PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DHULE

IIT – JEE: 2024 **PART TEST - 1** **DATE: 11/11/23**

MAIN

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	B	**31.**	A	**61.**	B
2.	B	**32.**	D	**62.**	B
3.	C	**33.**	D	**63.**	A
4.	C	**34.**	B	**64.**	C
5.	B	**35.**	A	**65.**	A
6.	D	**36.**	B	**66.**	C
7.	B	**37.**	B	**67.**	B
8.	B	**38.**	B	**68.**	B
9.	A	**39.**	D	**69.**	C
10.	B	**40.**	D	**70.**	B
11.	B	**41.**	A	**71.**	A
12.	A	**42.**	B	**72.**	D
13.	B	**43.**	D	**73.**	D
14.	B	**44.**	D	**74.**	C
15.	D	**45.**	D	**75.**	B
16.	B	**46.**	D	**76.**	D
17.	B	**47.**	A	**77.**	C
18.	B	**48.**	B	**78.**	B
19.	B	**49.**	B	**79.**	B
20.	A	**50.**	C	**80.**	B
21.	8	**51.**	20	**81.**	7
22.	5	**52.**	70	**82.**	1
23.	10	**53.**	72	**83.**	0
24.	17	**54.**	83	**84.**	1
25.	3	**55.**	9	**85.**	4
26.	10	**56.**	2	**86.**	6.25
27.	5	**57.**	8	**87.**	0
28.	3	**58.**	3	**88.**	22.50
29.	1.41	**59.**	20	**89.**	8
30.	2	**60.**	5	**90.**	98

PART (A) : PHYSICS

ANSWER KEY

1.	(B)	2.	(B)	3.	(C)	4.	(C)	5.	(B)
6.	(D)	7.	(B)	8.	(B)	9.	(A)	10.	(B)
11.	(B)	12.	(A)	13.	(B)	14.	(B)	15.	(D)
16.	(B)	17.	(B)	18.	(B)	19.	(B)	20.	(A)
21.	(8)	22.	(5)	23.	(10)	24.	(17)	25.	(3)
26.	(10)	27.	(5)	28.	(3)	29.	(1.41)	30.	(2)

SOLUTIONS

1. (B)

Distance of particle from earth's centre at Q and P are

$r_Q = 4R$ ...(i)

$r_P = R$ ...(ii)

By conservation of angular momentum between positions Q and P,

$mv_P r_P = mv_Q r_Q \sin 150°$

$\Rightarrow v_P = 2v_Q$ ...(iii) [using Eqs. (i) and (ii)]

By conservation of energy between Q and P,

$$\frac{1}{2}mv_P^2 - \frac{GMm}{R} = \frac{1}{2}mv_Q^2 - \frac{GMm}{4R}$$

$$\Rightarrow \frac{1}{2}m(2v_Q)^2 - \frac{GMm}{R} = \frac{1}{2}mv_Q^2 - \frac{GMm}{4R}$$ [using Eq. (iii)]

$$\Rightarrow v_Q = \sqrt{\frac{GM}{2R}} = \frac{v_e}{2}$$

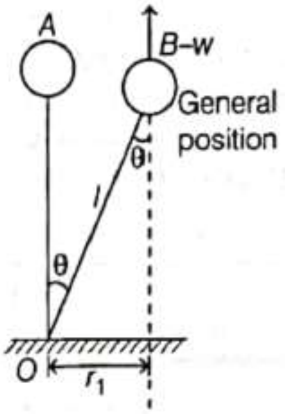
(as escape velocity is given by formula $v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$)

2. (B)

Let, volume of the balloon is V.

Weight of the balloon = $w = \rho Vg$

Also, buoyancy by surrounding air $= 3\rho Vg$



Net force F on the balloon providing torque will be equal to
$B - w = 3\rho Vg - \rho Vg = 2\rho Vg$ as shown in the figure.

Perpendicular distance r_1 of line of action of force from axis O at general position is $r_1 = l\sin\theta \approx l\theta$ as θ is given to be small. Therefore, restoring torque, $\rho = F \cdot r_1 = 2\rho Vgl\theta$

$\Rightarrow$ Angular acceleration of balloon (having moment of inertia I equal to ml^2) is

$$\alpha = \frac{\tau}{I} = \frac{2\rho Vgl\theta}{ml^2} = \frac{2\rho Vg\theta}{ml}$$

As mass m of balloon / gas is ρV, therefore

$$\alpha = \frac{2\rho Vg}{ml}\theta = \frac{2\rho Vg}{\rho Vl}\theta = \frac{2g}{l}\theta$$

As in angular oscillation, $\alpha = \omega^2\theta$, where, ω is angular frequency, we get

$$\omega^2\theta = \frac{2g}{l}\theta \Rightarrow \theta = \sqrt{\frac{2g}{l}}$$

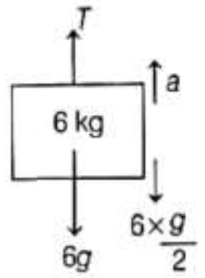
As, time to move from B (one extreme) to the other extreme is half of time period of oscillation.

$$\therefore \quad t = \frac{T}{2} = \frac{\frac{2\pi}{\omega}}{2}$$

$$= \frac{\pi}{\omega} = \pi\sqrt{\frac{l}{2g}}$$

3. (C)

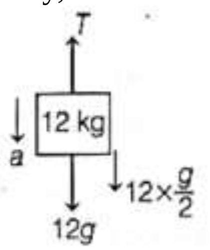
Let the tension in string be T and acceleration of block be a w.r.t. lift.
From FBD of 6 kg block in lift.



$$6 \times a = T - 6g - 6 \times \frac{g}{2}$$

$$6 \times a = T - 9g \qquad \text{...(i)}$$

Similarly,



$$12 \times a = 12g + 12 \times \frac{g}{2} - T$$

$$12a = 18g - T \qquad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$6a+12a=18g-9g$$

or $18a=9g$

or $a=\frac{9g}{18}=\frac{9\times10}{18}=5\ \text{m/s}^2$

Putting this value in Eq. (i), we get

$$6\times5=T-9\times10$$

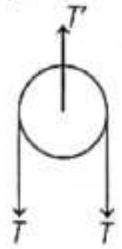
$\Rightarrow$ $T=30+90=120\ \text{N}$

Spring force, $T'=T+T=2T$

or $=2\times120=240\ \text{N}=240\ \text{N}$

As we know that, spring balance always measure tension force across its one end.

FBD of pulley,



Reading of spring balance $=\frac{\text{Spring force}}{g}$

$$=\frac{240}{10}=24\ \text{kg}$$

4. (C)

Speed of maximum height $=u\cos\theta=\frac{u}{2}\ \Rightarrow\ \theta=60^\circ$

$$R=\frac{u^2\sin120^\circ}{g}=\frac{\sqrt{3}u^2}{2g}$$

5. (B)

Point on periphery of a rolling sphere follows cycloid, as shown in figure-1

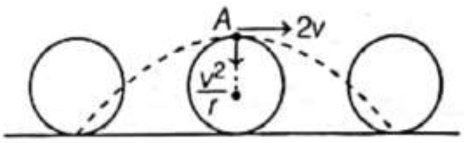


Fig.1

While at top (position A) velocity of the point is $2v$ while acceleration is $\frac{v^2}{r}$ in terms of radius r of the sphere. Radius of curvature of a path is given by

$$R=\frac{(\text{Speed})^2}{\text{Normal acceleration}}$$

Therefore, $R_1=\frac{(2v)^2}{\frac{v^2}{r}}=4r$ ...(i)

As in elastic collision of identical objects velocities are exchanged.
So, on collision, sphere P will come at rest while sphere Q will start moving with v but as normal force (during collision) will pass through centre of mass of sphere (as shown in figure-2) it will provide no torque.

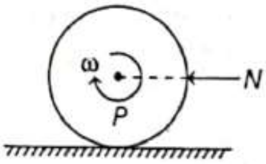


Fig.2

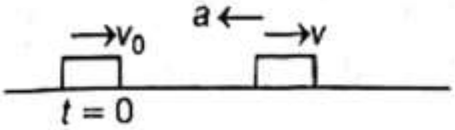
So, angular speed will remain unchanged. Thus, after collision sphere P will execute pure rotation about axis through centre of mass. Therefore, top particle A will move on circle of radius r.

Therefore, $R_2 = r$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{R_1}{R_2} = 4$$

6. (D)
A block on rough horizontal surface is as shown below



According to the question, retardation,
$a = -bv$ (Here, b = constant)

$$\Rightarrow \frac{dv}{dt} = -bv$$

Integrating both sides, we get

$$\Rightarrow \int_{v_0}^{v} \frac{dv}{v} = -b\int_0^t dt \Rightarrow v = v_0 e^{-bt}$$

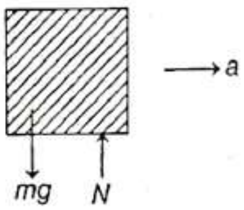
At $t = 1\,\text{s}$, $v = \frac{v_0}{2}$ (given)

$$\Rightarrow e^{-b} = \frac{1}{2}$$

At $t = 3\text{s}, v = v_0\left(e^{-b}\right)^3 = \frac{v_0}{8}$

As $a \propto v$, it will also become (1/8)th.

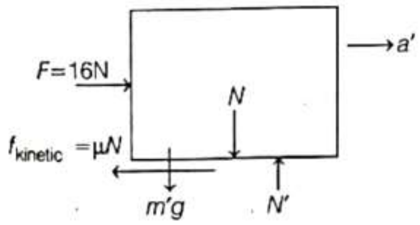
7. (B)
FBD of block is as shown
Note that there is no spring force, as spring is in natural length



For vertical equilibrium,

$N = mg = 2\times10 = 20\,\text{N}$

FBD of box is as shown.



For vertical equilibrium,

$N' = N + m'g$

$= 20 + 30 = 50$ N

Liming friction between box and ground $= \mu N' = 0.2\times50 = 10$ N

As, external force, $F = 16\,\text{N} > f_{\text{limiting}}$ block will slip and therefore it will experience kinetic friction as given by

$f_K = \mu N' = 0.2\times50 = 10$ N

Therefore, acceleration of box,

$$a' = \frac{F - f_K}{\text{Mass of mass}} = \frac{16-10}{3} = 2\,\text{m/s}^2$$

While acceleration (a) of block is zero, as there is not horizontal force on it.

8. (B)

From the given U *versus* x graph, between $x = 0$ and $x = x_1$, assuming $U = kx^2$ (since, the graph is parabolic in nature)

So, force, $F = -\frac{dU}{dx} = -\frac{d(kx^2)}{dx} = -2kx$

$\Rightarrow$ $F \propto -x$

$\Rightarrow$ So, the graph between F and x between $x = 0$ and $x = x_1$ will be a straight line with negative slope.

Between $x = x_1$ to $x = x_2$.

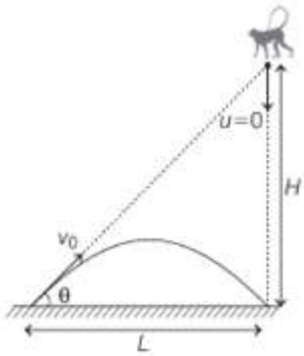
$F = -\frac{dU}{dx} = -$(slope of U-x graph)

$= -(-k') = k' =$ constant

$\Rightarrow$ Force is a positive constant.

Thus, the conservative force acting on a system as a function of x is correctly represented in option (B).

9. (A)



For minimum speed,arrow will hit the monkey just at ground.

$$T = \frac{2v_0 \sin\theta}{g} = \sqrt{\frac{2H}{g}}$$

$$\Rightarrow \quad v_0 \sin\theta = \sqrt{\frac{gH}{2}} \qquad \text{...(i)}$$

$$R = L = (v_0 \cos\theta)\sqrt{\frac{2H}{g}}$$

$$v_0 \cos\theta = \sqrt{\frac{gL^2}{2H}} \qquad \text{...(ii)}$$

Adding and squaring Eqs. (i) and (ii), we get

$$v_0 = \sqrt{\frac{g(H^2 + L^2)}{2H}}$$

10. (B)

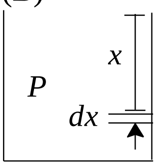
$$\frac{Gm(8m)}{3R} = \frac{1}{2}\frac{(8m)m}{9m}v_0^{\ 2}$$

$$v_0 = \sqrt{\frac{3Gm}{4R}}$$

$$\frac{G(m)(8m)}{3x} == \frac{1}{2}\left(\frac{8m}{9}\right)\frac{1}{4}v_0^{\ 2}$$

Solving $x = 4R$

11. (B)



$$P = P_0 + \rho g x$$

$$F = (P_0 + \rho g x)\ell dx$$

$d\tau = x\left(P_0 + \rho g x\right)\ell dx$

$\tau = \int_0^4 \times\left(P_0 + \rho g x\right)\ell dx$

$= P_0\ell\int_0^4 x\,dx + \rho g\ell\int_0^4 x^2 dx$

$= P_0\ell\left[\frac{4^2}{2}\right] + \rho g\ell\left[\frac{4^3}{3}\right]$

$= P_0\ell[8] + \rho g\ell\left[\frac{64}{3}\right]$

Torque due to P_{atm} will Cancel out

$\Rightarrow \quad \tau = \rho g\ell\left[\frac{64}{3}\right]$

Torque due to applied force

$\Rightarrow F \times 3$

$\therefore\ 3F = \rho g\ell\left(\frac{64}{3}\right)$

$F = \rho g\ell \times \frac{64}{9} \qquad (\because\ l = 4\,\text{m})$

$\Rightarrow \left(\rho g\,\frac{256}{9}\right)$

12. (A)

The force which increases the length of the spring by x = 2.5 cm is F = mg $\sin\theta$. Therefore, the spring constant is

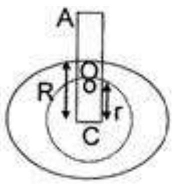
$k = \frac{F}{x} = \frac{mg\sin\theta}{x}$

Now time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg\sin\theta / x}} = 2\pi\sqrt{\frac{x}{g\sin\theta}}$

Putting x = 2.5 cm = 2.5 $\times 10^{-2}$m, g = 9.8 ms^{-2} and $\theta = 30^o$, we get $T = \pi / 7$ second, which is choice (A).

13. (B)

$a = -\frac{GM}{R^3}x$



$a = -\omega^2 x$

So, $\omega = \sqrt{\frac{GM}{R^3}}$

$V_C = \omega A = \sqrt{\frac{GM}{R^3}} \times R$

$V_c = \sqrt{\frac{GM}{R}}$

After collision velocity of ball towards.

$A, V_c' = eV_c = 0.2\sqrt{\frac{GM}{R}} = \frac{1}{5}\sqrt{\frac{GM}{R}}$

Let now amplitude be A', then

$A' = \frac{V_c'}{\omega} = \frac{\frac{1}{5}\sqrt{\frac{GM}{R}}}{\sqrt{\frac{GM}{R^3}}} = \frac{R}{5}$

Net distance = $R + (R/5) + (R/5) = (7/5)R$

14. (B)

$[a] = [t^2] = [T^2]$ and $\left[\frac{t^2}{bx}\right] = [p]$

$\Rightarrow \quad [b] = \left[\frac{t^2}{px}\right] \Rightarrow [b] = \left[\frac{T^2}{ML^{-1}T^{-2}L}\right] = [M^{-1}T^4]$

So, $\left[\frac{a}{b}\right] = \left[\frac{T^2}{M^{-1}T^4}\right] = [MT^{-2}]$

15. (D)

$F - f = ma_{com}$ ...(i)

Torque about centre

$f \times R = \frac{2}{5}mR^2\alpha$ ...(ii)

No slipping

$a = \alpha R$ ...(iii)

From (ii)

$f \times R = \frac{2}{5}mR^2\frac{a}{R}$

$f = \frac{2}{5}ma$ ____(A)

Also from (i)

$F - f = ma \qquad \because\ f = 4mg$

$\Rightarrow\ F - f = ma$

$\Rightarrow\ F - \mu mg = m\left(\frac{5f}{2m}\right)$

$F = \mu mg + \frac{5}{2}\mu mg = \frac{7}{2}\mu mg$

16. (B)

For limiting condition, $\mu = \frac{m_B}{m_A + m_C}$

$\Rightarrow\ 0.25 = \frac{5}{10 + m_C}$

$\Rightarrow\ m_C = 10\text{ kg}$

17. (B)

Even if tangential acceleration is decreasing speed of particle can increase resulting in increase of centripetal acceleration. Hence if speed increases we can't say acceleration of particle is increasing or decreasing. Therefore, both statements are independently true.

18. (B)

As there is no external horizontal force on the system, centre of mass of the system will not accelerate and so will remain at rest. Therefore, displacement of CM will be zero. So, statement given in option (C) is correct. Also, displacement of CM can be written as $s_{CM} = \frac{m_p s_p + m_s s_s}{m_p + m_s}$

Assuming displacement of sled equal to x as shown in the figure-1, we can put respective values. Using the figure, we get

$\Rightarrow\ 0 = \frac{-m(l - x) + 3mx}{m + 3m}$

$\Rightarrow\ x = \frac{l}{4} =$ Displacement of sled

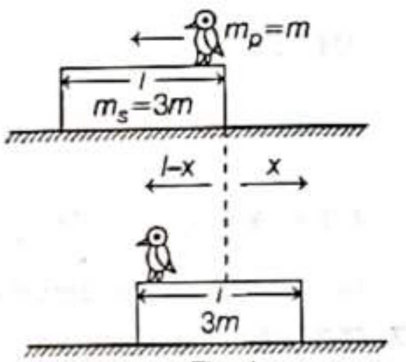
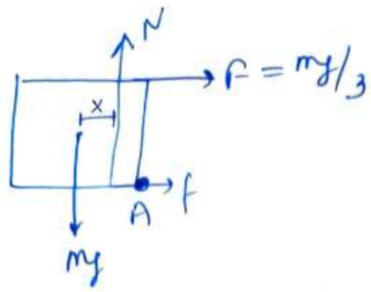


Fig. 1

Therefore, displacement of penguin is

$l - x = l - \frac{l}{4} = \frac{3l}{4}$

19. (B)



Torque balance about A

$$N\left(\frac{a}{2}-x\right)+\frac{mg}{3}(a)=mg\times\left(\frac{a}{2}\right)$$

$$N\left(\frac{a}{2}-x\right)=\frac{mg\,a}{6}$$

$$mg\left(\frac{a}{2}-x\right)=mg\,\frac{a}{6}$$

$$x=\frac{a}{3}$$

20. (A)

$$y=\frac{t^2+t^3}{8}$$

$$\frac{dy}{dt}=\frac{2t+3t^2}{8}$$

$$dy=\left(\frac{2t+3t^2}{8}\right)dt$$

$$100\times\frac{dy}{y}=\frac{\left(\frac{2t+3t^2}{8}\right)t}{y}\left(\frac{dt}{t}\right)\times 100$$

$$\%\text{ error in } y=\left(\frac{2t^2+3t^3}{8y}\right)\left(\frac{dt}{t}\times 100\right)$$

$$=\frac{32}{8\left(\frac{12}{8}\right)}\times(0.1\%)=0.267\%$$

21. (8)

First we have to find a point where the resultant field due to both is zero. Let the point P be at a distance x from centre of bigger star.

$$\Rightarrow\quad \frac{G(16M)}{x^2}=\frac{GM}{(10a-x)^2}$$

$\Rightarrow \quad x = 8a \qquad\qquad$ (from O_1)

i.e. Once the body reaches P, the gravitational pull of attraction due to M takes the lead to make m move towards it automatically. So, a minimum KE or velocity has to be imparted to m from surface of 16 M such that it is just able to cross P. By law of conservation of energy, total mechanical energy at surface of bigger star

= Total mechanical energy at P

$$\Rightarrow \quad k_{min} + \left[-\frac{G(16M)m}{2a} - \frac{GMm}{8a}\right] = 0 + \left[-\frac{GMm}{2a} - \frac{G(16M)m}{8a}\right]$$

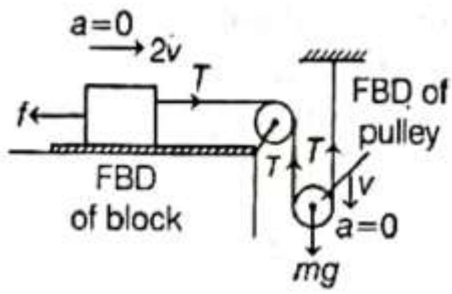
$$\Rightarrow \quad |k_{min}| = \frac{GMm}{8a}(45)$$

Comparing with given value, we get

$k = 8$

22. (5)

Let velocity of pulley be v. by pulley constraint relation, velocity of block will be $2v$. FBD of block and disc are as shown. As acceleration for block and disc is zero, from FBD of disc, $2T = mg$



$$\Rightarrow T = \frac{mg}{2} \qquad \ldots(i)$$

From FBD of block, we get

$$T = f \qquad \ldots(ii)$$

$$\frac{mg}{2} = f$$

$$\Rightarrow \frac{mg}{2} = \eta A \frac{2v}{d} \Rightarrow \eta = \frac{mgd}{4Av}$$

$$= \frac{20\times10^{-3}\times10\times0.2\times10^{-3}}{4\times10^{3}\times10^{-4}\times2\times10^{-2}}$$

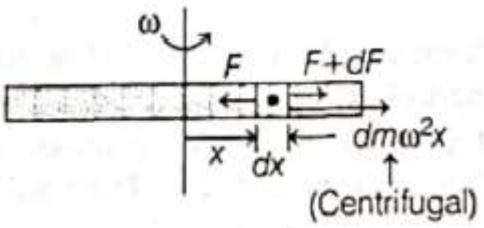
$$= 5\times10^{-3} \text{ Pa-s}$$

$\Rightarrow \quad x = 5$

23. (10)

Stress is zero at free ends and maximum at middle so rod will rupture at middle.

By Newton's second law for the element shown in the figure,



$$F-(F+dF)=dmx\omega^2$$
$$-\int dF=\int \rho A\omega^2 x dx$$

Where, ω is the speed of rotation and F is internal force.

$$\Rightarrow \quad -[F]_0^F=\rho A\omega^2\left[\frac{x^2}{2}\right]_{I/2}^{x}$$

$$\Rightarrow \quad F=\rho\frac{A\omega^2}{2}\left[\left(\frac{I}{2}\right)^2-x^2\right]$$

At $x=0,\ F=\frac{\rho A\omega^2 I^2}{8} \Rightarrow \frac{F}{A}=\frac{\rho\omega^2 I^2}{8}$

For rupture, $\frac{F}{A}=\sigma \Rightarrow \frac{\rho\omega^2 I^2}{8}=\sigma$

$$\Rightarrow \quad \omega=\sqrt{\frac{8\sigma}{\rho I^2}}$$

$\Rightarrow$ Rotation frequency,

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{8\sigma}{\rho I^2}}$$

$$\Rightarrow \quad f=\frac{1}{2\pi}\sqrt{\frac{8\times 2.5\times 10^8}{9\times 10^3\times\left(\frac{2\sqrt{2}}{3\pi}\right)^2}}=250 s^{-1}$$

$$=25n\,(\text{given}) \Rightarrow n=10$$

24. (17)

If centre of mass is at A,

$$A_{\text{square}}\, x_{\text{square}}=A_{\text{triangle}}\, x_{\text{triangle}}$$

$$a^2\frac{a}{2}=\frac{1}{2}ab\sin\theta\times\frac{1}{3}b\sin\theta$$

or $\frac{b}{a}=\sqrt{\frac{13}{4}} \qquad \left[\because \sin^2\theta=\left(1-\frac{a^2}{4b^2}\right)\right]$

$\therefore \quad m+n=13+4=17$

25. (3)

Given that, $a_C=kt$

$$\Rightarrow \quad \frac{v^2}{r}=kt \Rightarrow v=\sqrt{krt}$$

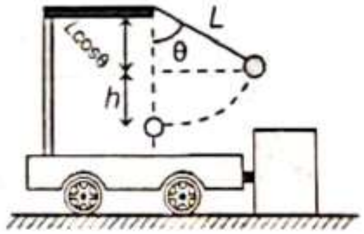
Tangential acceleration force, $P=\mathbf{F}\cdot\mathbf{v}$

$$=[ma_C+ma_t]\cdot\mathbf{v}=0+ma_t v$$

$$=m\frac{\sqrt{kr}}{2\sqrt{t}}\cdot\sqrt{krt}=\frac{mkr}{2}=\frac{2\times 1\times 3}{2}=3\text{W}$$

26. (10)

As string does no work on the ball, energy conservation can be applied. i.e. Loss in kinetic energy = Gain in potential energy



$\Rightarrow \frac{1}{2}mv^2 = mgh = mg(L - L\cos\theta)$

$\Rightarrow v = \sqrt{2gL(1-\cos\theta)}$

On putting values, we get

$c = \sqrt{10}$ m/s .

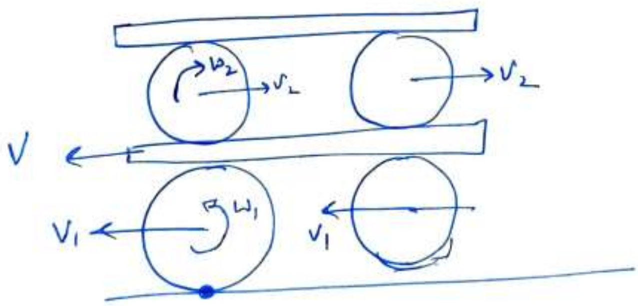
Therefore, square of speed in $10\ \text{m}^2/\text{s}^2$.

27. (5)

$R = \sqrt{2g(H-y)}\sqrt{\frac{2y}{g}} = y \Rightarrow \frac{4H}{5}$

$\therefore$ Rate of addition = $Av = a\sqrt{\frac{2gH}{5}}$

28. (3)



$V_1 = \omega_1 R$

$V_1 + \omega_1 R = V$

$\Rightarrow 2\omega_1 R = V$

$\Rightarrow \omega_1 = \frac{V}{2R}$

$V_2 + \omega_2 R = 2V$ .....(i)

$\omega_2 R - V_2 = V$ .....(ii)

(i)+(ii)

$2\omega_2 R = 3v$

$\Rightarrow \omega_2 = \frac{3v}{2R}$

$$\frac{\omega_2}{\omega_1}=\frac{3v/2R}{v/2R}=\frac{3}{1}$$

29. (1.41)

We use

$$f_0=\frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$$

Where, l is distance between point of suspension and centre of mass of the body

Thus, for the stick of length L and mass m frequency is

$$f_0=\frac{1}{2\pi}\sqrt{\frac{m.g.\frac{L}{2}}{\left(mL^2/3\right)}}=\frac{1}{2\pi}\sqrt{\frac{3g}{2L}}$$

When bottom half of the stick is cut off we use

$$f_0=\frac{1}{2\pi}\sqrt{\frac{\frac{m}{2}.g.\frac{L}{4}}{\frac{m}{2}\frac{(L/2)^2}{3}}}=\frac{1}{2\pi}\times\sqrt{\frac{3g}{L}}=\sqrt{2}f_0$$

30. (2)

$$I=mr^2$$

PART (B) : CHEMISTRY

ANSWER KEY

31.	(A)	32.	(D)	33.	(D)	34.	(B)	35.	(A)
36.	(B)	37.	(B)	38.	(B)	39.	(D)	40.	(D)
41.	(A)	42.	(B)	43.	(D)	44.	(D)	45.	(D)
46.	(D)	47.	(A)	48.	(B)	49.	(B)	50.	(C)
51.	(20)	52.	(70)	53.	(72)	54.	(83)	55.	(9)
56.	(2)	57.	(8)	58.	(3)	59.	(20)	60.	(5)

SOLUTONS

31. (A)

$$N_1 \times \frac{hc}{4000} \times 0.5 = N_2 \times \frac{hc}{5000} \Rightarrow \frac{N_2}{N_1} = \frac{5}{8}$$

32. (D)

Separation energy = $13.6 \times \frac{Z^2}{n^2}$

33. (D)

$$Q = \frac{(0.1)^3}{1^2 \times 1^2} = 10^{-3} < K,$$

Hence, reaction moves in forward direction.

34. (B)

$$K_{P_1} = K_{P_2}$$

$$\frac{\alpha_1^2}{1-\alpha_1^2} \times P_1 = \frac{\alpha_2^2}{1-\alpha_2^2} \times P_2$$

$$\frac{P_1}{P_2} = \frac{0.25}{0.75} \times \frac{0.84}{0.16} = \frac{7}{4}$$

35. (A)

As $P \uparrow$, ice melts because $H_2O(l)$ occupy lesser volume.

36. (B)

37. (B)

$$\left[H^+\right] = \frac{10^{-1} \times 50 + 10^{-2} \times 50}{100} = \frac{5+0.5}{100} = \frac{5.5}{100}$$

$$= 5.5 \times 10^{-2}$$

$$pH = 2 - \log 5.5 = 1.26$$

38. (B)

$K_h = ch^2 = 0.1 \times 10^{-4} = 10^{-5}$

$10^{-5} = \frac{[OH^-]^2}{0.4} \Rightarrow [OH^-] = 2 \times 10^{-3}$

or $[H^+] = 5 \times 10^{-12}$

39. (D)

$pH_1 = pK_a + \log\left(\frac{\frac{1}{3}}{\frac{2}{3}}\right)$

$pH_2 = pK_a + \log\left(\frac{\frac{2}{3}}{\frac{1}{3}}\right)$

$pH_2 - pH_1 = 2\log 2$

40. (D)

Same species undergoes oxidation as well as reduction in disproportionation reaction.

41. (A)

Valency factor of $Cl_2 = \frac{2 \times 10}{2+10} = \frac{5}{3}$

E.W. $= \frac{71}{5/3} = 42.6$

42. (B)

$\omega_A = 2\omega_B ; \Delta T_A = \Delta T_B$

$\Rightarrow \Delta U_A = \Delta U_B$

$\Rightarrow C_A.\Delta T - \omega_A = C_B \Delta T - \frac{\omega_A}{2}$

$\Rightarrow C_A > C_B$

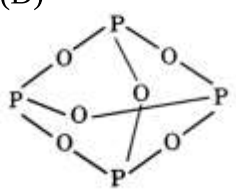
43. (D)

In (D), product formed is liquid while in other cases it is gas.

44. (D)

$q_{rev} = 0$

45. (D)



Enthalpy of atomisation = $x + y \times 12$

46. (D)

$$\frac{1}{[A]_t^{n-1}} = \frac{1}{[A]_0^{n-1}} + (n-1)kt$$

$$t_{7/8} = \frac{1}{(n-1)k}\left[8^{n-1} - 1\right].[A]_0^{1-n}$$

$$t_{1/2} = \frac{1}{(n-1)k}\left[2^{n-1} - 1\right].[A]_0^{1-n}$$

47. (A)

Highest *i* is of $Al_2(SO_4)_3 = (1 + 4 \times 0.9)$

48. (B)

$i \rightarrow NaHSO_4 = 3;\ i \rightarrow NaCl = 2$

49. (B)

$P_{ideal} = 0.92 \times 800 + 0.08 \times 300 = 760 \text{ or } 1\text{atm} > P_{Obs}$

$\Rightarrow\ -ve$ deviation solution.

$\Rightarrow\ \Delta V_{mix} < 0$ and $\Delta H_{mix} < 0$

50. (C)

$$\frac{0.224}{11.2} = \frac{V}{5.6} \Rightarrow V = 0.112\,L$$

51. (20)

$$\frac{30 \times 6.023 \times 10^{23} \times 2}{18.069 \times 10^{23}} = 20\,g$$

52. (70)

$$\frac{70}{100} \times \frac{W}{98} \times 2 = 1 \Rightarrow 70\ g$$

53. (72)

$0.9 \times 0.8 = 0.72$

So, overall yield = 72%

54. (83)

$$\Delta\upsilon = 150\left[\frac{1}{1^2} - \frac{1}{1.5^2}\right] = 83\,V$$

55. (9)

$$E_n = -\frac{Rhc}{9} \text{ for } n = 3$$

For single electron species; energy of 3s = 3p = 3d

So, degeneracy = 1 + 3 + 5 = 9

56. (2)
$x^x.S^{x+1} = 4\times10^{-12}; S = 10^{-4};$
x = 2 satisfies the equation.

57. (8)
$2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

58. (3)
Rate $= k\,P_{NO}^2\,P_{H_2}$
Order = 2 + 1 = 3

59. (20)
$\Delta H = E_{af} - E_{ab} = 180 - 200 = -20\,kJ/mol$

60. (5)
$\alpha = \frac{\wedge}{\wedge^{\circ}} = \frac{10}{200}$ or 5%

PART (A) : MATHEMATICS

ANSWER KEY

61.	(B)	62.	(B)	63.	(A)	64.	(C)	65.	(A)
66.	(C)	67.	(B)	68.	(B)	69.	(C)	70.	(B)
71.	(A)	72.	(D)	73.	(D)	74.	(C)	75.	(B)
76.	(D)	77.	(C)	78.	(B)	79.	(B)	80.	(B)
81.	(7)	82.	(1)	83.	(0)	84.	(1)	85.	(4)
86.	(6.25)	87.	(0)	88.	(22.50)	89.	(8)	90.	(98)

SOLUTIONS

61. (B)

$\alpha^2 = 2\alpha + 1$

$\alpha^3 = 5\alpha + 2$

$\alpha^4 = 12\alpha + 5$

$5\alpha^4 = 5(12\alpha + 5) + 12(5\beta + 2)$

$\Rightarrow\ 60(\alpha + \beta) + 49 = 169$

62. (B)

$x = \dfrac{-3 \pm \sqrt{3}\,i}{2} = -1 + \omega, -1 + \omega^2$

Let $\alpha = -1 + \omega$ and $\beta = -1 + \omega^2$

$\Rightarrow\ (\alpha + 1)^{100} + (\beta + 2) = \omega^{100} + (-\omega)^{104} = -1$

63. (A)

$\left(2e^{i\pi/6}\right)^{100} = 2^{99}(p + iq)$

$p + iq = 2\left(\cos\dfrac{50\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$

$p = -1, q = \sqrt{3}$

$x^2 - \left(\sqrt{3} - 1\right)x - \sqrt{3} = 0$

64. (C)

$\vec{\alpha} + \vec{\beta} + \vec{\gamma} + \vec{\delta} = (a + 1)\vec{\delta} = (b + 1)\vec{\alpha}\ \Rightarrow\ a + 1 = b + 1 = 0$

65. (A)

$\vec{r}$ is perpendicular to $\vec{a} \times \vec{b}$ and to $\vec{c}$

$\vec{r} = \lambda\left(\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right)$

$\vec{r} = \lambda\left((\vec{a} \cdot \vec{c}) \times \vec{b} - \left(\vec{b} \cdot \vec{c}\right)\vec{a}\right)$-

$\vec{r} = \lambda\left(4 < 1, 2, 1 > 4 < 1, 1, 2 >\right)$

$\vec{r} = \lambda\left(4 < 0, 1, -1 >\right)$

$|\vec{r}| = |\lambda| 4\sqrt{2} = \sqrt{2}$

$|\lambda| = \frac{1}{4}$

$\vec{r} = \pm\left(\hat{j} - \hat{k}\right)$

66. (C)

Total Number of Triangles $= {}^{15}C_3$

$i + j + k = 15$ (Given)

5 Cases

i	j	k
1	2	12
1	3	11
1	4	10
1	5	9
1	6	8

4 Cases

i	j	k
2	3	10
2	4	9
2	5	8
2	6	7

3 Cases

i	j	k
3	4	8
3	5	7

1 Cases

i	j	k
4	5	6

Number of Possible triangles using the vertices P_i, P, P_k such that $i + j + k \neq 15$ is equal to ${}^{15}C_3 - 12 = 443$

67. (B)

$A^2 + 2I = 0 \qquad \left|2C - A^2\right| = 32$

$A^5 - 2A^3C + BA^2 - 2BC = 0 \quad \Rightarrow \left(A^3 + B\right)\left(A^2 - 2C\right) = 0$

$\left|2C - A^2\right| = 32 \Rightarrow A^2 - 2C$ possesses inverse

$\therefore\ A^3 + B = 0 \ \Rightarrow B = -A^3$

$B = -A[-2I] = 2A$

$B - A = 2A - A \Rightarrow |B - A|^2 = \left|A^2\right| = -8$

68. (B)

$\left|\left(Z^3 - 1\right)\left(Z + 1\right)\right| = \left|\left(Z^3 + 1\right)\left(Z - 1\right)\right|$

$\Rightarrow \left|\frac{Z^3 - 1}{Z - 1}\right| = \left|\frac{Z^3 + 1}{Z + 1}\right|$

$\Rightarrow \left|Z^2 + Z + 1\right| = \left|Z^2 - Z + 1\right|$

If $Z = 1$ or -1

$\Rightarrow Z^2 = 1$

$\left|Z^2 - 1 - 2i\right|$ min 2

Squaring and simplify

$\left|Z^2\right|\left(Z+\bar{Z}\right)+Z=0$

$\left|Z^2\right|\left(2x\right)+Z=0$

$Z^2=4x^2\left|Z\right|^4$ purely real least value of $\left|Z^2-\left(1+2i\right)\right|$ is 2

69. (C)

Image of $Q\left(0,-1,-3\right)$ in plane is, $\frac{\left(x-0\right)}{3}=\frac{\left(y+1\right)}{-1}=\frac{z+3}{+4}=\frac{-2\left(1-12-2\right)}{9+1+16}=1$

$\Rightarrow x=3, y=-2, z=1$

$\Rightarrow P\left(3,-2,1\right), Q\left(0,-1,-3\right), R\left(3,-1,-2\right)$

$\therefore$ Area of ΔPQR is $\frac{1}{2}\left|\vec{QP}\times\vec{QR}\right|=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 4\\ 3 & 0 & 1\end{vmatrix}$

$=\frac{1}{2}\left|\left\{\hat{i}\left(-1\right)-\hat{j}\left(3-12\right)+\hat{k}\left(3\right)\right\}\right|$

$=\frac{1}{2}\sqrt{\left(1+81+9\right)}=\frac{\sqrt{91}}{2}$

70. (B)

Since, m = number of ways the committee is formed with at least 6 males

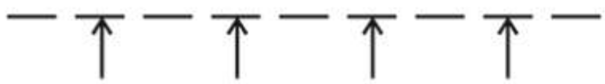
$= {}^8C_6\cdot{}^5C_5+{}^8C_7\cdot{}^5C_4+{}^8C_8\cdot{}^5C_3=78$ and n = number of ways the committee is formed with at least 3 females

$= {}^5C_3\cdot{}^8C_8+{}^5C_4\cdot{}^8C_7+{}^5C_5\cdot{}^8C_6=78$

Hence $m=n=78$

71. (A)

$\because$ There are total 9 digits and out of which only 3 digits are odd.



$\therefore$ Number of ways to arrange odd digits first

$= {}^4C_3\cdot\frac{3!}{2!}$

Hence, total number of 9 digit numbers

$=\left({}^4C_3\cdot\frac{3!}{2!}\right)\cdot\frac{6!}{2!\,4!}=180$

72. (D)

Since the system of linear equations are

$x+y+z=2$ …… (1)

$2x+3y+2z=5$ …… (2)

$2x+3y+\left(a^2-1\right)z=a+1$ …… (3)

Now, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix}$

$\Rightarrow \quad \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & a^2-3 \end{vmatrix}$ (Applying $R_3 \to R_3 - R_2$)

$= a^2 - 3$

When, $\Delta = 0 \Rightarrow a^2 - 3 = 0 \Rightarrow |a| = \sqrt{3}$

If $a^2 = 3$, then plane represented by equation (2) and equation (3) are parallel.
Hence, the given system of equation is inconsistent.

73. (D)
$S \cap T = \{-5, -4, 3\}$

74. (C)
$\left(\left(2^1 2^2 \ldots . 2^{60}\right)\left(4^1 . 4^2 \ldots . 4^n\right)\right)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}$

$\left(2^{30 \times 61} 4^{\frac{n(n+1)}{2}}\right)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}$

$2^{1830+n^2+n} = 2^{\frac{(225)(60+n)}{8}}$

$= 8n^2 - 217n + 1140 = 0$

$n = 20, \frac{57}{8}$

$\sum_{k=1}^{n} nk - k^2 = \frac{n^2(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$

$= 1330$

75. (B)
$\sum_{x=1}^{20} \left(r^2 + 2\right) r!$

$\sum_{x=1}^{20} \left((r+1)^2 - 2r\right) r!$

$\sum_{x=1}^{20} \left((r+1)(r+1)! - r.r!\right) - \sum_{r=1}^{20} r.r!$

$\sum_{x=1}^{202} \left((r+1)(r+1)! - r.r!\right) - \sum_{r=1}^{20} \left((r+1)! - r!\right)$

$= \left(21.\underline{|21} - 1\right) - \left(\underline{|21} - 1\right)$

$= 20.21! = 22! - 2.21!$

76. (D)

77. (C)

$$S = 0.\left({}^{30}C_0\right)^2 + 1.\left({}^{30}C_1\right)^2 + 2.\left({}^{30}C_2\right)^2 + ... + 30.\left({}^{30}C_{30}\right)^2$$

$$S = 30.\left({}^{30}C_0\right)^2 + 29.\left({}^{30}C_1\right)^2 + 28.\left({}^{30}C_2\right) + ... + 0.\left({}^{30}C_0\right)^2$$

$$2S = 30.\left({}^{30}C_0{}^2 + {}^{30}C_1{}^2 + ... + {}^{30}C_{30}{}^2\right)$$

$$S = 15.{}^{60}C_{30} = 15.\frac{60!}{(30!)^2}$$

$$\frac{15\cdot 10!}{(30!)^2} = \frac{\alpha\cdot 60!}{(30!)^2}$$

$$\Rightarrow \alpha = 15$$

78. (B)

Either all outcomes are positive or any two are negative or any 4 are negative.

Now, $p = P$ (positive) $= \frac{3}{6} = \frac{1}{2}$

$q = p$ (negative) $= \frac{2}{6} = \frac{1}{3}$

Required probability

$$= {}^5C_5\left(\frac{1}{2}\right)^5 + {}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3 + {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{1}{2}\right)^1$$

$$= \frac{521}{2592}$$

$\therefore$ Option (B) is correct.

79. (B)

$$np + npq = 5,\ np\cdot npq = 6$$

$$np(1+q) = 5,\ n^2p^2q = 6$$

$$n^2p^2(1+q)^2 = 25,\ n^2p^2q = 6$$

$$\frac{6}{q}(1+q)^2 = 25$$

$$6q^2 + 12q + 6 = 25q$$

$$6q^2 - 13q + 6 = 0$$

$$6q^2 - 9q - 4q + 6 = 0$$

$$(3q-2)(2q-3) = 0$$

$q = \frac{2}{3}, \frac{3}{2}, q = \frac{2}{3}$ is accepted

$$p = \frac{1}{3} \Rightarrow n\cdot\frac{1}{3} + n\cdot\frac{1}{3}\cdot\frac{2}{3} = 5$$

$$\frac{3n + 2n}{9} = 5$$

$n = 9$

So, $6(n + p - q) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right) = 52$

80. (B)

$\sqrt{(x-1)(x-3)} + \sqrt{(x+3)(x+3)}$

$= \sqrt{4\left(x - \frac{12}{4}\right)\left(x - \frac{2}{4}\right)}$

$\Rightarrow \sqrt{x-3} = 0 \Rightarrow x = 3$ which is in domain

or $\sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2}$

$2\sqrt{(x-1)(x+3)} = 2x - 4$

$x^2 + 2x - 3 = x^2 - 4x + 4$

$6x = 7$

$x = \frac{7}{6}$ (rejected)

81. (7)

$\frac{4}{T_n} = \sqrt{2n^2 + 2n + 1} - \sqrt{2n^2 - 2n + 1}$

$4S = \sum_{r=1}^{20} \frac{4}{T_n} = \sqrt{841} - 1 = 28$

$\Rightarrow S = 7$

82. (1)

$\lambda \in \left[\frac{1}{7}, 7\right]$

For $3x - 7 + 4z = 3$

$x + 2y - 3z = -2$

$6x + 5y + \lambda z = -3$

$\Delta = 7(\lambda + 5) > 0$ i.e. unique solution

83. (0)

$a + b + c = 0$

$\Rightarrow \bar{a} + \bar{b} + \bar{c} = 0$

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

$ab + bc + ca = 0$

$\Rightarrow a^2 + b^2 + c^2 = 0$

84. (1)

$\left[\overline{AB}\ \overline{AC}\ \overline{AD}\right] = 0$

85. (4)

Clearly B is skew symmetric matrix of order 3

$\therefore\ |B| = 0 \Rightarrow |adj\ B| = 0$

$|adj\ A| = |A|^2$

$|A| = (2k-1)\left\{-1+4k^2\right\} - 2\sqrt{k}\left\{-2\sqrt{k} - 2\sqrt{k}(2k)\right\} + 2\sqrt{k}\left\{4k\sqrt{k} + 2\sqrt{k}\right\}$

$= \left(4k^2-1\right)(2k+1) + 4k + 8k^2 + 8k^2 + 4k$

$= 8k^3 - 4k^2 - 2k + 1 + 4k + 8k^2 + 8k^2 + 4k$

$8k^3 + 12k^2 + 6k + 1$

$= (2k+1)^3 = 10^3$

$2K + 1 = 10$

$K = \dfrac{9}{2} = 4.5 \Rightarrow [k] = 4$

86. (6.25)

$|2z_1 + \bar{z}_2|^2 - |1 + 2z_1z_2|^2 = 8 - 9$

$\Rightarrow 4|z_1|^2 + |z_2|^2 - 1 - 4|z_1|^2|z_2|^2 = -1$

$\Rightarrow \dfrac{4}{|z_2|^2} + \dfrac{1}{|z_1|^2} = 4$

A.M. $\geq$ H.M.

$\dfrac{|z_1|^2 + 4|z_2|^2}{5} \geq \dfrac{5}{\dfrac{1}{|z_1|^2} + \dfrac{4}{|z_2|^2}}$

$\Rightarrow |z_1|^2 + 4|z_2|^2 \geq \dfrac{25}{4}$

87. (0)

$A^2 = I$

$\Rightarrow B = 1010I + 1010A$

$B = \begin{bmatrix} 1010 & 1010 \\ 1010 & 1010 \end{bmatrix} \Rightarrow |B| = 0$

88. (22.50)

$a - 2d + a - d + a + a + d + a + 2d = 10$

$\Rightarrow\ a = 2\ \ \&\ \dfrac{1}{a-2d} + \dfrac{1}{a+2d} + \dfrac{1}{a-d} + \dfrac{1}{a+d} + \dfrac{1}{a} = \dfrac{29}{10}$

$\Rightarrow\ \dfrac{4}{4-4d^2} + \dfrac{4}{4-d^2} + \dfrac{1}{2} = \dfrac{29}{10}$

$\Rightarrow\ d^2 = \dfrac{1}{4}$ or $\dfrac{8}{3}$ (reject)

$\Rightarrow$ numbers are $1, \dfrac{3}{2}, 2, \dfrac{5}{2}, 3$

89. (8)

Since two balls are drawn and they are found to be white, the urn must contain at least two white balls.
Let us define the following events:

$E_i\left(i=2,3,4\right)$: The event that the urn contains i white balls.

E : The event that two white balls are drawn.

Since the events E_2, E_3 and E_4 are equally likely, we have:

$$P\left(E_2\right)=P\left(E_3\right)=P\left(E_4\right)=\frac{1}{3} \qquad ...(i)$$

$P\left(E|E_2\right)=$ Probability of drawing two white balls, given that the urn contains 2 white balls.

$$=\frac{{}^2C_2}{{}^4C_2}=\frac{1}{6} \qquad ...(ii)$$

Similarly, we have: $P\left(E|E_3\right)=\frac{{}^3C_2}{{}^4C_2}=\frac{3}{6}=\frac{1}{2}$ and $P\left(E|E_4\right)=\frac{{}^4C_2}{{}^4C_2}=1$ ….. (iii)

We want the conditional probability $P\left(E_4|E\right)$.

$$P\left(E_4|E\right)=\frac{P\left(E_4\right)P\left(E|E_4\right)}{P\left(E_2\right)P\left(E|E_2\right)+P\left(E_3\right)P\left(E|E_3\right)+P\left(E_4\right)P\left(E|E_4\right)} \qquad \text{[By Bayes' Rule]}$$

$$=\frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1}=\frac{\frac{1}{3}}{\frac{1}{18}+\frac{1}{6}+\frac{1}{3}}=\frac{\frac{1}{3}}{\frac{10}{18}}=\frac{3}{5} \qquad \text{(From (i), (ii) and (iii)]}$$

90. (98)

Let $z=x+iy$

$$\arg\left(\frac{x-2+iy}{x+2+iy}\right)=\frac{\pi}{4}$$

$$\arg\left(x-2+iy\right)-\arg\left(x+2+iy\right)=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y}{x-2}\right)-\tan^{-1}\left(\frac{y}{x+2}\right)=\frac{\pi}{4}$$

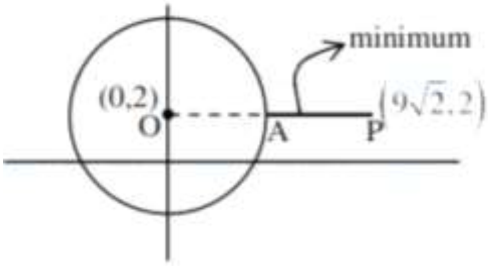
$$\frac{\frac{y}{x-2}-\frac{y}{x+2}}{1+\left(\frac{y}{x-2}\right)\cdot\left(\frac{y}{x+2}\right)}=\tan\frac{\pi}{4}=1$$

$$\frac{xy+2y-xy+2y}{x^2-4+y^2}=1$$

$$4y=x^2-4+y^2$$

$$x^2+y^2-4y-4=0$$

Locus is a circle with centre (0, 2) & radius $=2\sqrt{2}$



$$\text{min. value} = (AP)^2 = (OP - OA)^2$$
$$= \left(9\sqrt{2} - 2\sqrt{2}\right)^2$$
$$= \left(7\sqrt{2}\right)^2 = 98$$