1\right) - 10\cos x + 7 = 0$

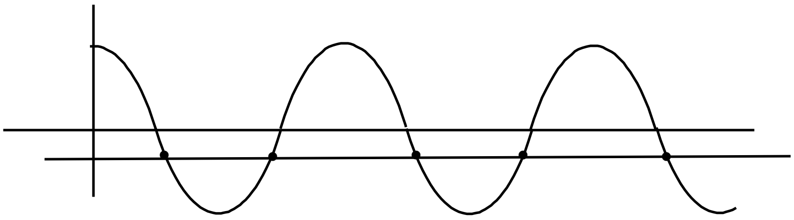
$6\cos^2 x - 10\cos x + 4 = 0$

$3\cos^2 x - 5\cos x + 2 = 0$

$3\cos^2 x - 6\cos x + \cos x - 2 = 0$

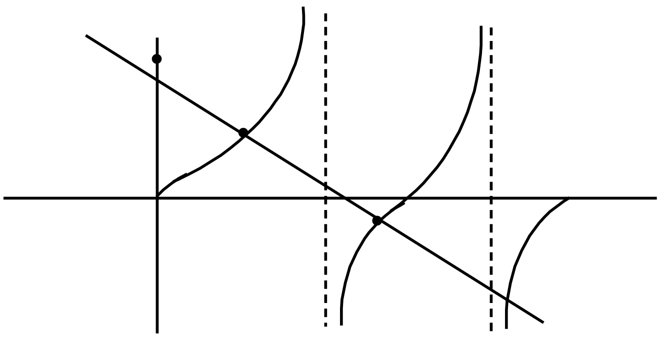
$(3\cos x + 1)(\cos x - 2) = 0$

$\cos x = \frac{-1}{3}$



37. **(3)**

$\tan x = \frac{5\pi}{4} - \frac{3}{2}x$



3 soln.

38. **(4)**

$|\cos x| + \cos^2 x = 0$

$0 \le x \le 4\pi$

$\cos x \ge 0 \Rightarrow \cos x + \cos^2 x = 0$

$\cos x = 0 \Rightarrow \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\cos x < 0$

$-\cos x+\cos^2 x=0$

$\cos x=0, \qquad \cos x=1$

4 soln.

39. **(3)**

$\sec^2(a+2)=1-a^2$

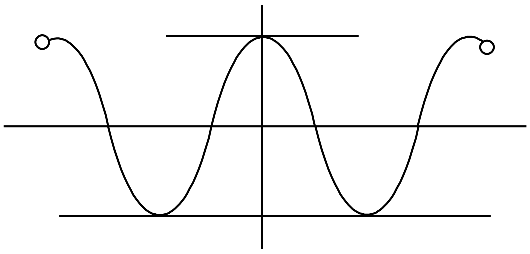
Only possible of a = 0

$\Rightarrow \qquad \sec^2 2x=1$

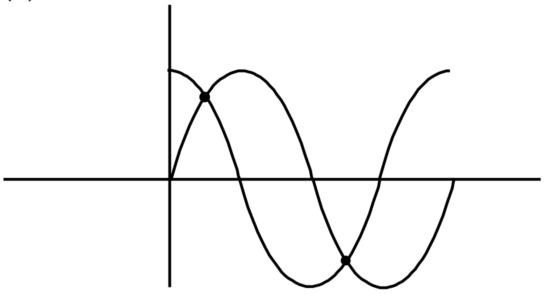
or $\qquad \sec^2 x=1 \qquad$ or $\qquad$ -1

$\Rightarrow \cos 2x=1$ or $\qquad$ -1

Total 3 soln.



40. **(4)**



Only after $x>\frac{5\pi}{4}$

i.e. $\quad x = 4$

JEE Advanced : PYQ

1. (c)

$\sqrt{3}\sec x+\operatorname{cosec} x+2(\tan x-\cot x)=0$

$\Rightarrow \frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos=\cos^2 x-\sin^2 x$

$\Rightarrow \cos\left(x-\frac{\pi}{3}\right)=\cos 2x \Rightarrow x-\frac{\pi}{3}=2n\pi\pm 2x$

$\Rightarrow x=\frac{2n\pi}{3}+\frac{\pi}{9}$ or $x=-2n\pi-\frac{\pi}{3}$

For $x\in S, n=0 \Rightarrow x=\frac{\pi}{9},-\frac{\pi}{3}$

Now, $n=1 \Rightarrow x=\frac{7\pi}{9}$ and $n=-1 \Rightarrow x=\frac{-5\pi}{9}$

Hence, sum of all values of $x=\frac{\pi}{9}-\frac{\pi}{3}+\frac{7\pi}{9}-\frac{5\pi}{9}=0$

2. (d)

$\sin x+2\sin 2x-\sin 3x=3$

$\Rightarrow \sin x+4\sin x\cos x-3\sin x+4\sin^3 x=3$

$\Rightarrow \sin x\left(-2+4\cos x+4\sin^2 x\right)=3$

$\Rightarrow \sin\left(-2+4\cos x+4-4\cos^2 x\right)=3$

$2+4\cos x-4\cos^2 x=\dfrac{3}{\sin x}$ $\quad [\because 0\le \sin x\le 1]$

$\Rightarrow 2-4\left(\cos^2 x-2\cos x\cdot\dfrac{1}{2}+\dfrac{1}{4}\right)+1=\dfrac{3}{\sin x}$

$\Rightarrow 3-4\left(\cos x-\dfrac{1}{2}\right)^2=\dfrac{3}{\sin x}$

$\therefore$ L.H.S. ≤ 3 and R.H.S ≥ 3

Hence, the equation hs no solution.

3. (d)

Given: $\cos(\alpha-\beta)=1$ and $\cos(\alpha+\beta)=\dfrac{1}{e}$, where $\alpha,\beta\in[-\pi,\pi]$

Now, $\cos(\alpha-\beta)=1 \Rightarrow \alpha-\beta=0 \Rightarrow \alpha=\beta$ and $\cos\alpha+\beta=\dfrac{1}{e} \Rightarrow \cos 2\alpha=\dfrac{1}{e}$

$\because 0<\dfrac{1}{e}<1$

Now, $2\alpha\in[-2\pi,2\pi]$

$\Rightarrow$ There will be two values of 2α in $[-2\pi,0]$ satisfying $\cos 2\alpha=\dfrac{1}{e}$ and two values in $[0,2\pi]$.

$\Rightarrow$ There will be four values of α in $[-\pi,\pi]$ and corresponding four values of β.

Hence there are four sets of (α,β).

4. (d)

$(\cos p-1)x^2+(\cos p)x+\sin p=0$

For real roots $D\ge 0$

$\Rightarrow \cos^2 p-4\sin p(\cos p-1)\ge 0$

$\Rightarrow \cos^2 p-4\sin p\cos p+4\sin^2 p+4\sin p-4\sin^2 p\ge 0$

$\Rightarrow (\cos p-2\sin p)^2+4\sin p(1-\sin p)\ge 0$

Since, $(\cos p-2\sin p)^2\ge 0$ and $1-\sin p\ge 0$

$\therefore D\ge 0,\ \forall\ p\in(0,\pi)$

5. (b)

$\sin x-3\sin 2x+\sin 3x=\cos x-3\cos 2x+\cos 3x$

$\Rightarrow 2\sin 2x\cos x-3\sin 2x=2\cos 2x\cos x-3\cos 2x$

$\Rightarrow \sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3)$

$\Rightarrow \sin x = \cos 2x \qquad \left[\because \cos x \neq \frac{3}{2}\right]$

$\Rightarrow \tan 2x = 1$

$\Rightarrow 2x = n\pi + \frac{\pi}{4}$

$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}$

6. (a)

Given: $2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2}$ where $0 < x \leq \frac{\pi}{2}$

LHS $= \cos^2\frac{x}{2}\sin^2 x = (1+\cos x)\sin^2 x$

$\because 1+\cos x < 2$ and $\sin^2 x \leq 1$ for $0 < x \leq \frac{\pi}{2}$

$\therefore (1+\cos x)\sin^2 x < 2$

And R.H.S. $= x^2 + \frac{1}{x^2} \geq 2$

Thus for $0 < x \leq \frac{\pi}{2}$, given equation is not possible

7. (a, c)

If we consider $\tan\frac{\alpha}{2} = x$ and $\tan\frac{\beta}{2} = y$, then $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$

$\Rightarrow 2\left[\frac{1-y^2}{1+y^2} - \frac{1-x^2}{1+x^2}\right] = 1 - \frac{(1-x^2)(1-y^2)}{(1+x^2)(1+y^2)}$

$\Rightarrow 2\left[(1+x^2)(1-y^2) - (1-x^2)(1+y^2)\right] = (1+x^2)(1+y^2) - (1-x^2)(1-y^2)$

$\Rightarrow 4(x^2 - y^2) = 2(x^2 + y^2)$

$\Rightarrow x^2 = 3y^2 \Rightarrow x = \pm\sqrt{3}\,y \Rightarrow \tan\frac{\alpha}{2} \pm \sqrt{3}\tan\frac{\beta}{2} = 0$

8. (c)

Let $f(x) = x^2 - x\sin x - \cos x$

$\therefore f'(x) = 2x - x\cos x = x(2 - \cos x)$

$\therefore f$ is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$

Also $\lim\limits_{x\to\infty} f(x) = \infty, \lim\limits_{x\to\infty} f(x) = \infty$ and $f(0) = -1$

$\therefore y = f(x)$ meets x-axis twice.

i.e., $f(x) = 0$ has two points in $(-\infty, \infty)$.

9. (c, d)

$$\sum_{m=1}^{6} \operatorname{cosec}\left[\theta+\frac{(m-1)\pi}{4}\right]\operatorname{cosec}\left[\theta+\frac{m\pi}{4}\right]=4\sqrt{2}$$

$$\Rightarrow \sum_{m=1}^{6} \frac{\sin\frac{\pi}{4}}{\sin\left[\theta+\frac{(m-1)\pi}{4}\right]\sin\left[\theta+\frac{m\pi}{4}\right]}=4$$

$$\Rightarrow \sum_{m=1}^{6} \frac{\sin\left[\left(\theta+\frac{m\pi}{4}\right)-\left(\theta+\frac{(m-1)\pi}{4}\right)\right]}{\sin\left(\theta+\frac{(m-1)\pi}{4}\right)\sin\left(\theta+\frac{m\pi}{4}\right)}=4$$

$$\Rightarrow \sum_{m=1}^{6} \frac{\sin\left[\sin\left(\theta+\frac{m\pi}{4}\right)\cos\left(\theta+\frac{(m-1)\pi}{4}\right)-\cos\left(\theta+\frac{m\pi}{4}\right)\sin\left(\theta+\frac{(m-1)\pi}{4}\right)\right]}{\sin\left(\theta+\frac{(m-1)\pi}{4}\right)\sin\left(\theta+\frac{m\pi}{4}\right)}=4$$

$$\Rightarrow \sum_{m=1}^{6}\left[\cot\left(\theta+\frac{(m-1)\pi}{4}\right)-\cot\left(\theta+\frac{m\pi}{4}\right)\right]=4$$

$$\Rightarrow \left[\cot\theta-\cot\left(\theta+\frac{\pi}{4}\right)\right]+\left[\cot\left(\theta+\frac{\pi}{4}\right)-\cot\left(\theta+\frac{2\pi}{4}\right)\right]+\ldots+\left[\cot\left(\theta+\frac{5\pi}{4}\right)-\cot\left(\theta+\frac{6\pi}{4}\right)\right]=4$$

$$\Rightarrow \cot\theta-\cot\left(\theta+\frac{3\pi}{2}\right)=4 \Rightarrow \cot\theta+\tan\theta=4$$

$$\Rightarrow \cos^2\theta+\sin^2\theta=4\sin\theta\cos\theta$$

$$\Rightarrow \sin 2\theta=\frac{1}{2} \Rightarrow 2\theta=\frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

$$\Rightarrow \theta=\frac{\pi}{12} \text{ or } \frac{5\pi}{12}$$

10. (b)

We have (I)

$$\left\{x\in\left[\frac{-2\pi}{3},\frac{2\pi}{3}\right]:\cos x+\sin x=1\right\}$$

$$\cos x+\sin x=1$$

$$\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x=\frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{4}+x\right)=\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{4}+x=n\pi+(-1)^n\frac{\pi}{4}$$

$$x=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4} \qquad \therefore\ x \text{ has 2 elements} \rightarrow (P)$$

We have (II)

$$\left\{x \in \left[\frac{-5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}; \sqrt{3}\tan 3x = 1$$

$$\Rightarrow \tan 3x = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = (6n+1)\frac{\pi}{18}; n \in Z \Rightarrow 3x = n\pi + \frac{\pi}{6} \text{ or } x = \frac{n\pi}{3} + \frac{\pi}{18}$$

$\therefore$ x has 2 elements $\rightarrow (P)$

We have (III)

$$\left\{x \in \left[\frac{-6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos 2x = \sqrt{3}\right\}$$

$$2\cos 2x = \sqrt{3}$$

$$\Rightarrow \cos 2x = \frac{\sqrt{3}}{2} \Rightarrow 2x = 2n\pi \pm \frac{\pi}{6}; n \in Z$$

or $x = n\pi \pm \frac{\pi}{12}; n \in Z$

$$\Rightarrow x \in \left\{\pm\frac{\pi}{12}, -\pi \pm \frac{\pi}{12}, \pi \pm \frac{\pi}{12}\right\}$$

$\therefore$ x has 6 elemetns $\rightarrow$ (**T**)

We have (IV)

$$\left\{x \in \left[\frac{-7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$$

$$\sin x - \cos x = 1$$

$$\frac{1}{\sqrt{2}}\sin(x) - \frac{1}{\sqrt{2}}\cos(x) = \frac{1}{\sqrt{2}}$$

$$\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$x - \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$$

$$x = n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{4}$$

$$x \in \left\{\frac{\pi}{2}, \frac{-3\pi}{2}, -\pi, \pi\right\}$$

$\therefore$ x has 4 elements $\rightarrow$ **R**

11. (a)

$$f(x) = 0 \Rightarrow \sin(\pi\cos x) = 0 \Rightarrow \pi l \cos x = n\pi$$

$$\Rightarrow \cos x = n \Rightarrow \cos x = -1, 0, 1$$

$$\Rightarrow x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, 3\pi, \frac{7\pi}{2}, 4\pi......$$

$$\therefore X = \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, 3\pi, \frac{7\pi}{2}, 4\pi, ...\right\}$$

$\therefore$ (I) - P, Q

$f'(x)=0 \Rightarrow \cos(\pi\cos x)(-\pi\sin x)=0$

$\Rightarrow \cos(\pi\cos x)=0,\ \sin x=0$

$\Rightarrow \pi\cos x=(2n-1)\dfrac{\pi}{2},\ x=n\pi$

$\Rightarrow \cos x=(2n-1)\dfrac{1}{2},\ \ x=\pi, 2\pi, 3\pi, ...$

$\Rightarrow \cos x=\dfrac{-1}{2},\dfrac{1}{2}.$

$\Rightarrow x=\dfrac{\pi}{3},\dfrac{2\pi}{3},\dfrac{4\pi}{3},\dfrac{5\pi}{3},\dfrac{7\pi}{3},\dfrac{8\pi}{3},\dfrac{10\pi}{3},\dfrac{11\pi}{3},\dfrac{13\pi}{3},....$

$\therefore\ Y=\left\{\dfrac{\pi}{3},\dfrac{2\pi}{3},\pi,\dfrac{4\pi}{3},\dfrac{5\pi}{3},2\pi,......\right\}.$

$\therefore\ (\text{II})$ - Q, T .

$g(x)=0 \Rightarrow \cos(2\pi\sin x)=0$

$\Rightarrow 2\pi\sin x=(2n-1)\dfrac{\pi}{2} \Rightarrow \sin x=\dfrac{2n-1}{4}$

$\Rightarrow \sin x=\dfrac{1}{4},\dfrac{-1}{4},\dfrac{3}{4},\dfrac{-3}{4}$

$\therefore\ Z=\left\{-\sin^{-1}\dfrac{3}{4},-\sin^{-1}\dfrac{1}{4},\sin^{-1}\dfrac{1}{4},\sin^{-1}\dfrac{3}{4}\right\}.$

(III) – R.

$g'(x)=0 \Rightarrow -\sin(2\pi\sin x).2\pi\cos x=0$

$\Rightarrow \sin(2\pi\sin x)=0, \cos x=0$

$\Rightarrow 2\pi\sin x=n\pi, x=(2n-1)\dfrac{\pi}{2}.$

$\Rightarrow \sin x=\dfrac{n}{2}, x=\dfrac{\pi}{2},\dfrac{3\pi}{2},\dfrac{5\pi}{2},,\dfrac{7\pi}{2},......$

$\Rightarrow \sin x=-1,-\dfrac{1}{2},0,\dfrac{1}{2},1.$

$\Rightarrow x=\dfrac{\pi}{6},\dfrac{\pi}{2},\dfrac{5\pi}{6},\pi,\dfrac{3\pi}{2},\dfrac{11\pi}{6},2\pi,\dfrac{13\pi}{6},.....$

$\therefore\ W=\left\{\dfrac{\pi}{6},\dfrac{\pi}{2},\dfrac{5\pi}{6},\pi,\dfrac{3\pi}{6},\dfrac{11\pi}{6},2\pi,\dfrac{13\pi}{6},......\right\}.$

(IV) – P, R, S.

12. (8)

$\dfrac{5}{4}\cos^2 2x+\cos^4 x+\sin^4 x+\cos^6 x+\sin^6 x=2$

$\Rightarrow \dfrac{5}{4}\cos^2 2x+1-\dfrac{1}{2}\sin^2 2x+1-\dfrac{3}{4}\sin^2 2x=2$

$$\Rightarrow \frac{5}{4}\left(\cos^2 2x - \sin^2 2x\right) = 0 \Rightarrow \cos 4x = 0$$

$$\Rightarrow 4x = (2n-1)\frac{\pi}{2} \text{ or } x = (2n+1)\frac{\pi}{8}$$

For $x \in [0, 2\pi], n$ can take values 0 to 7

Hence, there are 8 solutions.

13. (7)

$$\frac{1}{\sin\frac{\pi}{n}} - \frac{1}{\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}}$$

$$\Rightarrow \frac{\sin\frac{3\pi}{n} - \sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} \Rightarrow \frac{2\cos\frac{2\pi}{n}\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}}$$

$$\Rightarrow 2\sin\frac{2\pi}{n}\cos\frac{2\pi}{n} = \sin\frac{3\pi}{n} \Rightarrow \sin\frac{4\pi}{n} - \sin\frac{3\pi}{n} = 0$$

$$\Rightarrow 2\cos\frac{7\pi}{2n}\sin\frac{\pi}{2n} = 0 \Rightarrow \cos\frac{7\pi}{2n} = 0 \text{ or } \sin\frac{\pi}{2n} = 0$$

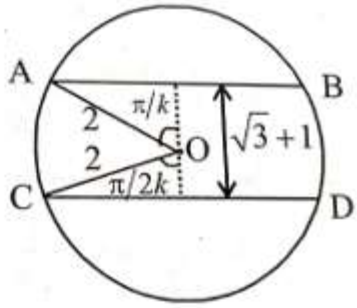
$$\Rightarrow \frac{7\pi}{2n} = (2k+1)\frac{\pi}{2} \text{ or } \frac{\pi}{2n} = 2k\pi, \text{ where } k \in Z$$

$$\Rightarrow n = \frac{7}{2k+1} \text{ or } n = \frac{1}{4k} \quad \left(n = \frac{1}{4k} \text{ not possible for any integral value of } k\right)$$

As $n > 3$; for $k = 0$, we get $n = 7$.

14. (3)

From the figure,



$$2\cos\frac{\pi}{k} + 2\cos\frac{\pi}{2k} = \sqrt{3}+1$$

$$\Rightarrow 2\times 2\cos^2\frac{\pi}{2k} + 2\cos\frac{\pi}{2k} - 2$$

$$= \sqrt{3}+1$$

$$\Rightarrow 4\cos^2\frac{\pi}{2k} + 2\cos\frac{\pi}{2k} - \left(3\sqrt{3}\right) = 0$$

$$\Rightarrow \cos\frac{\pi}{2k} = \frac{-2 \pm \sqrt{4+16\left(3+\sqrt{3}\right)}}{8} = \frac{-1 \pm \sqrt{13+4\sqrt{3}}}{4}$$

$$= \frac{-1 \pm \left(2\sqrt{3}+1\right)}{4} = \frac{\sqrt{3}}{2} \text{ or } -\left(\frac{\sqrt{3}+1}{2}\right)$$

As $\frac{\pi}{2k}$ is in acute angle, $\cos\frac{\pi}{2k} = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \Rightarrow k = 3$

15. (3)

$$\tan\theta = \cot 5\theta, \theta \neq \frac{n\pi}{5}$$

$\Rightarrow \cos\theta\cos 5\theta - \sin 5\theta\sin\theta = 0 \Rightarrow \cos 6\theta = 0$

$\Rightarrow 6\theta = \frac{-5\pi}{2}, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow 0 = \frac{-5\pi}{12}, \frac{-\pi}{4}, \frac{-\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$

Again $\sin 2\theta = \cos 4\theta = 1 - 2\sin^2 2\theta$

$\Rightarrow 2\sin^2\theta + \sin 2\theta - 1 = 0 \Rightarrow \sin 2\theta = -1, \frac{1}{2}$

$\Rightarrow 2\theta = \frac{-\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{-\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$

So, common solutions are $\theta = \frac{-\pi}{4}, \frac{\pi}{12}$ and $\frac{5\pi}{12}$

$\therefore$ Number of solutions = 3.

16. (0.5)

Given : $\sqrt{3}a\cos x + 2b\sin x = c$

Which has two roots α and β, such that $\alpha + \beta = \frac{\pi}{3}$

$\therefore \sqrt{3}a\cos\alpha + 2b\sin\alpha = c \qquad ...(i)$

and $\sqrt{3}a\cos\beta + 2b\sin\beta = c \qquad ...(ii)$

On subtracting equation (ii) from (i),

$\sqrt{3}a(\cos\alpha - \cos\beta) + 2b(\sin\alpha - \sin\beta) = 0$

$\Rightarrow -\sqrt{3}a\, 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} + 2b.2\cos\frac{\alpha+b}{2}\sin\frac{\alpha-\beta}{2} = 0$

$\Rightarrow -2\sqrt{3}a\sin\frac{\pi}{6} + 4b\cos\frac{\pi}{6} = 0 \quad \left(\because \sin\frac{\alpha-\beta}{2} \neq 0\right)$

$\Rightarrow -2\sqrt{3}a \times \frac{1}{2} + 4b\frac{\sqrt{3}}{2} = 0 \Rightarrow \frac{b}{a} = \frac{1}{2} = 0.5$