IIT – JEE: 2025 **TW TEST (MAIN)** **DATE: 25/11/23**

ANSWER KEY

CENTRE OF MASS

1.	(D)	2.	(C)	3.	(A)	4.	(D)	5.	(D)
6.	(A)	7.	(C)	8.	(B)	9.	(A)	10.	(D)
11.	(B)	12.	(B)	13.	(A)	14.	(D)	15.	(B)
16.	(A)	17.	(C)	18.	(A)	19.	(A)	20.	(B)
21.	(24)	22.	(20)	23.	(4)	24.	(2)	25.	(2)
26.	(3)	27.	(5)	28.	(6)	29.	(2)	30.	(8)

ENERGETICS

31.	(A)	32.	(B)	33.	(A)	34.	(C)	35.	(A)
36.	(D)	37.	(B)	38.	(C)	39.	(C)	40.	(B)
41.	(A)	42.	(C)	43.	(C)	44.	(A)	45.	(C)
46.	(D)	47.	(C)	48.	(A)	49.	(B)	50.	(C)
51.	(300)	52.	(6)	53.	(425)	54.	(196)	55.	(286)
56.	(4)	57.	(239)	58.	(6)	59.	(5)	60.	(2000)

BINOMIAL THEOREM

61.	(C)	62.	(B)	63.	(D)	64.	(C)	65.	(B)
66.	(B)	67.	(C)	68.	(C)	69.	(D)	70.	(D)
71.	(B)	72.	(A)	73.	(A)	74.	(B)	75.	(A)
76.	(D)	77.	(A)	78.	(D)	79.	(B)	80.	(C)
81.	(5)	82.	(6)	83.	(4)	84.	(0)	85.	(9)
86.	(9)	87.	(8)	88.	(1)	89.	(6)	90.	(1)

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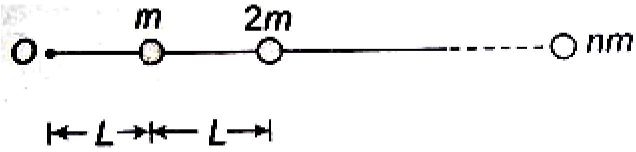
TOPIC: CENTRE OF MASS

SOLUTIONS

1. (D)



$$x_{c.m.} = \frac{mL + 2m\times 2L + ... + nm \times nL}{m + 2m + ... + nm}$$

$$= \frac{L\left(1^2 + 2^2 + ... + n^2\right)}{\left(1 + 2 + ... + n\right)}$$

$$= L\frac{\sum n^2}{\sum n} = L\frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{(2n+1)L}{3}$$

2. (C)

$$\Delta x_G = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2}$$

$$0 = \frac{10\times 6 + 30(\Delta x_2)}{40}$$

$$\Delta x_2 = -2\text{ cm}$$

Block of mass 30 kg will to move towards 10 kg.

3. (A)

$$\overrightarrow{r_{COM}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{1\left(\hat{i} + 2\hat{j} + \hat{k}\right) + 3\left(-3\hat{i} - 2\hat{j} + \hat{k}\right)}{1+3}$$

$$= -2\hat{i} - \hat{j} + \hat{k}$$

$$\left|2\hat{i} - \hat{j} + \hat{k}\right| = \sqrt{(2)^2 + (1)^2 + (1)^2} = \sqrt{6}$$

4. (D)

5. (D)

6. (A)

$$x_{CM} = \frac{0\times m + a\times m + \frac{a}{2}\times m}{m+m+m} = \frac{a}{2},\ y_{CM} = \frac{0\times m + 0\times m + \frac{a\sqrt{3}}{2}\times m}{m+m+m} = \frac{a\sqrt{3}}{6}$$

7. (C)

Treating the line joining the two particles as x axis

$$x_{CM} = \frac{m_1\times 0 + m_2\times L}{m_1+m_2} = \frac{m_2L}{m_1+m_2},\quad y_{CM} = 0\ z_{CM} = 0$$

8. (B)

CM of rod OA is at $\left(\frac{a}{2}, 0\right)$, CM of rod OB is at $\left(0, \frac{a}{2}\right)$ and CM of rod AB is at $\left(\frac{a}{2}, \frac{a}{2}\right)$

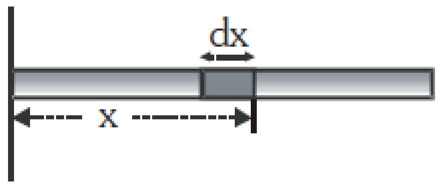
For the system, $x_{cm} = \frac{m\times\frac{a}{2} + m\times 0 + m\times\frac{a}{2}}{m+m+m} = \frac{a}{3} \Rightarrow y_{cm} = \frac{m\times 0 + m\times\frac{a}{2} + m\times\frac{a}{2}}{m+m+m} = \frac{a}{3}$

9. (A)

Let the X-axis be along the length of the rod with origin at one of its end as shown in figure. As the rod is along x-axis, so, $y_{CM} = 0$ and $z_{CM} = 0$ i.e., centre of mass will be on the rod.

Now consider an element of rod of length dx at a distance x from the origin, mass of this element $dm = \lambda dx = (A+Bx)dx$ so,

$$x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(A+Bx)dx}{\int_0^L (A+Bx)dx} = \frac{\frac{AL^2}{2} + \frac{BL^2}{3}}{AL + \frac{BL^2}{2}} = \frac{L(3A+2BL)}{3(2A+BL)}$$



10. (D)

We treat the hole as a 'negative mass' object that is combined with the original uncut disc. (When the two are overlapped together, the hole region then has zero mass). By symmetry, the CM lies along the +y-axis in figure, so $x_{CM} = 0$. With the origin at the centre of the original circle whose mass is assumed to be m.

Mass of original uncut circle $m_1 = m$ & Location of CM = (0, 0)

Mass of hole of negative mass : $m_2 = \frac{m}{4}$; Location of CM $= \left(0, \frac{R}{2}\right)$

Thus $y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1+m_2} = \frac{m(0) + \left(-\frac{m}{4}\right)\frac{R}{2}}{m + \left(-\frac{m}{4}\right)} = -\frac{R}{6}$

So the centre of mass is at the point $\left(0, -\frac{R}{6}\right)$.

Thus, the required distance is R/6.

11. (B)

From eq. corresponding to CM, we have

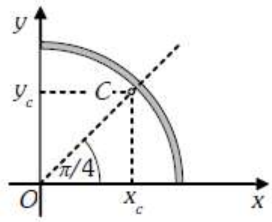
$$\vec{r}_c = \frac{\sum m_i\vec{r}_i}{M}$$

$$\vec{r}_c = \frac{1\left(4\hat{i}+2\hat{j}-3\hat{k}\right)+2\left(\hat{i}-4\hat{j}+2\hat{k}\right)+3\left(2\hat{i}-2\hat{j}+\hat{k}\right)}{1+2+3} = \left(2\hat{i}-2\hat{j}+\frac{2}{3}\hat{k}\right)\text{m}$$

12. (B)

Making use of the result of circular arc, distance OC of the center of mass from the center is $OC = \frac{r\sin(\pi/4)}{\pi/4} = \frac{2\sqrt{2}r}{\pi}$.

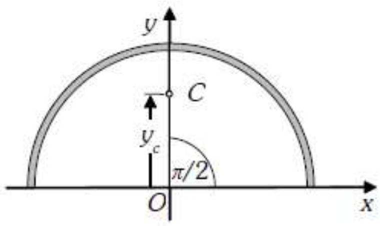
Coordinates of the center of mass (x_c, y_c) are $\left(\frac{2r}{\pi}, \frac{2r}{\pi}\right)$



13. (A)

The y-axis is the line of symmetry, therefore center of mass of the ring lies on it making x-coordinate zero.
Distance OC of center of mass from center is given by the result obtained for circular arc

$OC = \frac{r\sin\theta}{\theta} \Rightarrow y_c = \frac{r\sin(\pi/2)}{\pi/2} = \frac{2r}{\pi}$, So coordinates are $\left(0, \frac{2r}{\pi}\right)$

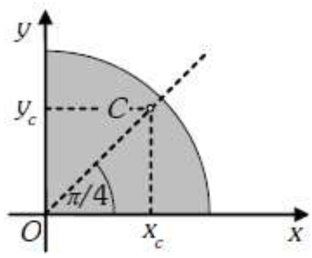


14. (D)

From the result obtained for sector of circular plate distance OC of the center of mass form the center is

$$OC = \frac{2r\sin(\pi/4)}{3\pi/4} = \frac{4\sqrt{2}r}{3\pi}$$

Coordinates of the center of mass (x_c, y_c) are $\left(\frac{4r}{3\pi}, \frac{4r}{3\pi}\right)$



15. (B)

16. (A)

17. (C)

Velocity of centre of mass of the system $\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$.

Since the two particles are moving in same direction, $m_1\vec{v}_1$ and $m_2\vec{v}_2$ are parallel.

$\Rightarrow \quad \left|m_1\vec{v}_1 + m_2\vec{v}_2\right| = m_1v_1 + m_2v_2$.

Therefore, $v_{cm} = \frac{\left|m_1\vec{v}_1 + m_2\vec{v}_2\right|}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(1)(2)+\left(\frac{1}{2}\right)(6)}{\left(1+\frac{1}{2}\right)} = 3.33\text{ m/s}$.

18. (A)

The acceleration of centre of mass of the system $\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} \Rightarrow a_{cm} = \frac{\left|m_1\vec{a}_1 + m_2\vec{a}_2\right|}{m_1 + m_2}$

Since, $\vec{a}_1$ and $\vec{a}_2$ are anti-parallel, so $a_{cm} = \frac{\left|m_1a_1 - m_2a_2\right|}{m_1 + m_2} = \frac{\left|(2)(1)-(4)(2)\right|}{2+4} = 1\text{ m/s}^2$.

Since $m_2a_2 < m_1a_1$ so the direction of acceleration of centre of mass is along in the direction of a_2.

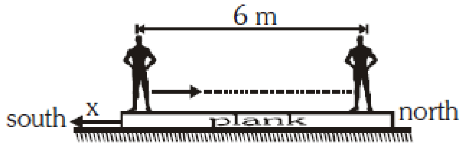
19. (A)
As net force on the system = 0 (after being released)
So centre of mass of the system remains stationary.

20. (B)

21. (24)

22. (20)
If the bigger block moves toward right by a distance (x) then the smaller block will move toward left by a distance (2.2 – x)
Now considering both the blocks together as a system, horizontal position of CM remains same.
As the sum of mass moments about centre of mass is zero i.e. $\sum m_1 x_{1/cm} = 0$.
$M(2.2 - x) = 10\,Mx \Rightarrow x = 0.2\text{ m}$.

23. (4)
Since net force is zero so centre of mass remains stationary
Let x be the displacement of the plank.
Since CM of the system remains stationary
So 80 (6 – x) = 40 x ⇒ 12 – 2x = x ⇒ x = 4 m.



24. (2)
If $\vec{a}$ is the acceleration of m_1, then $-\vec{a}$ is the acceleration of m_2 then

$$\vec{a}_{cm} = \frac{m_1\vec{a} + m_2(-\vec{a})}{m_1 + m_2} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{a}$$

But $\vec{a} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{g}$ so $\vec{a}_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \vec{g}$.

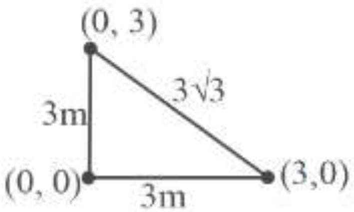
25. (2)

26. (3)

27. (5)

28. (6)

29. (2)



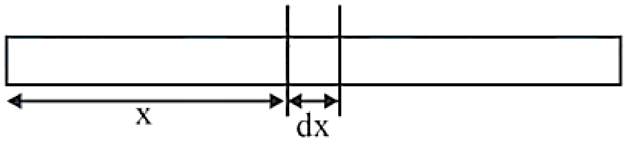
$$\vec{r}_{com} = \frac{M(0\hat{i} + 0\hat{j}) + M(3\hat{i}) + M(3\hat{j})}{3M}$$

$\vec{r}_{com} = \hat{i} + \hat{j}$

$|\vec{r}_{com}| = \sqrt{2} = \sqrt{x}$

$x = 2$

30. (8)



$$dm = \lambda \cdot dx = \lambda_0\left(1 - \frac{x^2}{\ell^2}\right)$$

$$X_{\text{cm}} = \frac{\int x\,dm}{\int dm_\ell}$$

$$= \frac{\lambda_0 \int_0^\ell \left(x - \frac{x^3}{\ell^2}\right)dx}{\int_0^\ell \lambda_0\left(1 - \frac{x^2}{\ell^2}\right)dx} = \frac{\frac{\ell^2}{2} - \frac{\ell^4}{4\ell^2}}{\ell - \frac{\ell^3}{3\ell^2}} = \frac{3\ell}{8}$$

SOLUTIONS

31. **(A)**
The given diagram represents that the process is carried out in infinite steps. Hence, it is isothermal reversible expansion of the ideal gas from pressure 2.0 atm to 1.0 atm at 298 K.

$$W = -2.303nRT\log\frac{p_1}{p_2}$$
$$W = -2.303\times1\times8.314\times298\times0.3010J$$
$$W = -1717.46J$$

32. **(B)**
$$C_2H_4 + 3O_2 \longrightarrow 2CO_2 + 2H_2O$$
$$\Delta H_{reaction} = \left[2\times\Delta H_f^o(CO_2) + 2\times\Delta H_f^o(H_2O)\right] - \left[\Delta H_f^o(C_2H_4) + 3\times\Delta H_f^o(O_2)\right]$$
$$= \left[2(-394) + 2(-286) - [52+0]\right]$$
$$= -1412\,kJ\,mol^{-1}$$

33. **(A)**
$$\frac{1}{2}N_2(g) + \frac{3}{2}H_2(g) \longrightarrow NH_3(g) \qquad \Delta_f H^\circ = -91.8kJ\,mol^{-1}$$
($\Delta_f H^\circ$ means enthalpy of formation of 1 mole of NH_3)
$\therefore$ Enthalpy change for the formation of 2 moles of NH_3
$$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$$
$$= 2\times-91.8 = -183.6\,kJ\,mol^{-1}$$

34. **(C)**
$$C + O_2 \longrightarrow CO_2(g); \Delta_r H = xkJ\,mol^{-1}$$
$$C_{(graphite)} + \frac{1}{2}O_2(g) \longrightarrow CO(g); \Delta_r H = ykJ\,mol^{-1}$$
$$CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g); \Delta_r H = zkJ\,mol^{-1}$$
$$(i) = (ii) + (iii) \Rightarrow \quad x = y + z$$

35. **(A)**
At transition point (373 K, 1.0 bar), liquid remains in equilibrium with vapour phase,
therefore ΔG =0. As
vaporisation occurs, degree of randomness increases, hence $\Delta S > 0$ (i.e + ve)

36. **(D)**

ΔH° and ΔS° remain constant in the given temperature range.

$$\Delta S^\circ = -\frac{\Delta G^\circ - \Delta H^\circ}{T}$$

$$= -\left(\frac{-66.9 + 41.8}{300}\right) = 0.08367\,\text{kJ K}^{-1}\,\text{mol}^{-1}$$

$$\therefore \Delta G^\circ_{330} = \Delta H^\circ - T\Delta S^\circ = -41.8 - 330 \times 0.08367$$

$$= -69.4\,\text{kJ mol}^{-1}$$

37. **(B)**

$$\Delta H = \Delta U + \Delta n_g RT$$

$$\Delta U = 2.1\text{kcal} = 2.1 \times 10^3\text{cal} \qquad \left[1\text{kcal} = 10^3\,\text{cal}\right]$$

$$\Delta H = \left(2.1 \times 10^3\right) + \left(2 \times 2 \times 300\right) = 3300\,\text{cal}$$

Hence $\Delta G = \Delta H - T\Delta S$

$$= (3300) - (300 \times 20)$$

$$= -2700\,\text{cal}$$

$$= -2.7\,\text{kcal}$$

38. **(C)**

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$

Given, $\Delta H_{vap} = 30\,\text{kJ mol}^{-1}$

$\Delta G^\circ = 0$ at equilibrium,

$$\Delta S_{vap} = \frac{\Delta H_{vap}}{T}$$

$$= \frac{30 \times 10^3\,\text{J mol}^{-1}}{300\,\text{K}} = 100\,\text{J mol}^{-1}\text{K}^{-1}$$

39. **(C)**

$$\Delta G = \Delta H - T\Delta S$$

For spontaneous reaction

$$\Delta G < 0$$

$$170 \times 10^3 - T \times 170 < 0$$

$$T > 1000$$

$$\therefore \; T = 1110\,\text{K}$$

40. **(B)**

$$\Delta G = \Delta H - T\Delta S$$

$$= -382.64 + (298 \times 145.6 \times 10^{-3})$$

$$= -339.3\,\text{kJ mol}^{-1}$$

41. **(A)**

Entropy change for isothermal expansion is

$$\Delta S = 2.303nR \log \frac{V_2}{V_1}$$

$= 2.303 \times 2 \times 8.314 \log \frac{100}{10}$

$= 38.294 = 38.3 J\ mol^{-1} K^{-1}$

42. **(C)**

Entropy change of fusion, $\Delta_f S^\circ = \frac{\Delta H^\circ f}{T}$

$\Delta_f S = \frac{6 \times 10^3}{273} = 21.97\ JK^{-1} mol^{-1}$

43. **(C)**

Molar heat capacity of ice $= 37.8 J\ mol^{-1}$

Molar heat capacity of water $= 75.6 J\ mol^{-1}$

Enthalpy of fusion of ice $= 6.012\ kJ\ mol^{-1}$

$= 6.012 \times 10^3 J\ mol^{-1}$

The conversion of 10g of ice at $-10^\circ C$ to water at $10^\circ C$ involves following steps

$10 g$ of ice at $-10^\circ C \xrightarrow{I} 10 g$ ice at $0^\circ C$

$10 g$ ice at $0^\circ \xrightarrow{II} 10 g$ water at $0^\circ C$

10 g water at $0^\circ C \xrightarrow{III} 10 g$ water at $10^\circ C$

Heat required for I step $= 37.8 \times \frac{10}{18} \times (10)$

$= \frac{37.8 \times 10 \times 10}{18} = 210 J$

Heat required for II step $= \frac{6.012 \times 10^3 \times 10}{18} = 3340 J$

Heat required for III step $= 75.6 \times \frac{10}{18} \times 10 = 420 J$

Total heat required $= 210 + 3340 + 420 = 3970 J$

44. **(A)**

Given that, 1 mole of $CCl_4 = 154 g$

$\Delta_{vap}H$ for $154 g\ CCl_4 = 30.5 kJ$

$\therefore \Delta_{vap}H$ for $284 g\ CCl_4 = \frac{30.5 \times 284}{154} kJ = 56.25 kJ$

45. **(C)**

Correct equation is (c), because $\Delta_c H =$ negative value which is showing exothermic reaction and stoichiometic constants are also balanced.

46. **(D)**

$CH_3 - \underset{|}{\overset{CH_3}{C}} = CH_2(g) + 6O_2(g) \longrightarrow 4CO_2(g) + 4H_2O(l)$

Here $\Delta n_g = 4 - 7 = -3$

$\because \Delta H^\circ = \Delta E^\circ + \Delta n_g RT$

$\therefore \Delta H^\circ = \Delta E^\circ - 3RT$

$\Delta H^\circ < \Delta E^\circ$

47. **(C)**

At constant p or T

$\Delta H = \Delta U + \Delta nRT$

Here, $\Delta n = n_p - n_R = 2 - 4 = -2$

$\Delta H < \Delta U$

48. **(A)**

$W = 2.303\, nRT \log \frac{p_2}{p_1}$

$= 2.303 \times 2 \times 300 \log \frac{10}{2} = 965.84$

At constant temperature , $\Delta E = 0$

$\Delta E = q + W$

$\therefore q = -W = -965.84\, \text{cal}$

49. **(B)**

$W = -2.303 nRT \log \frac{V_2}{V_1}$

$= -2.303 \times 1 \times 8.314 \times 10^7 \times 298 \log \frac{20}{10}$

$= -298 \times 10^7 \times 8.314 \times 2.303 \log 2$

50. **(C)**

For the process to occur under adiabatic conditions, $q = 0$ i.e. heat cannot flow from system to surrounding or vice versa.

51. **(300)**

$\Delta G = \Delta H - T\Delta S$

For a spontaneous reaction, ΔG must be negative

Given $\Delta H = 3\text{kJ} = 3000\, \text{J}$

$\Delta S = +10 \text{J} / \text{K}$

(a) If $T = 300\, \text{K}, \Delta G = 3000 - 300 \times 10 = 0$

(b) If $T = 373\text{K}, \Delta G = 3000 - 373 \times 10 = -730\, \text{J}$

Hence, beyond 300 K temperature, the reaction will be spontaneous

52. **(6)**

Volume, Heat capacity, Internal energy, Enthalpy, Entropy and free energy are extensive. The remaining are intensive.

53. **(425)**

$H_2 + Cl_2 \rightarrow 2HCl$

$\Delta H_{reaction} = \Delta H_{H-H} + \Delta H_{Cl-Cl} - 2\Delta H_{H-Cl}$

Or $-180 = 430 + 240 - 2\Delta H_{H-Cl}$

$\therefore \Delta H_{H-Cl} = \frac{430+240-(-180)}{2} = \frac{850}{2} = 425\ kJ\ mol^{-1}$

54. **(196)**

$H_2 + \frac{1}{2} O_2 \longrightarrow H_2O$; $\Delta H = -188\, \text{kJ mol}^{-1}$ ....(i)

$H_2 + O_2 \longrightarrow H_2O_2$; $\Delta H = -286\, \text{kJ mol}^{-1}$ ....(ii)

$2H_2 + O_2 \longrightarrow 2H_2O;\ \Delta H = -376\,kJ\,mol^{-1}$ ……..(iii)

$2H_2 + 2O_2 \longrightarrow 2H_2O_2;\ \Delta H = -572\,kJ\,mol^{-1}$ …….(iv)

By (iii) – (iv)

$2H_2O_2 \longrightarrow 2H_2O + O_2; \Delta H = +196\,kJ$

55. **(286)**

At equilibrium $\Delta G = 0$

$\Delta G = \Delta H - T\Delta S$

$0 = 30.03 \times 10^3 - T \times 105 JK^{-1}mol^{-1}$

$T = \frac{30.03\times10^3}{105} K = 286\ K$

56. (4)

$\frac{1}{2}X_2 + \frac{1}{2}Y_2 \rightarrow XY; \Delta H = -100\,kJ$

Let the bond dissociation energy of $X_2, Y_2,$ and XY be

$a : \frac{a}{2} : a$ (the given ratio) kJ mol^{-1}, respectively.

$\therefore \frac{a}{2} + \frac{a}{4} - a = -100$

$\therefore\ a = 400$

$100x = 400$

$x = 4$

57. (239)

Initial volume, $V_1 = 10L$

$V_2(\text{final}) = \frac{nRT}{P} = \frac{10 \times 0.083 \times 300}{1} = 249\,L$

$w = P\,\Delta V = 1 \times (249 - 10) = 239\,L\,bar$

58. (6)

$KCl(s) \rightarrow K^{\oplus} + Cl^{\ominus}(g), \quad \Delta H_1 = 181\,kcal\,mol^{-1}$

$KCl(s) + aq \rightarrow K^{\oplus}(aq) + Cl^{\ominus}(aq), \quad \Delta H_2 = 1.0\,kcal\,mol^{-1}$

Let the enthalpy of hydration of $K^{\oplus}$ is 2a kcal mol^{-1}

$K^{\oplus}(g) + aq \rightarrow K^{\oplus}(aq),\ \Delta H_3 = 2a$

$Cl^{\ominus}(g) + aq \rightarrow Cl^{\ominus}(aq), \quad \Delta H_4 = a$

$\therefore\ \Delta H_3 = -\Delta H_1 + \Delta H_2 - \Delta H_4$

$2a = -181 + 1 - a$

$3a = -180, a = -60$

$\therefore\ \Delta_{hyd}H^{\ominus}$ of $K^{\oplus} = 2a = -60 \times 2 = -120$

$\therefore -20x = -120$

$x = 6$

59. (5)

$$\Delta_{sys}S = \frac{q_{sys}}{T_{sys}} = -\frac{300}{273+127}$$

$$= \frac{-300}{400} = \frac{-3}{4} J\,K^{-1}$$

$$\Delta_{surr}S = \frac{-q_{sys}}{T_{surr}} = -\frac{300}{273+27}$$

$$= \frac{300}{300} = +1 J\,K^{-1}$$

$$\Delta_{total}S \text{ or } \Delta_{universe}S = \Delta_{sys}S + \Delta_{surr}S$$

$$= \frac{-3}{4} + 1 = \frac{1}{4} = 0.25\,JK^{-1}$$

$$\therefore 0.05x = 0.25$$

$$x = 5$$

60. (2000)

$$\Delta_{melting}S = \frac{\Delta_{melting}H}{T}$$

$$\text{Or } T = \frac{\Delta_{melting}H}{\Delta_{melting}S} = \frac{30\times10^3}{15} = 2000\text{ K}$$

IIT – JEE: 2025 **TW TEST (MAIN)** **DATE: 25/11/23**

TOPIC: BINOMIAL THEOREM

SOLUTIONS

61. **(C)**

We have $7^2 = 49 = 50 - 1$

Now, $7^{300} = (7^2)^{150} = (50-1)^{150}$

$= {}^{150}C_0(50)^{150}(-1)^0 + {}^{150}C_1(50)^{149}(-1)^1 + ... + {}^{150}C_{150}(50)^0(-1)^{150}$

Thus the last digits of 7^{300} are ${}^{150}C_{150}.1.1$ *i.e.*, 1.

62. **(B)**

Applying $T_{r+1} = {}^nC_r x^{n-r} a^r$ for $(x+a)^n$

Hence $T_6 = {}^{10}C_5(2x^2)^5\left(-\frac{1}{3x^2}\right)^5$

$$= -\frac{10!}{5!5!}32 \times \frac{1}{243} = -\frac{896}{27}$$

63. **(D)**

$T_3 = {}^nC_2(x)^{n-2}\left(\frac{1}{2x}\right)^2$ and $T_4 = {}^nC_3(x)^{n-3}\left(\frac{1}{2x}\right)^3$

But according to the condition,

$$\frac{n(n-1)\times 3\times 2\times 1\times 8}{n(n-1)(n-2)\times 2\times 1\times 4} = \frac{1}{2} \Rightarrow n = 10$$

64. **(C)**

${}^{20}C_{r-1} = {}^{20}C_{r+3} \Rightarrow 20 - r + 1 = r + 3 \Rightarrow r = 9$.

65. **(B)**

r^{th} term of $(a+2x)^n$ is ${}^nC_{r-1}(a)^{n-r+1}(2x)^{r-1}$

$$= \frac{n!}{(n-r+1)!(r-1)!}a^{n-r+1}(2x)^{r-1}$$

$$= \frac{n(n-1).....(n-r+2)}{(r-1)!}a^{n-r+1}(2x)^{r-1}$$

66. **(B)**

$7 - 2r = 3 \Rightarrow r = 2$

$\therefore$ The coefficient is ${}^7C_2 = 21$.

67. **(C)**

$(1+x)^m(1-x)^n$

$$=\left(1+mx+\frac{m(m-1)x^2}{2!}+....\right)\left(1-nx+\frac{n(n-1)}{2!}x^2-....\right)$$

$$=1+(m-n)x+\left[\frac{n^2-n}{2}-mn+\frac{(m^2-m)}{2}\right]x^2+.........$$

Given, $m - n = 3$ or $n = m - 3$

Hence $\frac{n^2-n}{2}-mn+\frac{m^2-m}{2}=-6$

$\Rightarrow \frac{(m-3)(m-4)}{2}-m(m-3)+\frac{m^2-m}{2}=-6$

$\Rightarrow m^2-7m+12-2m^2+6m+m^2-m+12=0$

$\Rightarrow -2m+24=0 \Rightarrow m=12$

68. **(C)**

$T_{r+1} = {}^{2n}C_r x^{2n-r}\left(\frac{1}{x^2}\right)^r = {}^{2n}C_r x^{2n-3r}$,

This contains x^m, if $2n - 3r = m$

i.e. if $r=\frac{2n-m}{3}$

$\therefore$ Coefficient of $x^m = {}^{2n}C_r, r=\frac{2n-m}{3}$

$$=\frac{2n!}{(2n-r)!r!}=\frac{2n!}{\left(2n-\frac{2n-m}{3}\right)!\left(\frac{2n-m}{3}\right)!}$$

$$=\frac{2n!}{\left(\frac{4n+m}{3}\right)!\left(\frac{2n-m}{3}\right)!}.$$

69. **(D)**

Coefficients of 2nd, 3rd and 4th terms are respectively ${}^nC_1, {}^nC_2$ and ${}^nC_3$ are in A.P.

$\Rightarrow 2.{}^nC_2={}^nC_1+{}^nC_3$

$\Rightarrow \frac{2n!}{2!(n-2)!}=\frac{n!}{(n-1)!}+\frac{n!}{3!(n-3)!}$

On solving, $n^2-9n+14=0 \Rightarrow n^2-9n=-14$.

70. **(D)**

$(1+x+x^3+x^4)^{10}=(1+x)^{10}(1+x^3)^{10}$

$=(1+{}^{10}C_1.x+{}^{10}C_2.x^2+.....)(1+{}^{10}C_1.x^3+{}^{10}C_2.x^6+.....)$

$\therefore$ Coefficient of $x^4={}^{10}C_1.{}^{10}C_1+{}^{10}C_4=310$.

71. **(B)**

If n is even.

$\therefore$ Middle term is 10th term.

$\Rightarrow T_{10} = {}^{18}C_9 (x)^9 . \left(-\frac{1}{x}\right)^9 = -{}^{18}C_9$.

72. **(A)**

We know that

$2^{n-1} = {}^nC_0 + {}^nC_2 + {}^nC_4 + ... = {}^nC_1 + {}^nC_3 + {}^nC_5 + ...$

So, ${}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + ... + {}^{10}C_9 = 2^{10-1} = 2^9$

73. **(A)**

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + ... + C_r x^r + ...$ .....(i)

$\left(1+\frac{1}{x}\right)^n = C_0 + C_1 \frac{1}{x} + C_2 \frac{1}{x^2} + ... + C_r \frac{1}{x^r} + ...$ .....(ii)

Multiplying both sides and equating coefficient of x^r in $\frac{1}{x^n}(1+x)^{2n}$ or the coefficient of x^{n+r} in $(1+x)^{2n}$ we get the value of required expression

$= {}^{2n}C_{n+r} = \frac{(2n)!}{(n-r)!(n+r)!}$

74. **(B)**

Expansion of $(1-2x)^{3/2}$

$= 1 + \frac{3}{2}(-2x) + \frac{3}{2}.\frac{1}{2}.\frac{1}{2}(-2x)^2 + \frac{3}{2}.\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{6}(-2x)^3 + ...$

Hence 4^{th} term is $\frac{x^3}{2}$.

75. **(A)**

$(217)^{1/3} = (6^3+1)^{1/3} = 6\left(1+\frac{1}{6^3}\right)^{1/3}$

On expansion by binomial theorem

$= 6\left(1 + \frac{1}{3\times 216} - \frac{1\times 2}{3\times 3\times 2}\left(\frac{1}{216}\right)^2 + ...\right) = 6.01$

76. **(D)**

The given expression can be written as $4^{-1/2}\left(1-\frac{3}{4}x\right)^{-1/2}$

and it is valid only when $\left|\frac{3}{4}x\right| < 1 \Rightarrow -\frac{4}{3} < x < \frac{4}{3}$.

77. **(A)**

$(a+bx)^{-2} = \frac{1}{a^2}\left(1+\frac{b}{a}x\right)^{-2} = \frac{1}{a^2}\left[a + \frac{(-2)}{1!}\left(\frac{b}{a}\right)x + ...\right]$

Equating it to $\frac{1}{4} - 3x + ...$, we get $a = 2, b = 12$.

78. **(D)**

In the expression $(1+x-3x^2)^{3148}$ the sum of coefficients is obtained by putting $x = 1$.

$\therefore$ Sum of coefficients $=(1+1-3.1^2)^{3148}$

$=(2-3)^{3148}=(-1)^{3148}=1$.

79. **(B)**

$$\sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \frac{1}{k(k+1)}$$

$$= \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+...+\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=1-\frac{1}{n+1}=\frac{n}{n+1}$$

$$\left(\sum_{k=1}^{n} a_k\right)^2 = \left(\frac{n}{n+1}\right)^2.$$

80. **(C)**

Sum of the coefficients $=(1+1)^5 = 2^5 = 32$.

81. **(5)**

Sum of the coefficient of odd powers of x

$= C_1 + C_3 + C_5 + ... = 2^{n-1}$

82. **(6)**

$$T_{r+1} = {}^{10}C_r (x^2)^{10-r} \left(\frac{-3\sqrt{3}}{x^3}\right)^r$$

For term independent of x, $20-2r-3r=0 \Rightarrow r=4$

$\therefore$ $T_{4+1} = {}^{10}C_4 (-3)^4 (\sqrt{3})^4 = {}^{10}C_4 (3)^6$

83. **(4)**

Middle term $= T_{\frac{2n+2}{2}} = T_{n+1} = {}^{2n}C_n x^n = \dfrac{2n!}{(n!)^2}.x^n$.

84. **(0)**

Greatest coefficient of $(1+x)^{2n+2}$ is ${}^{2n+2}C_{n+1}$

85. **(9)**

Let $(r + 1)^{\text{th}}$ term be the greatest term. Then

$$T_{r+1} = \sqrt{3}.\,{}^{20}C_r \left(\frac{1}{\sqrt{3}}\right)^r \text{ and } T_r = \sqrt{3}.\,{}^{20}C_{r-1}\left(\frac{1}{\sqrt{3}}\right)^{r-1}$$

Now $\dfrac{T_{r+1}}{T_r} = \dfrac{20-r+1}{r}\left(\dfrac{1}{\sqrt{3}}\right)$

$\therefore$ $T_{r+1} \geq T_r \Rightarrow 20-r+1 \geq \sqrt{3}r$

$\Rightarrow 21 \ge r(\sqrt{3}+1) \Rightarrow r \le \dfrac{21}{\sqrt{3}+1} \Rightarrow r \le 7.686 \Rightarrow r = 7$

Hence the greatest term is

$T_8 = \sqrt{3}\ {}^{20}C_7 \left(\dfrac{1}{\sqrt{3}}\right)^7 = \dfrac{25840}{9}$

86. (9)

(9) According to the question,

${}^{14}C_{r-1}, {}^{14}C_r, {}^{14}C_{r+1}$ are in A.P., so $\left\{b = \dfrac{a+c}{2}\right\}$

$\Rightarrow \quad 2\ {}^{14}C_r = {}^{14}C_{r-1} + {}^{14}C_{r+1}$

or $\quad \dfrac{2 \times 14!}{(14-r)!r!} = \dfrac{14!}{(14-r+1)!(r-1)!} + \dfrac{14!}{(14-r-1)!(r+1)!}$

or $\quad \dfrac{2}{(14-r)(13-r)!r(r-1)!} = \dfrac{1}{(15-r)(14-r)(13-r)!(r-1)!} + \dfrac{1}{(13-r)!(r+1)r(r-1)!}$

or $\quad \dfrac{2}{(14-r)r} = \dfrac{1}{(15-r)(14-r)} + \dfrac{1}{r(r+1)}$

or $\quad \dfrac{2}{(14-r)r} - \dfrac{1}{r(r+1)} = \dfrac{1}{(15-r)(14-r)}$

or $\quad \dfrac{3r-12}{r(r+1)} = \dfrac{1}{(15-r)}$

$\Rightarrow \quad r = 5$ or 9

87. (8)

Let the three consecutive coefficients be ${}^nC_{r-1} = 28$, ${}^nC_r = 56$ and ${}^nC_{r+1} = 70$,

so that $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r} = \dfrac{56}{28} = 2$ and

$\dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{n-r}{r+1} = \dfrac{70}{56} = \dfrac{5}{4}$

This gives $n+1 = 3r$ and $4n - 5 = 9r$. Therefore,

$\dfrac{4n-5}{n+1} = 3$ or $n = 8$

88. (1)

$$\left(5^{\frac{2}{5}\log_5 \sqrt{4^x+44}} + \frac{1}{5^{\log_5 \sqrt[3]{2^{x-1}+7}}}\right)^8$$

$$= \left((\sqrt{4^x+44})^{2/5} + \left(\frac{1}{\sqrt[3]{2^{x-1}+7}}\right)\right)^8$$

$$= \left((4^x+44)^{1/5} + \frac{1}{(2^{x-1}+7)^{1/3}}\right)^8$$

Now $T_4 = T_{3+1} = {}^8C_3\,((4^x+44)^{1/5})^{8-3}\dfrac{1}{((2^{x-1}+7)^{1/3})^3}$

Given $336 = {}^8C_3\left(\dfrac{4^x+44}{2^{x-1}+7}\right)$

Let $2^x = y$

$\Rightarrow\quad 336 = {}^8C_3\left(\dfrac{y^2+44}{(y/2)+7}\right)$

or $\quad 336 = \dfrac{8\times7\times6}{3\times2\times1}\left(\dfrac{2(y^2+44)}{y+14}\right)$

$\Rightarrow\quad y^2 - 3y + 2 = 0$ or $y = 0, 2$

89. (6)

$\left(1-2x+5x^2-10x^3\right)\left[{}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ...\right]$

$= 1 + a_1x + a_2x^2 + ...$

$\Rightarrow a_1 = n-2$ and $a_2 = \dfrac{n(n-1)}{2} - 2n + 5$

Given that $a_1^2 = 2a_2$

$\Rightarrow (n-2)^2 = n(n-1) - 4n + 10$

or $n^2 - 4n + 4 = n^2 - 5n + 10$

or $n = 6$

90. (1)

$$\sum_{k=0}^{4}\left(\frac{3^{4-k}}{(4-k)!}\right)\left(\frac{x^k}{k!}\right)$$

$$= \sum_{k=0}^{4}\left(\frac{3^{4-k}}{(4-k)!}\right)\left(\frac{x^k}{k!}\right)\frac{4!}{4!}$$

$$= \sum_{k=0}^{4} \frac{{}^4C_k \cdot 3^{4-k} \cdot x^k}{4!} = \frac{(3+x)^4}{4!}$$

According to the equation,

$$\frac{(3+x)^4}{4!} = \frac{32}{3}$$

or $\quad (3+x)^4 = 256$

or $\quad x+3=4$ or $x=1$