

Trigonometric Ratio

EXERCISE - 1 [A]

1. (B)

$$\pi \text{ radian} = 180 \text{ degree} \Rightarrow 1 \text{ radian} = \frac{180}{\pi} \text{ degree} \approx 57.3 \text{ degree, hence } \sin 1^\circ > \sin 1^0.$$

2. (B)

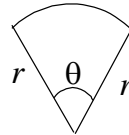
Length of arc of circle of radius r subtending θ at the center = $r\theta$.

$$\text{Hence } 15 = r \times \frac{3}{4} \text{ or } r = 20 \text{ cm}$$

3. (D)

$$\text{Perimeter} = r\theta + 2r = m \cdot r$$

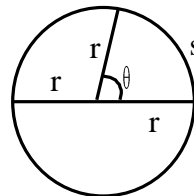
$$\theta = (m-2)^c$$



4. (C)

$$\pi r = S + 2r$$

$$\Rightarrow S = (\pi - 2)r$$



5. (B)

At half past 4, hour hand will be at $4\frac{1}{2}$ and minute hand will be at 6 (for 30 minutes). In 12 hours,

angle made by hour hand

$$\frac{360}{12} \times \frac{9}{2} = 135^\circ$$

In 60 minutes, angle made by minute hand

$$= 360^\circ$$

In 30 minutes, angle made by minute hand

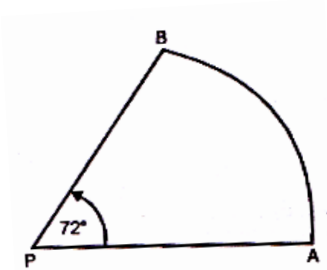
$$= \frac{360^\circ}{60} \times 30 = 180^\circ$$

$$\therefore \text{reqd. angle} = 180^\circ - 135^\circ = 45^\circ = \frac{\pi}{4}$$

Hence (B) holds.

6. (B)

Let the post be at point P and let PA be the length of the rope in tight position. Suppose the horse moves along the arc AB so that $\angle APB = 72^\circ$ and arc AB = 88m. Let r be the length of the rope i.e PA = r metres.



$$\text{Now, } \theta = 72^\circ = \left(72 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{5}\right)^c$$

$$\text{And } s = 88\text{m} \quad \therefore \theta = \frac{\text{arc}}{\text{radius}}$$

$$\Rightarrow \frac{2\pi}{5} = \frac{88}{r} \Rightarrow r = 88 \times \frac{5}{2\pi} = 70\text{m}$$

Hence (B) holds.

7. (C)

Since $180^\circ = \pi$ radians

$$\therefore 1^\circ = \frac{\pi}{180} \text{ radians} \quad \therefore p^\circ = \frac{p\pi}{180} \text{ radians}$$

$$\therefore \frac{p\pi}{180} = q$$

[\because p degrees = q radians]

$$\Rightarrow \frac{p}{180} = \frac{q}{\pi} \quad \text{i.e. } \frac{p}{90} = \frac{2q}{\pi}$$

Hence (C) holds.

8. (B)

Clearly length of the wire
= circumference of circle of radius 7
 $= 2\pi(7) = 14\pi$

$$\therefore l = 14\pi, r = 12\text{cm}$$

$$\text{Since } \theta = \frac{l}{r} = \frac{14\pi}{12} = \frac{7\pi}{6} = \frac{7}{6} \times 180^\circ = 210^\circ$$

Hence angle subtended at the centre = 210°

9. (A)

$$\tan(90^\circ - \theta) = \cot \theta \Rightarrow \tan 89^\circ = \cot 1^\circ, \tan 88^\circ = \cot 2^\circ, \tan 87^\circ = \cot 3^\circ, \dots \text{ etc.}$$

$$\text{Also, } \tan \theta \times \cot \theta = 1, \text{ hence } \tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ = 1$$

10. (A)

$$\sin \theta - \cos \theta = 1 \Rightarrow \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta = 1$$

$$\Rightarrow \sin \theta \cos \theta = 0$$

11. (C)

$$\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta} = \frac{5 \tan \theta - 3}{5 \tan \theta + 2}$$
$$5 \tan \theta = 4 \Rightarrow \frac{5 \tan \theta - 3}{5 \tan \theta + 2} = \frac{4 - 3}{4 + 2} = \frac{1}{6}$$

12. (A)

$$\sin x + \operatorname{cosec} x = 2 \Rightarrow \sin^2 x - 2 \sin x + 1 = 0$$
$$\Rightarrow \sin x = 1 \Rightarrow \sin^n x + \operatorname{cosec}^n x = 2$$

13. (C)

$$x \sin 45^\circ \cos^2 60^\circ = \frac{\tan^2 60^\circ \operatorname{cosec} 30^\circ}{\sec 45^\circ \cot^2 30^\circ} \Rightarrow x \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{2} \right)^2 = \frac{(\sqrt{3})^2 (2)}{(\sqrt{2})(\sqrt{3})^2} \Rightarrow x = 8$$

14. (A)

$$\tan^2 30 + 4 \sin^2 45 + \frac{1}{3} \cos^2 30 = \frac{1}{3} + \frac{4}{2} + \frac{1}{3} \times \frac{3}{4}$$
$$= \frac{1}{3} + \frac{1}{4} + 2$$
$$= 2 \frac{7}{12}$$

15. (D)

$$\frac{\tan^2 60 - 2 \tan^2 45 + \sec^2 30}{3 \sin^2 45 - \sin 90 + \cos^2 60 \cdot \cos^3 0} = \frac{3 - 2 + \frac{4}{3}}{3 \cdot \frac{1}{2} + \frac{1}{4}}$$
$$= \frac{1 + \frac{4}{3}}{\frac{1}{2} \left(3 + \frac{1}{2} \right)} = \frac{\frac{7}{3}}{\frac{1}{2} \cdot \frac{7}{2}} = \frac{4}{3}$$

16. (C)

$$2P_6 - 3P_4 + 1$$
$$= 2(1 - 3 \sin^2 x \cos^2 x) - 3 \left[(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x \right] + 1$$
$$= 2 - 6 \sin^2 x \cos^2 x - 3(1 - 2 \sin^2 x \cos^2 x) + 1 = 0$$

17. (A)

$$90^\circ < 130^\circ < 135^\circ \text{ hence } \sin A > 0 \text{ \& } \cos A < 0 \text{ \& } |\sin A| > |\cos A|$$
$$\Rightarrow \sin A + \cos A > 0$$

18. (A)

$$\cos 24^\circ + \cos 5^\circ + \cos 175^\circ + \cos 204^\circ + \cos 300^\circ$$
$$= \cos 24^\circ + \cos 5^\circ + \cos(180^\circ - 5^\circ) + \cos(180^\circ + 24^\circ) + \cos(360^\circ - 60^\circ)$$

$$\begin{aligned}
&= \cos 24^\circ + \cos 5^\circ - \cos 5^\circ - \cos 24^\circ + \cos 60^\circ \\
&= \cos 60^\circ = \frac{1}{2}
\end{aligned}$$

19. (C)
 $\sin \theta < 0$ & $\tan \theta > 0 \Rightarrow \theta$ lies in 3rd quadrant.

20. (C)
 Given expression

$$\begin{aligned}
&= \frac{-\sin(660^\circ) \tan(1050^\circ) \sec(420^\circ)}{\cos(180^\circ + 45^\circ) \operatorname{cosec}(360^\circ - 45^\circ) \cos(360^\circ + 150^\circ)} \\
&= \frac{-\sin(7 \times 90 + 30^\circ) \tan(3 \cdot 360^\circ - 30^\circ) \sec(360^\circ + 60^\circ)}{(-\cos 45^\circ) - \operatorname{cosec}(45^\circ) \cos 150^\circ} \\
&= \frac{\cos(30^\circ) (-\tan 30^\circ) \sec 60^\circ}{-(\cos 45^\circ) (-\operatorname{cosec} 45^\circ) (-\cos 30^\circ)} \\
&= \frac{\cos 30^\circ \tan 30^\circ \sec 60^\circ}{\cos 45^\circ \operatorname{cosec} 45^\circ \cos 30^\circ} \\
&= \frac{\frac{1}{\sqrt{3}} \cdot 2}{\frac{1}{\sqrt{2}} \cdot 1} = \frac{2}{\sqrt{3}}
\end{aligned}$$

Hence (C) holds

21. (C)
 Given expression

$$= \frac{(-\sin \theta)(\sec \theta)(-\tan \theta)}{(\sin \theta)(-\sec \theta) \tan \theta} = -1$$

Hence (C) holds.

22. (B)
 Since $\sec \theta$ i.e $\cos \theta$ is negative.
 $\therefore \theta$ lies in II nd or III rd quadrant. Since $\sin \theta$ is positive.
 $\therefore \theta$ lies in Ist or II nd quadrant. Hence θ lies in II nd quadrant.

23. (A)
 $\cot A + \tan(180^\circ + A) + \tan(90^\circ + A) + \tan(360^\circ - A)$
 $= \cot A + \tan A - \cot A - \tan A = 0$

24. (D)
 $\sin[-870^\circ] = -\sin(870^\circ) = -\sin(9 \times 90^\circ + 60^\circ)$
 $= -\cos 60^\circ = -\frac{1}{2}$

25. (A)

$$\tan \theta = -\frac{1}{\sqrt{5}}$$

$$\sec^2 \theta = 1 + \tan^2 \theta = 1 + \frac{1}{5} = \frac{6}{5}$$

$$\sec \theta = +\sqrt{\frac{6}{5}} \quad [\because \theta \text{ lies in the IV th quadrant}]$$

$$\therefore \cos \theta = \sqrt{\frac{5}{6}}$$

26. (C)

$$\sin 15^\circ \cos 15^\circ = \frac{1}{2} \sin 30^\circ = \frac{1}{4}$$

27. (A)

$$\frac{\cos 17^\circ + \sin 17^\circ}{\cos 17^\circ - \sin 17^\circ} = \frac{1 + \tan 17^\circ}{1 - \tan 17^\circ}$$
$$= \tan(45^\circ + 17^\circ) = \tan 62^\circ$$

28. (A)

$$\tan 75^\circ - \cot 75^\circ = -2 \cot 150^\circ$$
$$= 2 \tan 60^\circ = 2\sqrt{3}$$

29. (B)

$$\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$$
$$\Rightarrow \cos^2 48^\circ - \sin^2 12^\circ = \cos 60^\circ \cos 36^\circ = \frac{\sqrt{5}+1}{8}$$

30. (D)

As A & B are the acute angles hence,

$$\sin A = \frac{1}{\sqrt{10}} \Rightarrow \cos A = \frac{3}{\sqrt{10}} \quad \& \quad \sin B = \frac{1}{\sqrt{5}} \Rightarrow \cos B = \frac{2}{\sqrt{5}}$$

Now $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\Rightarrow \cos(A+B) = \frac{3}{\sqrt{10}} \times \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{10}} \times \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow A+B = \frac{\pi}{4}$$

31. (A)

$$\cos^2\left(\frac{\pi}{6} + \theta\right) - \sin^2\left(\frac{\pi}{6} - \theta\right)$$
$$= \cos\left(\frac{\pi}{6} + \theta + \frac{\pi}{6} - \theta\right) \cos\left(\frac{\pi}{6} + \theta - \frac{\pi}{6} + \theta\right)$$
$$[\because \cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B]$$

$$= \cos\left(\frac{\pi}{3}\right)\cos(2\theta) = \frac{1}{2}\cos 2\theta$$

Hence (A) holds.

32. (C)

$$\begin{aligned} & (1 + \tan A)(1 + \tan B) \\ &= (1 + \tan A)\left(1 + \tan\left(\frac{\pi}{4} - A\right)\right) \\ &= (1 + \tan A)\left(1 + \frac{1 - \tan A}{1 + \tan A}\right) \\ &= (1 + \tan A)\left(\frac{1 + \tan A + 1 - \tan A}{1 + \tan A}\right) = 2 \end{aligned}$$

\therefore (C) holds

33. (B)

$$\begin{aligned} \sin 163^\circ &= \sin(180^\circ - 17^\circ) = \sin 17^\circ \\ \cos 347^\circ &= \cos(360^\circ - 13^\circ) = \cos 13^\circ \\ \sin 73^\circ &= \sin(90^\circ - 17^\circ) = \cos 17^\circ \\ \sin 167^\circ &= \sin(180^\circ - 13^\circ) = \sin 13^\circ \\ \therefore \text{ given expression} \\ &= \sin 17^\circ \cos 13^\circ + \cos 17^\circ \sin 13^\circ \\ &= \sin(17^\circ + 13^\circ) = \sin(30^\circ) = \frac{1}{2} \end{aligned}$$

34. (D)

$$\begin{aligned} \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1 \\ \therefore A + B &= \frac{\pi}{4} \end{aligned}$$

35. (B)

$$\begin{aligned} \sin(\theta + \phi) &= \sin \theta \cos \phi + \cos \theta \sin \phi \\ \left[\cos \theta &= \sqrt{1 - \frac{144}{165}} = \frac{5}{13} \because 0 < \theta < \frac{\pi}{2} \quad \sin \phi = -\sqrt{1 - \frac{9}{25}} = -\frac{4}{5} \because \pi < \theta < \frac{3\pi}{2} \right] \\ &= \frac{12}{13}\left(-\frac{3}{5}\right) + \frac{5}{13}\left(-\frac{4}{5}\right) \\ &= -\frac{36}{65} - \frac{20}{65} = -\frac{56}{65} \end{aligned}$$

36. (D)

$$\begin{aligned}A + B = 45^\circ &\Rightarrow \frac{\cot A \cot B - 1}{\cot B + \cot A} = 1 \\(\cot A - 1)(\cot B - 1) \\&= \cot A \cot B - (\cot A + \cot B) + 1 \\&= 1 + 1 = 2 \\&\therefore \text{(D) holds}\end{aligned}$$

37. (C)

$$\begin{aligned}32 \sin\left(\frac{A}{2}\right) \sin\left(\frac{5A}{2}\right) &= 16(\cos 2A - \cos 3A) \\&= 16(2\cos^2 A - 1 - 4\cos^3 A + 3\cos A) \\&= 16\left(2 \times \frac{9}{16} - 1 - 4 \times \frac{27}{64} + 3 \times \frac{3}{4}\right) = 11\end{aligned}$$

38. (B)

$$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = -2\cos 60^\circ \sin 10^\circ + \sin 10^\circ = 0$$

39. (D)

$$\begin{aligned}\text{The given expression} &= \cos 20^\circ + \cos(180^\circ - 80^\circ) + \cos(180^\circ - 40^\circ) \\&= \cos 20^\circ - (\cos 80^\circ + \cos 40^\circ) \\&= \cos 20^\circ - 2\cos 60^\circ \cos 40^\circ = 0 \\&\therefore \text{(D) holds}\end{aligned}$$

40. (A)

$$\begin{aligned}\cos 52^\circ + \cos 68^\circ + \cos 172^\circ \\&= \cos 52^\circ + \cos 68^\circ + \cos(180^\circ - 8^\circ) \\&= \cos 52^\circ + \cos 68^\circ - \cos 8^\circ \\&= 2\cos \frac{52^\circ + 68^\circ}{2} \cos \frac{68^\circ - 52^\circ}{2} - \cos 8^\circ \\&= 2\cos 60^\circ \cos 8^\circ - \cos 8^\circ \\&= \cos 8^\circ - \cos 8^\circ = 0\end{aligned}$$

41. (C)

$$\frac{\sin(2A + B)}{\sin B} = \frac{5}{1}. \text{ By componendo and dividendo we shall get } \frac{\tan(A + B)}{\tan A} = \frac{3}{2}$$

42. (C)

$$\text{Given value} = (\cos 12^\circ + \cos 132^\circ) + (\cos 84^\circ + \cos 156^\circ) = -\frac{1}{2}$$

43. (B)

$$\begin{aligned}\frac{\sin 70^\circ + \cos 40^\circ}{\cos 70^\circ + \sin 40^\circ} &= \frac{\sin 70^\circ + \sin 50^\circ}{\cos 70^\circ + \cos 50^\circ} \\ &= \frac{2 \sin 60^\circ \cos 10^\circ}{2 \cos 60^\circ \cos 10^\circ} = \tan 60^\circ = \sqrt{3}\end{aligned}$$

44. (D)

$$\begin{aligned}\frac{\sin 55^\circ - \cos 55^\circ}{\sin 10^\circ} &= \frac{\sin 55^\circ - \sin 35^\circ}{\sin 10^\circ} \\ &= \frac{2 \cos 45^\circ \sin 10^\circ}{\sin 10^\circ} = 2 \cos 45^\circ = 2 \frac{1}{\sqrt{2}} = \sqrt{2} \\ \therefore (D) \text{ holds}\end{aligned}$$

45. (D)

$$\begin{aligned}1 - 2 \sin^2 \left(\frac{\pi}{4} + \theta \right) &= \cos 2 \left(\frac{\pi}{4} + \theta \right) \\ &= \cos \left(\frac{\pi}{2} + 2\theta \right) = -\sin 2\theta\end{aligned}$$

46. (B)

$$\begin{aligned}2 \cos^2 \theta - 2 \sin^2 \theta = 1 &\Rightarrow \cos 2\theta = \frac{1}{2} \\ \therefore \theta = 30^\circ\end{aligned}$$

47. (B)

$$\begin{aligned}2 \sin A \cos^3 A - 2 \sin^3 A \cos A &= 2 \sin A \cos A (\cos^2 A - \sin^2 A) \\ &= \sin 2A \cos 2A = \frac{\sin 4A}{2}\end{aligned}$$

48. (C)

$$\begin{aligned}\sin A + \cos A = 1, \text{ we get } 1 + \sin 2A &= 1 \\ \Rightarrow \sin 2A = 0\end{aligned}$$

49. (A)

$$\begin{aligned}\frac{\tan 80^\circ - \tan 10^\circ}{\tan 70^\circ} &= \frac{\cot 10^\circ - \tan 10^\circ}{\tan 70^\circ} \\ &= \frac{2 \cot 20^\circ}{\tan 70^\circ} = 2\end{aligned}$$

50. (A)

$$\text{Use } \cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

51. (D)

$$\begin{aligned} & \tan 67 \frac{1^\circ}{2} + \cot 67 \frac{1^\circ}{2} \\ &= \tan 67 \frac{1^\circ}{2} + \frac{1}{\tan 67 \frac{1^\circ}{2}} \\ &= \frac{1 + \tan^2 67 \frac{1^\circ}{2}}{2 \tan 67 \frac{1^\circ}{2}} \cdot 2 \\ &= 2 \cdot \frac{1}{\sin 2 \left(67 \frac{1^\circ}{2} \right)} \\ & \left[\because \sin 2A = \frac{2 \tan A}{1 + \tan^2 A} \right] \\ &= \frac{2}{\sin(135^\circ)} = \frac{2}{\sin(180^\circ - 45^\circ)} \\ &= \frac{2}{\sin 45^\circ} = \frac{2}{\frac{1}{\sqrt{2}}} = 2\sqrt{2} \end{aligned}$$

\therefore (D) holds

52. (B)

$$\begin{aligned} & \sqrt{2 + \sqrt{2 + 2 \cos 4\theta}} = \sqrt{2 + \sqrt{2(1 + \cos 4\theta)}} \\ &= \sqrt{2 + \sqrt{2 \cdot 2 \cos^2 2\theta}} \\ &= \sqrt{2 + 2 \cos 2\theta} = \sqrt{2(1 + \cos 2\theta)} \\ &= \sqrt{2(2 \cos^2 \theta)} = 2 \cos \theta \end{aligned}$$

\therefore (B) holds

53. (B)

$$\begin{aligned} & \cos 15^\circ \cos 7 \frac{1^\circ}{2} \sin 7 \frac{1^\circ}{2} \\ &= \frac{1}{2} \cos 15^\circ \sin 15^\circ = \frac{1}{4} \sin 30^\circ \\ &= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \end{aligned}$$

\therefore (B) holds

54. (A)

$$\frac{1-t^2}{1+t^2} = \frac{1-\tan^2 \frac{\theta}{2}}{1+\tan^2 \frac{\theta}{2}} = \cos \theta$$

55. (D)

$$\text{Since } 1 + \cos x = K \quad \therefore 2 \cos^2 \frac{x}{2} = K$$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{K}{2} \quad \Rightarrow 1 - \sin^2 \frac{x}{2} = \frac{K}{2}$$

$$\Rightarrow \sin^2 \frac{x}{2} = 1 - \frac{K}{2} = \frac{2-K}{2}$$

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{2-K}{2}}$$

56. (A)

$$\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

57. (A)

$$\frac{\sin 3\theta - \cos 3\theta}{\sin \theta + \cos \theta} + 1 = \frac{3 \sin \theta - 4 \sin^3 \theta - 4 \cos^3 \theta + 3 \cos \theta}{\sin \theta + \cos \theta} + 1$$

$$= 4 \frac{\sin \theta + \cos \theta - \sin^3 \theta - \cos^3 \theta}{\sin \theta + \cos \theta}$$

$$= 4 \frac{\sin \theta + \cos \theta - (\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta)}{\sin \theta + \cos \theta}$$

$$= 4 \sin \theta \cos \theta = 2 \sin 2\theta$$

58. (C)

$$\cos^2 A (3 - 4 \cos^2 A)^2 + \sin^2 A (3 - 4 \sin^2 A)^2 = (3 \cos A - 4 \cos^3 A)^2 + (3 \sin A - 4 \sin^3 A)^2$$
$$= \cos^2 3A + \sin^2 3A = 1$$

59. (C)

$$\frac{3 \cos \theta + \cos 3\theta}{3 \sin \theta - \sin 3\theta} = \frac{3 \cos \theta + (4 \cos^3 \theta - 3 \cos \theta)}{3 \sin \theta - (3 \sin \theta - 4 \sin^3 \theta)} = \cot^3 \theta$$

60. (C)

$$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = (\sin (60^\circ - 20^\circ) \sin 20^\circ \sin (60^\circ + 20^\circ)) \sin 60^\circ$$
$$= \frac{1}{4} \sin(3 \times 20^\circ) \sin 60^\circ = \frac{1}{4} \sin^2 60^\circ = \frac{3}{16}$$

61. (D)

$$\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{2 \sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 80^\circ}{2 \sin 20^\circ}$$
$$= \frac{2 \sin 40^\circ \cos 40^\circ \cos 80^\circ}{4 \sin 20^\circ} = \frac{2 \sin 80^\circ \cos 80^\circ}{8 \sin 20^\circ}$$

$$= \frac{\sin 160^\circ}{8 \sin 20^\circ}, \text{ but } \sin 160^\circ = \sin (180^\circ - 20^\circ) = \sin 20^\circ, \text{ hence}$$

$$\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$$

Alternately

$$\cos 20^\circ \cos 40^\circ \cos 80^\circ = \cos (60^\circ - 20^\circ) \cos 20^\circ \cos (60^\circ + 20^\circ)$$

$$\frac{1}{4} \cos(3 \times 20^\circ) = \frac{1}{8}$$

62. (D)

$$\begin{aligned} & \frac{1}{4} \left(4 \cos \theta \cos \left(\frac{\pi}{3} + \theta \right) \cos \left(\frac{\pi}{3} - \theta \right) \right) \\ &= \frac{\cos 3\theta}{4} \end{aligned}$$

63. (C)

$$\begin{aligned} & \tan \theta + \tan \left(\theta + \frac{\pi}{3} \right) + \tan \left(\theta - \frac{\pi}{3} \right) \\ &= \tan \theta + \frac{\tan \theta + \sqrt{3}}{1 - \tan \theta \cdot \sqrt{3}} + \frac{\tan \theta - \sqrt{3}}{1 + \tan \theta \cdot \sqrt{3}} \\ &= \tan \theta + \frac{\tan \theta + \sqrt{3} + \sqrt{3} \tan^2 \theta + 3 \tan \theta + \tan \theta - \sqrt{3} - \tan^2 \theta \sqrt{3} + 3 \tan \theta}{1 - 3 \tan^2 \theta} \\ &= \frac{\tan \theta - 3 \tan^3 \theta + 8 \tan \theta}{1 - 3 \tan^2 \theta} \\ &= \frac{9 \tan \theta - 3 \tan^3 \theta}{1 - 3 \tan^2 \theta} \\ &= \frac{3[3 \tan \theta - \tan^3 \theta]}{1 - 3 \tan^2 \theta} = 3 \tan 3\theta \end{aligned}$$

$$\therefore K = 3$$

Hence (C) holds.

64. (A)

$$x = \tan 15^\circ = 2 - \sqrt{3}$$

$$y = \operatorname{cosec} 75^\circ = \sqrt{6} - \sqrt{2}$$

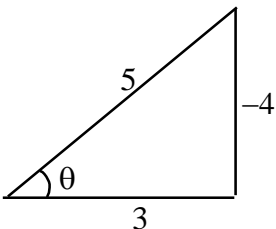
$$(z) = 4 \sin 18^\circ = \sqrt{5} - 1$$

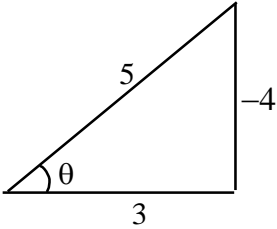
It follows that $x < y < z$

\therefore (D) holds

65. (D)

$$\begin{aligned} & -\sqrt{3^2 + 4^2} + 8 \leq 3 \cos x + 4 \sin x + 8 \leq \sqrt{3^2 + 4^2} + 8 \\ & \Rightarrow 3 \leq 3 \cos x + 4 \sin x + 8 \leq 13 \end{aligned}$$

66. (A)
 $\sin\left(\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{\pi}{6}\right)$ acquires maximum at $\alpha + \frac{\pi}{6} = \frac{\pi}{4}$ i.e. $\theta = \frac{\pi}{12}$
67. (D)
 $5\sin^2 \theta + 4\cos^2 \theta = 4 + \sin^2 \theta \geq 4$
68. (B)
 Maximum value of $\sin \theta \cos \theta$
 = Maximum value of $\left(\frac{\sin 2\theta}{2}\right)$
 = $\frac{1}{2}$ Maximum Value of $\sin 2\theta = \frac{1}{2}(1) = \frac{1}{2}$
69. (C)
 Minimum value of $\sin \theta \cos \theta$
 = $\frac{1}{2}$ Minimum value of $\sin 2\theta = \frac{1}{2}(-1) = -\frac{1}{2}$
70. (C)
 $\sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right) \leq \sqrt{2}$
 \therefore Maximum value = $\sqrt{2}$
71. (D)
 Put $\begin{cases} 3 = r \cos \alpha \\ 4 = r \sin \alpha \end{cases} \Rightarrow r^2 = 9 + 16 = 25$
 $\Rightarrow r = 5$
 $\therefore 3 \cos \theta + 4 \sin \theta = r(\cos \theta \cos \alpha + \sin \theta \sin \alpha)$
- 


- $= r \cos(\theta - \alpha) = 5 \cos(\theta - \alpha)$
- Its Maximum value = $5(1) = 5$
 And Minimum value = -5
72. (A)
 Given, $P = \frac{1}{2} \sin^2 \theta + \frac{1}{3} \cos^2 \theta$
 $\frac{1}{2}(1 - \cos^2 \theta) + \frac{1}{3} \cos^2 \theta = \frac{1}{2} - \frac{1}{6} \cos^2 \theta$
 Since, $0 \leq \cos^2 \theta \leq 1 \Rightarrow -\frac{1}{6} \leq -\frac{1}{6} \cos^2 \theta \leq 0$

$$\Rightarrow \frac{1}{3} \leq \frac{1}{2} - \frac{1}{6} \cos^2 \theta \leq \frac{1}{2} \Rightarrow \frac{1}{3} \leq P \leq \frac{1}{2}$$

\therefore (A) holds

73. (C)

$$A + B + C = \pi$$

$$\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

74. (A)

$$\frac{A + B + C}{2} = \frac{\pi}{2}$$

$$\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$\frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \tan \frac{B}{2}} = \frac{1}{\tan \frac{C}{2}}$$

$$\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{A}{2} \tan \frac{C}{2} = 1$$

$$\tan \frac{C}{2} = 1 - \frac{2}{9} = \frac{7}{9}$$

75. (D)

$$A + B + C = \pi$$

$$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$$

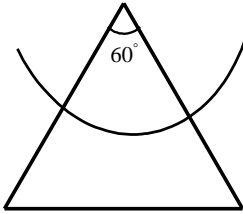
$$\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$\frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \tan \frac{B}{2}} = \frac{1}{\tan \frac{C}{2}}$$

$$\Rightarrow \sum \tan \frac{A}{2} \tan \frac{B}{2} = 1$$

EXERCISE - 1 [B]

1. (A)



$$\text{Area of } \Delta = \frac{\sqrt{3}}{4} \cdot 4^2 = 4\sqrt{3}$$

$$2\pi \text{ --- } \pi r^2$$

$$1 \text{ --- } \frac{\pi r^2}{2\pi}$$

$$\frac{\pi}{3} \text{ --- } \frac{\pi r^2}{2\pi} \cdot \frac{\pi}{3}$$

$$\text{Given, } \frac{\pi r^2}{6} = \frac{4\sqrt{3}}{2}$$

$$\pi r^2 = 12\sqrt{3}$$

$$r = \sqrt{\frac{12\sqrt{3}}{\pi}}$$

2. (C)

Angle covered from 6 A.M. to 3.15 P.M.

$$= 277 \frac{1^\circ}{2} = \frac{555}{2} \times \frac{\pi}{180} = \frac{111}{2} \times \frac{\pi}{36}$$

$$\therefore \theta = \frac{37\pi}{24} \text{ radians}$$

Length of hour hand = 12cm

i.e. $r = 12\text{cm}$

$$\text{Since } \theta = \frac{l}{r} \quad \therefore l = r\theta = \frac{12 \times 37}{24} = \frac{37\pi}{2}$$

$$\text{Hence reqd. distance} = \frac{37\pi}{2} \text{ cm.}$$

3. (D)

$$\text{An hour hand in 1 hour i.e., 60 minute traces} = \frac{360^\circ}{12} = 30^\circ$$

$$\therefore \text{An hour hand in 1 minute traces} = \frac{30^\circ}{60} = \left(\frac{1}{2}\right)^\circ$$

\therefore An hour hand in 40 minutes traces

$$= \left(\frac{1}{2} \times 40\right)^\circ = 20^\circ$$

$$\therefore \text{reqd. angle} = 90^\circ - 20^\circ = 70^\circ$$

$$= \frac{70 \times \pi}{180} = \frac{7\pi}{18}$$

∴ (D) holds

4. (B)

$$\begin{aligned} & \sin^2 x + \operatorname{cosec}^2 x + 2 + \cos^2 x + \sec^2 x + 2 - \tan^2 x - \cot^2 x - 2 \\ &= 3 + (\operatorname{cosec}^2 x - \cot^2 x) + (\sec^2 x - \tan^2 x) \\ &= 5 \end{aligned}$$

5. (D)

$$\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} = \operatorname{cosec}^2 \theta \sec^2 \theta$$

6. (A)

$$\text{Let } \lambda = 3\cos\beta - 5\sin\beta$$

$$\text{Let } S = 3\sin\beta + 5\cos\beta$$

$$\lambda^2 + S^2 = 9 + 25$$

$$\therefore \lambda^2 = 9$$

7. (A)

$$\tan^2 \alpha + \cot^2 \alpha + 2 = m^2$$

$$(\tan^2 \alpha + \cot^2 \alpha)^2 = (m^2 - 2)^2$$

$$\tan^4 \alpha + \cot^4 \alpha + 2 = m^4 + 4 - 4m^2$$

$$\tan^4 \alpha + \cot^4 \alpha = m^4 - 4m^2 + 2$$

8. (B)

$$\frac{\cot \theta + \operatorname{cosec} \theta - 1}{\cot \theta - \operatorname{cosec} \theta + 1}$$

$$= \frac{\frac{1}{\operatorname{cosec} \theta - \cot \theta} - 1}{\cot \theta - \operatorname{cosec} \theta + 1} \quad (\because (\operatorname{cosec} \theta + \cot \theta)(\operatorname{cosec} \theta - \cot \theta) = 1)$$

$$= \frac{1 - \operatorname{cosec} \theta + \cot \theta}{(\operatorname{cosec} \theta - \cot \theta)(\cot \theta - \operatorname{cosec} \theta + 1)}$$

$$= \frac{1}{\operatorname{cosec} \theta - \cot \theta} = \operatorname{cosec} \theta + \cot \theta$$

$$= \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 - \cos \theta}$$

9. (D)

$$\text{Since } \sin x + \sin^2 x = 1$$

$$\therefore \sin x = 1 - \sin^2 x = \cos^2 x$$

$$\therefore \sin^2 x = \cos^4 x$$

$$\Rightarrow 1 - \cos^2 x = \cos^4 x$$

$$\Rightarrow 1 = \cos^4 x + \cos^2 x$$

$$\Rightarrow 1 = (\cos^4 x + \cos^2 x)^2$$

$$= \cos^8 x + 2\cos^6 x + \cos^4 x$$

$$\text{Thus } \cos^8 x + 2\cos^6 x + \cos^4 x = 1$$

10. (C)

$$\begin{aligned} \sin(-566^\circ) &= -\sin 566^\circ = -\sin(6 \times 90^\circ + 26^\circ) \\ &= -(-\sin 26^\circ) = \sin 26^\circ \end{aligned}$$

11. (C)

$$\begin{aligned} f(x) &= 3 \left[\sin^4 \left(\frac{\pi}{2} - x \right) + \sin^4 x \right] - 2 [\cos^6 x + \sin^6 x] \\ &= 3 [(\cos^2 x)^2 + (\sin^2 x)^2] - 2 [(\sin^2 x)^3 + (\cos^2 x)^3] \\ &= 3 [1 - 2\sin^2 x \cos^2 x] - 2 [1 - 3\sin^2 x \cos^2 x] \\ &= 1 \end{aligned}$$

12. (A)

$$\begin{aligned} &2\cos 10 + \sin(90+10) + \sin(3 \times 360 - 80) + \sin(27 \times 360 + 280) \\ &= 2\cos 10 + \cos 10 - 2\sin 80 \\ &= \cos 10 + 2(\cos 10 - \sin(90-10)) \\ &= \cos 10 + 2(\cos 10 - \cos 10) = \cos 10 \end{aligned}$$

13. (C)

$$\begin{aligned} &\cos^2(90-17) + \cos^2 47 - \sin^2(90-47) + \sin^2(90+17) \\ &= \sin^2 17 + \cos^2 47 - \cos^2 47 + \cos^2 17 \\ &= 1 \end{aligned}$$

14. (D)

$$\begin{aligned} \text{(i) } \sin(2 \times 360 + 45) &= \sin 45 = \frac{1}{\sqrt{2}} \\ \text{(ii) } \frac{1}{-\sin(3 \times 360 + 330)} &= \frac{1}{-\sin(360 - 30)} = \frac{1}{\sin 30} = 2 \\ \text{(iii) } \tan\left(4\pi + \frac{\pi}{3}\right) &= \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \\ \text{(iv) } \frac{-1}{\tan\left(4\pi - \frac{\pi}{4}\right)} &= 1 \end{aligned}$$

15. (C)

$$\begin{aligned} \ell^2 &= \left[\sin \theta - \sin \left(\frac{\pi}{2} - \theta \right) \right]^2 + \left[\cos \theta + \cos \left(\frac{\pi}{2} - \theta \right) \right]^2 \\ &= (\sin \theta - \cos \theta)^2 + (\cos \theta + \sin \theta)^2 \\ &= 1 + 1 = 2 \\ \ell &= \sqrt{2} \end{aligned}$$

16. (B)
 $\sin(180+20) + \cos(180+20)$
 $= -\sin 20 - \cos 20 < 0$

17. (B)
 $a = \sin 170^\circ + \cos 170^\circ$
 $= \sin(180-10) + \cos(180-10)$
 $= \sin 10 - \cos 10$
 $\Rightarrow a = -ve$
 $\{\because \cos 10^\circ > \sin 10^\circ\}$

18. (C)
 $\cos\left(\frac{\pi}{2} - x\right) = \sin x, \cos\left(\frac{3\pi}{2} + x\right) = -\sin x$
 $\cos(\pi - x) = -\cos x, \cos(2\pi - x) = \cos x$
 $\therefore \text{required} = \sin x \cdot \sin x + \cos x \cdot \cos x$
 $= 1$

19. (B)
 $\cos \frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos \frac{\pi}{3}, \cos \frac{4\pi}{3} = \cos\left(\pi + \frac{\pi}{3}\right) = -\cos \frac{\pi}{3}$
 $\cos \frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos \frac{\pi}{3}, \cos \frac{4\pi}{3} = \cos\left(\pi + \frac{\pi}{3}\right) = -\cos \frac{\pi}{3}$
 $\Rightarrow x = y \cos \frac{2\pi}{3} = z \cos \frac{4\pi}{3} \Rightarrow x = -\frac{y}{2} = -\frac{z}{2} \Rightarrow y = z = -2x$
 Now, $xy + yz + zx = x(-2x) + (-2x)(-2x) + (-2x)x = 0$

20. (A)
 $\cos^2 \alpha + \cos^2(\alpha + 120^\circ) + \cos^2(\alpha - 120^\circ)$
 $= \frac{1 + \cos 2\alpha}{2} + \frac{1 + \cos 2(\alpha + 120^\circ)}{2} + \frac{1 + \cos 2(\alpha - 120^\circ)}{2}$
 $= \frac{3 + \cos 2\alpha + 2 \cos 2\alpha \cos 120^\circ}{2} = \frac{3 + \cos 2\alpha - \cos 2\alpha}{2} = \frac{3}{2}$

21. (B)
 $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \Rightarrow \tan(A+B) = \frac{-\frac{1}{2} - \frac{1}{3}}{1 - \left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)}$
 $\Rightarrow \tan(A+B) = -1$
 $\Rightarrow A+B = \frac{3\pi}{4}$

22. (D)

$$\begin{aligned} \tan 45^\circ &= \tan(180^\circ + 45^\circ) \Rightarrow \tan 225^\circ = \tan(100^\circ + 125^\circ) \\ &\Rightarrow \frac{\tan 100^\circ + \tan 125^\circ}{1 - \tan 100^\circ \tan 125^\circ} = 1 \\ &\Rightarrow \tan 100^\circ + \tan 125^\circ + \tan 100^\circ \tan 125^\circ = 1 \end{aligned}$$

23. (B)

$$\begin{aligned} \frac{\sin(B+A) + \cos(B-A)}{\sin(B-A) + \cos(B+A)} &= \frac{\sin(B+A) + \sin\left(\frac{\pi}{2} - (B-A)\right)}{\cos\left(\frac{\pi}{2} - (B-A)\right) + \cos(B+A)} \\ &= \frac{2 \sin\left(\frac{\pi}{4} + A\right) \cos\left(\frac{\pi}{4} - B\right)}{2 \cos\left(\frac{\pi}{4} + A\right) \cos\left(\frac{\pi}{4} - B\right)} = \tan\left(\frac{\pi}{4} + A\right) \\ &= \frac{1 + \tan A}{1 - \tan A} = \frac{\cos A + \sin A}{\cos A - \sin A} \end{aligned}$$

24. (C)

$$\begin{aligned} \tan 3A &= \tan(2A+A) \\ &= \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A} \\ &\Rightarrow \tan 3A - \tan 2A \tan A \\ &= \tan 2A + \tan A \\ \therefore \tan 3A - \tan 2A - \tan A \\ &= \tan A \tan 2A \tan 3A \end{aligned}$$

25. (C)

$$\begin{aligned} &\sin^2 A \cos^2 B - \cos^2 A \sin^2 B \\ &= \sin^2 A (1 - \sin^2 B) - \sin^2 B (1 - \sin^2 A) \\ &= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B \\ &= \sin^2 A - \sin^2 B \end{aligned}$$

26. (B)

$$\begin{aligned} &\sin^2 A + \sin^2(A-B) + 2 \sin A \cos B \sin(B-A) \\ &= \sin^2 A + \sin^2(A-B) + [\sin(A+B) + \sin(A-B)] \sin(B-A) \\ &= \sin^2 A + \sin^2(A-B) + \sin(A+B) \sin(B-A) - \sin^2(A-B) \\ &= \sin^2 A + \sin^2 B - \sin^2 A \\ &= \sin^2 B \end{aligned}$$

27. (A)

$$\frac{\frac{\sin\left(\frac{3\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \sin\left(\theta + \frac{2\theta}{2}\right)}{\frac{\sin\left(\frac{3\theta}{2}\right)}{\sin\frac{\theta}{2}} \cos\left(\theta + \frac{2\theta}{2}\right)} = \frac{\sin \alpha}{\cos \alpha}$$

$$\tan 2\theta = \tan \alpha$$

$$\theta = \frac{\alpha}{2}$$

28. (B)

$$\sin \alpha + \sin \beta = a \Rightarrow 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = a \quad \dots (i)$$

$$\& \cos \alpha - \cos \beta = b \Rightarrow 2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2} = -b \quad \dots (ii)$$

$$\text{From (i) \& (ii) } \tan \frac{\alpha - \beta}{2} = -\frac{b}{a}$$

29. (C)

$$\frac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta} = \frac{(\sin 3\theta + \sin 9\theta) + (\sin 5\theta + \sin 7\theta)}{(\cos 3\theta + \cos 9\theta) + (\cos 5\theta + \cos 7\theta)}$$

$$= \frac{2 \sin 6\theta \cos 3\theta + 2 \sin 6\theta \cos \theta}{2 \cos 6\theta \cos 3\theta + 2 \cos 6\theta \cos \theta} = \tan 6\theta$$

30. (D)

Given value

$$= (\sin 47^\circ + \sin 61^\circ) - (\sin 11^\circ + \sin 25^\circ)$$

$$= 2 \sin 54^\circ \cos 7^\circ - 2 \sin 18^\circ \cos 7^\circ$$

$$= 2 \cos 7^\circ (\sin 54^\circ - \sin 18^\circ)$$

$$= 2 \cos 7^\circ \frac{2 \sin 18^\circ \cos 18^\circ}{\cos 18^\circ} \cos 36^\circ$$

$$= \cos 7^\circ \frac{2 \sin 36^\circ \cos 36^\circ}{\cos 18^\circ}$$

$$= \cos 7^\circ \frac{\sin 72^\circ}{\cos 18^\circ} = \cos 7^\circ \left[\because \sin 72^\circ = \cos 18^\circ \right]$$

31. (D)

$$\cos 40^\circ + \cos 80^\circ + \cos 160^\circ + \cos 240^\circ$$

$$= 2 \cos 60^\circ \cos 20^\circ + \cos(180^\circ - 20^\circ) + \cos(180^\circ + 60^\circ)$$

$$= 2 \left(\frac{1}{2} \right) \cos 20^\circ - \cos 20^\circ - \cos 60^\circ$$

$$= -\cos 60^\circ = -\frac{1}{2}$$

32. (A)

$$\begin{aligned} & \cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 228^\circ \\ &= (\cos 12^\circ + \cos 228^\circ) + (\cos 84^\circ + \cos 156^\circ) \\ &= 2\cos(120^\circ)\cos(108^\circ) + 2\cos 120^\circ \cos 36^\circ \\ &= 2\cos(120^\circ) [\cos(108^\circ) + \cos 36^\circ] \\ &= 2\left(-\frac{1}{2}\right) \cdot 2\cos 72^\circ \cos(36^\circ) \\ &= -2\cos 72^\circ \cos 36^\circ = -2 \cdot \frac{\sqrt{5}-1}{4} \cdot \frac{\sqrt{5}+1}{4} \\ &= -2\left(\frac{5-1}{16}\right) = -\frac{1}{2} \end{aligned}$$

33. (B)

$$\begin{aligned} & \cot \theta - 2\cot 2\theta \\ &= \frac{1}{\tan \theta} - \frac{2 \times (1 - \tan^2 \theta)}{2 \tan \theta} \\ &= \frac{\tan^2 \theta}{\tan \theta} = \tan \theta \end{aligned}$$

34. (D)

$$\begin{aligned} & \frac{\cos 10^\circ - \sqrt{3} \sin 10^\circ}{\sin 10^\circ \cdot \cos 10^\circ} \\ &= 2 \cdot 2 \frac{\left(\frac{1}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ\right)}{2 \sin 10^\circ \cos 10^\circ} \\ &= \frac{4 \sin(30^\circ - 10^\circ)}{\sin 20^\circ} = 4 \end{aligned}$$

35. (B)

$$\begin{aligned} & \cos \alpha + \cos \beta = 0 \Rightarrow \cos^2 \alpha + \cos^2 \beta = -2\cos \alpha \cos \beta \\ & \& \sin \alpha + \sin \beta = 0 \Rightarrow \sin^2 \alpha + \sin^2 \beta = -2\sin \alpha \sin \beta \\ & \text{Now } \cos 2\alpha + \cos 2\beta = 2(\cos^2 \alpha + \cos^2 \beta - 1) = 2(1 - \sin^2 \alpha - \sin^2 \beta) \\ & \Rightarrow \cos 2\alpha + \cos 2\beta = 2(-2\cos \alpha \cos \beta - 1) \quad \dots \text{(i)} \\ & \& \cos 2\alpha + \cos 2\beta = 2(1 + 2\sin \alpha \sin \beta) \quad \dots \text{(ii)} \\ & \text{(i) + (ii)} \\ & \Rightarrow \cos 2\alpha + \cos 2\beta = 2\cos(\alpha - \beta) \end{aligned}$$

36. (C)

$$\begin{aligned}\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ &= \frac{\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ}{\frac{1}{2} \cos 20^\circ \sin 20^\circ} \\ &= \frac{4 \sin(60^\circ - 20^\circ)}{2 \cos 20^\circ \sin 20^\circ} = \frac{4 \sin 40^\circ}{\sin 40^\circ} = 4\end{aligned}$$

37. (C)

$$\begin{aligned}\sqrt{3}[\cot \theta + \tan \theta] &= 4 \\ \Rightarrow \sqrt{3}\left[\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}\right] &= 4 \\ \Rightarrow \frac{1}{\sin \theta \cos \theta} &= \frac{4}{\sqrt{3}} \Rightarrow \frac{1}{2 \sin \theta \cos \theta} = \frac{2}{\sqrt{3}} \\ \Rightarrow \sin 2\theta &= \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \\ \Rightarrow 2\theta &= \frac{\pi}{3} \quad \therefore \theta = \frac{\pi}{6}\end{aligned}$$

38. (B)

$$\begin{aligned}\text{Since, } \cos 20^\circ - \sin 20^\circ &= p \\ \Rightarrow \cos^2 20^\circ + \sin^2 20^\circ - 2 \sin 20^\circ \cos 20^\circ &= p^2 \\ \Rightarrow 1 - p^2 &= \sin 40^\circ \\ \Rightarrow 1 - p^2 &= \sqrt{1 - \cos^2 40^\circ} \\ \Rightarrow (1 - p^2)^2 &= 1 - \cos^2 40^\circ \\ \Rightarrow \cos^2 40^\circ &= 1 - (1 + p^4 - 2p^2) \\ \Rightarrow \cos 40^\circ &= \sqrt{2p^2 - p^4} \\ \Rightarrow \cos 40^\circ &= p\sqrt{2 - p^2} \\ \therefore \text{(B) holds}\end{aligned}$$

39. (B)

$$\begin{aligned}\text{LHS} &= \left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right) \\ &= \left(1 - \cos^2 \frac{\pi}{8}\right) \left(1 - \cos^2 \frac{3\pi}{8}\right) \\ &= \sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8} = \frac{1}{4} \left[\cos \frac{\pi}{4} - \cos \frac{\pi}{2}\right]^2 \frac{1}{8}\end{aligned}$$

40. (C)

$$\sin \theta = \frac{2t}{1+t^2} = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \text{ if } t = \tan \frac{\theta}{2}$$

$$\therefore \cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} = \frac{1 - t^2}{1 + t^2}$$

41. (D)

$$\begin{aligned} & \frac{1}{4} \left(4 \cos \theta \cos \left(\frac{\pi}{3} + \theta \right) \cos \left(\frac{\pi}{3} - \theta \right) \right) \\ &= \frac{\cos 3\theta}{4} \end{aligned}$$

42. (B)

$$\begin{aligned} & \frac{\cos 20 + 4(\cos 40 - \cos 60) \sin 70}{\cos^2 10} \\ &= \frac{\cos 20 + 4 \cos 40 \cos 20 - 2 \cos 20}{\cos^2 10} \\ &= \frac{2(\cos 20 + 2 \cos 60 + 2 \cos 20 - 2 \cos 20)}{2 \cos^2 10} \\ &= \frac{2(\cos 20 + 1)}{(1 + \cos 20)} = 2 \end{aligned}$$

43. (C)

$$\begin{aligned} \sin 12^\circ \sin 48^\circ \sin 54^\circ &= \sin(60^\circ - 12^\circ) \sin 12^\circ \sin(60^\circ + 12^\circ) \frac{\sin 54^\circ}{\sin 72^\circ} \\ &= \frac{\sin 36^\circ \sin 54^\circ}{4 \sin 72^\circ} = \frac{\sin 36^\circ \sin 54^\circ}{8 \sin 36^\circ \cos 36^\circ} \\ &= \frac{\sin 54^\circ}{8 \cos 36^\circ}, \text{ but } \sin 54^\circ = \sin(90^\circ - 36^\circ) = \cos 36^\circ \\ \therefore \sin 12^\circ \sin 48^\circ \sin 54^\circ &= \frac{1}{8} \end{aligned}$$

44. (B)

$$\begin{aligned} x + \frac{1}{x} &= 2 \cos \theta \quad \& \quad x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x} \right)^3 - 3 \left(x \right) \left(\frac{1}{x} \right) \left(x + \frac{1}{x} \right) \\ \Rightarrow x^3 + \frac{1}{x^3} &= 8 \cos^3 \theta - 6 \cos \theta = 2 \cos 3\theta \end{aligned}$$

45. (C)

$$\begin{aligned} & \sin \frac{\pi}{10} \sin \frac{13\pi}{10} \\ &= \sin \frac{\pi}{10} \sin \left(\pi + \frac{3\pi}{10} \right) \end{aligned}$$

$$= -\sin \frac{\pi}{10} \sin \frac{3\pi}{10} = -\sin 18^\circ \sin 54^\circ$$

$$= -\frac{\sqrt{5}-1}{4} \cdot \frac{\sqrt{5}+1}{4} = -\frac{5-1}{16} = -\frac{1}{4}$$

46. (C)

$$3 \sin 2\theta = 2 \sin 3\theta$$

$$\Rightarrow 6 \sin \theta \cos \theta = 2(3 \sin \theta - 4 \sin^3 \theta)$$

$$\Rightarrow 4 \cos^2 \theta - 3 \cos \theta - 1 = 0$$

$$\Rightarrow \cos \theta = 1, \frac{1}{4}$$

Since $\cos \theta \neq 1$ ($\because \theta \neq 0$)

$$\therefore \cos \theta = \frac{1}{4} \Rightarrow \sin \theta = \frac{\sqrt{15}}{4}$$

47. (C)

$$f(\theta) = \sin \theta (\sin \theta + 3 \sin \theta - 4 \sin^3 \theta)$$

$$f(\theta) = 4 \sin^2 \theta - 4 \sin^4 \theta$$

$$f(\theta) = 4 \sin^2 \theta (1 - \sin^2 \theta) \geq 0 \quad \forall \theta$$

48. (D)

$$f(\theta) = 5 \cos \theta + 3 \cos \left(\theta + \frac{\pi}{3} \right) + 3$$

$$= 5 \cos \theta + 3 \cos \theta \cdot \cos \frac{\pi}{3} - 3 \sin \theta \sin \frac{\pi}{3} + 3$$

$$= \frac{13}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta + 3$$

$$-\sqrt{\frac{13^2}{4} + \frac{27}{4}} + 3 \leq f(\theta) \leq \sqrt{\frac{13^2}{4} + \frac{27}{4}} + 3$$

$$-4 \leq f(\theta) \leq 10$$

49. (C)

$$\sin \theta + \cos \beta$$

Maximum Value = 2

50. (C)

$$y = \frac{12}{9 + 5 \left(\frac{3}{5} \cos x + \frac{4}{5} \sin x \right)}$$

$$= \frac{12}{9 + 5 \sin(\alpha + x)} \text{ let } \left(\sin \alpha = \frac{3}{5} \right)$$

$$y_{\text{maximum}} = \frac{12}{4} = 3$$

51. (D)

$$y_{\text{minimum}} = \frac{1}{9}$$

52. (A)

$$B = 2\sin^2 x - \cos 2x$$

$$= 2\sin^2 x - (1 - 2\sin^2 x)$$

$$= 4\sin^2 x - 1$$

$$\text{Since } 0 \leq \sin^2 x \leq 1 \quad \therefore 0 \leq 4\sin^2 x \leq 4$$

$$\Rightarrow 0 - 1 \leq 4\sin^2 x - 1 \leq 3 \Rightarrow -1 \leq B \leq 3$$

53. (B)

$$\sin \theta + \cos \theta = \sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right) \text{ is Max when } \theta + \frac{\pi}{4} = \frac{\pi}{2} \text{ i.e. when } \theta = \frac{\pi}{4} = 45^\circ$$

54. (B)

$$2\sin^2 \theta + 3\cos^2 \theta$$

$$= 2[\sin^2 \theta + \cos^2 \theta] + \cos^2 \theta = 2 + \cos^2 \theta$$

$$\text{Since least value of } \cos^2 \theta = 0$$

$$\therefore \text{least value of } 2\sin^2 \theta + 3\cos^2 \theta = 2 + 0 = 2$$

55. (C)

$$\text{Let } y = \cos \theta - \cos 2\theta$$

$$\frac{dy}{d\theta} = \sin \theta + 2\sin 2\theta = 2.2 \sin \theta \cos \theta - \sin \theta$$

$$= \sin \theta (4 \cos \theta - 1)$$

$$\text{For Max or Min of } y, \frac{dy}{d\theta} = 0$$

$$\therefore \sin \theta = 0 \text{ or } \cos \theta = \frac{1}{4}$$

$$\text{i.e. } \theta = 0, \pi, 2\pi, \dots \text{ or } \cos \theta = \frac{1}{4}$$

$$\frac{d^2y}{d\theta^2} = -\cos \theta + 4\cos 2\theta > 0 \text{ for } \theta = 0, \pi, 2\pi, \dots \text{ and } 4\cos 2\theta - \cos \theta = 4[2\cos^2 \theta - 1] - \cos \theta$$

$$= 8\left(\frac{1}{4}\right)^2 - 4 - \frac{1}{4} = \frac{1}{2} - 4 - \frac{1}{4} < 0$$

$$\therefore y \text{ max. when } \cos \theta = \frac{1}{4}$$

$$\therefore \text{max. } y = \cos \theta - (2\cos^2 \theta - 1)$$

$$= \frac{1}{4} - \left[\frac{2}{16} - 1\right]$$

$$= \frac{1}{4} + 1 - \frac{1}{8} = \frac{1}{4} + \frac{7}{8}$$

$$= \frac{2+7}{8} = \frac{9}{8}$$

∴ (C) holds

56. (D)

$$\frac{8 \sin x \cdot \cos x \cdot \cos x \cdot \cos x}{8 \cdot 8 \cdot 4 \cdot 2}$$

$$= \theta = \frac{x}{7}$$

$$8 \sin \theta \cdot \cos \theta \cdot \cos 2\theta \cdot \cos 4\theta$$

$$= 8 \sin \theta \cdot \frac{\sin(2^3 \theta)}{2^3 \sin \theta} = \sin 8\theta = \sin x$$

57. (C)

We know that

$$\cos A \cos 2A \cos 2^2 A \cos 2^3 A \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$$

$$\therefore \cos A \cos 2A \cos 2^2 A = \frac{\sin 2^3 A}{2^3 \sin A} \quad [\text{By putting } n = 3]$$

$$\therefore \cos A \cos 2A \cos 4A = \frac{\sin 8A}{8 \sin A}$$

$$\therefore \cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} = \frac{\sin \frac{8\pi}{7}}{8 \sin \frac{\pi}{7}}$$

$$= \frac{\sin \left(\pi + \frac{\pi}{7} \right)}{8 \sin \frac{\pi}{7}} = \frac{-\sin \frac{\pi}{7}}{8 \sin \frac{\pi}{7}} = -\frac{1}{8}$$

Hence (C) holds

58. (D)

The given expression

$$= \frac{1}{2 \sin \frac{\pi}{65}} \left(2 \sin \frac{\pi}{65} \cos \frac{\pi}{65} \right) \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

$$= \frac{1}{4 \sin \frac{\pi}{65}} \left(2 \sin \frac{2\pi}{65} \cos \frac{2\pi}{65} \right) \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

$$= \frac{1}{8 \sin \frac{\pi}{65}} \left(2 \sin \frac{4\pi}{65} \cos \frac{4\pi}{65} \right) \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

$$= \frac{1}{16 \sin \frac{\pi}{65}} \left(2 \sin \frac{8\pi}{65} \cos \frac{8\pi}{65} \right) \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

$$\begin{aligned}
&= \frac{1}{32 \sin \frac{\pi}{65}} \left(2 \sin \frac{16\pi}{65} \cos \frac{16\pi}{65} \right) \cos \frac{32\pi}{65} \\
&= \frac{1}{64 \sin \frac{\pi}{65}} \left(2 \sin \frac{32\pi}{65} \cos \frac{32\pi}{65} \right) \\
&= \frac{1}{64 \sin \frac{\pi}{65}} \sin \frac{64\pi}{65} \\
&= \frac{1}{64 \sin \frac{\pi}{65}} \sin \left(\pi - \frac{\pi}{65} \right) = \frac{\sin \frac{\pi}{65}}{64 \sin \frac{\pi}{65}} = \frac{1}{64}
\end{aligned}$$

59. (A)

$$\begin{aligned}
&\cos A \cos 2A \cos 4A \dots \cos 2^{n-1} A \\
&= \frac{\sin 2^n A}{2^n \sin A} \quad \text{[Standard Result]} \\
&\therefore \text{(A) hold.}
\end{aligned}$$

60. (B)

$$\begin{aligned}
A + B + C &= \frac{3\pi}{2} \\
\cos 2A + \cos 2B + \cos 2C + 4 \sin A \sin B \sin C \\
&= 2 \cos(A+B) \cos(A-B) + 2 \cos^2 C - 1 + 4 \sin A \sin B \sin C \\
&= 2 \cos \left(3\frac{\pi}{2} - C \right) \cos(A-B) + 2 \cos^2 C - 1 + 4 \sin A \sin B \sin C \\
&= -2 \sin C \cos(A-B) + 1 - 2 \sin^2 C + 4 \sin A \sin B \sin C \\
&= -2 \sin C [\cos(A-B) - \cos(A+B)] + 4 \sin A \sin B \sin C \\
&= -4 \sin A \sin B \sin C + 4 \sin A \sin B \sin C + 1
\end{aligned}$$

61. (C)

$$\begin{aligned}
&\text{If } A + B + C = n\pi \text{ then} \\
&\tan A + \tan B + \tan C = \tan A \tan B \tan C
\end{aligned}$$

62. (B)

$$\begin{aligned}
&\left(\frac{\cos B}{\cos B} + \frac{\cos C}{\sin C} \right) \left(\frac{\cos C}{\sin C} + \frac{\cos A}{\sin A} \right) \left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} \right) \quad \left| \begin{array}{l} A + B = \pi - C \\ \Rightarrow \sin(A+B) = \sin C \end{array} \right. \\
&= \frac{\sin(B+C) \cdot \sin(A+C) \cdot \sin(A+B)}{\sin^2 A \cdot \sin^2 B \cdot \sin^2 C} \\
&= \operatorname{cosec} A \cdot \operatorname{cosec} B \cdot \operatorname{cosec} C
\end{aligned}$$

63. (A)

$$A + B + C = 180^\circ \Rightarrow \frac{A}{2} + \frac{B}{2} = 90^\circ - \frac{C}{2}$$

$$\therefore \tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(90^\circ - \frac{C}{2}\right) = \cot \frac{C}{2}$$

$$\therefore \frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \tan \frac{B}{2}} = \frac{1}{\tan \frac{C}{2}}$$

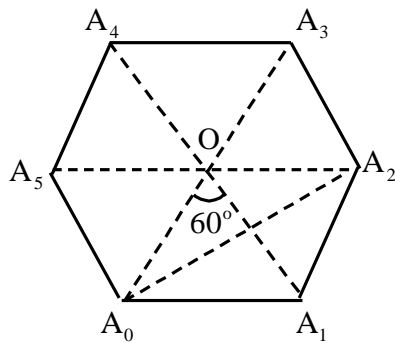
$$\Rightarrow \tan \frac{A}{2} \tan \frac{C}{2} + \tan \frac{B}{2} \tan \frac{C}{2} = 1 - \tan \frac{A}{2} \tan \frac{B}{2}$$

$$\Rightarrow \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} + \tan \frac{A}{2} \tan \frac{B}{2} = 1$$

Divide thro' out by $\tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2}$, we get

$$\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}$$

64. (C)



OA_0A_1 is equilateral triangle

$$A_0A_1 = 1$$

$$\cos 120^\circ = \frac{1^2 + 1^2 - A_0A_2^2}{2 \cdot 1 \cdot 1} \quad (\text{cosine law})$$

$$-\frac{2}{2} = 2 - A_0A_2^2$$

$$A_0A_2^2 = 3$$

$$A_0A_2 = \sqrt{3}$$

$$A_0A_4 = \sqrt{3}$$

$$A_0A_1 \times A_0A_2 + A_0A_4 = 1 \cdot \sqrt{3} \cdot \sqrt{3} = 3$$

65. (C)

$$\frac{b}{y} = \cot \theta \quad \& \quad \frac{a}{x} = \operatorname{cosec} \theta$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = \frac{a^2}{x^2} - \frac{b^2}{y^2}$$

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$$

66. (A)

$$\text{Required number of sides} = \frac{360^\circ}{36^\circ} = 10^\circ$$

\therefore (A) holds

67. (A)

$$\cos A = m \cos B \Rightarrow \frac{\cos A}{\cos B} = m$$

$$\text{Apply componendo to get } \frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$$

$$\Rightarrow -\frac{2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}} = \frac{m+1}{m-1}$$

$$\Rightarrow \cot \frac{A+B}{2} = \frac{m+1}{m-1} \tan \frac{B-A}{2}$$

68. (A)

$$\frac{\sin 2x}{\sin 2y} = n \Rightarrow \frac{\sin 2x + \sin 2y}{\sin 2x - \sin 2y} = \frac{n+1}{n-1} \quad \{\text{By componendo \& dividendo}\}$$

$$\Rightarrow \frac{2 \sin(x+y) \cos(x-y)}{2 \cos(x+y) \sin(x-y)} = \frac{n+1}{n-1}$$

$$\Rightarrow \frac{\tan(x+y)}{\tan(x-y)} = \frac{n+1}{n-1}$$

69. (D)

$$\frac{\sin \alpha}{\sin \beta} = \frac{5}{3}$$

$$\frac{\sin \alpha + \sin \beta}{\sin \alpha - \sin \beta} = \frac{8}{2} = 4 \quad (\text{C \& D})$$

$$\frac{2 \sin\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right)}{2 \cos\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\alpha-\beta}{2}\right)} = 4$$

$$\frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4$$

70. (B)

$$\operatorname{cosec} A (\sin B \cos C + \cos B \sin C)$$

$$= \operatorname{cosec} A \sin(B+C)$$

$$= \operatorname{cosec} A \sin(180^\circ - A) \quad \left[\because A+B+C = 180^\circ \right]$$

$$= \operatorname{cosec} A \sin A = 1$$

Hence, (B) holds

EXERCISE - 1 [C]

1. (0.625)

$$\sin 2\theta + \sin 2\phi = \frac{1}{2} \quad \& \quad \cos 2\theta + \cos 2\phi = \frac{3}{2}$$

$$\Rightarrow 2 \sin(\theta + \phi) \cdot \cos(\theta - \phi) = \frac{1}{2} \quad \dots(1)$$

$$2 \cos(\theta + \phi) \cdot \cos(\theta - \phi) = \frac{3}{2} \quad \dots(2)$$

Square & add (1) & (2)

$$\Rightarrow 4 \cos^2(\theta - \phi) \cdot [\sin^2(\theta + \phi) + \cos^2(\theta + \phi)] = \frac{1}{4} + \frac{9}{4}$$

$$\Rightarrow \cos^2(\theta - \phi) = \frac{10}{16}$$

2. (0.0625)

$$\begin{aligned} & [\cos 20^\circ \cdot \cos(60 - 20^\circ) \cdot \cos(60 + 20^\circ)] \cdot \cos 60^\circ \\ &= \frac{1}{4} \cdot \cos 60^\circ \cdot \cos 60^\circ = \frac{1}{16} \end{aligned}$$

3. (0.78)

$$\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\sin A = \frac{1}{\sqrt{5}} \Rightarrow \cos A = \sqrt{1 - \sin^2 A} = \frac{2}{\sqrt{5}}$$

$$\cos B = \frac{3}{\sqrt{10}} \Rightarrow \sin B = \sqrt{1 - \cos^2 B} = \frac{1}{\sqrt{10}}$$

$$\therefore \cos(A + B) = \frac{2}{\sqrt{5}} \cdot \frac{3}{\sqrt{10}} - \frac{1}{\sqrt{5}} \cdot \frac{1}{\sqrt{10}} = \frac{1}{\sqrt{2}}$$

$$\therefore A + B = \frac{\pi}{4}$$

4. (2)

$$A - B = \frac{\pi}{4} \Rightarrow \tan(A - B) = 1$$

$$\Rightarrow \tan A - \tan B = 1 + \tan A \cdot \tan B$$

$$\begin{aligned} \therefore (1 + \tan A) \cdot (1 - \tan B) &= 1 + (\tan A - \tan B) - \tan A \tan B \\ &= 2 \end{aligned}$$

5. (0.57)

Divide numerator & denominator by $\cos \theta$

$$\frac{2 \tan \theta - 5}{4 - 5 \tan \theta} = \frac{3 - 5}{4 - 5 \cdot \frac{3}{2}} = \frac{-2}{-\frac{7}{2}} = \frac{4}{7}$$

6. (0.26)

$$\frac{2 \sin\left(x + \frac{\pi}{6}\right) \cdot \cos\left(x + \frac{\pi}{6}\right)}{2}$$
$$= \frac{\sin\left(2x + \frac{\pi}{3}\right)}{2}$$

$$\text{Maximum} = \frac{1}{2} \text{ if } \sin\left(2x + \frac{\pi}{3}\right) = 1$$

$$\Rightarrow x = \frac{\pi}{12}$$

7. (0.75)

$$\cos^2 108^\circ + \cos^2 144^\circ$$
$$= \sin^2 18^\circ + \cos^2 36^\circ$$
$$= \left(\frac{\sqrt{5}-1}{4}\right)^2 + \left(\frac{\sqrt{5}+1}{4}\right)^2 = \frac{12}{16} = \frac{3}{4}$$

8. (1)

$$x \cdot 1 \cdot \frac{1}{2} = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}}$$
$$\Rightarrow x = 1$$

9. (1.5)

$$\cos^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{4} + \cos^2 \frac{5\pi}{12}$$
$$= \cos^2 15^\circ + \cos^2 45^\circ + \cos^2 75^\circ$$
$$= (\sin^2 75^\circ + \cos^2 75^\circ) + \cos^2 45^\circ$$
$$= 1 + \frac{1}{2} = \frac{3}{2}$$

10. (1)

$$\frac{\sin 85^\circ - \sin 35^\circ}{\cos 65^\circ}$$
$$= \frac{2 \sin 25^\circ \cdot \cos 60^\circ}{\cos 65^\circ} \quad \{\sin 25^\circ = \cos 65^\circ\}$$
$$= 1$$

11. (0.19)

$$\cos\left(\frac{7\pi}{6}\right) = \cos\left(\pi + \frac{\pi}{6}\right) = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$
$$\cos \frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos \frac{\pi}{3} = -\frac{1}{2}$$
$$\therefore \text{Ans} = \left(1 + \frac{\sqrt{3}}{2}\right) \left(1 + \frac{1}{2}\right) \cdot \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{\sqrt{3}}{2}\right)$$

$$\begin{aligned}
&= \left(1 - \frac{\sqrt{3}^2}{2^2}\right) \cdot \left(1 - \frac{1}{2^2}\right) \\
&= \left(1 - \frac{3}{4}\right) \cdot \left(1 - \frac{1}{4}\right) = \frac{3}{16}
\end{aligned}$$

12. (2)

$$\cos 54^\circ = \cos(90^\circ - 36^\circ) = \sin 36^\circ$$

13. (1)

$$\cot(102^\circ) = \cot(90^\circ + 12^\circ) = -\tan 12^\circ$$

$$\begin{aligned}
\therefore \text{Required} &= \cot 12^\circ (-\tan 12^\circ) - \tan 12^\circ \cdot \cot 66^\circ + \cot 66^\circ \cot 12^\circ \\
&= -1 + \cot 66^\circ \cdot [\cot 12^\circ - \tan 12^\circ] \\
&= -1 + \cot 66^\circ (2 \cot 24^\circ) \\
&= -1 + (\tan 24^\circ) \cdot (2 \cot 24^\circ) \\
&= -1 + 2 = 1 \qquad \qquad \qquad \{\cot 66^\circ = \tan 24^\circ\}
\end{aligned}$$

14. (1.69)

$$\begin{aligned}
\tan 2\alpha &= \tan(\alpha + \beta + \alpha - \beta) \\
&= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)} \qquad \dots(1)
\end{aligned}$$

$$\begin{aligned}
\cos(\alpha + \beta) &= \frac{4}{5} \\
\Rightarrow \tan(\alpha + \beta) &= \sqrt{\sec^2(\alpha + \beta) - 1} \\
&= \frac{3}{4} \qquad \dots(2)
\end{aligned}$$

$$\begin{aligned}
\sin(\alpha - \beta) &= \frac{5}{13} \\
\Rightarrow \tan(\alpha - \beta) &= \frac{\sin(\alpha - \beta)}{\sqrt{1 - \sin^2(\alpha - \beta)}} \\
&= \frac{5}{12} \qquad \dots(3)
\end{aligned}$$

Put (2) & (3) in (1)

$$\tan 2\alpha = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{\frac{14}{12}}{1 - \frac{5}{16}} = \frac{56}{33}$$

15. (0.40)

$$\begin{aligned}
\cos^2 A - \sin^2 B &= \cos(A + B) \cdot \cos(A - B) \\
&= \cos(48 + 12) \cdot \cos(48 - 12) \\
&= \cos 60^\circ \cdot \cos 36^\circ
\end{aligned}$$

$$= \frac{1}{2} \times \left(\frac{\sqrt{5}+1}{4} \right)$$

16. (1.73)

$$A = \frac{2\pi}{5}, B = \frac{\pi}{15}$$

$$A - B = \frac{2\pi}{5} - \frac{\pi}{15} = \frac{\pi}{3}$$

$$\Rightarrow \tan(A - B) = \sqrt{3}$$

$$\Rightarrow \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B} = \sqrt{3}$$

$$\Rightarrow \tan A - \tan B = \sqrt{3} + \sqrt{3} \tan A \tan B$$

$$\Rightarrow \text{Ans.} = \sqrt{3}$$

17. (3.87)

$$\frac{\sin x}{\sin y} \cdot \frac{\cos y}{\cos x} = \frac{1/2}{3/2} \quad \& \quad \frac{\tan x}{\tan y} = \frac{1}{3}$$

$$\Rightarrow \tan x = \frac{\tan y}{3} \quad \dots(1)$$

Square & add

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \frac{\sin^2 y}{4} + \frac{9}{4} \cos^2 y = 1$$

$$\Rightarrow 1 - \cos^2 y + 9 \cos^2 y = 4$$

$$\Rightarrow \cos^2 y = \frac{3}{8}$$

$$\Rightarrow \tan y = \sqrt{\sec^2 y - 1}$$

$$\tan y = \sqrt{\frac{5}{3}} \quad \dots(2)$$

(1) & (2)

$$\therefore \tan x = \frac{1}{3} \sqrt{\frac{5}{3}}$$

$$\therefore \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$= \frac{\frac{4 \tan y}{3}}{1 - \tan^2 y} = \frac{4 \tan y}{3 - \tan^2 y}$$

$$= \frac{4 \sqrt{5}/3}{3 - \frac{5}{3}} = \sqrt{15}$$

18. (1.5)

$$\sin \frac{7\pi}{8} = \sin \left(\pi - \frac{\pi}{8} \right) = \sin \frac{\pi}{8}$$

$$\sin \frac{5\pi}{8} = \sin \left(\pi - \frac{3\pi}{8} \right) = \sin \frac{3\pi}{8}$$

$$\begin{aligned}\text{Further } \sin \frac{3\pi}{8} &= \sin \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \\ &= \cos \frac{\pi}{8}\end{aligned}$$

$$\begin{aligned}\therefore \text{ Required} &= \sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \\ &= 2\end{aligned}$$

19. (0.5)

$$\begin{aligned}A + B &= 90^\circ \\ \therefore \sin A \sin B &= \sin A \cdot \sin(90 - A) \\ &= \sin A \cos A \\ &= \frac{1}{2} \sin 2A \\ \therefore \text{ Maximum} &= \frac{1}{2}\end{aligned}$$

20. (2)

$$\begin{aligned}\frac{\cot x - \tan x}{\cot 2x} &= \frac{\cos^2 x - \sin^2 x}{\left(\frac{2 \cos x \cdot \sin x}{2} \right) \cdot \cos 2x} = \frac{2 \cos 2x}{\sin 2x \cot 2x} = 2\end{aligned}$$

JEE Main : PYQ

1. (c)

$$\text{Take } \cos 72^\circ = \frac{\sqrt{5}-1}{4}$$

$$\text{Let } \sin 36^\circ = \alpha$$

$$\Rightarrow 1 - 2 \sin^2 36^\circ = \frac{\sqrt{5}-1}{4} \Rightarrow 4 - 8\alpha^2 = \sqrt{5} - 1$$

$$\Rightarrow 5 - 8\alpha^2 = \sqrt{5}$$

Take square both sides,

$$\Rightarrow (5 - 8\alpha^2)^2 = 5 \Rightarrow 25 + 64\alpha^4 - 80\alpha^2 = 5$$

$$\Rightarrow 64\alpha^4 - 80\alpha^2 + 20 = 0 \Rightarrow 16\alpha^4 - 20\alpha^2 + 5 = 0$$

2. (c)

$$\begin{aligned}\text{Given, } &2 \sin \left(\frac{\pi}{8} \right) \sin \left(\frac{2\pi}{8} \right) \sin \left(\frac{3\pi}{8} \right) \sin \left(\frac{5\pi}{8} \right) \sin \left(\frac{6\pi}{8} \right) \sin \left(\frac{7\pi}{8} \right) \\ &= 2 \sin^2 \frac{\pi}{8} \sin^2 \frac{2\pi}{8} \sin^2 \frac{3\pi}{8} = 2 \sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8} \cdot \frac{1}{2}\end{aligned}$$

$$\begin{aligned}
&= \sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8} = \sin^2 \frac{\pi}{8} \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \\
&= \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} = \frac{1}{4} \sin^2 \left(\frac{\pi}{4} \right) = \frac{1}{8}
\end{aligned}$$

3. (b)

$$\begin{aligned}
&3(\sin \theta - \cos \theta)^4 + 6(\sin \theta + \cos \theta)^2 + 4\sin^6 \theta \\
&= 3(1 - 2\sin \theta \cos \theta)^2 + 6(1 + 2\sin \theta \cos \theta) + 4\sin^6 \theta \\
&= 9 + 12\sin^2 \theta \cos^2 \theta + 4\sin^6 \theta \\
&= 9 + 12\cos^2 \theta(1 - \cos^2 \theta) + 4(1 - \cos^2 \theta)^3 \\
&= 9 + 12\cos^2 \theta - 12\cos^4 \theta + 4(1 - \cos^6 \theta - 3\cos^2 \theta + 3\cos^4 \theta) \\
&= 9 + 4 - 4\cos^6 \theta \\
&= 13 - 4\cos^6 \theta
\end{aligned}$$

4. (b)

$$\text{Given } 2\cos \theta + \sin \theta = 1$$

$$\text{Squaring both sides, we get } (2\cos \theta + \sin \theta)^2 = 1^2$$

$$\Rightarrow 4\cos^2 \theta + \sin^2 \theta + 4\sin \theta \cos \theta = 1$$

$$\Rightarrow 3\cos^2 \theta + (\cos^2 \theta + \sin^2 \theta) + 4\sin \theta \cos \theta = 1$$

$$\Rightarrow 3\cos^2 \theta + 1 + 4\sin \theta \cos \theta = 1$$

$$\Rightarrow 3\cos^2 \theta + 4\sin \theta \cos \theta = 0$$

$$\Rightarrow \cos \theta(3\cos \theta + 4\sin \theta) = 0$$

$$\Rightarrow 3\cos \theta + 4\sin \theta = 0$$

$$\Rightarrow 3\cos \theta = -4\sin \theta$$

$$\Rightarrow \frac{-3}{4} = \tan \theta = -\sqrt{\sec^2 \theta - 1} = \frac{-3}{4} \quad \left(\because \tan \theta = \sqrt{\sec^2 \theta - 1} \right)$$

$$\Rightarrow \sec^2 \theta - 1 = \left(\frac{-3}{4} \right)^2 = \frac{9}{16}$$

$$\Rightarrow \sec^2 \theta = \frac{9}{16} + 1 = \frac{25}{16} \Rightarrow \sec \theta = \frac{5}{4}$$

$$\text{or } \cos \theta = \frac{4}{5} \quad \dots(\text{i})$$

$$\text{Now, } \sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta + \left(\frac{4}{5} \right)^2 = 1$$

$$\sin^2 \theta + \left(\frac{4}{5} \right)^2 = 1 \Rightarrow \sin^2 \theta + \left(\frac{4}{5} \right) = 1$$

$$\sin \theta = \pm \frac{3}{5} \quad \dots(\text{ii})$$

Taking $\left(\sin \theta = -\frac{3}{5}\right)$ because $\left(\sin \theta = \frac{3}{5}\right)$ cannot satisfy the given equation,

$$\text{Therefore; } 7 \cos \theta + 6 \sin \theta = 7 \times \frac{4}{5} - 6 \times \frac{3}{5} = \frac{28}{5} - \frac{18}{5} = 2$$

5. (b)

Given : $\sin \theta = \frac{1}{2}$ and θ is acute angle

$$\therefore \theta = \frac{\pi}{6}$$

Also given, $\cos \phi = \frac{1}{3}$ and ϕ is acute angle.

$$\therefore 0 < \frac{1}{3} < \frac{1}{2}$$

$$\Rightarrow \cos \frac{\pi}{2} < \cos \phi < \cos \frac{\pi}{3} \text{ or } \frac{\pi}{3} < \phi < \frac{\pi}{2}$$

$$\therefore \frac{\pi}{3} + \frac{\pi}{6} < \theta < \phi < \frac{\pi}{2} + \frac{\pi}{6} \text{ or } \frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$$

$$\Rightarrow \theta + \phi \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$$

6. (b)

$$\text{Since, } \sin\left(\frac{\pi}{22}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{22}\right) = \cos \frac{5\pi}{11} = -\cos \frac{16\pi}{11}$$

$$\sin \frac{3\pi}{22} = \cos \frac{4\pi}{11}, \sin \frac{5\pi}{22} = \cos \frac{3\pi}{11} = -\cos \frac{8\pi}{11}$$

$$\sin \frac{7\pi}{22} = \cos \frac{2\pi}{11}, \sin \frac{9\pi}{22} = \cos \frac{\pi}{11}$$

$$\text{Now, } 2 \sin \frac{\pi}{22} \cdot \sin \frac{3\pi}{22} \cdot \sin \frac{5\pi}{22} \cdot \sin \frac{7\pi}{22} \cdot \sin \frac{9\pi}{22}$$

$$= 2 \cos \frac{\pi}{11} \cdot \cos \frac{2\pi}{11} \cdot \cos \frac{4\pi}{11} \cdot \cos \frac{8\pi}{11} \cos \frac{16\pi}{11}$$

$$= \frac{2 \times \sin 2^5 \frac{\pi}{11}}{2^5 \sin \frac{\pi}{11}} = \frac{1}{16} \quad \left(\because \sin 2^5 \frac{\pi}{11} = \sin \frac{\pi}{11} \right)$$

7. (a)

We are given that

$$\cot \alpha = 1, \sec \beta = \frac{-5}{3},$$

$$\Rightarrow \cos \beta = -\frac{3}{5}, \tan \beta = -\frac{4}{3}, \tan \alpha = 1$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{1 - \frac{4}{3}}{1 + \frac{4}{3} \times 1} = \frac{-1}{7}$$

$$\therefore \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{-1}{7} \text{ lies in IVth quadrant}$$

8. (b)

Given trigonometric equation is

$$\begin{aligned} & \cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right) \\ &= \left(\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) \right) \\ &= \cos\left(\frac{4\pi}{7}\right) \left[2\cos\left(\frac{2\pi}{7}\right) + 1 \right] \\ &= \cos\left(\frac{4\pi}{7}\right) \left[2\left(1 - 2\sin^2\left(\frac{\pi}{7}\right)\right) + 1 \right] \\ &= \cos\left(\frac{4\pi}{7}\right) \left[3 - 4\sin^2\left(\frac{\pi}{7}\right) \right] = \frac{\sin\left(3 \times \frac{\pi}{7}\right)}{\sin \frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right) \end{aligned}$$

Multiply both sides by 2

$$= \frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin \frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right)$$

Apply $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$$= \frac{\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2\sin \frac{\pi}{7}}, \text{ Here, } \sin(\pi) = 0.$$

$$= \frac{-\sin \frac{\pi}{7}}{2\sin \frac{\pi}{7}} = -\frac{1}{2}$$

9. (b)

$$\begin{aligned} & 16 \sin 20^\circ \sin 40^\circ \sin 80^\circ \\ &= 16 \sin 40^\circ \sin 20^\circ \sin 80^\circ \\ &= 4(4 \sin(60 - 20) \sin(20) \sin(60 + 20)) \\ &= 4 \times \sin(3 \times 20^\circ) \quad \left[\because \sin 3\theta = 4 \sin(60 - \theta) \times \sin \theta \times \sin(60 + \theta) \right] \end{aligned}$$

$$= 4 \times \sin 60^\circ$$

$$= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

10. (d)

Given trigonometric equation is

$$\begin{aligned} & \sin 12^\circ + \sin 12^\circ - \sin 72^\circ \\ &= \sin 12^\circ - 2 \cos 42^\circ \sin 30^\circ \\ &= \sin 12^\circ - \sin(90^\circ - 48^\circ) = \sin 12^\circ - \sin 48^\circ \\ &= -2 \cos 30^\circ \sin 18^\circ = -2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{5}-1}{4} = \frac{\sqrt{3}}{4} (1 - \sqrt{5}) \end{aligned}$$

11. (d)

$$x = \sum_{n=0}^{\infty} \cos^{2n} \theta = 1 + \cos^2 \theta + \cos^4 \theta + \dots \quad (\text{G.P.})$$

$$= \frac{1}{1 - \cos^2 \theta} = \operatorname{cosec}^2 \theta \Rightarrow \sin^2 \theta = \frac{1}{x} \quad \text{and} \quad y = \sum_{n=0}^{\infty} \sin^{2n} \phi = 1 + \sin^2 \phi + \sin^4 \phi + \dots \quad (\text{G.P.})$$

$$= \frac{1}{1 - \sin^2 \phi} = \sec^2 \phi \Rightarrow \cos^2 \phi = \frac{1}{y}$$

$$\text{Now, } z = \sum_{n=0}^{\infty} \cos^{2n} \theta \sin^{2n} \phi$$

$$= 1 + \cos^2 \theta \sin^2 \phi + (\cos^2 \theta \sin^2 \phi)^2 + \dots \quad (\text{G.P.})$$

$$= \frac{1}{1 - \cos^2 \theta \sin^2 \phi}$$

$$\Rightarrow x = \frac{1}{1 - \left(1 - \frac{1}{x}\right) \left(1 - \frac{1}{y}\right)} = \frac{xy}{xy - (x-1)(y-1)}$$

$$= \frac{xy}{x+y-1} \Rightarrow z(x+y) - z = xy \Rightarrow z(x+y) = xy + z$$

12. (d)

$$L + M = 1 - 2 \sin^2 \frac{\pi}{8} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad \dots(\text{i})$$

$$\text{and } L - M = -\cos \frac{\pi}{8} \quad \dots(\text{ii})$$

From equations (i) and (ii),

$$L = \frac{1}{2} \left(\frac{1}{\sqrt{2}} - \cos \frac{\pi}{8} \right) = \frac{1}{2\sqrt{2}} - \frac{1}{2} \cos \frac{\pi}{8} \quad \text{and} \quad M = \frac{1}{2} \left(\frac{1}{\sqrt{2}} + \cos \frac{\pi}{8} \right) = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

13. (b)

$$\begin{aligned} & \cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ \\ &= \left(\frac{1 + \cos 20^\circ}{2} \right) + \left(\frac{1 + \cos 100^\circ}{2} \right) - \frac{1}{2} (2 \cos 10^\circ \cos 50^\circ) \\ &= 1 + \frac{1}{2} (\cos 20^\circ + \cos 100^\circ) - \frac{1}{2} [\cos 60^\circ + \cos 40^\circ] \\ &= \left(1 - \frac{1}{4} \right) + \frac{1}{2} [\cos 20^\circ + \cos 100^\circ - \cos 40^\circ] \\ &= \frac{3}{4} + \frac{1}{2} [2 \cos 60^\circ \times \cos 40^\circ - \cos 40^\circ] = \frac{3}{4} \end{aligned}$$

14. (a)

$$\begin{aligned} f_k(x) &= \frac{1}{k} (\sin^k x + \cos^k x); \quad f_4(x) = \frac{1}{4} [\sin^4 x + \cos^4 x] \\ &= \frac{1}{4} \left[(\sin^2 x + \cos^2 x)^2 - \frac{(\sin 2x)^2}{2} \right] = \frac{1}{4} \left[1 - \frac{(\sin 2x)^2}{2} \right] \\ f_6(x) &= \frac{1}{6} [\sin^6 x + \cos^6 x] \\ &= \frac{1}{6} \left[(\sin^2 x + \cos^2 x)^3 - \frac{3}{4} (\sin 2x)^2 \right] = \frac{1}{6} \left[1 - \frac{3}{4} (\sin 2x)^2 \right] \\ \text{Now } f_4(x) - f_6(x) &= \frac{1}{4} - \frac{1}{6} - \frac{(\sin 2x)^2}{8} + \frac{1}{8} (\sin 2x)^2 = \frac{1}{12} \end{aligned}$$

15. (a)

We have

$$\begin{aligned} 5 \tan^2 x - 5 \cos^2 x &= 2(2 \cos^2 x - 1) + 9 \\ \Rightarrow 5 \tan^2 x - 9 \cos^2 x &= -2 + 9 \\ \Rightarrow 5 \tan^2 x &= 9 \cos^2 x + 7 \\ \Rightarrow 5(\sec^2 x - 1) &= 9 \cos^2 x + 7 \\ \text{Let } \cos^2 x = t &\Rightarrow \frac{5}{t} - 9t - 12 = 0 \\ \Rightarrow 9t^2 + 12t - 5 &= 0 \Rightarrow 9t^2 + 15t - 3t - 5 = 0 \\ \Rightarrow (3t - 1)(3t + 5) &= 0 \Rightarrow t = \frac{1}{3} \text{ as } t \neq -\frac{5}{3}. \\ \cos 2x = 2 \cos^2 x - 1 &= 2 \left(\frac{1}{3} \right) - 1 = -\frac{1}{3} \\ \cos 4x = 2 \cos^2 2x - 1 &= 2 \left(-\frac{1}{3} \right)^2 - 1 = -\frac{7}{9} \end{aligned}$$

16. (a)

$$4 + \frac{1}{2} \sin^2 2x - 2 \cos^4 x$$

$$4 + 2(1 - \cos^2 x) \cos^2 x - 2 \cos^4 x = -4 \left\{ \cos^4 x - \frac{\cos^2 x}{2} - 1 + \frac{1}{16} - \frac{1}{16} \right\}$$

$$= -4 \left\{ \left(\cos^2 x - \frac{1}{4} \right)^2 - \frac{17}{16} \right\}; \quad 0 \leq \cos^2 x \leq 1$$

$$\Rightarrow -\frac{1}{4} \leq \cos^2 x - \frac{1}{4} \leq \frac{3}{4} \Rightarrow 0 \leq \left(\cos^2 x - \frac{1}{4} \right)^2 \leq \frac{9}{16}$$

$$-\frac{17}{16} \leq \left(\cos^2 x - \frac{1}{4} \right)^2 - \frac{17}{16} \leq \frac{9}{16} - \frac{17}{16}$$

$$\frac{17}{4} \geq -4 \left\{ \left(\cos^2 x - \frac{1}{4} \right)^2 - \frac{17}{16} \right\} \geq \frac{4}{2}$$

$$M = \frac{17}{4} \Rightarrow m = 2; \quad M - m = \frac{17}{4} - 2 = \frac{9}{4}$$

17. (b)

$$\text{Let } \cos \alpha + \cos \beta = \frac{3}{2}$$

$$\Rightarrow 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = \frac{3}{2} \quad \dots(\text{i})$$

$$\text{and } \sin \alpha + \sin \beta = \frac{1}{2}$$

$$\Rightarrow 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = \frac{1}{2} \quad \dots(\text{ii})$$

$$\text{On dividing (ii) by (i), we get } \tan \left(\frac{\alpha + \beta}{2} \right) = \frac{1}{3}$$

$$\text{Given: } 0 = \frac{\alpha + \beta}{2} \Rightarrow 2\theta = \alpha + \beta$$

$$\text{Consider } \sin 2\theta + \cos 2\theta = \sin(\alpha + \beta) + \cos(\alpha + \beta)$$

$$= \frac{\frac{2}{3}}{1 + \frac{1}{9}} + \frac{1 - \frac{1}{9}}{1 + \frac{1}{9}} = \frac{6}{10} + \frac{8}{10} = \frac{7}{5}$$

18. (9)

$$\text{Let } f(x) = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$$

$$\Rightarrow f'(x) = 0 \Rightarrow \sin x = \frac{2}{3} \quad \left(\because x \in \left(0, \frac{\pi}{2} \right) \cos x \neq 0 \right)$$

$$\therefore f(x)_{\min} = \frac{4}{2/3} + \frac{1}{1-2/3} = 9; f(x)_{\max} \rightarrow \infty$$

(x) is continuous function $\therefore \alpha_{\min} = 9$

19. (80)

$$\begin{aligned} & \sin 10^\circ \left(\frac{1}{2} \cdot 2 \sin 20^\circ \sin 40^\circ \right) \cdot \sin 10^\circ \sin (60^\circ - 10^\circ) \sin (60^\circ + 10^\circ) \\ &= \sin 10^\circ \frac{1}{2} (\cos 20^\circ - \cos 60^\circ) \cdot \frac{1}{4} \sin 30^\circ \\ &= \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \sin 10^\circ \left(\cos 20^\circ - \frac{1}{2} \right) \\ &= \frac{1}{16} \cdot \frac{1}{2} (2 \sin 10^\circ \cos 20^\circ - \sin 10^\circ) \\ &= \frac{1}{32} (\sin 30^\circ - \sin 10^\circ - \sin 10^\circ) \\ &= \frac{1}{32} \left(\frac{1}{2} - 2 \sin 10^\circ \right) = \frac{1}{64} (1 - 4 \sin 10^\circ) \\ &= \frac{1}{64} - \frac{1}{16} (\sin 10^\circ) \end{aligned}$$

$$\text{Hence, } \alpha = \frac{1}{64} \Rightarrow \alpha^{-1} = 64$$

$$\text{Hence, } 16 + \alpha^{-1} = 80$$

20. (1)

$$\frac{\sqrt{2} \sin \alpha}{\sqrt{1 + \cos 2\alpha}} = \frac{1}{7} \text{ and } \sqrt{\frac{1 - \cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$$

$$\Rightarrow \frac{\sqrt{2} \sin \beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$$

$$\therefore \tan \alpha = \frac{1}{7} \text{ and } \sin \beta = \frac{1}{\sqrt{10}}; \tan \beta = \frac{1}{3}$$

$$\therefore \tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2 \cdot \frac{1}{3}}{1 - \frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan \alpha + \tan 2\beta}{1 - \tan \alpha \tan 2\beta} = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \cdot \frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$$