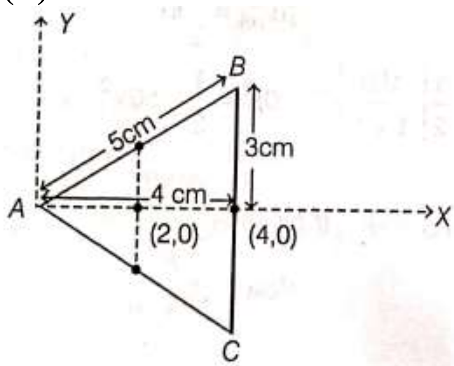
JEE Main Exercise

1. (A)



$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{\lambda(5)(2) + \lambda(5)(2) + (2\lambda)(6)(4)}{\lambda(5) + \lambda(5) + 2\lambda(6)} = \frac{34}{11}\text{ cm}$$

2. (C)

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 - m_2\mathbf{r}_2 - m_3\mathbf{r}_3}{m_1 - m_2 - m_3}$$

$$= \frac{[\sigma\pi(4R)^2)](0\hat{i} + 0\hat{j}) - (\sigma\pi R^2)(3R\,\hat{i}) - (\sigma\pi R^2)(3R\hat{j})}{\sigma\pi(4R)^2 - \sigma\pi R^2 - \sigma\pi R^2}$$

$$= \frac{-3R}{14}(\hat{i} + \hat{j})$$

3. (D)

$$s_{AC} = 2L \Rightarrow s_A = 2L + s_C$$

$$s_{BC} = -2L \Rightarrow s_B = -2L + s_C$$

$$s_{CM} = \frac{m_As_A + m_Bs_B + m_Cs_C}{m_A + m_B + m_C} = 0$$

$$\Rightarrow \quad \frac{m(2L + s_C) + 2m(-2L + s_C) + 3m\,s_C}{6m} = 0$$

$$\Rightarrow \quad s_C = \frac{L}{3}$$

4. (B)

$s_{AP} = +4 \Rightarrow s_A = 4 + s_P$

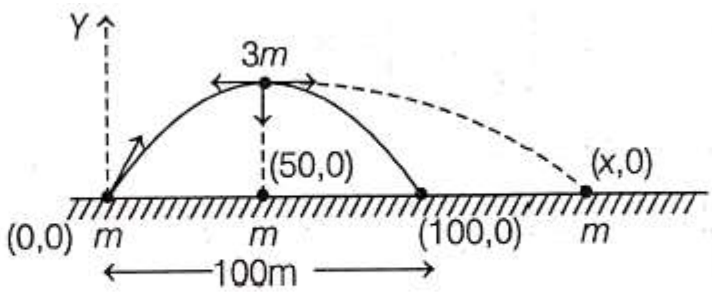
$s_{CP} = -4 \Rightarrow s_C = -4 + s_P$

$$s_{CM} = \frac{m_1 s_1 + m_2 s_2 + m_3 s_3 + m_4 s_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow 0 = \frac{40(4 + s_p) + 60(-4 + s_p) + 50 s_p + 90 s_p}{40 + 60 + 50 + 90}$$

$$\Rightarrow s_p = \frac{1}{3}\text{m towards right}$$

5. (C)



$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$\Rightarrow \quad 100 = \frac{m(0) + m(50) + mx}{3m} \Rightarrow x = 250 \text{ m}$$

6. (B)

Friction on wedge will be acting towards right. Due to friction, centre of mass of (wedge + block) system will move rightward and due to gravity centre of mass will move downward.

7. (D)

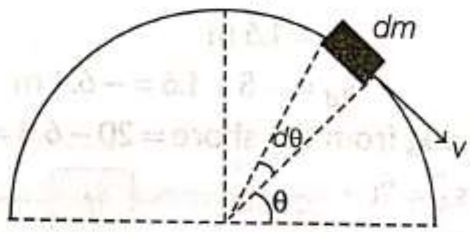
External forces on the (Earth + ball) system is zero. So to keep centre of mass stationary both will move away from each other.

8. (B)

$$dm = \frac{M}{\pi R} R d\theta = \frac{M}{\pi} d\theta$$

$$dp = d\,mv \sin\theta\,\hat{i} + dmv\cos\theta\,\hat{j}$$

$$\Rightarrow \int d\mathbf{p} = \frac{Mv}{\pi}\int_0^{\pi} \sin\theta\, d\theta\,\hat{i} + \frac{Mv}{\pi}\int_0^{\pi}\cos\theta\, d\theta\,\hat{j}$$



$$\Rightarrow \quad \mathbf{p} = \frac{2Mv}{\pi}\hat{i} \Rightarrow p = \frac{2Mv}{\pi}$$

9. (A)

Applying linear momentum conservation in horizontal

$$0 + 0 = 1v_1 + 2(-v_2)$$

$$\Rightarrow \quad v_1 = 2v_2$$

Applying work-energy theorem,

$$W_{\text{gravity}} + W_{\text{internal normal}} + W_{\text{normal from ground}} = \Delta K$$

$$\Rightarrow \quad 1 \times 10\,(1.1 - 0.1) + 0 + 0$$

$$= \left(\frac{1}{2}\times 1 v_1^2 + \frac{1}{2}\times 2v_2^2\right) - (0+0)$$

$$\Rightarrow \quad 10 = \frac{1}{2}(2v_2)^2 + v_2^2$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{10}{3}}\ \text{m/s}$$

10. (A)

m_1 will break off the wall when the spring acquires natural length,

Applying work-energy theorem,

$$W_{spring} + W_{mg} + W_N = \Delta K$$

$$\Rightarrow \quad \frac{1}{2}K(x^2 - 0^2) + 0 + 0 = \frac{1}{2}m_2v^2 - 0$$

$$\Rightarrow \quad v = \left(\sqrt{\frac{K}{m_2}}\right)x$$

$$v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$= \frac{m_1(0) + m_2\left(\sqrt{\frac{K}{m_2}}\right)x}{m_1 + m_2} = \frac{(\sqrt{Km_2})x}{m_1 + m_2}$$

11. (C)

Using energy conservation in CM reference frame,

$$K_1 + U_1 = K_2 + U_2$$

$$\Rightarrow \quad \frac{1}{2}\left[\frac{3(6)}{3+6}\right][2-(-1)]^2 + 0 = \frac{1}{2}\left[\frac{3(6)}{3+6}\right](0)^2 + \frac{1}{2}\times 200\,x^2$$

$$\Rightarrow \quad x = 30 \text{ cm}$$

12. (B)

$$F = u\left(\frac{dm}{dt}\right) = 400 \times 0.05 = 20 \text{ N}$$

13. (B)

$$F = \frac{dm}{dt}v$$

$$F_{avg} = \frac{\Delta m}{\Delta t}v = \frac{5}{2.5}\times 4 = 8 \text{ dyne}$$

14. (A)

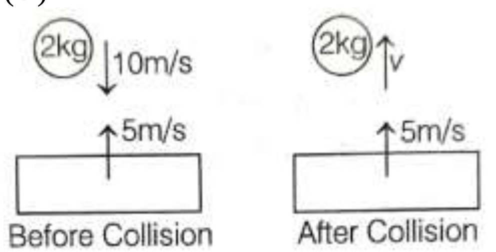
2. (a) $I_1 = m\left[\sqrt{2gh} - \left\{-\sqrt{2g\left(\frac{h}{4}\right)}\right\}\right] = \frac{3m}{2}\sqrt{2gh}$

$$I_2 = m\left[\sqrt{2gh} - \left\{-\sqrt{2g\left(\frac{h}{16}\right)}\right\}\right] = \frac{5}{4}m\sqrt{2gh}$$

$$m\sqrt{2gh} = \frac{2I_1}{3} = \frac{4}{5}I_2$$

$$\Rightarrow \quad 5I_1 = 6I_2$$

15. (D)



$$e = 1 = \frac{v-5}{10-(-5)}$$

$$\Rightarrow \quad v = 20 \text{ m/s}$$

$$I = m(v_2 - v_1) = 2[20-(-10)]$$

$$= 60 \text{ N-s}$$

16. (C)

5. (c) Let time be t when string again gets taut

$$s_1 = s_2 \Rightarrow 4t = 2t + \frac{1}{2}(10)t^2 \Rightarrow t = 0.4 \text{ s}$$

At $t = 0.4$ s, velocity of lower block

$$v = u + at = 2 + (10)(0.4) = 6 \text{ m/s}$$

Using linear momentum conservation,

$$4(4) + 4(6) = (4+4)v$$

$$v = 5 \text{ m/s}$$

$$I = \int T dt = 4(5-4) = 4 \text{ N-s}$$

17. (A)

7. (a) $mu + m(0) = mv + m(2v)$

$$\Rightarrow \quad v = \frac{u}{3}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{2v - v}{u - 0} = \frac{v}{u} = \frac{1}{3}$$

18. (A)

11. (a) $mu + m(0) = mv_1 + mv_2$

$$\Rightarrow \quad v_1 + v_2 = u \quad \text{...(i)}$$

$$e = \frac{v_2 - v_1}{u - 0} \Rightarrow v_2 - v_1 = eu \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_1 = \frac{(1-e)u}{2}, v_2 = \frac{(e+1)u}{2}$$

$$\frac{v_1}{v_2} = \left(\frac{1-e}{1+e}\right)$$

19. (A)

Speed of 2 kg just before collision,

$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad v^2 = (1)^2 + 2(-0.2 \times 10)(0.16)$$

$$\Rightarrow \quad v = 0.6 \text{ m/s}$$

Using linear momentum conservation,

$$2(0.6) + 4(0) = 2v_1 + 4v_2$$

$$v_1 + 2v_2 = 0.6 \quad \text{... (i)}$$

$$e = \frac{v_2 - v_1}{0.6 - 0} = 1$$

$$\Rightarrow \quad v_2 - v_1 = 0.6 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_1 = -0.2 \text{ m/s and } v_2 = 0.4 \text{ m/s}$$

Separation between them when they come to rest,

$$= \frac{v_1^2}{2\mu g} + \frac{v_2^2}{2\mu g} = 0.05 \text{ m} = 5 \text{ cm}$$

20. (A)

13. (a) When A collides with B, they will exchange their velocities.

For collision between B and C

$$mv + 4m(0) = mv_1 + 4mv_2$$

$$\Rightarrow \quad v_1 + 4v_2 = v \quad \text{...(i)}$$

$$e = \frac{v_2 - v_1}{v - 0} = 1 \Rightarrow v_2 - v_1 = v \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_1 = -0.6v,\ v_2 = 0.4\,v$$

Now, B will collide with A and they will exchange their velocities. So, finial velocity of A will be $0.6\,v$, left.

21. (B)

17. (b) Velocity of ball B just after collision $= \sqrt{2gh}$

$$= \sqrt{2 \times 10 \times 5}$$

$$= 10\ \text{m/s}$$

Using linear momentum conservation,

$$m(16) + m(0) = mv_1 + m(10)$$

$$\Rightarrow \quad v_1 = 6\ \text{m/s}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{10 - 6}{16 - 0} = \frac{1}{4}$$

22. (D)

21. (d) $T = \sqrt{\frac{2h}{g}} + \frac{2(e\sqrt{2gh})}{g} + \frac{2(e^2\sqrt{2gh})}{g} + \frac{2(e^3\sqrt{2gh})}{g} + \ldots$

$$= \sqrt{\frac{2h}{g}}(1 + 2e + 2e^2 + 2e^3 + \ldots)$$

$$= \sqrt{\frac{2h}{g}}[1 + 2e(1 + e + e^2 + \ldots)]$$

$$= \sqrt{\frac{2h}{g}}\left(1 + 2e\left[\frac{1}{1-e}\right]\right)$$

$$= \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$

23. (D)

27. (d) $mu + 0 = mv_1 + nmv_2$

$\Rightarrow \quad v_1 + nv_2 = u \quad$ ...(i)

$$e = \frac{v_2 - v_1}{u - 0} = 1$$

$\Rightarrow \quad v_2 - v_1 = u \quad$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$v_2 = \frac{2u}{(n+1)}$$

$$K_2 = \frac{1}{2}(nm)v_2^2$$

Fraction of incident energy transferred to the heavier ball

$$= \frac{\frac{1}{2}(nm)\left(\frac{2u}{n+1}\right)^2}{\frac{1}{2}mu^2} = \frac{4n}{(1+n)^2}$$

24. (A)

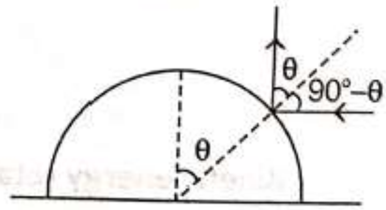
31. (a) $\tan\phi = \frac{\tan\theta}{e} = \frac{\tan 45°}{\left(\frac{1}{\sqrt{2}}\right)}$

$\Rightarrow \quad \phi = \tan^{-1}(\sqrt{2})$

$$v' = \sqrt{(ev\cos\theta)^2 + (v\sin\theta)^2} = \frac{\sqrt{3}}{2}v$$

25. (D)

32. (d) $\tan\phi = \dfrac{\tan\theta}{e}$



$$\Rightarrow \quad \tan\theta = \frac{\tan(90^\circ - \theta)}{(1/3)}$$

$$\Rightarrow \quad \tan^2\theta = 3$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60^\circ$$

26. (A)

$$\mathbf{n} = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) = -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

$$e = \frac{v_{\text{sep}}}{v_{\text{app}}} = \frac{|\mathbf{v}\cdot\mathbf{n}|}{|\mathbf{u}\cdot\mathbf{n}|}$$

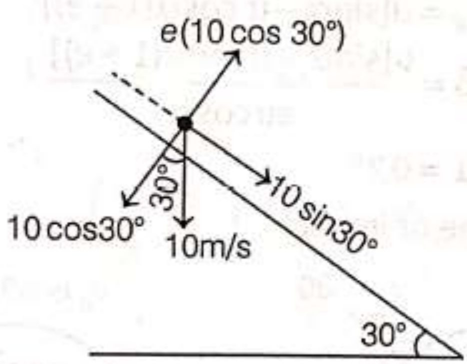
$$= \frac{-4 - 2 + 24}{|-8 - 3 - 40|}$$

$$= \frac{18}{51} = \frac{6}{17}$$

27. (A)

Velocity of ball just before hitting the inclined plane

$$= \sqrt{2gh} = 10\text{ m/s}$$



$$T = \frac{2u_y}{g\cos 30^\circ} = \frac{2[0.5(10\cos 30^\circ)]}{10\cos 30^\circ} = 1\text{ s}$$

28. (B)

(b) $F_{ext} + F_{Thrust} = ma$

$\Rightarrow \quad -mg + \frac{dm}{dt} v_{rel} = ma$

$\Rightarrow \quad -5000 \times 10 + \frac{dm}{dt}(800) = 5000 \times 20$

$\Rightarrow \quad \frac{dm}{dt} = 187.5 \text{ kgs}^{-1}$

29. (C)

(c) $F_{Thrust} = m\frac{dv}{dt}$

$\Rightarrow \quad -\frac{dm}{dt} v_{rel} = m\frac{dv}{dt}$

$\Rightarrow \quad \int_0^v dv = -\int_{m_0}^{m_0/2} \frac{dm}{m} v_{rel}$

$\Rightarrow \quad v = 2\ln 2$

30. (9)

$$x_{CM} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2} = \frac{\sigma\pi(28)^2(0) - \sigma\pi(21)^2(7)}{\sigma\pi(28)^2 - \sigma\pi(21)^2} = -9 \text{ cm}$$

31. (4)

$$x_{CM} = \frac{\int x\,dm}{\int dm} = \frac{\int x\rho\pi y^2 dx}{\int \rho\pi y^2 dx} = \frac{\int_0^6 x\rho\pi\left(\frac{x}{k}\right)dx}{\int_0^6 \rho\pi\left(\frac{x}{k}\right)dx} = 4m$$

$y_{CM} = 0$

So, $\quad \alpha + \beta = 4 + 0 = 4$

32. (3)

$$(s_{CM})_x = \frac{m_1 s_1 + m_2 s_2}{m_1 + m_2}$$

$\Rightarrow \quad 0 = \frac{2s_m + 1(S_m - 9)}{2+1} \Rightarrow s_m = 3\text{m}$

33. (8)

$$(s_{CM})_x = \frac{m_1 s_1 + m_2 s_2}{m_1 + m_2}$$

$$\Rightarrow \quad 0 = \frac{50(-3 + s_p) + 100 s_p}{50 + 100}$$

$$\Rightarrow \quad s_p = 1\text{ m}$$

$$s_m = -3 + 1 = -2\text{m}$$

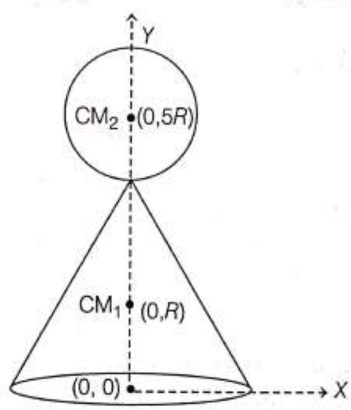
Distance travelled by plank in 5 s $= 2 \times 5 = 10$ m

Distance travelled by man w.r.t. ground $= 10 - 2 = 8$ m

34. ***4R* from *O***

Mass of cone $= \rho\left[\frac{1}{3}\pi(2R)^2(4R)\right] = \frac{16}{3}\rho\pi R^3$

Mass of sphere $= 12\rho\left(\frac{4}{3}\pi R^3\right) = 16\rho\pi R^3$



Centre of mass of solid cone is $\frac{H}{4} = \frac{4R}{4} = R$ above base.

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$= \frac{\frac{16}{3}\rho\pi R^3(R) + 16\rho\pi R^3(5R)}{\frac{16}{3}\rho\pi R^3 + 16\rho\pi R^3} = 4R$$

35. $\dfrac{2r}{3(4-\pi)}$

Let mass per unit area of plate be σ.

m_1 = mass of rectangular plate $= \sigma(2r \times r) = 2\sigma r^2$,

m_2 = mass of semi-circular plate $= \sigma\left(\frac{\pi r^2}{2}\right)$

$$x_{CM} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

$$= \frac{(2\sigma r^2)\left(\frac{r}{2}\right) - \sigma\left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)}{2\sigma r^2 - \sigma\left(\frac{\pi r^2}{2}\right)}$$

$$= \frac{2r}{3(4-\pi)}$$

36. **g/4**

37. **1670 N**

$\Sigma F_{ext} = Ma_{CM} = m_1a_1 + m_2a_2 + m_3a_3 + m_4a_4$

$\Rightarrow \quad T - 40g - 60g - 50g - 20g$

$= 40(-2) + 60(0) + 50(1) + 20(0)$

$\Rightarrow \quad T = 1670\text{ N}$

38. ***L*/4**

Let displacement of plank $= s_p$

Displacement of man w.r.t. plank $= s_{mp} = L$

$\Rightarrow \quad s_m - s_p = L \Rightarrow s_m = L + s_p$

Since, external forces on (man + plank) system are zero in horizontal direction and initial velocity of CM of (man + plank) system is zero,

$\Rightarrow \quad (s_{CM})_x = 0 \Rightarrow \frac{m_1s_p + m_2s_m}{m_1 + m_2} = 0$

$\Rightarrow \quad \frac{3Ms_p + M(L + s_p)}{4M} = 0$

$\Rightarrow \quad s_p = -\frac{L}{4} = \frac{L}{4}$, left

39. **40/3 cm**

Let displacement of boat $= s_b$

Displacement of Ram w.r.t. boat $= s_{Ab} = +2$

$\Rightarrow \quad s_A = 2 + s_b$

Displacement of Shyam w.r.t. boat $= s_{Bb} = -2$

$\Rightarrow \quad s_B = -2 + s_b$

$$(s_{CM})_x = 0$$

$$\Rightarrow \quad \frac{40s_b + 50(2+s_b) + 60(-2+s_b)}{40+50+60} = 0$$

$$\Rightarrow \quad s_b = \frac{2}{15}\text{ m} = \frac{40}{3}\text{ cm}$$

40. $\boldsymbol{h\cot\theta/6}$

Let displacement of wedge $= s_w$

Displacement of block w.r.t. wedge in horizontal

$= s_{bw} = -h\cot\theta$

$\Rightarrow \quad s_b = -h\cot\theta + s_w$

$$(s_{CM})_x = 0 \quad \Rightarrow \quad \frac{m(-h\cot\theta + s_w) + 5m\,s_w}{m+5m} = 0$$

$$\Rightarrow \quad s_w = \frac{h\cot\theta}{6}$$

41. (871.5)

For 8 kg block, $v^2 = u^2 + 2as$

$\Rightarrow \quad 0^2 = u^2 + 2(-0.5\times 9.8)(0.8)$

$\Rightarrow \quad u = 2.8\text{ m/s}$

For 6 kg block, $v^2 = u^2 + 2as$

$\Rightarrow \quad 0^2 = u^2 + 2(-0.5\times 9.8)(1.25)$

$\Rightarrow \quad u = 3.5\text{ m/s}$

Applying conservation of linear momentum,

$0.05\,v = 8\times 2.8 + 6.05\times 3.5$

$\Rightarrow \quad v = 871.5\text{ m/s}$

42. (32)

Applying linear momentum conservation for throwing of sack,

$$0+0=150\,v_{boat}+50(6)$$
$$v_{boat}=-2\text{ m/s}$$

Applying linear momentum conservation for landing of sack in the second boat,

$$50\times6+150\times0=200\,v'_{boat}$$
$$v'_{boat}=1.5\text{ m/s}$$
$$T=\frac{2u_y}{g}\Rightarrow 0.5=\frac{2u_y}{10}\Rightarrow u_y=2.5\text{ m/s}$$
$$R=\frac{2u_xu_y}{g}=\frac{2(6)(2.5)}{10}=3\text{ m}$$

Initial separation between boats $=3$m

Distance travelled by first boat while the sack is in the air $=2\times0.5=1$ m

Distance $=u_{rel}t=[2-(-1.5)](8)=28$ m

Total distance $=3+1+28=32$ m

43. (22)

$$T=\sqrt{\frac{2H}{g}}-\sqrt{\frac{2h}{g}}=\frac{1}{\sqrt{g}}(\sqrt{2\times50}-\sqrt{2\times18})=\frac{4}{\sqrt{g}}$$
$$s_x=u_xt\Rightarrow1=u_x\left(\frac{4}{\sqrt{g}}\right)\Rightarrow u_x=\frac{\sqrt{g}}{4}$$

Applying conservation of linear momentum in horizontal,

$$\Rightarrow\quad 0+0=-2v_{bag}+56\left(\frac{\sqrt{g}}{4}\right)$$
$$\Rightarrow\quad v_{bag}=7\sqrt{g}=7\times\frac{22}{7}=22\text{ m/s}$$

44. (60^o)

According to work-energy theorem

$W=$ Change in kinetic energy

$$Fs\cos\theta=\frac{1}{2}mv^2-\frac{1}{2}mu^2$$

Substituting the given values, we get;

$$20\times 4\times \cos\theta = 40-0 \qquad [\because\ u=0]$$

$$\cos\theta = \frac{40}{80} = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ .$$

45. **15 m**

$$u_{CM} = \frac{m_1u_1+m_2u_2}{m_1+m_2} = \frac{2(0)+4(15)}{2+4} = 10\text{ m/s}\uparrow$$

$$a_{CM} = \frac{m_1a_1+m_2a_2}{m_1+m_2} = \frac{2g+4g}{2+4} = g\downarrow$$

$$H_{max} = \frac{u_{CM}^2}{2a_{CM}} = \frac{(10)^2}{2(10)} = 5\text{ m}$$

Initial height of CM from ground

$$= \frac{m_1y_1+m_2y_2}{m_1+m_2}$$

$$= \frac{2(30)+4(0)}{2+4}$$

$$= 10\text{ m}$$

Maximum height attained by CM from the ground
$= 10+5 = 15$ m

46. (a) **288 J** (b) **384 J**

(a) Applying conservation of linear momentum,
$16(0) = 4v_1 + 12(4)$

$\Rightarrow \qquad v_1 = -12$ m/s

Kinetic energy of 4 kg mass $= \frac{1}{2}(4)(12)^2 = 288$ J

(b) Energy released $= K_1+K_2 = \frac{1}{2}(4)(12)^2 + \frac{1}{2}(12)(4)^2$

$= 384$ J

47. (a) $v/\sqrt{2}$ (b) $(3/2)mv^2$

(a) Applying conservation of linear momentum,

$$4m(0) = m(v\hat{i}) + m(v\hat{j}) + 2m\mathbf{v}$$

$$\Rightarrow \quad \mathbf{v} = -\frac{v}{2}\hat{i} - \frac{v}{2}\hat{j}$$

Speed of third fragment $= \sqrt{\left(\frac{v}{2}\right)^2 + \left(\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$

(b) Energy released

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)\left(\frac{v}{\sqrt{2}}\right)^2 = \frac{3}{2}mv^2$$

48. $3\sqrt{gl}$

Applying conservation of linear momentum

$$mv + 2m(0) = 3mv'$$

$$\Rightarrow \quad v' = \frac{v}{3}$$

Applying work-energy theorem after collision,

$W_{mg} + W_T = \Delta K$

$$\Rightarrow \quad -3mgl(1-\cos 60^\circ) + 0 = 0 - \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2$$

$$\Rightarrow \quad v = 3\sqrt{gl}$$

49. **220 m/s**

Speed of block just after collision $= \sqrt{2gh}$

$$= \sqrt{2 \times 9.8 \times 0.1}$$

$$= 1.4 \text{ m/s}$$

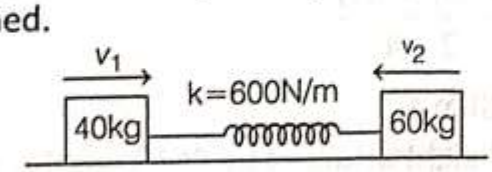
Applying conservation of linear momentum,

$$\left(\frac{10}{1000}\right)(500) + 2 \times 0 = \left(\frac{10}{1000}\right)v + 2 \times 1.4$$

$$\Rightarrow \quad v = 220 \text{ m/s}$$

50. $v_1 = 4.5\ \text{m/s},\ v_2 = 3\ \text{m/s}$

Let their velocities be v_1 and v_2 when spring becomes unstrectched.



Applying conservation of linear momentum,

$$40(0)+60(0)=40v_1+60(-v_2)$$

$$\Rightarrow \quad v_1=\frac{3v_2}{2} \quad \text{... (i)}$$

Applying conservation of mechanical energy,

$$K_1+U_1=K_2+U_2$$

$$\Rightarrow (0+0)+\frac{1}{2}\times 600\times(1.5)^2=\left(\frac{1}{2}\times 40v_1^2+\frac{1}{2}\times 60v_2^2\right)+0$$

$$\Rightarrow \quad 2v_1^2+3v_2^2=67.5 \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_1=4.5\text{ m/s and } v_2=3\text{ m/s}$$

51. **10 m/s**

Let velocity of platform $= v_p$

Velocity of man w.r.t. platform $= v_{mp}=30$

$$\Rightarrow \quad v_m-v_p=30$$

$$\Rightarrow \quad v_m=(30+v_p)$$

Applying linear momentum conservation for (man + platform) system in horizontal,

$$0+0=100(30+v_p)+200v_p$$

$$\Rightarrow \quad v_p=-10\text{ m/s}$$

52. $\dfrac{m^2u}{(M+2m)(M+m)}$

Applying conservation of linear momentum when A jumps

$$0+0=(M+m)v_c+m(-u+v_c)$$

$$\Rightarrow \quad v_c=\frac{mu}{M+2m}$$

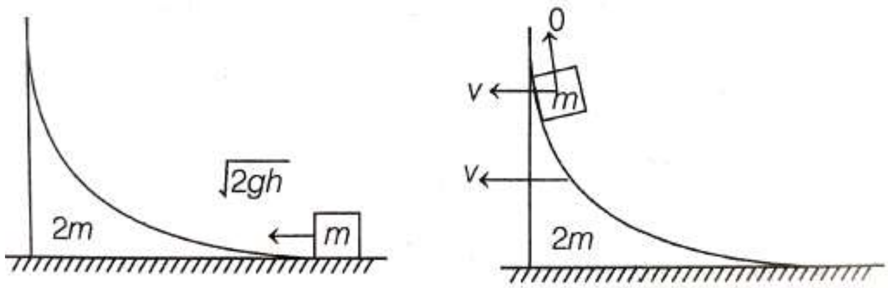
Applying conservation of linear momentum when B jumps

$$(M+m)\left(\frac{mu}{M+2m}\right)=Mv_c'+m(u+v_c')$$

$$\Rightarrow \quad v_c'=-\frac{m^2u}{(M+2m)(M+m)}$$

53. (a) $\sqrt{\frac{2gh}{9}}$ (b) $\frac{2h}{3}$

(a)



When the smaller block reaches maximum height from ground, its velocity w.r.t. the larger block is zero.

Using conservation of linear momentum,

$$2m(0) + m(\sqrt{2gh}) = 2mv + mv$$

$$\Rightarrow \qquad v = \frac{\sqrt{2gh}}{3}$$

(b) Using energy conservation,

$$K_1 + U_1 = K_2 + U_2$$

$$\Rightarrow \left[\frac{1}{2}m(\sqrt{2gh})^2 + 0\right] + 0 = \frac{1}{2}(3m)\left(\frac{\sqrt{2gh}}{3}\right)^2 + mgh_{max}$$

$$\Rightarrow \qquad h_{max} = \frac{2h}{3}$$

54. (3)

3. For ball B, $T - mg = \frac{mv_2^2}{l}$

$$\Rightarrow \qquad 40 - 10 = \frac{1v_2^2}{0.3} \Rightarrow v_2 = 3 \text{ m/s}$$

Using momentum conservation,

$$2v_0 + 0 = 2v_1 + 1(3)$$

$$2v_0 = 2v_1 + 3 \qquad \text{... (i)}$$

$$e = \frac{1}{2} = \frac{3 - v_1}{v_0 - 0}$$

$$\Rightarrow \qquad 6 - 2v_1 = v_0 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_0 = 3 \text{ m/s}$$

55. (4)

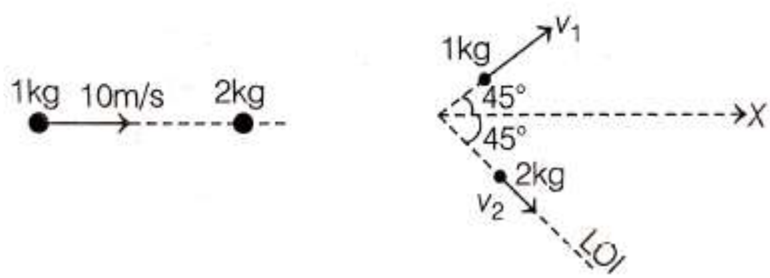
7. Collision doesn't affect the vertical component of velocity. So, total time of flight will be unchanged.

$$T = t_1 + t_2$$

$$\Rightarrow \quad \frac{2u\sin 30^\circ}{g} = \frac{50}{u\cos 30^\circ} + \frac{50}{eu\cos 30^\circ}$$

$$\Rightarrow \quad u = \frac{100}{\sqrt{7}}\ \text{m/s}$$

56. **1/2**



Using linear momentum conservation,

In X-direction,

$$1\times 10 + 2\times 0 = 1v_1\cos 45^\circ + 2v_2\cos 45^\circ$$

$$\Rightarrow \quad v_1 + 2v_2 = 10\sqrt{2} \quad \text{... (i)}$$

In Y-direction, $0 + 0 = 1v_1\sin 45^\circ - 2v_2\sin 45^\circ$

$$\Rightarrow \quad v_1 = 2v_2 \quad \text{... (ii)}$$

Solving Eqs. (i) and (ii), we get

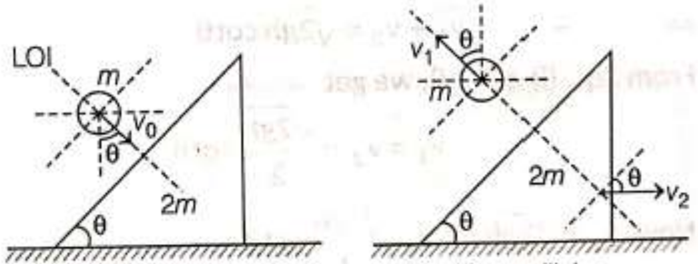
$$v_1 = \frac{10}{\sqrt{2}}\ \text{m/s and } v_2 = \frac{5}{\sqrt{2}}\ \text{m/s}$$

Line of impact will be along the line of motion of 2 kg after collision.

$$= \frac{v_2 - v_1\cos 90^\circ}{10\cos 45^\circ - 0} = \frac{5/\sqrt{2}}{10/\sqrt{2}} = \frac{1}{2}$$

57. $\dfrac{(1+e)v_0\sin\theta}{2+\sin^2\theta}$

Using linear momentum consevation for (wedge + ball) system is horizontal,



Before collision After collision

$$mv_0 \sin\theta = -mv_1 \sin\theta + 2mv_2$$

$$\Rightarrow \quad 2v_2 - v_1 \sin\theta = v_0 \sin\theta \quad \text{...(i)}$$

$$e = \frac{v_{sep}}{v_{app}} = \frac{v_1 + v_2 \sin\theta}{v_0}$$

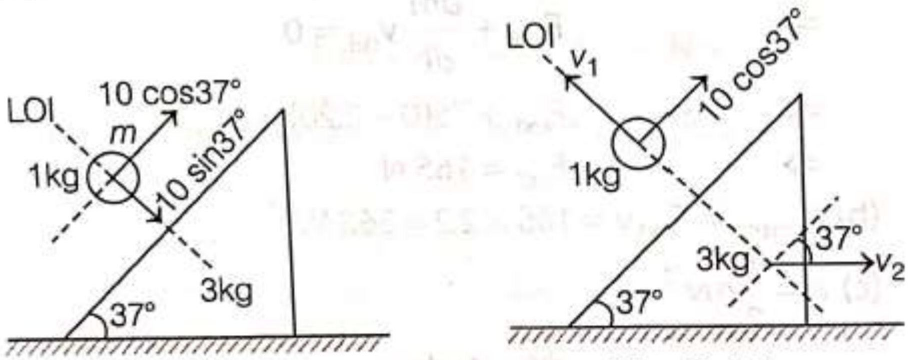
$$\Rightarrow \quad v_1 + v_2 \sin\theta = ev_0 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_2 = \frac{(e+1)v_0 \sin\theta}{2 + \sin^2\theta}$$

58. $v_{\text{Wedge}} = 1.5 \text{ m/s}$

8. Component of velocity of ball perpendicular to the line of impact remains unchanged.



Before Collision After Collision

Using conservation of momentum for (wedge + ball) system in horizontal

$$1(10)+3(0)=3v_2+1((10\cos 37°)\cos 37° - v_1 \sin 37°)$$

$$\Rightarrow \quad 5v_2 - v_1 = 6 \quad \text{...(i)}$$

$$e=\frac{v_2\sin 37° - (-v_1)}{10\sin 37°}=0.4$$

$$\Rightarrow \quad 5v_1 + 3v_2 = 12 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_2 = 1.5 \text{ m/s}$$

59. (a) 5.32×10^5 N (b) 3.22×10^5 N (c) $v_0 - gt + v_{rel}\ln\frac{M_0}{M}$ (d) 2718 m/s

(a) $F_{Thrust}=\frac{dm}{dt}v_{rel}=190\times2800$

$=5.32\times10^5$ N

(b) $F_{net}=F_{Thrust}-mg$

At blast off, $F_{net}=5.32\times10^5-(21000)(10)$

$=3.22\times10^5$N

(c) $\sum F = m\frac{dv}{dt}$

$$\Rightarrow \quad F_{ext} + F_{Thrust} = m\frac{dv}{dt}$$

$$\Rightarrow \quad -mg + \left(-\frac{dm}{dt}\right)v_{rel} = \frac{m\,dv}{dt}$$

$$\Rightarrow \quad -\int_0^t g\,dt - \int_{M_0}^{M}\frac{dm}{m}v_{rel} = \int_{v_0}^{v} dv$$

$$\Rightarrow \quad -gt + \left[\ln\left(\frac{M_0}{M}\right)\right]v_{rel} = v - v_0$$

$$\Rightarrow \quad v = v_0 - gt + v_{rel}\ln\left(\frac{M_0}{M}\right)$$

(d) Time taken to burn all the fuel $= \frac{15000}{190}\text{ s} = 78.95\text{ s}$

$$v = 0 - 10(78.95) + 2800\ln\left(\frac{21000}{6000}\right)$$

$$= 2718\text{ m/s}$$

60. (a) $\frac{M}{L}\left(gy + v_0^2\right)$ (b) $\frac{Myv_0^2}{2L}$

(a) $F_{Thrust} = \frac{dm}{dt}v_{rel} = \frac{d\left(\frac{M}{L}y\right)}{dt}(0 - v_0) = -\frac{M}{L}v_0^2$

$$F_{ext} + F_{Thrust} = m\frac{dv}{dt}$$

$$\Rightarrow \quad P - \frac{M}{L}yg - \frac{M}{L}v_0^2 = 0$$

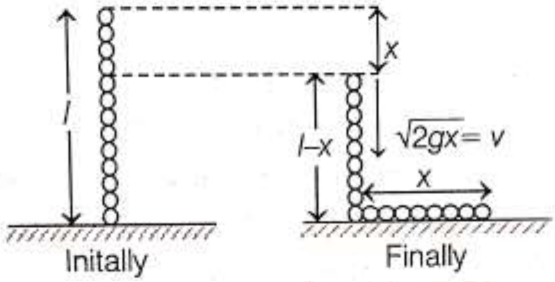
$$\Rightarrow \quad P = \frac{M}{L}(gy + v_0^2)$$

(b) Energy lost during the lifting = Work done by applied force – Increase in mechanical energy of chain

$$= \int_0^y P\,dy - \left(\left(\frac{m}{L}y\right)g\left(\frac{y}{2}\right) + \frac{1}{2}\left(\frac{M}{L}y\right)v_0^2\right)$$

$$= \frac{Myv_0^2}{2L}$$

61. $3mg\dfrac{x}{l}$



$$F_{Thrust} = \frac{dm}{dt}v_{rel} = \frac{d\left(\frac{m}{l}x\right)}{dt}(v-0)$$

$$= \frac{m}{l}\frac{dx}{dt}v = \frac{m}{l}v^2 = \frac{2mgx}{l}$$

$$N = F_{Thrust} + \left(\frac{m}{l}x\right)g$$

$$= \frac{2mgx}{l} + \frac{mgx}{l} = \frac{3mgx}{l}$$

PYQ : JEE Main

1. (b)

Given, $m_1 = 4g, u_1 = 300\,m/s$

$m_2 = 0.8\,kg = 800\,g, u_2 = 0\,m/s$

From law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Let the velocity of combined system $= v\,m/s$ then,

$$4\times300 + 800\times0 = (800+4)\times v \Rightarrow v = \frac{1200}{804} = 1.49m/s$$

Now, $\mu = 0.2(\text{given})$

$a = \mu g \Rightarrow a = 0.3\times10 \quad \left(\text{take}\,g = 10m/s^2\right)$

$= 3\,m/s^2$

Then, from $v^2 = u^2 + 2as$

$$(1.49)^2 = 0 + 2\times3\times s \Rightarrow s = \frac{(1.49)^2}{6} s = \frac{2.22}{6} = 0.379\,m$$

2. (a)

$$Z_0 = h - \frac{h}{4} = \frac{3h}{4}$$

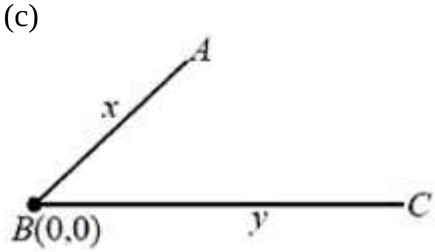
3. (b)

$$E_{\text{initial}} = \frac{1}{2}m(2v)^2 + \frac{1}{2}2nm(v)^2 = 3\,mv^2$$

$$E_{\text{final}} = \frac{1}{2}3m\left(\frac{4}{9}v^2 + \frac{4}{9}v^2\right) = \frac{4}{3}mv^2$$

$$\therefore \quad \text{Fractional loss} = \frac{3-\frac{4}{3}}{3} = \frac{5}{9} = 56\%$$

4. (c)



$$x_{cm} = \frac{x}{2}\frac{(\rho x)\left(\frac{x}{2}\right)\frac{1}{2} + \rho\,y^{\frac{x}{2}}}{\rho(x+y)}$$

$\Rightarrow \frac{1}{2}+\frac{y}{x}=\frac{y^2}{x^2}$

$\Rightarrow \frac{y}{x}=\frac{1+\sqrt{3}}{2}=1.37$

5. (b)
From impulse momentum theorem

$\int_0^1 6t\,dt = mv$

$\therefore\ v = 3\,m/s$

So, work done by the force $=\Delta K.E.=\frac{1}{2}(1)(3)^2=4.5\,J$

6. (a)
According to law of conservation of linear momentum vertical component,

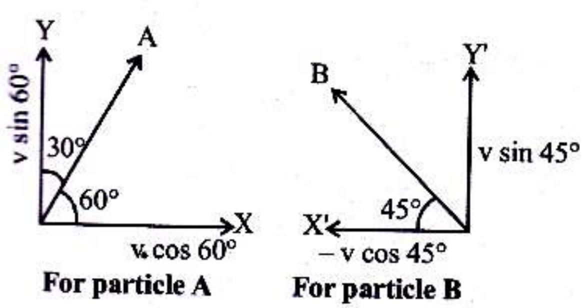
$2mv'\sin\theta = mv\sin 60^\circ + mv\sin 45^\circ$

$2mv'\sin\theta = \frac{mv}{\sqrt{2}}+\frac{mv\sqrt{3}}{2}$ ……(i)

Horizontal component,

$2mv'\cos\theta = mv\sin 60^\circ - mv\cos 45^\circ$

$2mv'\cos\theta = \frac{mv}{2}+\frac{mv}{\sqrt{2}}$ ……(ii)



Dividing equation (i) by equation(ii),

$\tan\theta = \frac{\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}}{\frac{1}{2}-\frac{1}{\sqrt{2}}}=\frac{\sqrt{2}+\sqrt{3}}{1-\sqrt{2}}$

7. (b)
Given : $m_1 = 5\text{ kg}; m_2 = 10\ \kappa\gamma; \mu = 0.15\,m$

FBD for m_1, $m_1g - T = m_1a$

$\Rightarrow \quad 50 - T = 5 \times a$ and $T - 0.15(m+10)g$

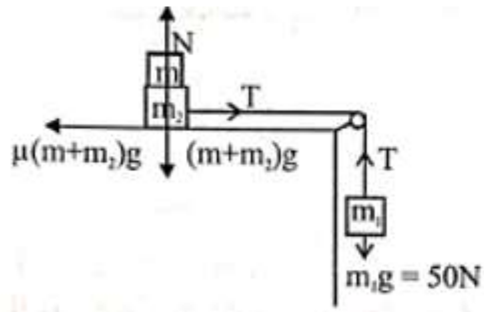
$= (10+m)a$

From rest $a = 0$

Or, $\quad 50 = 0.15(m+10)10$

$\Rightarrow \quad 5 = \frac{3}{20}(m+10)$

$\frac{100}{3} = m + 10 \quad \therefore \; m = 23.3\text{ kg}$; close to option (b)



8. (b)

$2MV, \cos 30° + mv_2 \cos 45° = 10\ M \cos 30° + 10 \cos 45°$

$\Rightarrow \quad v_1\sqrt{3} + \frac{v_2}{\sqrt{2}} = 5\sqrt{3} + 5\sqrt{2} \qquad \text{...(i)}$

$2MV, \sin 30° - mV_2 \sin 45° = -10\ m \sin 30° + 10 M \sin 45°$

$V_1 - \frac{V_2}{\sqrt{2}} = -5 + 5\sqrt{2} \qquad \text{...(ii)}$

$V_1 = \frac{5(\sqrt{3}-1) + 10\sqrt{2}}{\sqrt{3}+1} = \frac{17.5}{2.7} = 6.5\text{ m/s}$

$V_2 = 6.3\text{ m/s}$

9. (c)

$k_f = 1.5\, k_i$

$v_1^2 + v_2^2 = 1.5\, v_0^2$

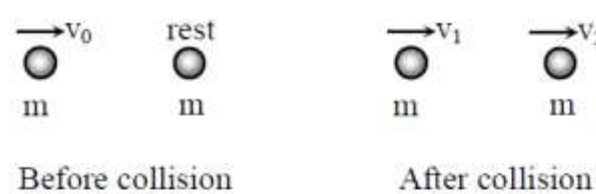
From conservation of momentum

$v_1 + v_2 = v_0$

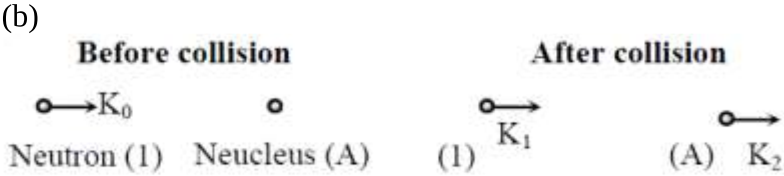
From (i) and (ii)

$2v_1v_2 = -0.5\, v_0^2$

So, $v_2 - v_1 = \sqrt{v_2^2 + v_1^2 - 2v_1v_2} = \sqrt{1.5v_0^2 + 0.5v_0^2} = \sqrt{2}\, v_0$



10. (b)



$\therefore \quad \sqrt{K_0} = \sqrt{K_1} + \sqrt{AK_2}$ (from conservation of momentum) and $K_0 = K_1 + K_2$ (for elastic collision)

So after solving

$(1+A)\frac{K_1}{K_2} - 2\sqrt{\frac{K_1}{K_0}} = (A-1)$

For Deuterium, $A = 2, 1-\frac{K_1}{K_0} = 0.89$

For Carbon, $A = 12, A-\frac{K_1}{K_0} = 0.28$

11. (b)

$$P = \frac{(2mv\cos 45°)n}{A} \qquad (P \to \text{Pressure, } A \to \text{Area})$$

$$= \frac{2\times 3.32\times 10^{-27}\times 10^{3}\times \frac{1}{\sqrt{2}}\times 10^{23}}{2\times 10^{-4}}$$

$$= 2.35\times 10^{3}\ N/m^{2}$$

12. (b)

$v_{s,m} = v_s - v_m \Rightarrow 0.7 = v_s - v_m$

$P_i = P_f$

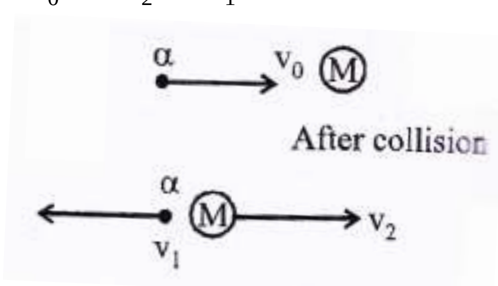
$0 = 20(0.7-\nu) - 50\nu$

Or $\nu = 0.2 m/s$

13. (d)

Using conservation of momentum

$mv_0 = Mv_2 - mv_1$



$\frac{1}{2}mv_1^2 = 0.36\times\frac{1}{2}mv_0^2\ [\because M = \text{mass of nucleus}]$

$\Rightarrow v_2 = \sqrt{\frac{m}{M}}\times 0.8v_0$

$mv_0 = \sqrt{mM}\times 0.8v_0 - m\times 0.6v_0$

$\Rightarrow 1.6m = 0.8\sqrt{mM} \Rightarrow 4m^2 = mM \qquad \therefore M = 4m$

14. (c)

Kinetic energy of block A

$k_1 = \frac{1}{2}mv_0^2$

$\therefore$ From principle of linear momentum conservation

$$mv_0 = (2m+M)v_f \Rightarrow v_f = \frac{mv_0}{2m+M}$$

According to question, if $\frac{5}{6}$th the initial kinetic energy is lost in while process.

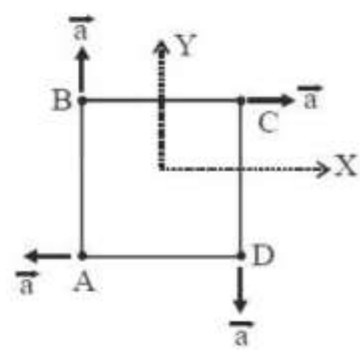
$$\therefore \frac{k_i}{k_f} = 6 \Rightarrow \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m+M)\left(\frac{mv_0}{2m+M}\right)^2} = 6$$

$$\Rightarrow \frac{2m+M}{m} = 6 \qquad \therefore \frac{M}{m} = 4$$

15. (a)

$\vec{a}_A = -a\hat{i}$; $\vec{a}_B = a\hat{j}$

$\vec{a}_c = a\hat{i}$; $\vec{a}_D = -a\hat{j}$



$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2m\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m} = \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j} = \frac{a}{5}(\hat{i} - \hat{j})$$

16. (c)

Applying linear momentum conservation

$$m_1v_1\hat{i} + m_2v_2\hat{i} = m_1v_3\hat{i} + m_2v_4\hat{i}$$

$$m_1v_1 + 0.5m_1v_2 = m_1(0.5v_1) + 0.5m_1v_4$$

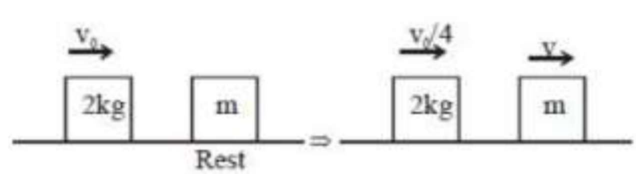
$$v_1 = v_4 - v_2$$

17. (b)

By conservation of linear momentum: n

$$2v_0 = 2\left(\frac{v_0}{4}\right) + mv \Rightarrow 2v_0 = \frac{v_0}{2} + mv\,2$$

$$\Rightarrow \frac{3v_0}{2} = mv \qquad ...(1)$$

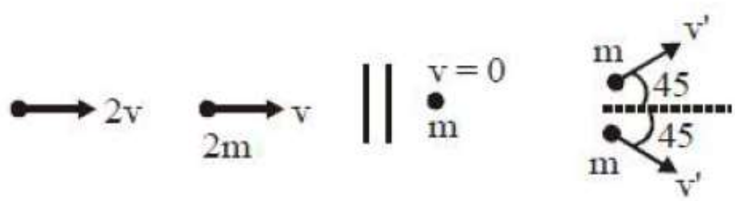


Since collision is elastic $\rightarrow$

$$V_{seperation} = v_{approch}$$

$$\Rightarrow v - \frac{v_0}{4} = v_0 \Rightarrow m = \frac{6}{5} = 1.2\text{ kg}$$

18. (b)



Linear momentum conservation

$$m\,2v + 2m\,v = m \times 0 + m\frac{v'}{\sqrt{2}} \times 2$$

$$v' = 2\sqrt{2}\,v\,.$$

19. (b)
Given,
Mass of block, $m_1 = 1.9\text{kg}$
Mass of bullet, $m_2 = 0.1\text{kg}$
Velocity of bullet, $\nu_2 = 20\,m/s$
Let ν be the velocity of the combined system. It is an inelastic collision.
Using conservation of linear momentum

$$m_1 \times 0 + m_2 \times \nu_2 = (m_1 + m_2)\nu$$

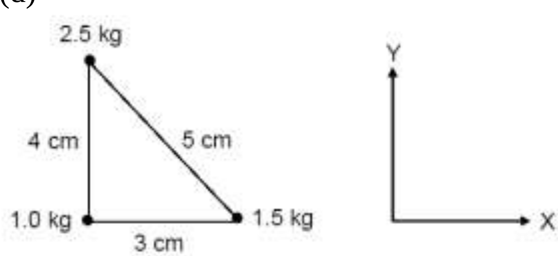
$$\Rightarrow 0.1 \times 20 = (0.1 + 1.9) \times \nu = 1m/s$$

Using work energy theorem
Work done = Change in Kinetic energy
Let K be the Kinetic energy of combined system. $(m_1 + m_2)gh$

$$= K - \frac{1}{2}(m_1 + m_2)v^2$$

$$\Rightarrow 2 \times g \times 1 = K - \frac{1}{2} \times 2 \times 1^2 \Rightarrow K = 21J$$

20. (d)

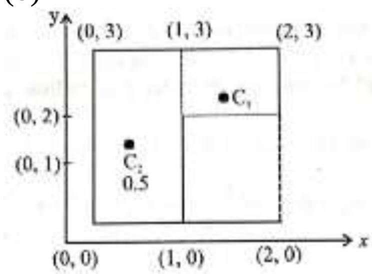


Take 1 kg mass at origin

$$X_{cm} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5} = 0.9\text{ cm}$$

$$Y_{cm} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5} = 2\text{ cm}$$

21. (b)



For given Lamina

$m_1 = 1, C_1 = (1.5, 2.5)$

$m_2, C_2 = (0.5, 1.5)$

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1m_2} = \frac{1.5+1.5}{4} = 0.75$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{2.5+4.5}{4} = 1.75$$

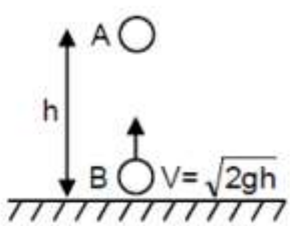
$\therefore$ Coordinate of centre of mass of flag shaped lamina (0.75, 1.75)

22. (d)

Time for collision $\qquad t_1 = \frac{h}{\sqrt{2gh}}$

After t_1 $\qquad V_A = 0 - gt_1 = -\sqrt{\frac{gh}{2}}$

and $\qquad V_B = \sqrt{2gh} - gt_1 = \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{}}\right]$



At the time f collision

$\vec{P}_i = \vec{P}_f$

$\Rightarrow \quad m\vec{V}_A + m\vec{V}_B = 2m\vec{V}_f$

$\Rightarrow \quad -\sqrt{\frac{gh}{2}} + \sqrt{gh}\left[\sqrt{2} - \frac{1}{\sqrt{2}}\right] = 2\vec{V}_f$

$V_f = 0$

and height from ground $= h - \frac{1}{2}gt_1^2 = h - \frac{h}{4} = \frac{3h}{4}$

So time $= \sqrt{2 \times \frac{\left(\frac{3}{4}\right)}{g}} = \sqrt{\frac{3h}{2g}}$

23. (b)

Conserving momentum

$$mv\hat{i}+m\left(\frac{u}{2}\hat{i}+\frac{u}{2}\hat{j}\right)=2m\left(u_1\hat{i}+u_2\hat{j}\right)$$

On solving

$$u_1=\frac{3u}{4}\text{ and }u_2=\frac{u}{4}$$

Change in K.E.

$$\left[\frac{1}{2}mu^2+\frac{1}{2}m\left(\frac{u}{2}\sqrt{2}\right)^2\right]-\left[\frac{1}{2}(2M)\left(\frac{9u^2}{16}+\frac{u^2}{16}\right)\right]$$

$$=\frac{3mu^2}{4}-\frac{5mu^2}{8}=\frac{mu^2}{8}$$

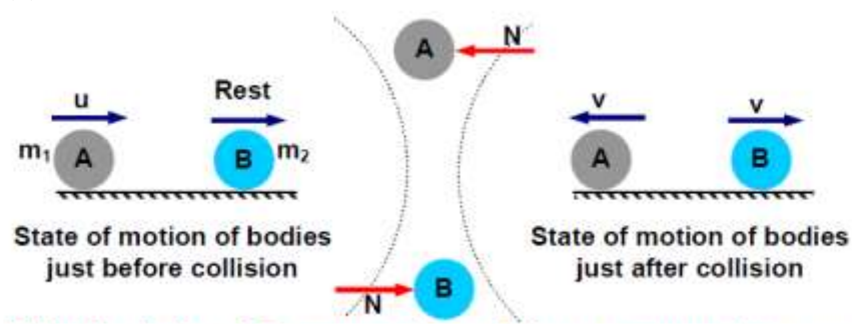
24. (d)

If collision is elastic, C comes to rest after collision. When compression in spring is maximum, velocities of A and B are same, (say v).

Using conversion of Mechanical Energy, we can write

$$\frac{1}{2}mv^2=2x\frac{1}{2}mv^2+\frac{1}{2}kx^2 \Rightarrow x=v\sqrt{\frac{m}{2k}}$$

25. (b)



With the help of Conservation of linear momentum, we can write

$$m_1u=\left(m_2-m_1\right)v \qquad ...(1)$$

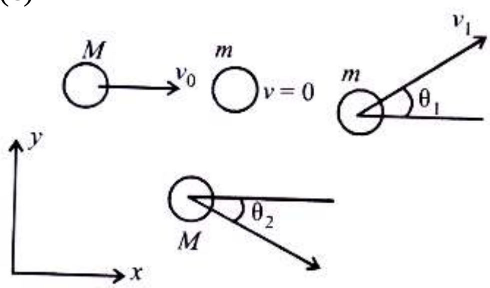
With the help of definition of e, we can write

$$e=\frac{v_s}{v_a}=\frac{2v}{u} \Rightarrow u=\frac{2v}{e} \qquad ...(2)$$

Putting the value of e in equation (1), we have

$$m_1\frac{2v}{e}=\left(m_2-m_1\right)v \Rightarrow 2m_1=em_2-em_1 \Rightarrow \frac{m_2}{m_1}=\frac{2+e}{e}=1+\frac{2}{e}>2$$

26. (c)



Let $\theta_1 = \theta_2 = \theta$. Then

$Mv_0 = mv_1 \cos\theta + Mv_2 \cos\theta$ ..... (i)

And, $0 = mv_1 \sin\theta - Mv_2 \sin\theta$ ….. (ii)

From (i) &(ii), we get

$$Mv_0 = mv_1 \cos\theta + M\left(\frac{mv_1}{M}\right)\cos\theta$$

$Mv_0 = 2mv_1 \cos\theta$ ….. (iii)

By conservation of K.E.

$$\frac{1}{2}Mv_0^2 = \frac{1}{2}Mv_1^2 + \frac{1}{2}Mv_2^2$$

$$\Rightarrow \frac{1}{2}M\left(\frac{2mv_1\cos\theta}{M}\right)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}M.\frac{m^2v_1^2}{M^2}$$

$$\Rightarrow \frac{1}{2}\left(\frac{4m^2v_1^2\cos^2\theta}{M}\right)^2 = \frac{1}{2}mv_1^2 + \frac{m^2v_1^2}{2M}$$

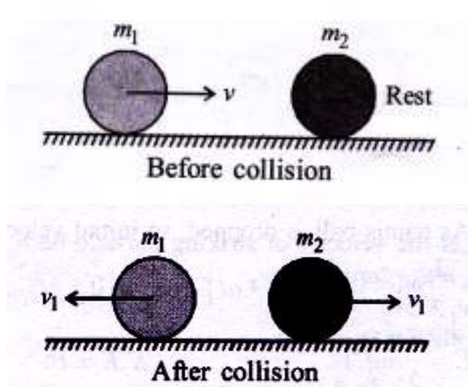
$$\Rightarrow \frac{4m\cos^2\theta}{M} = 1 + \frac{m}{M} \Rightarrow 4\cos^2\theta = \frac{M}{m} + 1$$

For largest $\frac{M}{m}$, $\cos\theta = 1$ So, $\frac{M}{m} = 3$

27. (b)

After the collision the objects move in opposite direction let with velocity v_1 then from law of conservation of momentum $P_i = P_f$

$$m_1 v = (m_2 - m_1) v_1$$



$$\Rightarrow v_1 = \frac{m_1 v}{(m_2 - m_1)} \qquad \ldots.(i)$$

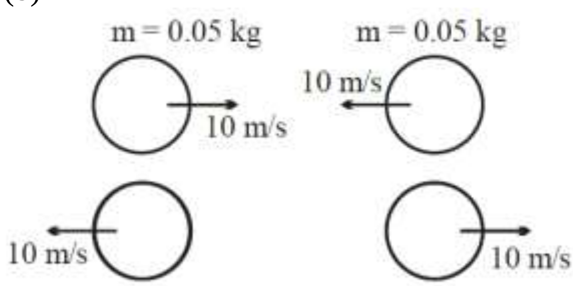
If collision is elastic then, $e = 1 = \frac{v_1 - (-v_1)}{v - 0}$

Also, $2v_1 = v \Rightarrow v_1 = \frac{v}{2} \qquad \ldots\ldots(ii)$

From equation (i) &(ii)

$$\frac{1}{2} = \frac{m_1}{m_2 - m_1} \qquad \therefore \frac{m_2}{m_1} = \frac{3}{1}$$

28. (b)



Change in momentum of any one ball

$$\left|\Delta\vec{P}\right| = 2 \times 0.05 \times 10$$

$$\left|\Delta\vec{P}\right| = 1$$

$$\left|\vec{F}_{av}\right| = \frac{\left|\Delta\vec{P}\right|}{\Delta t}$$

$$F_{av.} = 200\ \text{N}$$

29. (b)

$P_i = P_f$ (no any external force)

$$0.2 \times 10 = 10 \times v$$

$$v = 0.2\ \text{m/sec}$$

$$\text{Loss in K.E.} = \frac{1}{2} \times (0.2) \times 10^2 - \frac{1}{2} \times 10 (0.2)^2$$

$$= \frac{1}{2} \times 10 \times (0.2)[10 - 0.2]$$

$$= 9.8\ \text{J}$$

30. (b)

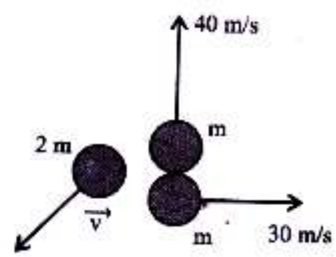
$$\vec{P}_i = 0.15 \times 12 (\hat{i})$$

$$\vec{P}_f = 0.15 \times 12 (-\hat{i})$$

$$\left|\overrightarrow{\Delta P}\right| = 3.6\ \text{kg} - \text{m/s}$$

$3.6 = F\Delta t$
$3.6 = 100\Delta t$
$\Delta t = 0.036\,\text{sec}$

31. (b)



By law of conservation of momentum

$$\left|\vec{P}_t\right| = \left|\vec{P}_f\right|$$

$$\theta = m\left(30\hat{i} + 40\hat{j}\right) + 2m\vec{v}$$

$$\vec{v} = -15\hat{i} - 20\hat{j}$$

So, $\left|\vec{v}\right| = \sqrt{-15^2 + (-20)^2}$

$$= \sqrt{625} = 25\,m/s$$

32. (c)

Let the velocity of striking particle by u_0. Then, $mu_0 = mv_1 + 5mv_2$

$u_0 = v_1 + 5v_2$ ……(i)

As, collision is elastic

So, $e = 1 \Rightarrow \dfrac{v_2 - v_1}{u_0} = 1$

$\Rightarrow v_2 - v_1 = u_0$ …..(ii)

Adding (i) ad (ii), we get

$$2u_0 = 6v_2 \Rightarrow v_2 = \frac{u_0}{3}$$

So, $\%\Delta K.E_2 = \dfrac{\frac{1}{2}(5m)\left(\frac{u_0}{3}\right)^2 - 0}{\frac{1}{2}mu_0^2} \times 100 = \dfrac{500}{9} \simeq 55.6\%$

33. (d)

$$20 \times 10^{-3} \times \frac{180}{60} \times 100 = 10\text{ V}$$

$\Rightarrow\ v = 0.6\,m/s$

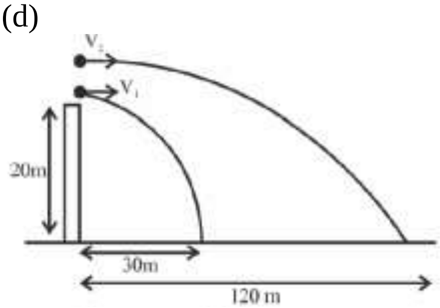
34. (c)

The velocities will be interchanged after collision.

Speed of P just before collision $= \sqrt{2gh}$

$$= \sqrt{2 \times 10 \times 0.2} = 2 \text{ m/s}$$

35. (d)

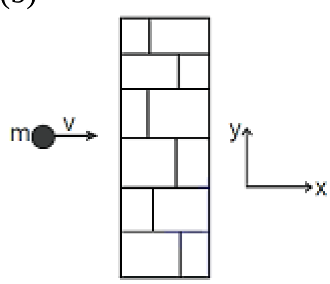


$$v_1 = \frac{30}{\sqrt{\frac{2h}{g}}}, \quad v_2 = \frac{120}{\sqrt{\frac{2h}{g}}}$$

$$(0.01)u = (0.2)\frac{30\sqrt{g}}{\sqrt{2h}} + (0.01)\frac{120\sqrt{g}}{\sqrt{2h}}$$

$$u = 300 + 60 = 360 \text{ ms}^{-1}$$

36. (b)



$P_i = Nm\upsilon\,\hat{i}$ $\qquad\qquad \vec{P}_f = -Nm\upsilon\,\hat{i}$

N is Number of balls strikes will wall N = 100

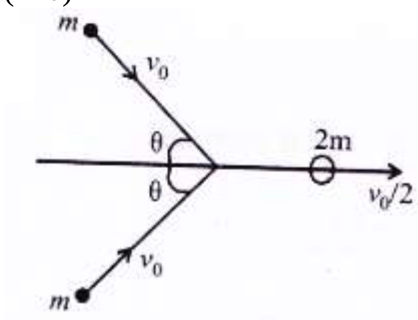
$$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = -2Nm\upsilon\,\hat{i}$$

$$= -200\ Nm\upsilon\,\hat{i}$$

$$\vec{F}_{Total} = \frac{\Delta\vec{P}}{\Delta t} = -\frac{200\,mvt}{t}$$

$$|\vec{F}| = \frac{200\,mv}{t}$$

37. (120)



Momentum conservation along x direction,

$$2mv_0\cos\theta = 2m\frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} \text{ or } \theta = 60^\circ$$

Hence angle between the initial velocities of the two bodies

$= \theta + \theta = 60^\circ + 60^\circ = 120^\circ$.

38. (10.00)

From momentum conservation in perpendicular direction of initial motion.

$mu_1 \sin\theta_1 = 10mv_1 \sin\theta_2$ ......(i)

It is given that energy of m reduced by half. If u_1 be velocity of m after collision, then

$$\left(\frac{1}{2}mu^2\right)\frac{1}{2} = \frac{1}{2}mu_1^2 \Rightarrow u_1 = \frac{u}{\sqrt{2}}$$

If v_1 be the velocity of mass 10 m after collision, then $\frac{1}{2}\times 10m\times v_1^2 = \left(\frac{1}{2}\right)m\left(\frac{u^2}{2}\right) \Rightarrow v_1 = \frac{u}{\sqrt{20}}$

From equation (i), we have

$\sin\theta_1 = \sqrt{10}\sin\theta_2$

39. (1)

For elastic collision $KE_i = KE_f$

$$\frac{1}{2}m\times 25 + \frac{1}{2}\times m\times 9 = \frac{1}{2}m\times 32 + \frac{1}{2}mv_B^2$$

$$34 = 32 + V_B^2 \Rightarrow V_B = \sqrt{2}$$

$$KE_B = \frac{1}{2}mv_B^2 = \frac{1}{2}\times 0.1\times 2 = 0.1J = \frac{1}{10}J$$

$\therefore x = 1$

40. (30)

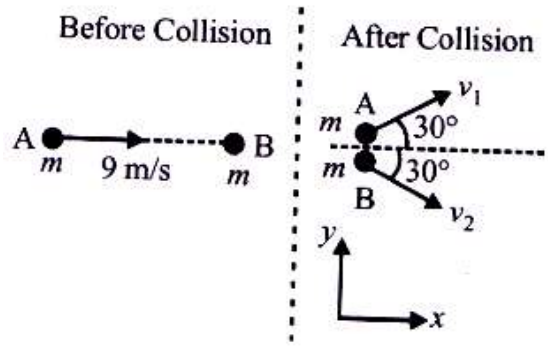
Using Conservation of linear momentum along X-axis, we can write

$$mv_0 = mv_2\cos\theta \Rightarrow \cos\theta = \frac{v_0}{v_2} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$$

41. (4)

$$x_{CM} = y_{CM} = \frac{4a}{3\pi}$$

42. (1)



From conservation of momentum along y-axis,

$$\vec{P}_{iy} = \vec{P}_{fy}$$

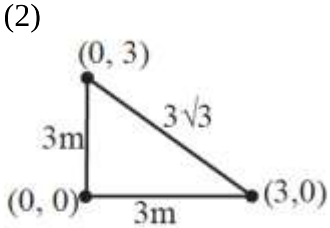
$$0 + 0 = mv_1 \sin 30^\circ \hat{j} + mv_2 \sin 30^\circ (-\hat{j})$$

$$mv_2 \sin 30^\circ = mv_1 \sin 30^\circ$$

$$v_2 = v_1 \text{ or } \frac{v_1}{v_2} = 1$$

43. (12)

44. (2)



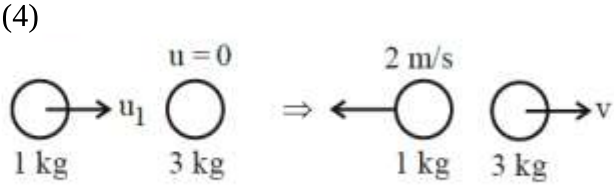
$$\vec{r}_{com} = \frac{M(0\hat{i} + 0\hat{j}) + M(3\hat{i}) + M(3\hat{j})}{3M}$$

$$\vec{r}_{com} = \hat{i} + \hat{j}$$

$$|\vec{r}_{com}| = \sqrt{2} = \sqrt{x}$$

$$x = 2$$

45. (4)



$$1 \times u_1 = -2 + 3v \quad \Rightarrow \quad u_1 = -2 + 3v \quad ...(1)$$

$1 = \dfrac{v + +2}{u_1} \qquad \Rightarrow \quad v + 2 = u_1 \qquad ...(2)$

Solving (1) and (2)

$u_1 = 4\ m/s$

COM & Conservation of P. Exercise #1.

(1).

$F_{cm} = M a_{cm}$

(D)

$\Rightarrow a_{cm} = \frac{F_{cm}}{M} = \frac{F}{M}$: independent of h (const)

(2) If ball and box is a system then there's no external force. Hence V_{cm} remains const.

(B)

(3) cm is at a distance r/2 from the centre of ring.

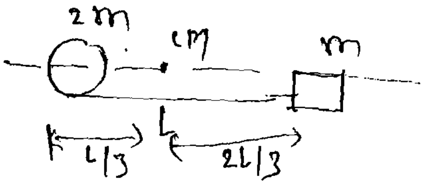
$F_{ext} = \frac{mv^2}{(r/2)} = \frac{2mv^2}{r}$

(C)

(4) (B) ~~use~~ moment of mass about cm must be neutralised.

(5)

(c)



C.M should not shift along x-axis.

$$2m\,x = m\cdot\frac{2L}{3}(1-\cos\theta)$$

$$x = \frac{L}{3}(1-\cos\theta).$$

(6) ~~Same as Q.No.~~

(b) At max^m compression both the blocks will have equal velocity

$$6*2 - 3*1 = (3+6)V$$

$$V = 1 \text{ m/s}$$

(7)

(d) Let x : Displacement of Boat (away from shore)

C.M should remain fixed.

$$5(4-x) = 20x$$

$$\boxed{x = 0.8 \text{ m}}$$

Distance of Dog from shore $= 6+0.8$
$= 6.8$ m

(8)

(c) $$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1+m_2} = \frac{m\vec{a}}{m+m} = \frac{\vec{a}}{2}$$

$$|\vec{a}_{cm}| = a/2,$$

(9) $\vec{V}_{cm} = \dfrac{m_1\vec{V_1} + m_2\vec{V_2}}{m_1+m_2} = \dfrac{10(2\hat{i}-7\hat{j}+3\hat{k}) + 2(-10\hat{i}+35\hat{j}-3\hat{k})}{10+2}$

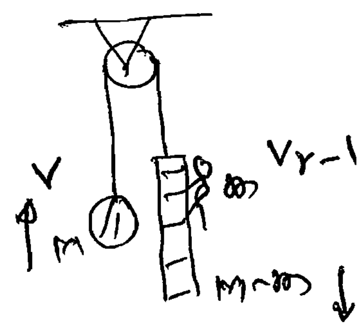
(b)

$= 2\hat{k}$ m/s

(10) $\vec{V}_{cm} = \dfrac{MV + m(V_r - V) - (M-m)V}{2M}$

(b)

$= \dfrac{mV_r}{2M}$



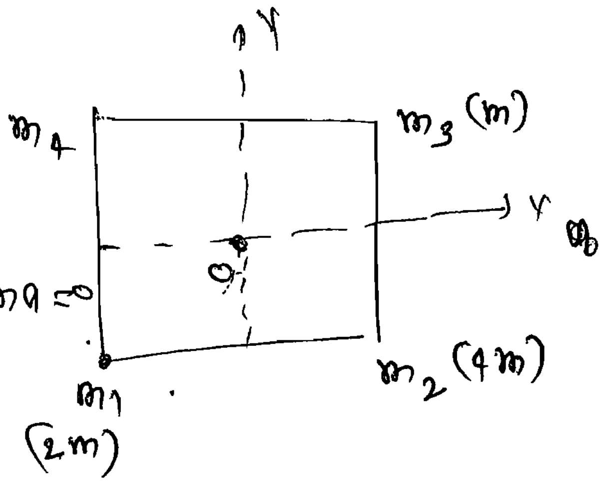
(11) motion of CM remains unaffected due to internal forces.

(a)

(12)

(b) c.m will follow the original Trajectory as if there is no explosion

(13) (d)



$x_{cm} = 0$

$\Rightarrow m_4 a + ma + 4ma - 2ma = 0$

$\Rightarrow m_4 + 5m = 2m$

$\boxed{m_4 = -3m}$

NOT possible

(14)

(b) cart and man will meet at cm of the system i.e. $x = 5$

(15)

(c) There is no force in horizontal dir^n for the "gun-shot" system. Hence $\vec{V}_{cm} = 0$

E

(16)
(c) K.E of a system of particles

$= $ K.E of CM + K.E. of different particles in the frame of CM

K.E of CM $= \frac{1}{2}mv^2$

$\therefore$ K.E of system of particles $\geq \frac{1}{2}mv^2$

(17)
(a) $$a = \frac{mg\sin 60^\circ - mg\sin 30^\circ}{2m} = \frac{(\sqrt{3}-1)g}{4}$$

$$\vec{a}_{cm} = \frac{m\vec{a}_1 + m\vec{a}_2}{m+m} = \frac{1}{2}(\vec{a}_1 + \vec{a}_2)$$

$$|\vec{a}_{cm}| = \frac{1}{2}|\vec{a}_1 + \vec{a}_2| = \sqrt{2}a = \frac{(\sqrt{3}-1)g}{4\sqrt{2}}$$

(18)
(c) $$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{\vec{v}_1+\vec{v}_2}{2} = (\hat{i}+\hat{j})\ m/s$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1+m_2} = \frac{\vec{a}_1+\vec{a}_2}{2} = \frac{3}{2}(\hat{i}+\hat{j})\ m/s^2$$

$\vec{V}_{cm} \parallel \vec{a}_{cm} \Rightarrow$ St. line path

(19)
(d) $$\left.\begin{aligned} u_x &= 20\sqrt{2}\cos 45^\circ = 20\ m/s \\ u_y &= 20\sqrt{2}\sin 45^\circ = 20\ m/s \end{aligned}\right]\ t=0$$

$t = 1$ sec, $$\left[\begin{aligned} V_x &= u_x = 20\ m/s \\ V_y &= u_y - gt = 10\ m/s \end{aligned}\right.$$

Due to explosion one part comes to rest. Hence from conservation linear momentum vertical component of second part will be $V_y' = 2V_y = 20\ m$

$$H_{max} = h_1 + h_2 = \left[20(1) - \frac{1}{2}g(1)^2\right] + \frac{V_y'^2}{2g}$$

$$= 20 + 15 = 35\ m$$

(20) (b) CM will move in a vertical line if $v_1 \cos\theta_1 = v_2 \cos\theta_2$. otherwise for any other values it will follow a parabolic path.

(21) (b) Initial x-coordinate of CM

$$x_i = \frac{4M(0) + M(5R)}{4M+M} = R \quad \text{(i)}$$

Let x_0 = x-coordinate of shell when the small sphere reaches the other extreme position

$$x_f = \frac{4M(x_0) + M(x_0 - 5R)}{4M+M} = x_0 - R \quad \text{(ii)}$$

surface is smooth $\Rightarrow x_{in} = x_f$

$$x_0 - R = R$$

$$\boxed{x_0 = 2R}$$

(22) (b) $a = \frac{2g - 1g}{2+1} = \frac{10}{3}\ m/s^2$

$$a_{cm} = \frac{2a - 1(a)}{2+1} = \frac{a}{3} = \frac{10}{9}\ m/s^2 \text{ (downward)}$$

$$\therefore S_{cm} = \frac{1}{2} a_{cm} t^2 = \frac{20}{9}\ m.$$

(23) (c) Range $= \frac{u^2 \sin 2\theta}{g} = 10\ m$

Net force in horizontal dir^n = 0

C.M remains stationary in horizontal dir^n

$$(60 + 40)\, x = 1(10)$$

$$\boxed{x = 0.1\ m}$$

(24) (d) CM cannot move towards left. It will always move towards right. Because wedge has a tendency to move left and only external force on the system is friction which will act towards right.

(25) (B) Example: Explosion

internal force can't change linear momentum

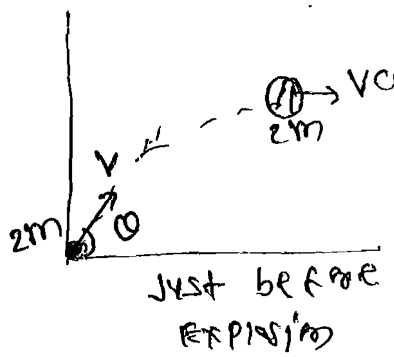
(26) (D)

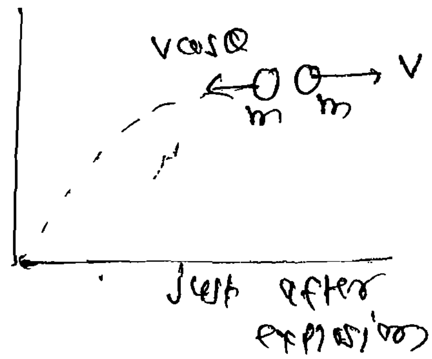
Emission is due to internal force, hence linear momentum must be conserved,

$$m\vec{v} = m_1\vec{v_1} + m_2\vec{v_2}$$

$\vec{v}$ must be parallel to $(m_1\vec{v_1} + m_2\vec{v_2})$

(27) (A)





Linear momentum conservation

$$2mv\cos\theta = mv' - mv\cos\theta$$

$$\boxed{v' = 3v\cos\theta.}$$

(28) (A)

Linear momentum conservation

$$mu = MV \quad (I)$$

Mechanical energy conservation

$$mg(R-r) = \frac{1}{2}mu^2 + \frac{1}{2}MV^2 \quad (II)$$

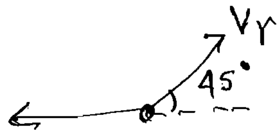
solving (I) & (II)

$$\boxed{V = \sqrt{10/3}}$$

$m = 10\,kg$
$M = 20\,kg$
$R = 1.1\,m$
$r = 0.1\,m.$

29

c



V : recoil speed of gun

$$\vec{V}_{bullet} = \vec{V}_r + \vec{V}_{gun}$$
$$= (V_r \cos 45° - V)\hat{i} + V_r \sin 45° \hat{j}$$
$$\tan\theta = \frac{V_r \sin 45°}{V_r \cos 45° - V} > 1$$
$$\boxed{\theta > 45°}$$

30

b

$h <<$ radius of Earth $\Rightarrow g$ can be assumed to be constt.

Let v = velocity of block at height $h/2$. Then velocity of Earth will be $v/3$ (conservation of $\vec{p}$)

conservation of mechanical energy:

$$\frac{M}{3} \cdot g \cdot \frac{h}{2} = \frac{1}{2}\frac{M}{3}v^2 + \frac{1}{2}M(v/3)^2$$
$$v = \frac{\sqrt{3gh}}{2}$$

31

b

Linear momentum conservation

$$mu = (m+2m)v \Rightarrow \boxed{v = u/3} \quad \text{(i)}$$

Mechanical energy conservation

$$\frac{1}{2}mu^2 = \frac{1}{2}(m+2m)v^2 + mgh \quad \text{(ii)}$$
$$\boxed{u = \sqrt{3gh}}$$

32

a

$$\frac{1}{2}\left(\frac{m \cdot 2m}{m+2m}\right)V_{rel}^2 = \frac{1}{2}kx^2$$
$$V_{rel} = \left(\frac{3k}{2m}\right)^{1/2} x$$

(33) (C)

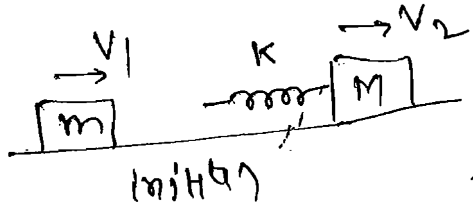
change in linear momentum, $\vec{\Delta P} = \vec{F}\cdot\Delta t$

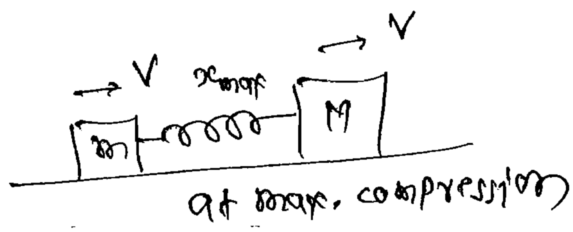
$= (m\vec{g})\,\Delta t$

$|\vec{\Delta P}| = mg\,(\Delta t)$

$= (1)\,(10)\,(1) = 10$ kg m/s.

(34) (C)





$mV_1 + MV_2 = (m+M)\,V \Rightarrow \boxed{V = \dfrac{mV_1 + MV_2}{m+M}}$ — (I)

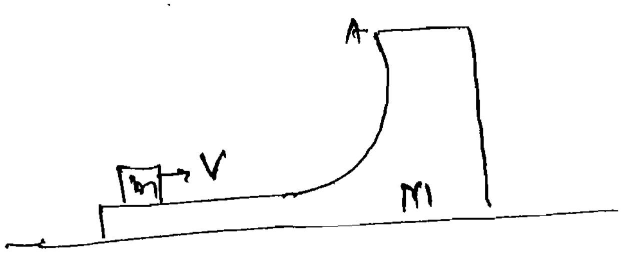
$\frac{1}{2}mV_1^2 + \frac{1}{2}MV_2^2 = \frac{1}{2}(m+M)V^2 + \frac{1}{2}K x_{max}^2$

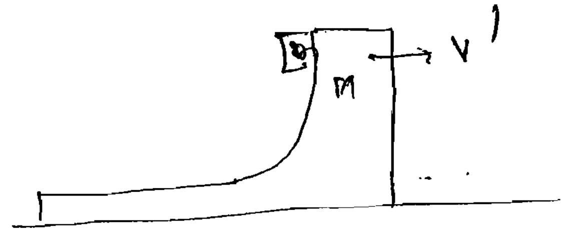
$mV_1^2 + MV_2^2 = (M+m)V^2 + \frac{1}{2}K x_{max}^2$ — (II)

solving (I) & (II)

$x_{max} = (V_1 - V_2)\sqrt{\dfrac{Mm}{(M+m)K}}$

(35) (C)





$mV = (m+M)\,V'$

$V' = \dfrac{mV}{M+m}$,

(36)
(c)

Linear momentum is conserved.

$$|\vec{P_1}| = |\vec{P_2}| = P$$

Energy released = Σ K.E of fragments

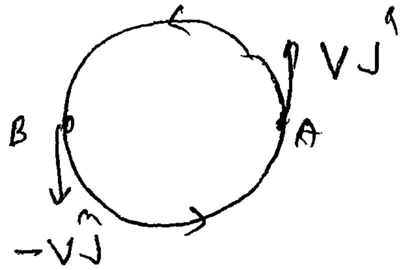
$$= \frac{P^2}{2m_1} + \frac{P^2}{2m_2}$$

(37)
(b)

$$\Delta\vec{P} = \vec{P_B} - \vec{P_A}$$

$$= -2mv\hat{j}$$

$$|\Delta\vec{P}| = 2mv$$

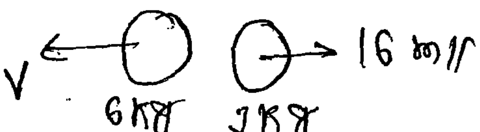


$\Delta KE = 0$

(38)
(a)

$$\Delta\vec{P} = \vec{P_f} - \vec{P_{in}} < \begin{matrix} |\Delta\vec{P}|_{lead} = mv \\ |\Delta\vec{P}|_{Tennis} = m(v+v') \end{matrix}$$

(39)
(c)



$$P_{3kg} = P_{6kg} = 48$$

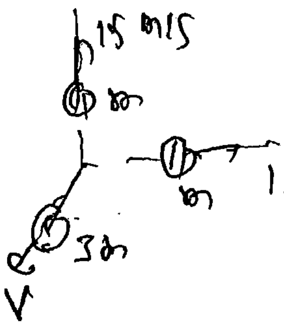
$$KE = \frac{P^2}{2m} = \frac{(48)^2}{2 \times 6} = 192\ J$$

(40)
(a)

Use linear momentum conservation

$$\vec{P_{in}} = \vec{P_f}$$

$$\Rightarrow \vec{P_f} = 0$$



$$15\sqrt{2}\,m = 3mV$$

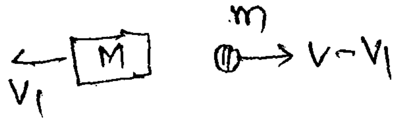
$$\boxed{V = 5\sqrt{2}\ m/s}$$

(41)
(c)

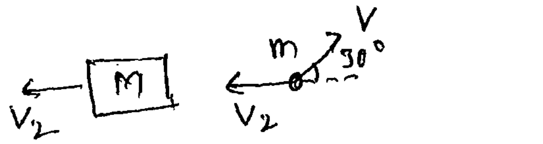
$$\vec{V_{cm}} = \frac{m_1\vec{V_1} + m_2\vec{V_2}}{m_1 + m_2} = \frac{2.48(10) + 4.0}{(10+4)} = 20\ m/s$$

(42)

(b)



case-I



case-2

$$m(v - v_1) = M v_1$$

$$\boxed{v_1 = \frac{mv}{M+m}}$$

$$m(v\cos 30^\circ - v_2) = M v_2$$

$$\boxed{v_2 = \frac{\sqrt{3}\, m v}{2(M+m)}}$$

$$v_1/v_2 = \frac{2}{\sqrt{3}}$$

(43)

(a) $\vec{F} = \frac{d\vec{P}}{dt} = 2\vec{B}t$

When $\vec{a}$ and $\vec{v}$ are at 45°, $\vec{F}$ and $\vec{p}$ will also be at 45° and this will happen

at $t = \sqrt{A/B}$

$$\vec{F} = 2\vec{B}\sqrt{A/B}$$

(44)

(a) At max^m extension velocity of both blocks will be equal.

$$(3+6)v = 6\times 2 - 3\times 1 = 9 \quad \text{(momentum conservation)}$$

$$\boxed{v = 1 \text{ m/s}}$$

Mechanical energy conservation

$$\frac{1}{2}\, 3.(1)^2 + \frac{1}{2}.6(2)^2 = \frac{1}{2}\, 200 .\, x_{max}^2 + \frac{1}{2}\, 9.(1)^2$$

$$\boxed{x_{max} = 0.3 \text{ m}}$$

(45)

(c) Impulse = change in linear momentum

$$\vec{F}.\Delta t = m(\vec{v_f} - \vec{v_{in}})$$

$$(2\hat{i} + \hat{j} + 3\hat{k})(2) = 1\left[\vec{v_f} - (2\hat{i} + \hat{j})\right]$$

$$\vec{v_f} = 6\hat{i} + 3\hat{j} + 6\hat{k}$$

$$|\vec{v_f}| = 9 \text{ m/s}$$

(46)
(b)

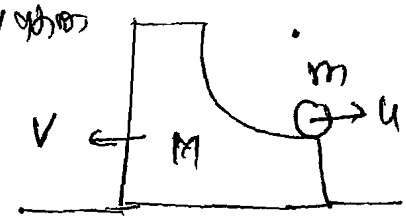
$F = 0 \Rightarrow t = 0.003$ sec

$$\text{Impulse} = \int_0^t F\,dt = 0.9 \text{ NS}$$

(47)
(d)

Linear momentum conservation

$mu = MV$ — (i)



conservation of energy

$mgR = \frac{1}{2}MV^2 + \frac{1}{2}mu^2$ — (ii)

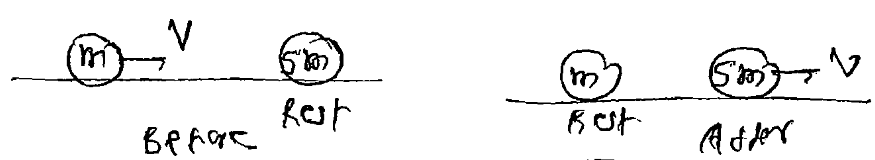
$$V = \sqrt{\frac{2mgR}{M+m}}$$

(48)
(d)

$$\text{Force} = V\frac{dm}{dt} = 5(1) = 5\text{ N}$$

$$a = \frac{F}{M} = \frac{5}{2} = 2.5\ m/s^2$$

(49)
(a)



$mV = 5mV' \Rightarrow \boxed{V' = V/5}$

$$e = \frac{V'-0}{V-0} = 1/5$$

(50)
(c)

$V_1 + 2V_2 = 36$ — (i)

$\frac{V_1 - V_2}{36} = \frac{2}{3}$ — (ii)

$V_1 = 28$ m/s
$V_2 = 4$ m/s

$$\text{loss} = \frac{1}{2}\cdot1\cdot(12)^2 + \frac{1}{2}\cdot2(24)^2 - \left[\frac{1}{2}\cdot1\cdot28^2 + \frac{1}{2}\cdot2\cdot4^2\right]$$

$= 240$ J

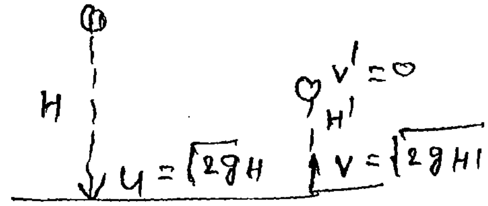
(51) (a) After collision

$$v_2 = \left(\frac{1+e}{2}\right) u$$

$$v_1 = \left(\frac{1-e}{2}\right) u$$

$$v_2 = 2v_1 \Rightarrow e = 1/3.$$

(52) (d)



$$v = eu \Rightarrow v^2 = e^2 u^2$$

$$H' = e^2 H.$$

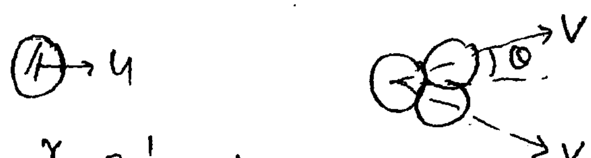
$$e^2 = H'/H = \frac{2.5}{10} = 1/4$$

$$\boxed{e = \frac{1}{2}}$$

$$I = F.\Delta t = \Delta p = mv + mu = m\left[\sqrt{2gH} + \sqrt{2gH'}\right]$$

$$F = \frac{m}{\Delta t}\sqrt{2g}\left[\sqrt{H} + \sqrt{H'}\right]$$

(53) (c)



$$\sin\theta = \frac{r}{2r} = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Linear momentum conservation

$$mu = 2mv\cos 30^\circ \Rightarrow v = u/\sqrt{3}$$

$$e = \frac{v}{u\cos\theta} = \frac{u/\sqrt{3}}{u\sqrt{3}/2} = \frac{2}{3}$$

(54) (b)

u = velocity before collision

v = velocity of ball after collision

$$= \sqrt{(u/\sqrt{2})^2 + (u/2\sqrt{2})^2} = \sqrt{5/8}\, u.$$

Fractional loss in KE

$$= \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = 1 - (v/u)^2$$

$$= 3/8.$$

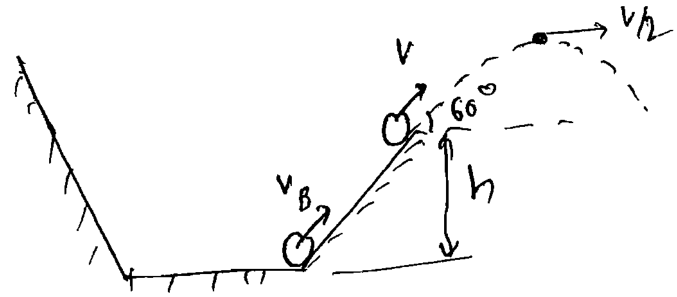
(55) After collision balls exchange their velocities

(c)

$$V_A = \sqrt{2gh}$$

$$V_B = \sqrt{2g(4h)} = 2\sqrt{2gh}$$

$$H_A = \frac{V_A^2}{2g} = h$$



$$H_B = \frac{13h}{4}$$

$$\frac{H_A}{H_B} = 4/13$$

(56) component of velocity parallel to wall remains unchanged whereas normal component will be revered and e times the initial component

(b)

$$\vec{v}_{in} = 2\hat{i} + 2\hat{j}$$

$$\vec{V}_f = -\tfrac{1}{2}(2\hat{i}) + 2\hat{j} = -\hat{i} + 2\hat{j}$$

(57)

(a)

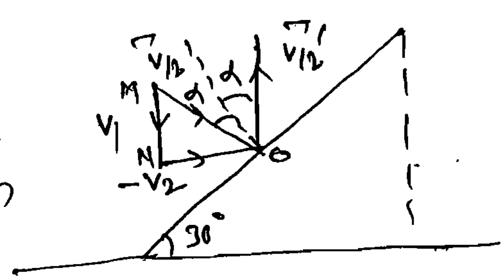
a = retardation due to friction $= \mu g = 2.5\ m/s^2$

$$S = \frac{v^2}{2a} = \frac{(5)^2}{2(2.5)} = 5$$

∴ Final separation $= 5 - 2 = 3\ m$.

(58) $\vec{v}_{12}$ = velocity of ball w.r.t wedge before collision

(b) $\vec{v}_{12}'$ = velocity of ball w.r.t wedge after collision



$$\angle MON = 30^\circ$$

$$v_1/v_2 = \tan 30^\circ = 1/\sqrt{3}$$

(59)

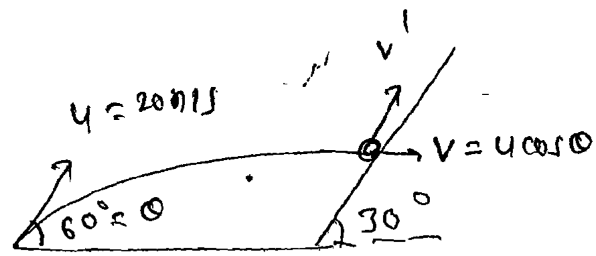
(b) $$T = \frac{d}{(v/\sqrt{2})} + \frac{d}{(ev/\sqrt{2})} = \left(1+\frac{1}{e}\right)\frac{\sqrt{2}\,d}{v}$$

$$\frac{2v \cdot \sin 45^\circ}{g} = \left(1+\frac{1}{e}\right)\frac{\sqrt{2}\,d}{v}$$

$$\boxed{e = \frac{gd}{v^2 - gd}}$$

(60) $v = u\cos 60^\circ = 10$ m/s

(a) since $e = 0$, ball will not bounce, will move along the plane with velocity $v' = v\cos 30^\circ$
$= 5\sqrt{3}$ m/s



H = max^m height attained

$$= \frac{u^2 \sin^2 60^\circ}{2g} + \frac{v'^2}{2g} = 18.75 \text{ m}.$$

Exercise #2

(1) (c, d) ~~zero~~, ~~zero~~

~~point~~

• Body may be along Y-axis $\boxed{x \geq 0}$

(2) (A, B) Non-uniform mass distribution around mid-point of rod.

(3) $F_{ext} \neq 0$, $a_{cm} = \frac{F_{ext}}{m} \neq 0$

(B, D) But V_{cm} may be zero (motion under gravity)

(4) A, B

speed of man w.r.t ground

$= V - V_{car}$

$KE = \frac{1}{2} m (V - V_{car})^2$

$W = \Delta KE < \frac{1}{2} m V^2$

If he walks normal to rail

$V_c = 0$

$KE = \frac{1}{2} m V^2$

$W = \Delta KE = \frac{1}{2} m V^2$.

(5) B, C, D

$(K.E)_{system} = (KE)_{cm} + \Sigma (K.E)$ of particle relative to cm.

$(K.E)_{system} \geq (KE)_{cm}$.

(6) (b, c) $a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{mg + mg}{2m} = g$ (downward)

$$(V_{cm})_x = (V_{cm})_y = 10 \text{ m/s}$$

$$H = \frac{(V_{cm})_y^2}{2g} = 5 \text{ m}$$

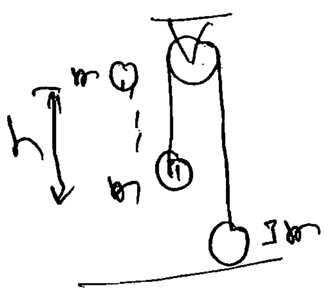
H_{max} of cm = 20 + 5 = 25 m

(7) (a, c) $\vec{a}_1 = \vec{a}_2 \Rightarrow \vec{a}_{cm} = \vec{a}_1 = \vec{a}_2$

(8) (a, c, d) $V_{cm} = \frac{mV_0 + M \cdot 0}{m + M} = \frac{mV_0}{M + m}$

At max. compression m & M will have equal velocity and will be rest in cm frame

(9) (a, b)



Just before striking: $u = \sqrt{2gh}$

After striking common speed $V = \frac{mu}{5m} = u/5$

system: $|a| = \frac{3mg - 2mg}{5m} = \frac{g}{5}$

$$H = \frac{V^2}{2 \cdot |a|} = h/5$$

(10) (a, d) Till the block B returns to its mean position spring is compressed and hence there is a force on block A. once A leaves contact with wall, net force on system becomes zero.

$V_B = \omega A = \sqrt{K/m} \cdot A = \sqrt{\frac{16}{1}} \cdot 1 = 4$ m/s (right)

$V_{cm} = 2$ m/s

(11) (a, b, d) momentum conservation

$\frac{m}{2}u = \left(m+\frac{m}{2}\right)v$

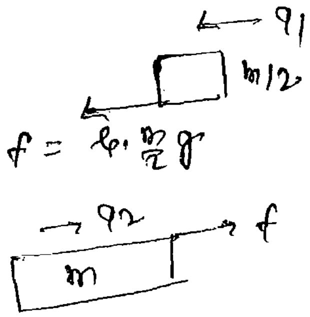
$\boxed{v = u/3}$

work done against friction

$= E_{in} - E_f$

$= \frac{1}{2}\cdot\frac{m}{2}\cdot u^2 - \frac{1}{2}\,\frac{3m}{2}\cdot\left(\frac{u}{3}\right)^2$

$= \frac{1}{6}mu^2 = \frac{2}{3}\left(\frac{1}{4}mu^2\right)$



$f = \mu\cdot\frac{m}{2}g$

$f = \mu\frac{m}{2}g$

$a_1 = \mu g$, $a_2 = \mu g/2$

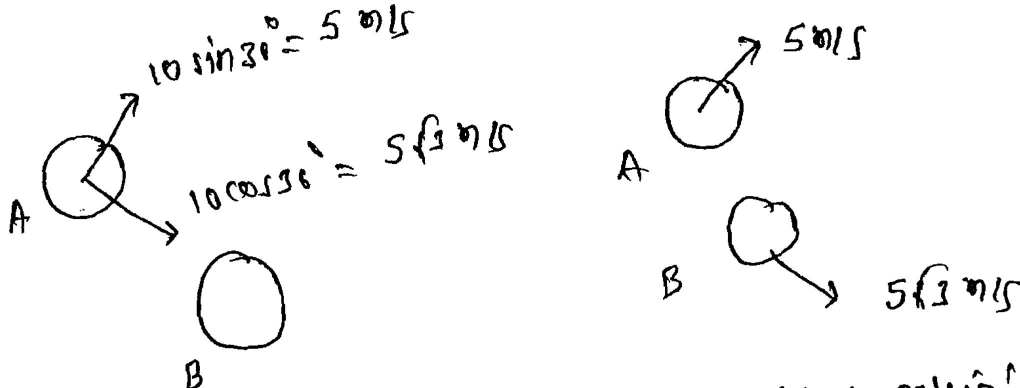
$a_{rel} = 3\mu g/2$.

(12) (b) (c)

B will give impulse −J to A

$e = \frac{J-(P-J)}{P} = \frac{2J}{P} - 1$.

(13) (a, b, c)



Before collision

After collision

Normal component exchanged in elastic collision with equal mass.

(14) (a, b)

Horizontal component of v remains unchanged while vertical component get multiplied by e.

$T_1 = \frac{2u_y}{g}$, $T_2 = \frac{2\cdot eu_y}{g}$

$R_1 = \frac{2}{g}u_x\cdot u_y$, $R_2 = \frac{2}{g}\cdot u_x(eu_y)$

$H_1 = \frac{u_y^2}{2g}$, $H_2 = \frac{(eu_y)^2}{2g}$

$\frac{T_1}{T_2} = 1/e = \frac{R_1}{R_2}$

$\frac{H_1}{H_2} = 1/e^2$

(15) (a,b,c)

Before collision $V_A = \sqrt{2gH}, \quad V_B = 0$

After collision $V_A' = \left(\frac{m_A - m_B}{m_A + m_B}\right) V_A = -\frac{\sqrt{2gH}}{2}$

$$V_B' = \left(\frac{2m_A}{m_A + m_B}\right) V_A = \frac{\sqrt{2gH}}{2}$$

$$H_A = V_A'^2 / 2g = H/4$$

$$H_B = V_B'^2 / 2g = H/4$$

collision: perfectly inelastic

$$V = \frac{m_A V_A}{m_A + m_B} = \frac{\sqrt{2gH}}{4}$$

$$H = \frac{V^2}{2g} = H/16.$$

(16) (b, c)

$$V_1' = \left(\frac{m - M}{m + M}\right) V = \left(\frac{1 - x}{1 + x}\right) V, \quad x = M/m$$

$$\boxed{V_1' = \pm V/3} \Rightarrow x = \frac{M}{m} = \frac{1}{2} \text{ or } 2$$

(17) (b, c)

Angle of incidence = Angle of reflection

$u = v$

(18) (b, c)

Momentum of the system is always conserved.
Potential energy is max^m when both particles have equal velocity.

$K.E = E$ (Before collision) $= \frac{1}{2} m v^2$

At max^m P.E, $v = u/2$

$$KE = \frac{1}{2} \cdot 2m \cdot (u/2)^2 = \frac{1}{2}\left(\frac{1}{2} m v^2\right) = E/2$$

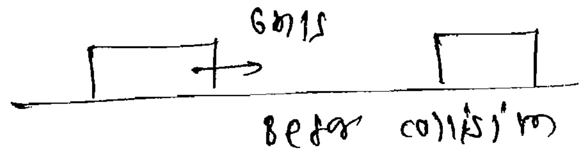
$(KE)_{min} = E - E/2 = E/2.$

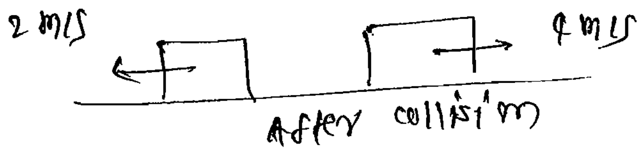
(19) (a, b, c) use definition of oblique collision

(20)

(20)

(a, c)



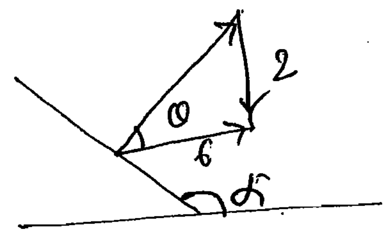


$$e = \frac{4-(-2)}{6-0} = 1$$

$$(v_{cm})_{in} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{1\times 6 + 2\times 0}{1+2} = 2\ m/s$$

(21) (a, c, d) Impulse = change in linear momentum

$$= 2(\vec{v_2} - \vec{v_1})$$
$$= 2(3\hat{i} + \hat{j})$$

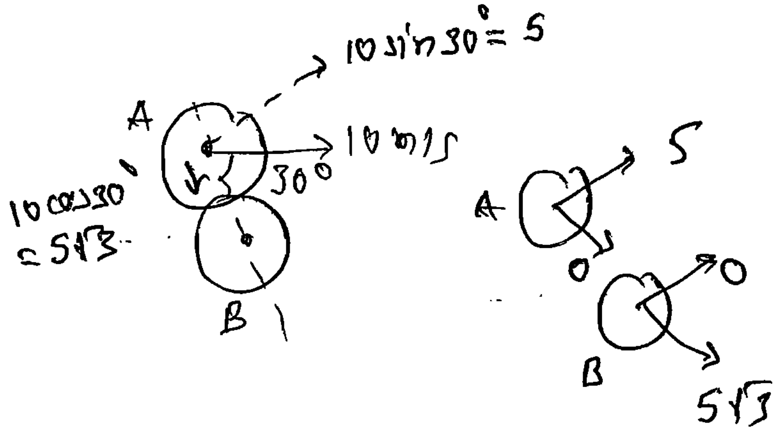


Impulse is in the normal to plane of collision

$\tan\theta = 2/6 = 1/3$

$\theta = \tan^{-1}(1/3) \Rightarrow \alpha = 90^\circ + \tan^{-1}(1/3)$

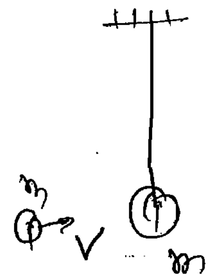
(22) (a, b, c)



(23) 23 (b, c, d)

K.E is not conserved during collision even in elastic collision.

24 (a, b, c, d)



Inelastic : $v' = v/2$

$H_{max} = \frac{v'^2}{2g} = \frac{v^2}{8g}$

$(KE)_{after} = \frac{1}{2} \cdot 2m \cdot v'^2 = \frac{1}{4}m$

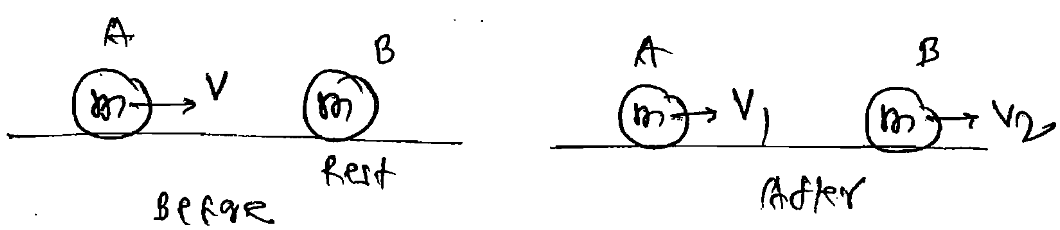
Elastic : $v' = v$

$H_{max} = v^2/2g$

(24)

25

(b, c)



$mV = mV_1 + mV_2 \Rightarrow \boxed{V_1 + V_2 = V}$ (I)

$\frac{V_2 - V_1}{V} = e \Rightarrow \boxed{V_2 - V_1 = eV}$ (II)

(25)

$V_1 = (1-e)V/2$

$V_2 = (1+e)V/2$

Exercise #3

(1). use std. result.

(A) correction

$A \to r$

$B \to p$

$C \to s$

$D \to q$

(2) use $x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$

$A \to q$
$B \to s$
$C \to p$
$D \to R$

(3) $\vec{v_1} = (2t)\hat{i}$, $\vec{a_1} = \frac{d\vec{v_1}}{dt} = 2\hat{i}$

$\vec{v_2} = (t^2)\hat{j}$, $\vec{a_2} = \frac{d\vec{v_2}}{dt} = 2t\hat{j}$

$\vec{F_{cm}} = m_1\vec{a_1} + m_2\vec{a_2} = 2\hat{i} + 2(2t)\hat{j} = 2\hat{i} + 4t\hat{j}$

$|\vec{F_{cm}}| = \sqrt{4 + 16t^2}$ $\quad t = 2\,sec,\ |\vec{F_{cm}}| = \sqrt{68}$ units

$\vec{v_{cm}} = \frac{m_1\vec{v_1} + m_2\vec{v_2}}{m_1 + m_2} = \frac{1(4\hat{i}) + 2(4\hat{j})}{1+2} = \frac{4\hat{i} + 8\hat{j}}{3}$

$|\vec{v_{cm}}| = \frac{1}{3}\sqrt{16 + 64} = \sqrt{80}/3$ m/s

$\vec{s_1} = \int_0^2 \vec{v_1}\,dt = 4\hat{i}$

$\vec{s_2} = \int_0^2 \vec{v_2}\,dt = \frac{8}{3}\hat{j}$

$\vec{s_{cm}} = \frac{m_1\vec{s_1} + m_2\vec{s_2}}{m_1 + m_2} = \frac{4}{3}\hat{i} + \frac{16}{9}\hat{j}$

$|\vec{s_{cm}}| = 20/9$

$A \to q$
$B \to R$
$C \to P$

(4) $V_{cm} = \dfrac{m_1 V_1 + m_2 V_2}{m_1 + m_2} = \dfrac{1(10) + (2)(-5)}{3} = 0$

$P_{cm} = 0$

Net force on system is zero, hence V_{cm} and P_{cm} will remain constant.

velocity of 1kg and 2kg blocks keep on decreasing initially and finally both of them stop as $V_{cm} = 0$.

A → R, S

B → R, S

C → φ

D → φ

(5) (A) $K = p^2/2m$, $K' = \dfrac{(3p)^2}{2m} = 9K$ (P)

% increase in K = 800%

(B) $p = \sqrt{2Km}$

$p' = \sqrt{2 \cdot 4K \cdot m} = 2p$ (T)

% increase in p = 100%

(C) $K = p^2/2m$

For small change

% increase in K = 2(% increase in p)

= 2% (S)

(D) $p = (2Km)^{1/2}$

% increase in p = $\frac{1}{2}$(% increase in K)

= 0.5% (R)

(6) (7), 8, 9

d, a, c

$(P_{in})_x = (P_f)_x \Rightarrow mu = mv_1\cos 30^\circ + mv_2 \cos 30^\circ$

$\boxed{v_1 + v_2 = \frac{2}{\sqrt{3}}u}$ (i)

$(P_{in})_y = (P_f)_y \Rightarrow 0 = mv_1 \sin 30^\circ - mv_2 \sin 30^\circ$

$\Rightarrow \boxed{v_1 = v_2}$ (ii)

$\boxed{v_1 = v_2 = \frac{u}{\sqrt{3}}}$

$(K.E)_{Before} = \frac{1}{2}mu^2$, $(K.E)_{after} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$= mv^2 = mu^2/3$,

(8) (10, 11, 12)

(c) Height after n^{th} collision. $H_n = He^{2n}$.

$S = H_0 + 2H_1 + 2H_2 + \dots \infty$

$= H_0 + 2e^2H + 2e^4H + \dots = h\left(\frac{1+e^2}{1-e^2}\right)$

(a) Time of ascent after n^{th} collision $= e^n t_0$

where $t_0 = \sqrt{2h/g}$

$T = t_0 + 2t_1 + 2t_2 + \dots \infty = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$

(b) $P = (\Delta P)_1 + (\Delta P)_2 + \dots$

$= (mu + meu) + (meu + me^2u) + \dots$

$= mu(1+e) + mue(1+e) + \dots$

$= mu(1+e)[1 + e + \dots \infty] = mu\left(\frac{1+e}{1-e}\right)$

$= m\sqrt{2gh}\left(\frac{1+e}{1-e}\right)$

(8)

(a) • At time t : $x_1 = 3+3t$, $y_1 = 0$

13, 14, 15

$x_2 = 0$, $y_2 = 9+6t$

(b)

$$x_{cm} = \frac{1 x_1 + 2 x_2}{1+2} = \frac{3+3t}{3} = 1+t$$

$$y_{cm} = \frac{1 \cdot y_1 + 2 \cdot y_2}{1+2} = \frac{18+12t}{3} = 6+4t$$

$$\boxed{y_{cm} = 4x_{cm} + 2}$$

(c) • First particle will stop at $t_1 = \frac{v_1}{\mu g} = 1.5 s$

second particle will stop at $t_2 = \frac{v_2}{\mu g} = 3 sec$,

Hence $v_{cm} = 0$ when both v_1 & v_2 are zero

$$\boxed{t = 3 sec}$$

(9) • 1kg will stop at $x_1 = 3 + \frac{v_1^2}{2\mu g} = 5.25 m$

$y_1 = 0$

(d) 2kg will stop at $y_2 = 9 + \frac{v_2^2}{2\mu g} = 18 m$

$x_2 = 0$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1 \cdot 5.25}{3} = 1.75 m$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{36}{3} = 12 m$$

$$\boxed{(1.75 m, 12 m)}$$

(10) (10) cm will follow the original parabolic traject

(16, 17, 18)

(a) $v^2 = u^2 - 2gh = (10)^2 - 2 \cdot 10 \cdot 1 = 80$

$v = 4\sqrt{5}$ m/s

(b) $T = \dfrac{2u\sin\theta}{g} = \sqrt{2}$ sec $\dfrac{2 \times 10 \cdot \frac{1}{\sqrt{2}}}{10}$

(c) speed of V_{cm} will be min at its heighest position

$(V_{cm})_{min} = u\cos\theta = 10 \cdot \frac{1}{\sqrt{2}}$

$= 5\sqrt{2}$

(19, 20, 21)

(11) • $P_1 = m_1 v_1 = 1(15 - \mu g t)$

$P_2 = m_2 v_2 = 2\left(\dfrac{\mu m_1 g}{m_2}\right)t = \mu g t$

$P_1 + P_2 =$ initial momentum of 1kg block $= 15$ kg·m/s (a)

but $P_2 > P_1$ since $m_2 > m_1$

• slope $= \dfrac{dP}{dt} =$ force

forces are equal and opposite (c)

$|F_1| = |F_2| = \mu m_1 g = 4$ N

• $15 - \mu g t = \mu g t$

$t = \dfrac{15}{2\mu g} = 1.875$ sec (b)

(12) ~~Let~~ ~~22, 23, 24~~ V = horizontal velocity of wedge at topmost point.
V_r = velocity of block relative to wedge.

From given condition $V_r = V$ (in magnitude)

Absolute velocity of block

$$V_b = 2V\cos 30^\circ = \sqrt{3}V$$

(22)

momentum conservation:

$$1\cdot V_0 = 2V + 1\cdot V_b\cos 30^\circ$$

$$V_0 = 2V + \frac{3}{2}V$$

$$\boxed{V_0 = \frac{7V}{2}}$$

mechanical Energy conservation

$$\frac{1}{2}\cdot 1\cdot V_0^2 = \frac{1}{2}\cdot 2\cdot V^2 + \frac{1}{2}\cdot 1\cdot(\sqrt{3}V)^2 + 1\cdot 10\cdot(1.45)$$

$$\boxed{V_0^2 - 5V^2 = 29}$$

(b)

$$\boxed{V_0 = 7\ m/s}$$

$$H = 1.45 + \frac{V_b^2\sin^2 30^\circ}{2g} = 1.6\ m$$

(c)

$$J_H = 2V = 4V_0/7 = 4\ N\text{-}s$$

$$\frac{J_H}{J_V} = \tan 60^\circ \Rightarrow J_V = J_H\cot 60^\circ = 4/\sqrt{3}\ N\text{-}s$$

$$J = \sqrt{J_H^2 + J_V^2}$$

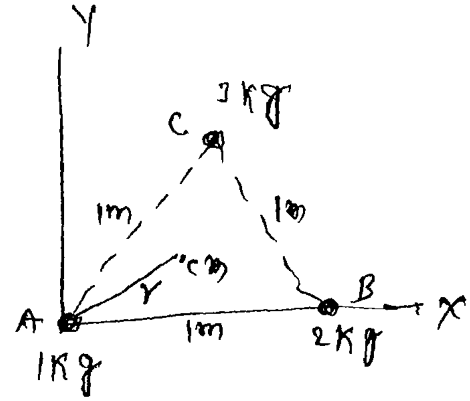
$$= 8/\sqrt{3}\ N\text{-}s$$

(a)

1.
$$x_{cm} = \frac{1\cdot 0 + 2\cdot 1 + 3\cdot \frac{1}{2}}{1+2+3} = \frac{7}{12}$$

$$y_{cm} = \frac{1\cdot 0 + 2\cdot 0 + 3\cdot \frac{\sqrt{3}}{2}}{1+2+3} = \frac{3\sqrt{3}}{12}$$

$$r = \sqrt{x_{cm}^2 + y_{cm}^2} = \frac{\sqrt{19}}{6}$$

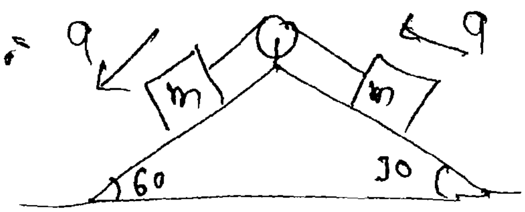


2. $\vec{F}_{ext} = 0$; CM remains at rest.

3. $$(\Delta x)_{cart} = \frac{m(\Delta x)_{\text{ball relative to cart}}}{m+M} = \frac{mL}{M+m}$$

4. $$(\Delta x)_{balloon} = \frac{m\cdot(\Delta x)_{rel}}{m+M} = \frac{mL}{m+M}$$

5. $$a_A = a_B = \frac{g\sin 60 - g\sin 30}{2} = \frac{g}{4}(\sqrt{3}-1)$$



$$\vec{a}_{cm} = \frac{m_A\vec{a}_A + m_B\vec{a}_B}{m_A+m_B} = \frac{1}{2}(\vec{a}_A + \vec{a}_B)$$

$$|\vec{a}_{cm}| = \frac{1}{2}|\vec{a}_A + \vec{a}_B| = \frac{1}{\sqrt{2}}a = \frac{g}{4\sqrt{2}}(\sqrt{3}-1)$$

6. ~~CM of man and bag will be falling vertically. Hence~~

$$\cancel{m x_{Bag} = M x}$$

$$\cancel{x_{Bag} = \frac{Mx}{m}} \; \text{~~(left)~~}$$

(7)

$m \cdot \frac{R}{2} = 3m \cdot x$

$x = \frac{R}{6} = 160\,m.$

$R = \frac{2u^2}{g} \sin\theta \cos\theta$

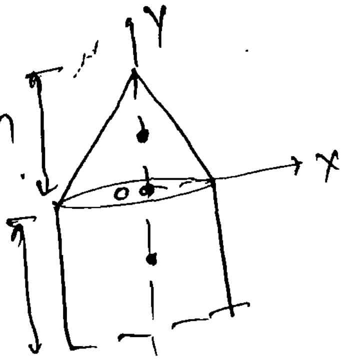
$= \frac{2}{10} (100)^2 \frac{3}{5} \cdot \frac{4}{5}$

$= 960\,m$

Distance of 3m from O $= 960 + 160$

$= 1120\,m.$

(8)



$Y_{cm} = \frac{m_1 Y_1 + m_2 Y_2}{m_1 + m_2}$

$= \frac{V_1 Y_1 + Y_2 \cdot V_2}{V_1 + V_2}$

$= \frac{\frac{1}{3}\pi R^2 h \cdot \frac{h}{4} + \left(-\pi R^2 h \cdot \frac{h}{2}\right)}{\frac{1}{3}\pi R^2 h + \pi R^2 h}$

$= -\frac{5h}{16}$

cm will be within cylinder.

(9)

$a_1 = g \sin 30^\circ = g/2$

$a_2 = g \sin 60 = \sqrt{3}g/2$

$\vec{a}_{cm} = \frac{m\vec{a}_1 + m\vec{a}_2}{2m} = \frac{\vec{a}_1 + \vec{a}_2}{2}$

$|\vec{a}_{cm}| = \frac{1}{2}|\vec{a}_1 + \vec{a}_2| = \frac{1}{2}\sqrt{a_1^2 + a_2^2} = g/2$

(10) $(x_{cm})_{in} = 44$ $\qquad (\Delta x)_{cm} = 44$

$(x_{cm})_f = 42$

since $(\vec{F}_{ext})_x = 0$, cm should not shift

hence hinge will shift toward right by 44.

(11) $u_{cm} = \frac{m_1 v_1 + m_2 u_2}{m_1 + m_2} = \frac{u_1 + v_2}{2} = \frac{50 + 30}{2} = 40\ m/s\ (\uparrow)$

$a_{cm} = 10\ m/s^2\ (\downarrow)$

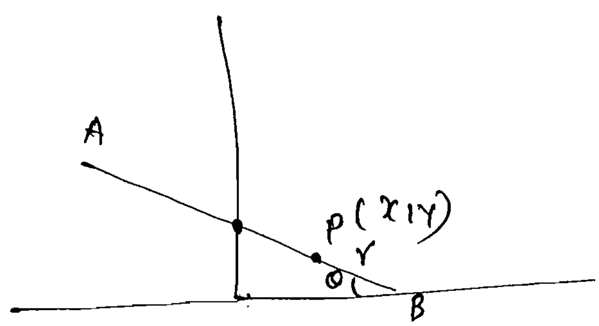
$(y_{cm})_{in} = 20\ m.$

Addition height gained $= \frac{(u_{cm})^2}{2 a_{cm}} = \frac{(40)^2}{2(10)} = 80\ m$

$(H_{cm})_{max} = 20 + 80 = 100\ m.$

(12) (a) No force on rod along x-axis. Hence cm will fall vertically downward along y-axis

(b) co-ordinates of point P

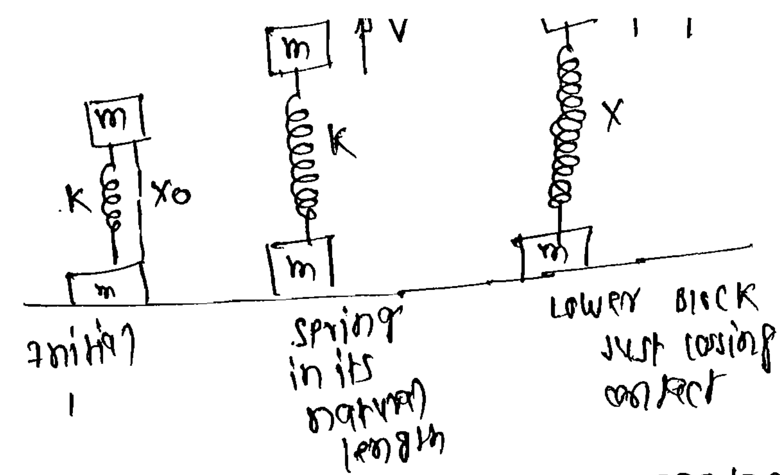


$x = \left(\frac{L}{2} - r\right)\cos\theta$

$y = r\sin\theta$

$$\boxed{\frac{x^2}{\left(\frac{L}{2} - r\right)^2} + \frac{y^2}{r^2} = 1}$$ Ellipse

Trajectory of point P.



When lower block just loses contact CM has already gained a height of $\frac{4mg}{K}$.

Now using conservation of mechanical energy

$$\frac{1}{2}Kx_0^2 = \frac{1}{2}Kx^2 + \frac{1}{2}mv_1^2 + mg(x+x_0)$$

also $Kx = mg$

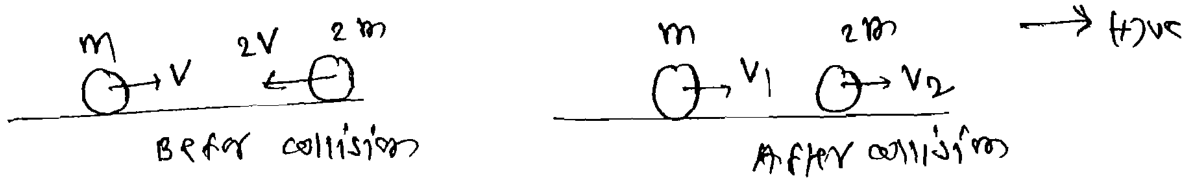
$$\boxed{v_1^2 = \frac{32(mg)^2}{Km}}$$

$$V_{cm} = \frac{m v_1 + m\cdot 0}{2m} = \frac{v_1}{2}$$

$$H = \frac{V_{cm}^2}{2g} = \frac{v_1^2}{8g} = \frac{4mg}{K}$$

$$(H_{cm})_{max} = \frac{4mg}{K} + \frac{4mg}{K} = \frac{8mg}{K}.$$

(14)



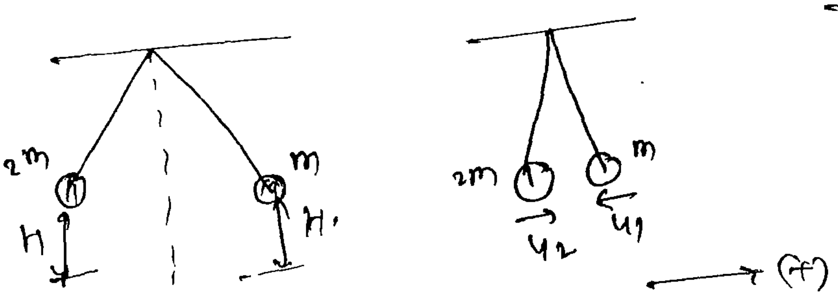
$$\vec{V_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{u_1} + \frac{2m_2}{m_1+m_2}\vec{u_2}$$

$$\vec{V_2} = \left(\frac{2m_1}{m_1+m_2}\right)\vec{u_1} + \left(\frac{m_2 - m_1}{m_1+m_2}\right)\vec{u_2}$$

$m_1 = m,\quad m_2 = 2m,\quad u_1 = V,\quad u_2 = -2V$

$$\boxed{\begin{aligned}\vec{V_1} &= -3V \\ \vec{V_2} &= 0\end{aligned}}$$

(18)



$u_1 = u_2 = \sqrt{2gH} = u$

$$\vec{V_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{u_1} + \left(\frac{2m_2}{m_1+m_2}\right)\vec{u_2}$$

$$\vec{V_2} = \left(\frac{2m_1}{m_1+m_2}\right)\vec{u_1} + \left(\frac{m_2 - m_1}{m_1+m_2}\right)\vec{u_2}$$

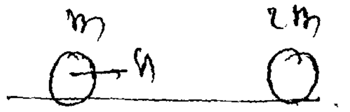
$u_1 = -u \quad | \quad m_1 = m$

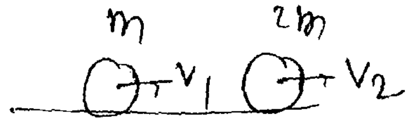
$u_2 = u \quad | \quad m_2 = 2m$

$\vec{V_1} = 5u/3 \quad \Rightarrow \quad H_1 = V_1^2/2g = \frac{25}{9}H$

$\vec{V_2} = -u/3 \qquad H_2 = V_2^2/2g = H/9.$

15





→ (4)

$$v_2 = \left(\frac{2m_1}{m_1+m_2}\right)v_1 = \frac{2 \cdot m}{m+2m}\,v = \frac{2v}{3}$$

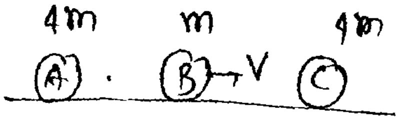
Loss in K.E of m = Gain in KE of 2m

$$= \frac{1}{2} \cdot 2m \cdot \left(\frac{2v}{3}\right)^2 = \frac{8}{9} \cdot \frac{1}{2} m v^2$$

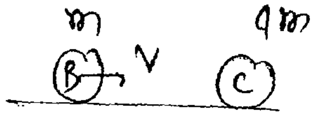
fractional loss in K.E of m

$$= \frac{\frac{8}{9} \cdot \frac{1}{2} m v^2}{\frac{1}{2} m v^2} = 8/9.$$

19



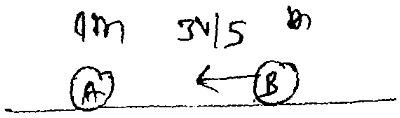
collision betn B and C.



Before

After

collision betn A and B

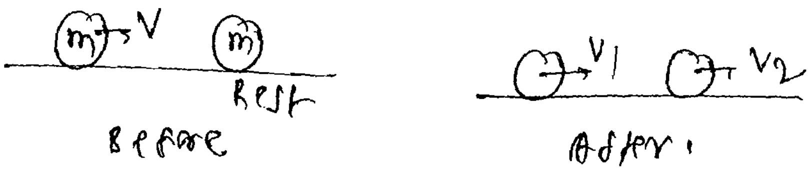


Before

After

velocity of B is less than C hence only 2 collisions will take place.

16



$$m v_1 + m v_2 = m v \Rightarrow \boxed{v_1 + v_2 = v} \quad (1)$$

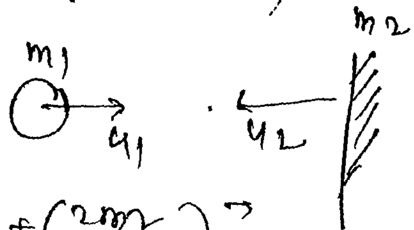
$$\frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 = \frac{3}{4}\left(\frac{1}{2} m v^2\right)$$

$$v_1^2 + v_2^2 = \frac{3}{4} v^2 \quad (11)$$

solving (1) & (11) $\boxed{v_2 - v_1 = \frac{v}{\sqrt{2}}}$

$$e = \frac{v_2 - v_1}{v} = \frac{1}{\sqrt{2}}$$

20



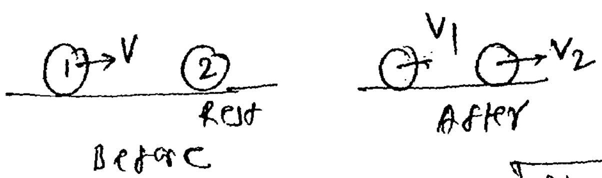
$m_2 >> m_1$

$$\vec{v_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{u_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{u_2} \longrightarrow (+)$$

$$= -\vec{u_1} + 2\vec{u_2}$$

$$= -2 + 2(-1) = -4$$

17



$$mv = m(v_1 + v_2) \Rightarrow \boxed{v_1 + v_2 = v} \quad (1)$$

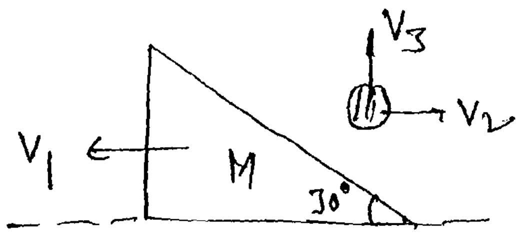
$$\boxed{v_2 - v_1 = ev} \quad (11)$$

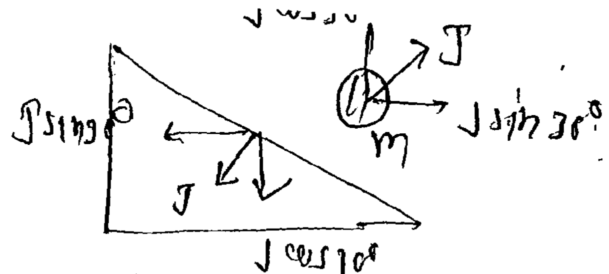
$$v_2 = \frac{(1+e)v}{2}$$

Generalise this

$$v_n = \frac{(1+e)^{n-1} \cdot v}{2^{n-1}}$$

21)





using impulse = change in linear momentum

$$J\sin 30^\circ = MV_1 = mV_2 \quad \text{(I)}$$

$$J\cos 30^\circ = m(V_3 + V_0) \quad \text{(II)}$$

using definition of e.

$$(V_1 + V_2)\sin 30^\circ + V_3\cos 30^\circ = \frac{1}{2}(V_0 \cos 30^\circ) \quad \text{(III)}$$

solving (I), (II) & (III)

$$V_1 = 1/\sqrt{3}, \quad V_2 = 2/\sqrt{3}, \quad V_3 = 0$$

22) component of velocity parallel to wall remains unchange i.e $2\hat{j}$

while component of velocity normal to wall gets reversed and multiplied by e.

i.e $-\frac{1}{2}(2\hat{i})$

$$\vec{V_f} = -\hat{i} + 2\hat{j}$$

23) $\vec{J} = \vec{P_f} - \vec{P_{in}} = m(\hat{i} + 3\hat{j}) - m(4\hat{i} - \hat{j})$

$$= m(-3\hat{i} + 4\hat{j})$$

Impulse direction is same as that of normal to wall

$$\hat{J} = \frac{\vec{J}}{|\vec{J}|} = \frac{1}{5}(-3\hat{i}+4\hat{j})$$

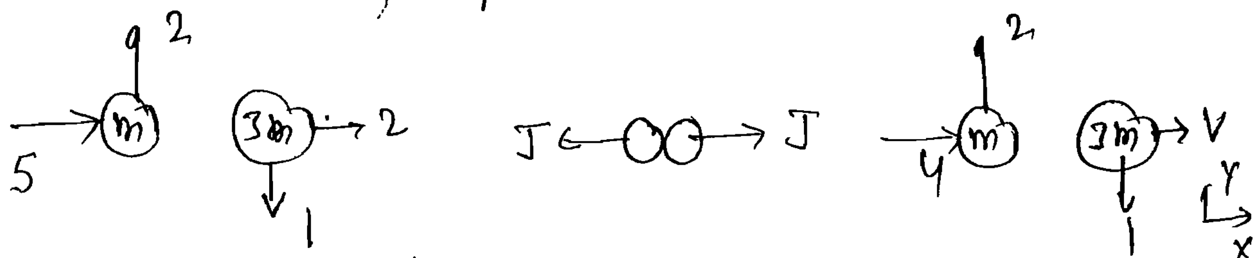
magnitude of velocity components in $\hat{J}$ dirn

$$= (4\hat{i}-\hat{j})\cdot\frac{1}{5}(-3\hat{i}+4\hat{j}) = -16/5 \text{ (Before)}$$

$$\& \quad (\hat{i}+3\hat{j})\cdot\frac{1}{5}(-3\hat{i}+4\hat{j}) = \frac{9}{5} \text{ (After)}$$

$$e = \frac{9/5}{|16/5|} \Rightarrow e = 9/16.$$

24)



conservation of linear momentum

$$5m + 3m(2) = mu + 3mV \quad \text{—(i)}$$

Definition of e: $\frac{5-2}{3} = V-u$ —(ii)

$$\boxed{u = 2,\ V = 3}$$

$$\vec{V}_A = 2\hat{i}+2\hat{j}, \quad \vec{V}_B = 3\hat{i}-\hat{j}.$$

Before collision $(KE)_A = \frac{1}{2}m(5^2+2^2) = 29m/2$

$(KE)_B = \frac{1}{2}\cdot 3m(1^2+2^2) = 15m/2$

After collision $(KE)_A = \frac{1}{2}m(2^2+2^2) = 4m$

$(KE)_B = \frac{1}{2}\cdot 3m(3^2+1^2) = 15m$

Loss in KE $= 3m.$

For sphere B: $J = 3m(3-2) = 3m$.

(25) unit vector along line of impact

$$\hat{a} = \frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$$

unit vector $\perp$r to line of impact

$$\hat{b} = \frac{1}{\sqrt{2}}(\hat{i}+\hat{j}).$$

Sphere A.

component of $\vec{V}_A$ along $\hat{a} = (\hat{i}+2\hat{j})\cdot\hat{a} = -\frac{1}{\sqrt{2}}$

component of $\vec{V}_A$ along $\hat{b} = (\hat{i}+2\hat{j})\cdot\hat{b} = 3/\sqrt{2}$

Sphere B

component of $\vec{V}_B$ along $\hat{a} = (-\hat{i}+3\hat{j})\cdot\hat{a} = -2\sqrt{2}$

component of $\vec{V}_B$ along $\hat{b} = (-\hat{i}+3\hat{j})\cdot\hat{b} = \sqrt{2}$

conservation of momentum & defn. of e along line of impact

$$m\cdot\frac{1}{\sqrt{2}} + 2m(2\sqrt{2}) = mu + 2mV$$

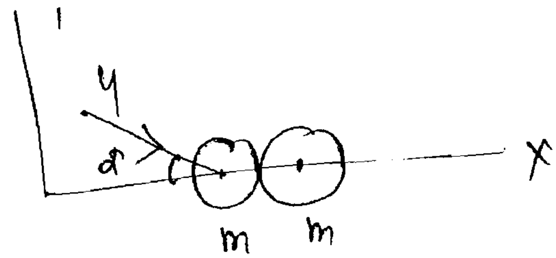
$$\frac{1}{2}\left(2\sqrt{2} - \frac{1}{\sqrt{2}}\right) = u - V$$

$$\Rightarrow \boxed{u = 2\sqrt{2},\ V = \frac{5}{4}\sqrt{2}}$$

$$(\vec{V}_A)_{\text{after collision}} = -u\hat{a} + \frac{3}{\sqrt{2}}\hat{b} = \frac{1}{2}(-\hat{i}+7\hat{j})$$

$$(\vec{V}_B)_{\text{after collision}} = -V\hat{a} + \sqrt{2}\,\hat{b} = \frac{1}{4}(-\hat{i}+9\hat{j})$$

(20)



momentum conservation along x-axis

$$mu\cos\alpha = mV_{1x} + mV_{2x}$$

$$\boxed{V_{1x} + V_{2x} = u\cos\alpha}$$

At the moment of max^m. deformation

$$V_{1x} = V_{2x}$$

$$\Rightarrow \boxed{V_{1x} = \frac{u\cos\alpha}{2}}$$

$$(KE)_{in} = \frac{1}{2}mu^2$$

$$(KE)_f = \frac{1}{2}mV_{1x}^2 + \frac{1}{2}mV_{1y}^2 + \frac{1}{2}mV_{2x}^2$$

$$= 2\cdot\frac{1}{2}mV_{1x}^2 + \frac{1}{2}mV_{1y}^2$$

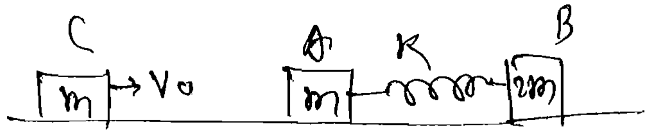
$$= m\frac{u^2\cos^2\alpha}{4} + \frac{mu^2\sin^2\alpha}{2}$$

$$= \frac{3mu^2}{8} \qquad (\alpha = 45^\circ)$$

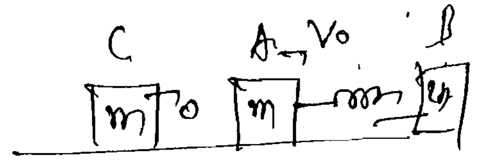
$$PE = (KE)_{in} - (KE)_f = \frac{mu^2}{2} - \frac{3mu^2}{8} = \frac{mu^2}{8}$$

$$\text{fraction} = \frac{mu^2/8}{mu^2/2} = 1/4 = 0.25$$

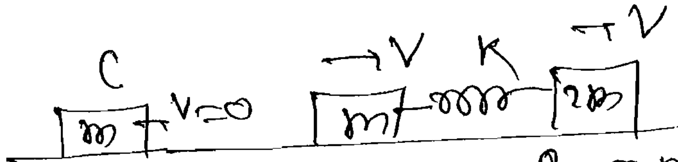
27)



Before collision



Just after collision



At max. compression

(i) conservation of linear momentum

$$m v_0 = (m+2m)V$$

$$V = v_0/3$$

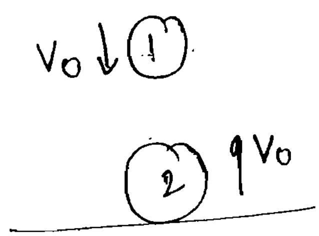
(ii) conservation of mechanical energy

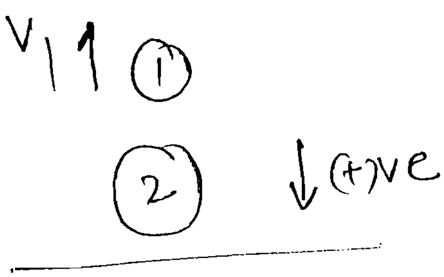
$$\frac{1}{2} m v_0^2 = \frac{1}{2}(m+2m)V^2 + \frac{1}{2} K x_0^2$$

$$\boxed{K = \frac{2}{3}\frac{m v_0^2}{x_0^2}}$$

28) Assuming balls are slightly separated when the superball hits the floor.

~~before the ball~~





$v_0 = \sqrt{2gh}$

V_1 = velocity of ball (1) just after collision

$$V_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2$$

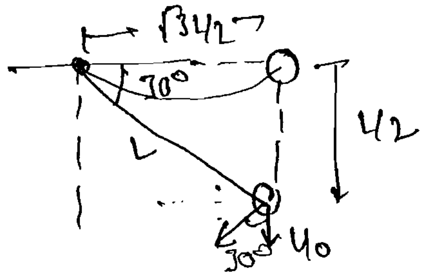
$$u_1 = V_0, \quad u_2 = -V_0, \quad \frac{m_1}{m_2} = \frac{m}{m} \approx 0$$

$$V_1 = -3V_0 = -3\sqrt{2gh}$$

H_1 = Ht. gained by ball 1

$$= \frac{V_1^2}{2g} = 9h.$$

29)



$$u_0 = \sqrt{2 \cdot g \cdot \frac{L}{2}} = \sqrt{gL}$$

(i)

An impulsive force will act when string gets its natural length. component of u_0 along string will be zero, while component of u_0 $\perp$r to string will be same

$$(u_0)_\perp = u_0 \cos 30^\circ = u_0 \cdot \frac{\sqrt{3}}{2}$$

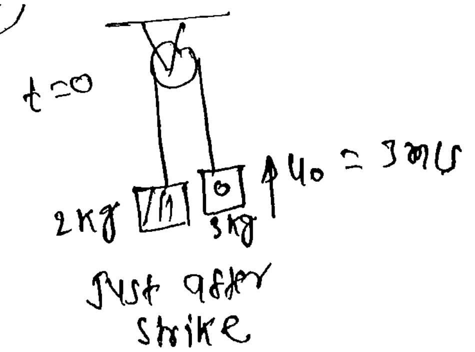
velocity of m_1 just before striking m_2 is

$$u_1 = \sqrt{\left(u_0 \frac{\sqrt{3}}{2}\right)^2 + 2 \cdot g \cdot \frac{L}{2}}$$

$$= \sqrt{\frac{7}{4} gL} = \sqrt{\frac{7}{4} \cdot 10 \cdot \frac{32}{5}} = 4 m/s$$

(ii) $$V_{3m} = \frac{2m}{m + 3m} \cdot 4 = 2 m/s$$

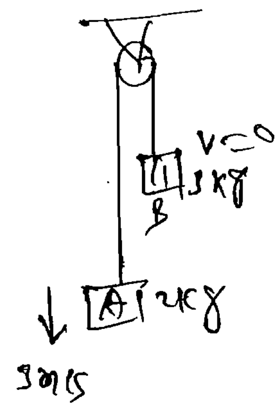
$$(H_{3m})_{max} = \frac{(2)^2}{2g} = 0.20 m.$$



velocity of 3kg will be zero at $t = \frac{u_0}{g} = 0.3$ sec

Ht. gained by B $= \frac{3^2}{2g} = 0.45$ m

velocity of A at $t = 0.3$ sec $= g(0.3) = 3$ m/s



An impulsive force (tension) will act and after that both A & B will be moving together

$$3(2) = (3+2)V$$

$$\Rightarrow \boxed{V = 1.2 \text{ m/s}}$$

System will under retardation

$$a = \frac{3g - 2g}{3+2} = 2 \text{ m/s}^2$$

velocity of B will become zero at $t = \frac{1.2}{2} = 0.6$ sec

Ht gained further $= \frac{(1.2)^2}{2(2)} = 0.36$ m

(i) Time when B starts moving down $= 0.3 + 0.6 = 0.9$ sec

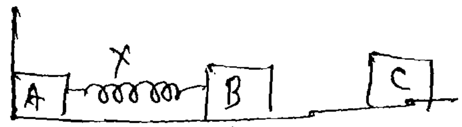
(ii) max Ht reached by B $= 0.45 + 0.36 = 0.81$ m

(iii) Loss $= \frac{1}{2} \cdot 1 \cdot (9)^2 - (3-2) \cdot 10 \cdot (0.81)$

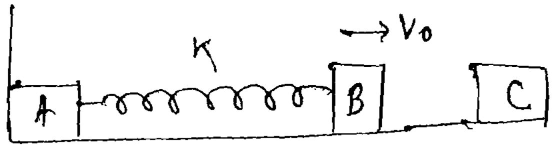
$= 40.5 - 8.1$

$= 32.4$ Joule

(vi)

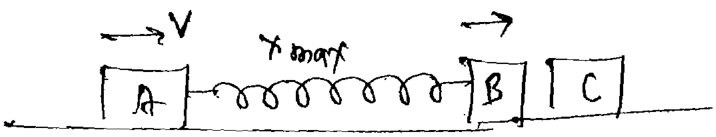


compressed



spring : Natural length

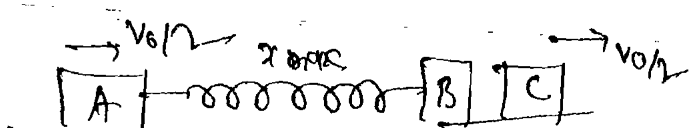
$\frac{1}{2} kx^2 = \frac{1}{2} m v_0^2$ (I)



maxm extension in spring

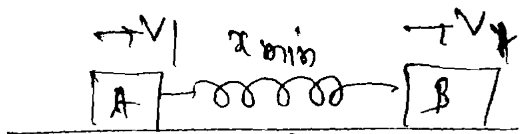
$m v_0 = (m+m)V$

$\boxed{V = v_0/2}$



just after collision

$\frac{1}{2} m v_1^2 = \frac{1}{2} \cdot 2m V^2 + \frac{1}{2} k x_{max}^2$ (II)



maxm compression in spring

$m \frac{v_0}{2} = (m+m) V_1$

$\boxed{V_1 = v_0/4}$

$\frac{1}{2} m \left(\frac{v_0}{2}\right)^2 + \frac{1}{2} k x_{max}^2 = \frac{1}{2} \cdot 2m \cdot \left(\frac{v_0}{4}\right)^2 + \frac{1}{2} k (x_{min})^2$ (III)

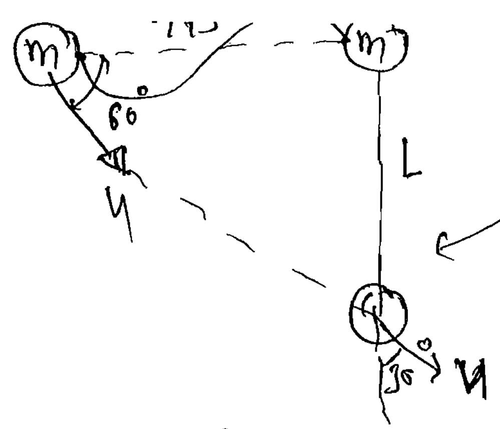
solving (I), (II) & (III)

$x_{min} = \sqrt{\frac{5}{8}}\, x$: maxm compression

and hence minm separation betn A & B

$= L - \sqrt{\frac{5}{8}}\, x$.

32)



string will be in its natural length.

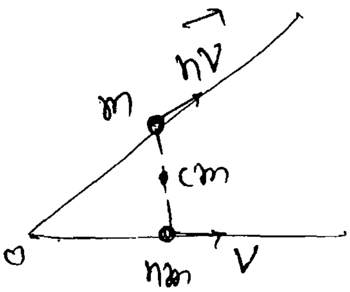
conservation of linear momentum along string

$$mu\cos 30^\circ = (m+m)V$$

$$V = \frac{u}{2}\cos 30^\circ = \frac{\sqrt{3}u}{4}$$

$$J = mV = \frac{\sqrt{3}mu}{4}$$ = impulse of the tension

33)



$v_1 = v_2 = v$

$$\vec{V}_{cm} = \frac{nm\vec{v_1} + m(n\vec{v_2})}{m+nm} = \frac{n}{n+1}(\vec{v_1}+\vec{v_2})$$

$$|\vec{V}_{cm}| = \frac{n}{n+1}|\vec{v_1}+\vec{v_2}|$$

$$= \frac{n}{n+1}\sqrt{v_1^2+v_2^2+2v_1v_2\cos\varphi}$$

$$= \frac{n}{n+1}\cdot 2V\cos(\varphi/2)$$

(34) For shell

$$H = \frac{u^2 \sin^2\theta}{2g} = 15m$$

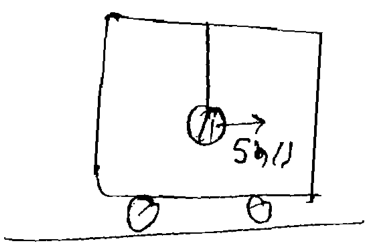
Hence it will strike the ball at its heighest position of its trajectory.

V_{shell} just before collision $= u\cos 60^\circ = 10$ m/s

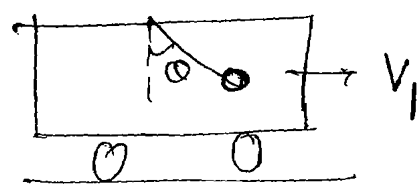
collision betn shell & ball

conservation of linear momentum

$$1 \cdot 10 = (1+1)V \Rightarrow \boxed{V = 5 \text{ m/s}}$$



Just after collision



At highest point combined mass is at rest relative to trolley. Let V_1 be the their common speed then

$$2 \times 5 = (2+18)V_1 \Rightarrow \boxed{V_1 = \frac{1}{2} \text{ m/s}}$$

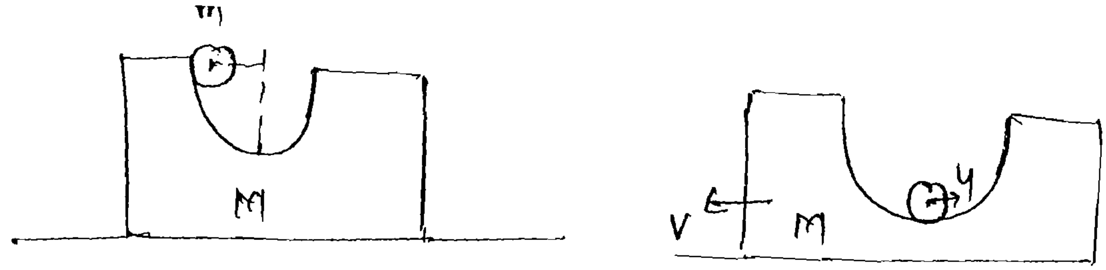
mechanical Energy conservation

$$\frac{1}{2} \cdot 2 \cdot (5)^2 - \frac{1}{2} (2+18)\left(\frac{1}{2}\right)^2 = 2 \cdot 10 \cdot 1(1-\cos\theta)$$

$$\cos\theta = 0.125$$

$$\boxed{\theta \simeq 82.82^\circ}$$

35)



(a) CM should not shift along horizontal direction

$m(R-r-x) = Mx$ x : movement of M towards left.

$$\Rightarrow \boxed{x = \frac{m(R-r)}{M+m}}$$

(b) conservation of mechanical energy

$$mg(R-r) = \frac{1}{2}mu^2 + \frac{1}{2}MV^2 \quad \text{— (I)}$$

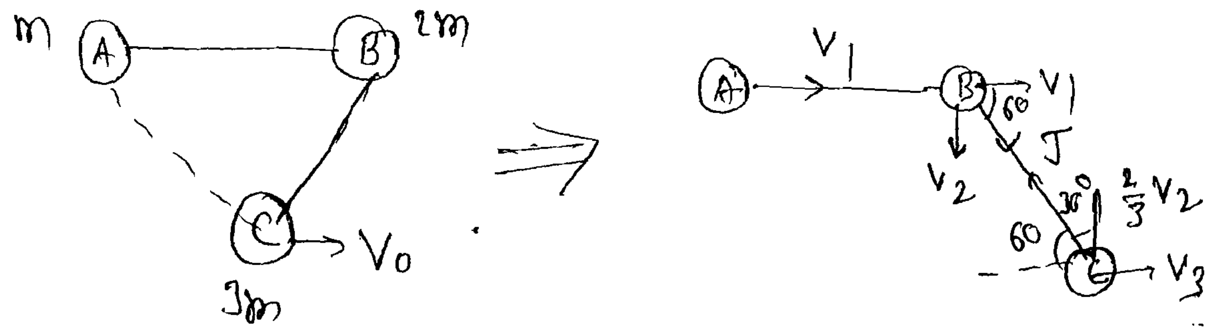
conservation of linear momentum

$$mu = MV \quad \text{— (II)}$$

solving (I) & (II)

$$V = m\sqrt{\frac{2g(R-r)}{M(m+M)}}$$

36)



momentum conservation along horizontal

$$3mV_0 = mV_1 + 2mV_1 + 3mV_3$$

$$V_0 = V_1 + V_3 \quad \text{— (I)}$$

component of velocity along AC of particle B

and C will be equal

$$v_1 \cos 60^\circ + v_2 \cos 30^\circ = v_3 \cos 60^\circ - \frac{2}{3} v_2 \cos 30^\circ$$

$$v_1 - v_3 + \frac{5}{\sqrt{3}} v_2 = 0 \quad \text{— (I)}$$

Impulse = change in linear momentum

$$J \cos 60^\circ = 3m(v_0 - v_3)$$

$$J/2 = 3m(v_0 - v_3) \quad \text{— (III)}$$

$$J \cos 30^\circ = 3m\left(\frac{2}{3} v_2\right)$$

$$\frac{\sqrt{3}J}{2} = 2mv_2 \quad \text{— (IV)}$$

Solving (I), (II), (III) & (IV)

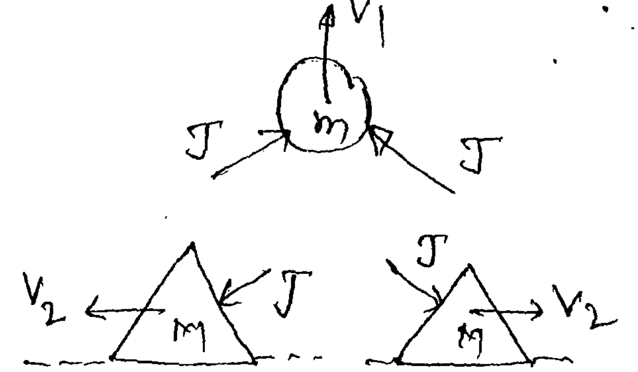
$$\boxed{v_1 = \frac{2v_0}{19}}$$

37)

$$2J\sin 30^\circ = mv_1 - (-mv_0)$$

$$J = m(v_1 + v_0) \quad \text{(I)}$$

$$J\cos 30^\circ = Mv_2 \quad \text{(II)}$$



From (I) & (II):

$$\boxed{\frac{2}{\sqrt{3}} Mv_2 = mv_1 + mv_0} \quad \text{(III)}$$

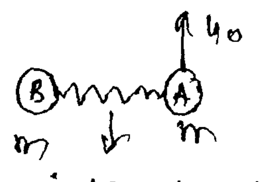
$$e = \frac{v_1\cos 60^\circ + v_2\cos 30^\circ}{v_0\cos 60^\circ} = \frac{v_1 + \sqrt{3}v_2}{v_0} \quad \text{(IV)}$$

solving (III) & (IV)

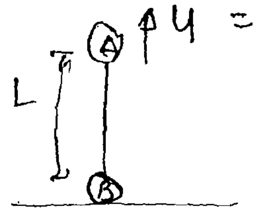
$$v_1 = \frac{(2eM - 3m)v_0}{2M + 3m}$$

$$v_2 = \frac{\sqrt{3}(1+e)mv_0}{2M + 3m}.$$

(38)

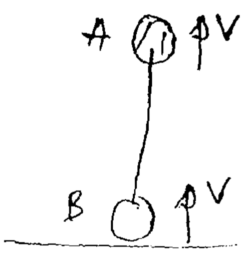


length of string = L



$$u = \sqrt{u_0^2 - 2gL} = \sqrt{8gL}$$

string just before tightening



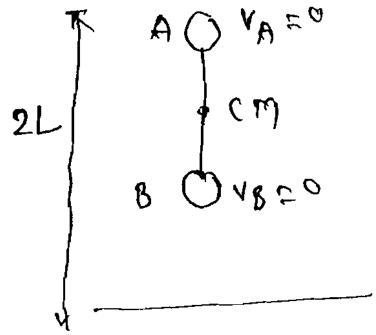
$$2mv = mu \Rightarrow v = \frac{u}{2} = \sqrt{2gL}$$

just after impulsive Tension

$V_{cm} = \frac{mv + mv}{m+m} = V$

Maximum height attained by cm $= \frac{V_{cm}^2}{2 g_{cm}}$

$= \frac{V^2}{2g} = L$



Height of A above ground surface $= 2L$

While returning velocity of A when it strikes the ground surface

$V_A = \sqrt{2g \cdot 2L} = \sqrt{4gL} = 2\sqrt{gL}$

39) (a) Let v_1 = velocity of block after bullet just emerges from it

$\frac{1}{2} M v_1^2 = \frac{1}{2} K x^2 \Rightarrow v_1 = x\sqrt{\frac{K}{M}} = 1.5 \text{ m/s}$

Now Let V_2 = speed of bullet when it comes out

$m u_0 = m V_2 + M V_1$

$5 \times 10^{-3} \times 400 = 5 \times 10^{-3} V_2 + 1 \times 1.5$

$\boxed{V_2 = 100 \text{ m/s}}$

(b) $E_{in} = \frac{1}{2} m u_1^2$

$E_f = \frac{1}{2} K x^2 + \frac{1}{2} m V_2^2$

Loss $= E_{in} - E_f$

$\approx 379 J$

(40) Net force on table by the chain

= Thrust force + wt. of portion chain at that moment

$= v_r \frac{dm}{dt}$ + weight of x length of chain

$= v_r(\lambda v_r) + \frac{Mgx}{L}$

$= \frac{M}{L} v_r^2 + \frac{Mgx}{L}$

$= \frac{M}{L}.2gx + \frac{Mgx}{L} = \frac{3Mgx}{L}$

The table will apply same force on chain in vertical upward direction.

(41) In variable mass system

$\vec{F}_{net} = \vec{F}_{ext} + \vec{F}_{Th}$.

$$\boxed{m\frac{d\vec{v}}{dt} = \vec{F}_{ext} + \vec{F}_{Th}}$$

$$\boxed{m\frac{dv}{dt} = \vec{F} - \cancel{m}u\mu.}$$

$\mu = \frac{dm}{dt}$

u : speed of jet relative to rocket.

where

$\vec{F}_{Th}$ = Thrust force $= \vec{v}_{rel}.\frac{dm}{dt}$

m = mass at time "t"

$\vec{F}_{ext}$: external force acting on main mass

(42) For rocket

$$\boxed{m\frac{dv}{dt} = v_r\left(-\frac{dm}{dt}\right) - mg}$$

$$dv = v_r\left(-\frac{dm}{m}\right) - g\,dt$$

$$\int_u^v dv = v_r\int_{m_0}^{m}\frac{-dm}{m} - g\int_0^t dt$$

$$\boxed{v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)}$$

In this question $u=0$, $g=0$, $v_r = u$

$$\boxed{v(t) = u \ln(m_0/m)}$$

(43) (a) sand's rate of change of linear momentum

$= v_{rel}\cdot\frac{dm}{dt} = (0.75)5 = 3.75\ kg\cdot m/s^2$

(b) since speed is const

$F_f = v_{rel}\cdot\frac{dm}{dt} = 3.75\ N$

(c) $F_{ext} = F_f = 3.75\ N$

(d) work done by F_{ext} in 1 sec

$= F_{ext}\cdot s = 3.75 \times 0.75 = 2.81\ J$

(e) K.E acquired by sand each sec

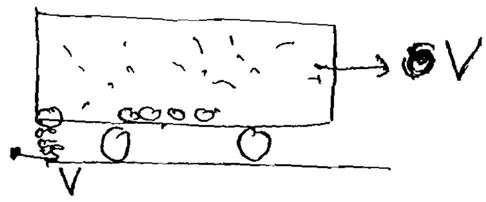
$= \frac{1}{2}\left(\frac{dm}{dt}\right)v^2$

$= \frac{1}{2}\cdot 5 \times (0.75)^2$

≈ 1.41 Joule.

(f) Energy gets converted into internal energy due to friction

(44)



$V_r = 0$

sand spills through a hole in the bottom of the cart hence relative velocity of sand, $V_r = 0 \Rightarrow F_{Th} = 0$

$$F_{net} = F$$

$$m\left(\frac{dv}{dt}\right) = F \quad , \text{where } m = m_0 - bt.$$

$$dv = \frac{F\,dt}{m} = \frac{F\,dt}{m_0 - bt}$$

$$\int_0^v dv = F\int_0^t \frac{dt}{m_0 - bt}$$

$$\boxed{v = \frac{F}{b}\ln\left(\frac{m_0}{m_0 - bt}\right)}$$

$$\boxed{a = \frac{dv}{dt} = \frac{F}{m} = \frac{F}{m_0 - bt}}$$

(45.)

(a) chain has a constant speed.
Net force on it should be zero.

F = Wt. of length y of chain + Thrust force

$$= \left(\frac{M}{L}\right) g y + \left(\frac{M}{L}\right) v_0^2$$

$$= \frac{M}{L}\left(g y + v_0^2\right)$$

(b) Reaction of the floor

= Wt. of length $(L-y)$ of chain

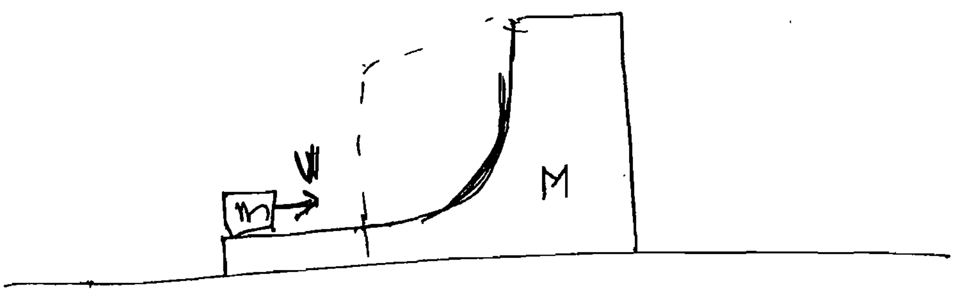
$$= Mg\left(1-\frac{y}{L}\right)$$

(c) Energy lost during the lifting

= workdone by applied force − increase in mechanical energy of chain

$$= Fy - \left(\frac{M}{L} y\right) g \left(\frac{y}{2}\right) - \frac{1}{2}\left(\frac{M}{L} y\right) v_0^2$$

$$= \frac{My}{2L}\left(g y + v_0^2\right).$$


m
v
M

PYQ : JEE Advanced

Only One Option Correct

1. (d)

2. (a)

3. (a)

4. (c)

5. (c)

6. (d)

7. (d)

Given : momentum $\vec{p}(t) = A\left[\hat{i}\cos(kt) - \hat{j}\sin(kt)\right]$

And, force, $\vec{F} = \dfrac{d\vec{p}}{dt} = Ak\left[-\hat{i}\sin(kt) - \hat{j}\cos(kt)\right]$

Here, $\vec{F}.\vec{P} = 0$ But $\vec{F}.\vec{p} = Fp\cos\theta$

$\therefore\ \cos\theta = 0 \quad \Rightarrow\ \theta = 90^\circ$.

Hence, angle between the force momentum, $\theta = 90^\circ$

8. (c)

9. (a)

10. (a)

11. (d)

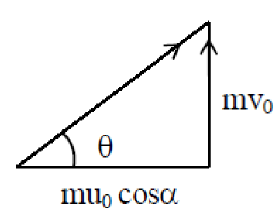
12. (A)

Velocity of particle performing projectile motion at highest point

$= v_1 = v_0\cos\alpha$

Velocity of particle thrown vertically upwards at the position of collision

$= v_2^2 = u_0^2 - 2g\dfrac{u^2\sin^2\alpha}{2g} = v_0\cos\alpha$



So, from conservation of momentum

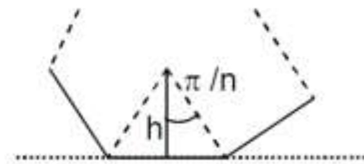
$$\tan\theta = \frac{mv_0\cos\alpha}{mu_0\cos\alpha} = 1$$

$$\Rightarrow\ \theta = \frac{\pi}{4}$$

13. (B)

For a regular polygon of n sides

$$\Delta = \frac{h}{\cos\frac{\pi}{n}} - h = h\left[\frac{1}{\cos\frac{\pi}{n}} - 1\right]$$



14. (B)

$$\frac{d(KE)}{dt} = mv\frac{dv}{dt}$$

One or More than One Option Correct

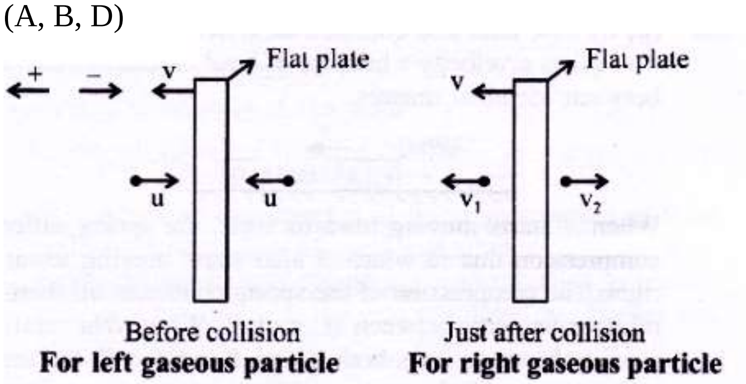
1. (c, d)

2. (b, d)

3. (A, D)

4. (A, C)

5. (A, B, D)



$1 = \frac{v_1 - v}{v + u}$ $\quad\quad$ $1 = \frac{v + v_2}{u - v}$

$\therefore v_1 = u + 2v$ $\quad\quad$ $\therefore v_2 = u - 2v$

$\therefore \Delta v_1 = 2u + 2v$ and $\Delta v_2 = 2u - 2v$

Now $F_1 = \frac{dp_1}{dt} = \rho A(u+v)(2u+2v)$

And $F_2 = \frac{dp_2}{dt} = \rho A(u-v)(2u-2v)$

$\therefore F_1 = 2\rho A(u+v)^2$ and $F_2 = 2\rho A(u-v)^2$

ΔF is the net force due to the air molecules on the plane.

The net force $F_{net} = F - \Delta F = ma$

$\therefore F - 8\rho Auv = ma$

Due to viscosity, plate will eventually reach terminal velocity. So now plate will move with constant Velocity.

6. (A, C)

Δx_{cm} of the block & point mass system = 0

$\therefore \quad m(x+R)+Mx=0$

Where x is the displacement of the block.

$x=-\frac{mR}{M+m}$

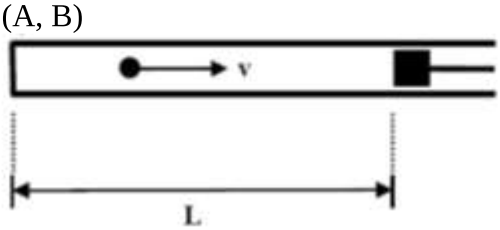
From conservation of momentum and mechanical energy of the combined system

$0=mv-MV$

$mgR=\frac{1}{2}mv^2+\frac{1}{2}MV^2$

$\therefore \quad v=\sqrt{\frac{2gR}{1+\frac{m}{M}}}$

7. (A, B)



The rate of collision of the particle with the piston is $\frac{1}{2L/v}=\frac{v}{2L}$

The speed of the particle after a collision with the piston is :

$v+2V$, when it collides with it with a speed v.

If the piston moves inward by dL the speed of the particle increase by

$$dv=2V\times\frac{dL}{V}\times\frac{v}{2L}$$

i.e. $\frac{dv}{v}=\frac{dL}{L}$

Since, kinetic energy, $K=\frac{1}{2}mv^2$, $\frac{dK}{K}=\frac{2(-dL)}{L}$ (after putting the proper sign)

Integrating, KL^2 = constant.

$\therefore \quad K_f=4K_i$

Hence, correct options are (A), (B).

Stem Type Questions

1. (0.50)

After splitting 1st mass takes 0.5 sec to reach ground.

Initial velocity is same for both mass at the highest point in vertical direction.

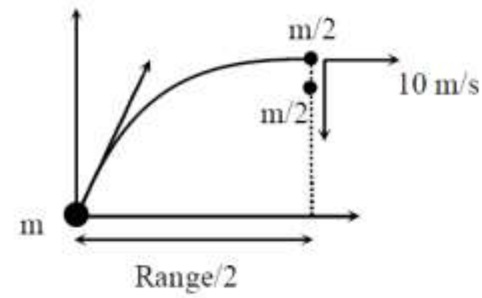
Displacement and acceleration in vertical direction is also same.

So, 2nd mass will also take 00.5 sec to reach ground.

2. (7.50)

Velocity of projectile at highest point 5 m/s $\hat{i}$

Since, there is no external force in horizontal direction so by conservation of momentum



$$m(5) = \frac{m}{2}(0) + \frac{m}{2}(v)$$

$$\vec{v} = 10\ m/s\,\hat{i}$$

Distance covered by second mass before landing

$$= \frac{Range}{2} + 10(t) = 7.5\ m$$

Fill in the Blanks

1. $3\ mv^2/2$

2. (0.005)

True / False

1. False

Numerical Value Answer

16. (5)

The initial speed of 1st bob (suspended by a string of length ℓ_1) is $\sqrt{5g\ell_1}$.

The speed of this bob at highest point will be $\sqrt{g\ell_1}$.

When this bob collides with the other bob there speeds will be interchanged.

$$\sqrt{g\ell_1} = \sqrt{5g\ell_1} \Rightarrow \frac{\ell_1}{\ell_2} = 5$$

7. (6.30)

Using impulse momentum theorem

$$J = mv_0$$

$$v_0 = \frac{J}{m} = \frac{1}{0.4} = 2.5\ m/s$$

$$v = v_0 e^{\frac{-t}{\pi}}$$

$$\int dx = v_0 \int_0^{\tau} e^{\frac{-t}{\tau}}\,dt$$

$$\Delta x = v_0\tau\left[1 - e^{\frac{-t}{\tau}}\right]$$

At $t = \tau$ sec

$$\Delta x = 6.30\ m$$

8. (30.00)

$$H_0 = \frac{(V_0 \sin 45°)^2}{2g} = \frac{v_g^2}{4g}$$

$$\frac{v_0^2}{4g} = 120 \qquad ...(i)$$

$$\text{Now, } \frac{1}{2}mv^2 = \frac{1}{2}\times\frac{1}{2}mv_0^2$$

$$v = \frac{v_0}{\sqrt{2}}$$

$$\text{Now, } H = \frac{(v\sin 30°)^2}{2g} = \frac{v^2}{8g} = \frac{v_0^2}{16g} = \frac{H_0}{4} = 30\text{ m}$$

Subjective

1. (a) $\theta = \tan^{-1}$ (MV/mv), $[1/(M+m)]\,[(mv)^2 + (MV)^2]^{1/2}$
   (b) (m M/ m + M) $[(V^2 + v^2)/(MV^2 + mv^2)]$

2. 25 %

3. No

4. 9 cm to left of centre of bigger circle

5. $10\sqrt{2}$ m/s

6. $v_c = v$ m/s

7. $m(R - r)/(M + m)$, $m\sqrt{[2g(R - r)/M(m + M)]}$

8. (i) $v_0/3$ (ii) $2mv_0^2/3x_0^2$

9. 6.53 sec

10. 4

11. (i) $\theta = 37^o$ (ii) 45m, 122.5 m

12. 44.25 m

13. $v_0 = (5/2)\sqrt{(6\mu gd)}$, $6d\sqrt{(3\mu)}$

14. (a) straight line (b) $[x/(L/2 - r)]^2 + [y/r]^2 = 1$, Ellipse

15. 10^5 m

16. (L + 2R, 0)

17. (i) 2.5 m/s (ii) 0.318 m

18. (a) $F = \frac{2mv}{\sqrt{3}\Delta t}\left(\sqrt{3}i - k\right)$, $N = \left(\frac{2mv}{\sqrt{3}\Delta t} + mg\right)k$

(b) $\left(\frac{4mvh}{\sqrt{3}\Delta t}\right)\vec{k}$

19. 0.84, 15.02 kg

20. 12 sec, 15.75 m/s

21. m [$v_2 \sin(v_2 t/R)$ (–i) + $v_2\cos(v_2 t/R) - v_1$ (j)]

22. (a) $v_0 t + (m_1 A/m_2)(1 - \cos \omega t)$ (b) $[(m_1/m_2) + 1]A$

23. 10 m/s

24. 4 m/s

25. N = 4