PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DHULE

IIT – JEE: 2024 **PART TEST - 1** **DATE: 12/11/23**

ADVANCED

ANSWER KEY

PAPER – 1

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	A, B, D	**18.**	A, D	**35.**	B, C
2.	A, B, C	**19.**	A, B, C	**36.**	C, D
3.	B	**20.**	A, B	**37.**	B, C
4.	B	**21.**	C	**38.**	A
5.	B	**22.**	A	**39.**	C
6.	B	**23.**	B	**40.**	B
7.	C	**24.**	B	**41.**	A
8.	6	**25.**	2	**42.**	5
9.	16	**26.**	1	**43.**	4
10.	4	**27.**	3	**44.**	12
11.	2	**28.**	3	**45.**	2
12.	26	**29.**	2	**46.**	10
13.	218	**30.**	6	**47.**	4
14.	C	**31.**	C	**48.**	C
15.	D	**32.**	D	**49.**	D
16.	B	**33.**	B	**50.**	B
17.	A	**34.**	D	**51.**	A

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ANSWER KEY

PAPER – 2

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	C	**18.**	C	**35.**	C
2.	A	**19.**	C	**36.**	B
3.	B	**20.**	B	**37.**	B
4.	B	**21.**	D	**38.**	C
5.	A, B, D	**22.**	A, C, D	**39.**	A, C, D
6.	A, B, C	**23.**	A, B, D	**40.**	A, D
7.	B, C	**24.**	A, C	**41.**	A, B
8.	4	**25.**	2	**42.**	7
9.	4	**26.**	7	**43.**	8
10.	45	**27.**	2	**44.**	1
11.	2	**28.**	5	**45.**	42
12.	12	**29.**	1	**46.**	5
13.	2	**30.**	8	**47.**	1
14.	0.75	**31.**	0	**48.**	3
15.	0.6	**32.**	5	**49.**	4
16.	0	**33.**	5.57	**50.**	8
17.	30	**34.**	15	**51.**	5

PART (A) : PHYSICS

1. (A, B, D)

Applying Bernoulli's equation at B and C,

$$p_0 + 0 + 10^3 \times 10(3.6) = p_0 + \frac{1}{2} \times 10^3 v_c^2 + 0$$

Discharge rate of flow

$$= Av = \pi\left(\frac{4}{\sqrt{\pi}} \times 10^{-2}\right)^2 \left(6\sqrt{2}\right)$$

$$= 96\sqrt{2} \times 10^{-4} m^3/s$$

Applying Bernoulli's equation at A and C.

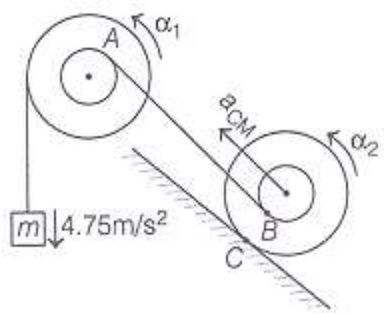
$$p_A + \frac{1}{2} \times 10^3 \times \left(6\sqrt{2}\right)^2 + 10^3 \times 10 \times (1.8 + 3.6)$$

$$= 10^5 + \frac{1}{2} \times 10^3 \times \left(6\sqrt{2}\right)^2 + 0$$

$$\Rightarrow p_A = 0.46 \times 10^5 \,Pa$$

2. (A, B, C)

$$4.75 = r_2\alpha_1$$



$$a_A = r_1\alpha_1$$

$$a_B = a_{CM} - r_1\alpha_2 = a_A$$

$$a_{CM} - r_2\alpha_2 = 0$$

Solving we get,

$$a_{CM} = \frac{r_1(4.75)}{r_2 - r_1} = 4.75 m/s^2$$

$$v = u + at \Rightarrow v_{CM} = (4.75)4 = 19\, m/s$$

$$\Rightarrow v_B = a_B t = \left(\frac{4.75}{2}\right) \times 4 = 9.5\, m/s$$

Work done by friction on pulley Q is zero.

3. (B)

$u_A =$ Velocity of A just before collision $= \sqrt{2gl_A\left(1 - \cos\theta_A\right)}$

$v_B =$ Velocity of B just after collision $= \sqrt{2gl_B\left(1 - \cos\theta_B\right)}$

Using linear momentum conservation,

$mu_A = mv_A + mv_B \Rightarrow v_A + v_B = u_A$ ……..(i)

$$e = \frac{v_B - v_A}{u_B - 0} \Rightarrow v_B - v_A = eu_A \qquad \text{.......(ii)}$$

Solving, we get

$$v_B = \left(\frac{e+1}{2}\right)u_A$$

$$\Rightarrow \sqrt{2gl_B(1-\cos\theta)} = \left(\frac{e+1}{2}\right)\sqrt{2gl_A(1-\cos\theta)}$$

$$\Rightarrow \frac{l_B}{l_A} = \frac{(1+e)^2}{4}$$

4. (B)

$$[X] = [M^a L^b T^{-c}]$$

$$\frac{\Delta x}{x}\times 100 = \left[a\frac{\Delta M}{M} + b\frac{\Delta L}{L} + \frac{c\Delta T}{T}\right]\times 100$$

$$= (\alpha a + \beta b + c\gamma)\%$$

5. (B)

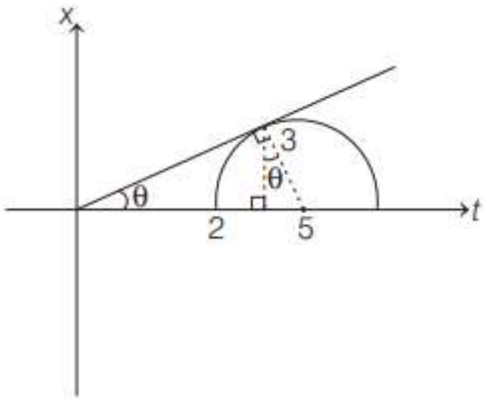
$$x = (1+3\cos\theta)t \qquad \text{......(i)}$$

$$y = (3\sin\theta)t \qquad \text{......(ii)}$$

From Eqs. (i) and (ii), we get

$$(x-1)^2 + y^2 = 9t^2$$

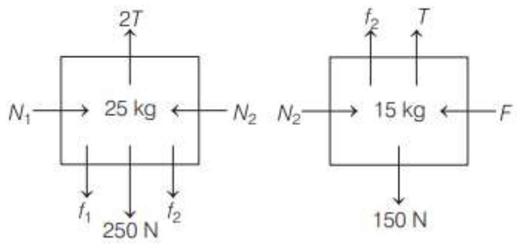
6. (B)



$$\sin\theta = \frac{3}{5}$$

$$\text{Time } = 5 - 3\sin\theta = 5 - 3\left(\frac{3}{5}\right) = \frac{16}{5} = 3.2\,\text{s}$$

7. (C)



$\sum F_y = 0$

For 25 kg

$\Rightarrow 2T = 250 + f_1 + f_2$ ....(i)

For 15 kg,

$\Rightarrow T + f_2 = 150$ ....(ii)

$f_1 = \mu F$ ....(iii)

$f_2 = \mu F$ ......(iv)

Solving Eqs. (i), (ii), (iii) and (iv), we get

$\mu F = \frac{50}{4}$

$\Rightarrow F = 125N$

8. (6)

Water will stop coming out of orifice when pressure at the bottom becomes equal to atmospheric pressure.

$p_{bottom} = p_{top} + \rho gh$

$\Rightarrow 10^5 = p_{top} + 10^3 \times 10 \times 0.2$

$\Rightarrow p_{top} = 9.8 \times 10^4 \text{Pa}$

For the air above water,

$p_1V_1 = p_2V_2$

$\Rightarrow 10^5 \times A(0.5 - H) = 9.8 \times 10^4 A(0.5 - 0.2)$

$\Rightarrow H = 0.206m = 206mm$

Fall in height $= 206 - 200 = 6mm$

9. (16)

$p_B = p_0 + \frac{2T}{3R}$,

$p_C = P_0 + \frac{2T}{3R} + \frac{2T}{R} + \frac{4T}{(R/2)} = p_0 + \frac{32T}{3R}$

$p_B' =$ Gauge pressure at $B = \frac{2T}{3R}$

$p_C' =$ Gauge pressure at $C = \frac{32T}{3R}$

$\frac{p_C'}{p_A'} = 16$

10. (4)

$$V = \frac{\sigma R}{\varepsilon_0} \Rightarrow \sigma = \frac{V\varepsilon_0}{R}$$

$$p_1V_1 = p_2V_2$$

$$\Rightarrow \left(p_0 + \frac{4T}{r}\right)\frac{4}{3}\pi r^3 = \left(p_0 + \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}\right)\frac{4}{3}\pi R^3$$

$$\Rightarrow p_0\left(R^3 - r^3\right) + 4T\left(R^2 - r^2\right) - \frac{1}{2}\varepsilon_0 V^2 R = 0$$

So, $\lambda = 4$

11. (2)

$$E^n = p^2c^2 + m_0^2c^4$$

$$\Rightarrow \left[E^n\right] = \left[p^2c^2\right] = \left[m_0^2c^4\right]$$

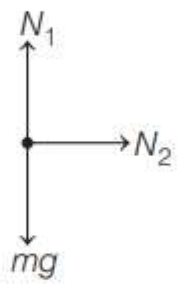
$$\Rightarrow \left[ML^2T^{-2}\right]^n = [M]^2\left[LT^{-1}\right]^4$$

$$\Rightarrow \left[ML^2T^{-2}\right]^n = \left[ML^2T^{-2}\right]^2 \Rightarrow n = 2$$

12. (26)

$$v = 5t\hat{i} + 2t\hat{j}$$

$$a = 5\hat{i} + 2\hat{j}$$



$$\sum F_x = ma_x \Rightarrow N_2 = m(5) = 10N$$

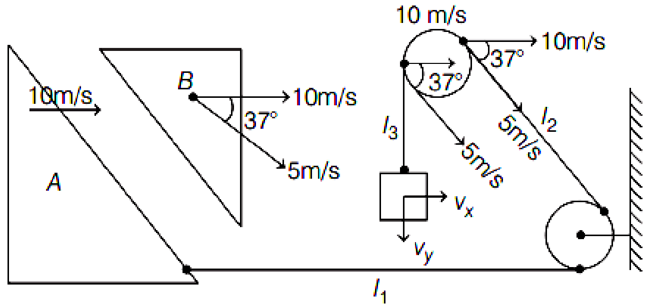
$$\sum F_y = ma_y \Rightarrow N_1 - mg = m(2) \Rightarrow N_1 = 24N$$

Net force by the cube on the sphere

$$= \sqrt{N_1^2 + N_2^2}$$

$$= \sqrt{10^2 + 24^2} = 26N$$

13. (218)



String constraint, $l_1 + l_2 + l_3 =$ constant

$\frac{dl_1}{dt} + \frac{dl_2}{dt} + \frac{dl_3}{dt} = 0$

$-10 + \left(-5 - 10\cos 37^\circ\right) + \left(-5\sin 37^\circ + v_y\right) = 0$

$\Rightarrow v_y = 26 m/s$

Wedge constraint for (B and C)

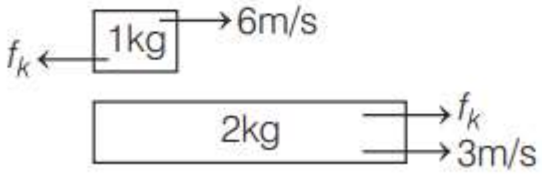
$10 + 5\cos 37^\circ = v_x$

$\Rightarrow v_x = 14 m/s$

$v_c = 14\hat{i} + 26\hat{j}$

$v_c = \sqrt{14^2 + 26^2} = \sqrt{872}\ m/s$

14. (C)



For 1 kg block,

$\sum F_y = 0$

$\Rightarrow N_1 = 10N$

$F_k = \mu_k N_1 = (0.04)(10) = 0.4N$

$\sum F_x = ma_x$

$\Rightarrow -0.4 = 1a_1$

$\Rightarrow a_1 = -0.4\, m/s^2$

$v = u + at$

$\Rightarrow v_1 = 6 - 0.4t$

For 2 kg

$\sum F_x = ma_x$

$\Rightarrow 0.4 = 2a_2$

$\Rightarrow a_2 = 0.2\, m/s^2$

$v = u + at$

$\Rightarrow v_2 = 3 + 0.2t$

(A) For relative motion to stop $v_1 = v_2$

$\Rightarrow 6 - 0.4t = 3 + 0.2t$

$\Rightarrow t = 5s$

And $v_1 = v_2 = 4m/s$

(B) Work done by friction on the block

$= \Delta K_{block} = \frac{1}{2} \times 1 \times (4)^2 - \frac{1}{2} \times 1 \times (6)^2$

$= -10J$

(C) Work done by friction on the plank $= \Delta K_{plank}$

$= \frac{1}{2} \times 2 \times 4^2 - \frac{1}{2} \times 2 \times 3^2 = 7J$

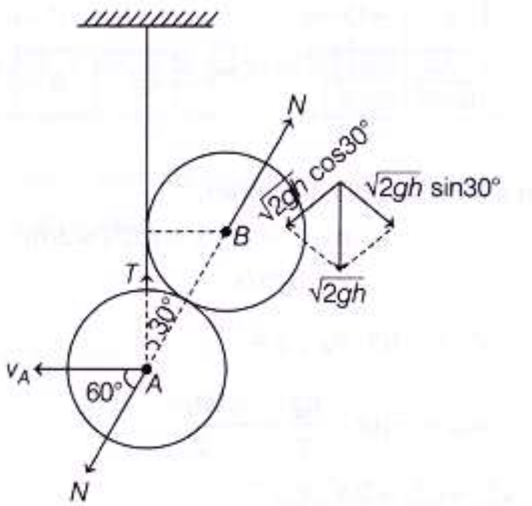
(D) Work done by friction on system $= -10 + 7 = -3J$

$W = \Delta E$

$\Rightarrow \Delta E = -3J$

$\Rightarrow$ Loss in mechanical energy $= 3J$

15. (D)



Lets take ball B is rebounded along the line of impact with velocity v.

For B, $\int N dt = m\left[v - \left(-\sqrt{2gh}\cos 30^\circ\right)\right]$ ……(i)

For A, $\int N dt \cos 60^\circ = m v_A$ …….(ii)

$e = 1 - \frac{v + v_A \cos 60^\circ}{\sqrt{2gh}\cos 30^\circ}$ ……..(iii)

Solving , we get $v_A = \frac{2\sqrt{6gh}}{5}$

(B) $\int T dt = \int N \sin 60^\circ dt = \frac{6m\sqrt{2gh}}{5}$

(C) $e = 0 = \frac{v + v_A \cos 60^\circ}{\sqrt{2gh}\cos 30^\circ}$

$\Rightarrow v = -\frac{v_A}{2}$ …………….(iv)

Solving Eqs. (i), (ii) and (iv), we get $v_A = \frac{\sqrt{6gh}}{5}$

(D) $\int T dt = \int N \sin 60^\circ dt = \frac{3m}{5}\sqrt{2gh}$

16. (B)

(A) Applying Bernoulli's theorem,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \left(p_0 + \frac{Mg}{A} + \rho gH + 2\rho gH\right) + 0 = p_0 + \frac{1}{2}(2\rho)v_A^2$$

$$\Rightarrow v_A = \sqrt{\frac{Mg}{\rho A} + 3gH}$$

(B) Applying Bernoulli's theorem,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \left(p_0 + \frac{Mg}{A} + \rho gH\right) + 0 = p_0 + \frac{1}{2}\rho v_B^2$$

$$\Rightarrow v_B = \sqrt{\frac{2Mg}{\rho A} + 2gH}$$

(C) $F_A = (2\rho)av_A^2 = (2\rho)\left(\frac{A}{10}\right)\left(\frac{Mg}{\rho A} + 3gH\right) = \frac{Mg + 3\rho AgH}{5}$

(D) $F_B = \rho\left(\frac{A}{10}\right)v_B^2 = \rho\left(\frac{A}{10}\right)\left(\frac{2Mg}{\rho A} + 2gH\right) = \frac{Mg + \rho AgH}{5}$

17. (A)

(A) $K_1 + U_1 = K_2 + U_2 \Rightarrow \frac{1}{2}m\left(\sqrt{gR}\right)^2 - mgR = 0 - \frac{mgR}{1 + \frac{h}{R}}$

$$\Rightarrow h = R$$

(B) $K_1 + U_1 = K_2 + U_2$

$$\Rightarrow \frac{1}{2}m\left(\sqrt{gR}\right)^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$\Rightarrow v = \sqrt{\frac{GM}{Rr}(2R - r)}$$

$$\Rightarrow \int_R^{2R} \frac{\sqrt{r}\,dr}{\sqrt{2R - r}} = \sqrt{\frac{GM}{R}}\int_0^T dt$$

Substitute $r = 2R\sin^2\theta$

$$\Rightarrow dr = 4R\sin\theta\cos\theta d\theta$$

$$\Rightarrow \int_{\pi/4}^{\pi/2} \frac{\sqrt{2}R\sin\theta(4R\sin\theta\cos\theta d\theta)}{\sqrt{2R}\cos\theta} = \sqrt{\frac{GM}{R}}\int_0^T dt$$

$$\Rightarrow T = \left(\frac{\pi}{2} + 1\right)\sqrt{\frac{R^3}{GM}}$$

(C) $g' = \frac{GM}{(2R^2)} = \frac{GM}{4R^2}$

(D) $v = \sqrt{\dfrac{GM}{2R}}$

PART (B) : CHEMISTRY

18. (A, D)

19. (A, B, C)

20. (A, B)

21. (C)

22. (A)

23. (B)

24. (B)

25. (2)

26. (1)

27. (3)

28. (3)

29. (2)

30. (6)

31. (C)

32. (D)

33. (B)

34. (D)

PART (C) : MATHEMATICS

35. (B, C)

$x, 12, y$ in H.P.

So, $12 = \dfrac{2xy}{x+y}$

$\Rightarrow 12(x+y) = 2xy$ …….(1)

$x, 12, z, y$ are in increasing A.P.

So, $24 = x + z$ (option B) …….(2)

$2z = 12 + y$ ……..(3)

$12 + z = x + y$ ……..(4)

From (2), (3) and (4) put values in (1)

$12(12+z) = 2(24-z)(2z-12)$

We get , $z = 15$

Put $z = 15$ in equation (2) and (3), we get

$x = 9, y = 18$

So, maximum value of

$\sqrt{(x-3)\sin\alpha - (y-10)\cos\beta + 2} = \sqrt{6\sin\alpha - 8\cos\beta + 2}$

Maximum when $\sin\alpha = 1$ and $\cos\beta = -1$ is 4.

36. (C, D)

No. of basic colours $= 5$

No. of colours =5

No. of colours formed by mixing

$= {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 26$

If n is even

No. of ways: $5^{n/2}.(26)^{n/2} + 5^{n/2}.(26)^{n/2} = 2(130)^{n/2}$

If n is odd

No. of ways: $5^{\frac{n+1}{2}}.(26)^{\frac{n-1}{2}} + 5^{\frac{n-1}{2}}.(26)^{\frac{n+1}{2}}$

$= 31(130)^{\frac{n-1}{2}}$

37. (B, C)

Observe that the lines $L_1, L_2 \& L_3$ are parallel to the vector $\hat{i} - \hat{j} - \hat{k}$.

Also, $\Delta = 0 = \Delta_1 \& b_1c_2 \neq b_1c_1$. Hence the three planes intersect in a line

$P_1 = 2x + y + z + 4 = 0$

$P_2 = 0x + y - z + 4 = 0$

$P_3 = 3x + 2y + z + 8 = 0$

P_2 and P_3 gives line L_1

Vector parallel to line $L_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{vmatrix}$

$=3\hat{i}-3\hat{j}-3\hat{k}$

$=3\left[\hat{i}-\hat{j}-\hat{k}\right]$

Similarly

Vector parallel to L_2, P_3 and $P_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix}$

$=-\hat{i}+\hat{j}+\hat{k} = -\left[\hat{i}-\hat{j}-\hat{k}\right]$

Similarly

Vector parallel to L_3, P_1 and $P_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix}$

$=(-2)\hat{i}-(-2)\hat{j}+2\hat{k}$

$=-2\hat{i}+2\hat{j}+2\hat{k}$

$=-2\left(\hat{i}-\hat{j}-\hat{k}\right)$

We can see all the lines are parallel to vector

$\left(\hat{i}-\hat{j}-\hat{k}\right)$

Also $2x+y+z=-4$

$0x+y-z=-4$

$3x+2y+z=-8$

$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{vmatrix}$

$\Rightarrow 2(1+2)-1(0+3)+1(0-3)$

$\Rightarrow 2(3)-3-3=0 \quad \Delta_1 = \begin{vmatrix} -4 & 1 & 1 \\ -4 & 1 & -1 \\ -8 & 2 & 1 \end{vmatrix} = 0$

$\Delta_2 = \begin{vmatrix} 2 & -4 & 1 \\ 0 & -4 & -1 \\ 3 & -8 & 1 \end{vmatrix} = 0$

$=-4\begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{vmatrix} = 0$

$\Delta_3 = 0$

So all planes intersection in line L.

38. (A)

$(a_1)^{1/x_1} = (a_2)^{1/x_2} = (a_3)^{1/x_3} = ... = (a_n)^{1/x_n} = k, (\text{Let})$

$\therefore a_1 = k^{x_1}, a_2 = k^{x_2}, a_3 = k^{x_3}......., a_n = k^{x_n}$

Since $a_1, a_2, a_3, ..., a_n$ are in G.P.

$k^{x_1}, k^{x_2}, k^{x_3}........k^{x_n}$ are in G.P.

$\therefore x_1, x_2, x_3, ..., x_n$ are in A.P.

39. (C)

G.M of roots $=$ H.M. of roots

$\Rightarrow \alpha = \beta = \gamma = \delta = 2(\text{each})$

Where $\alpha, \beta, \gamma, \delta$ are roots of given equation

$\therefore p = -16, q = 24$

40. (B)

Putting $\lambda = 0$, in the given identity, we get

$$\begin{vmatrix} 0 & -1 & 3 \\ 1 & 1 & -4 \\ -2 & 4 & 0 \end{vmatrix} = t$$

$\Rightarrow \quad t = 0 + 1(0-8) + 3(4+2) = 10$

41. (A)

If *n* is odd, then $3^n = 4\lambda_1 - 1, 5^n = 4\lambda_2 + 1$

$\Rightarrow \quad 2^n + 3^n + 5^n$ is divisible by 4 if $n \geq 2$

Thus $n = 3, 5, 7, ...99$

$\Rightarrow$ Total number of numbers = 49

If *n* is even, then $3^n = 4\lambda_1 + 1, 5^n = 4\lambda_2 + 1$

$\Rightarrow 2^n + 3^n + 5^n$ will be in the form of $4\lambda + 2$ which is not divisible by 4.

42. (5)

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{m \cdot m!n!(-1)^m}{(m+1)^2 (m+n+1)!}$$

Notice that for $m = 0$ the first term is zero, then

$$\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{m!n!(-1)^m}{(m+1)^2 (m+n+1)!} \times ((m+n+1)-(n+1))$$

$$\sum_{m=1}^{\infty} \frac{(-1)^m}{(m+1)^2} \cdot m! \left(\sum_{n=0}^{\infty} \frac{n!}{(m+n)!} - \frac{(n+1)!}{(m+n+1)!} \right)$$

It is a telescopic sum,

$$\sum_{m=1}^{\infty} \frac{(-1)^m}{(m+1)^2} \cdot m! \cdot \frac{1}{m!} = \sum_{m=1}^{\infty} \frac{(-1)^m}{(m+1)^2} = \frac{\pi^2}{12} - 1$$

43. (4)

Let $z = x + iy$

$\text{Re}(z^2) = 0 \Rightarrow x^2 = y^2$

$|z| = a\sqrt{2} \Rightarrow x^2 + y^2 = 2a^2$

On solving $x = \pm a$ and $y = \pm a$.

Hence 4 possible solutions.

44. (12)

Perfect square >1 $[\sqrt{100}]-1=9$ (where [.] denotes integral part)

Perfect cubes >1 $[\sqrt[3]{100}]-1=3$ (in which 4^3 is already in included)

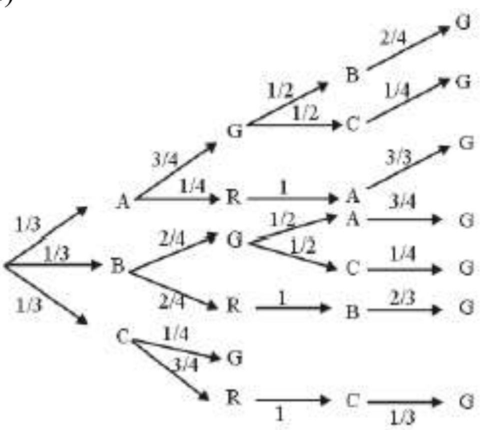
Perfect 4^{th} power >1 $[\sqrt[4]{100}]-1=2$ (already included in squares)

Perfect 5^{th} power >1 $[\sqrt[5]{100}]-1=1$

Perfect 6^{th} power >1 $[\sqrt[6]{100}]-1=1$ (already included in cubes)

$\Rightarrow$ 9 + 2 + 1 = 12

45. (2)



Required Probability

$$=\frac{1}{3}\left(\frac{3}{4}\cdot\frac{1}{2}\left(\frac{2}{4}+\frac{1}{4}\right)+\frac{1}{4}\cdot1\cdot1\right)+\frac{1}{3}\left(\frac{2}{4}\cdot\frac{1}{2}\left(\frac{3}{4}+\frac{1}{4}\right)+\frac{2}{4}\cdot1\cdot\frac{2}{3}\right)+\frac{1}{3}\left(\frac{1}{4}\cdot\frac{1}{2}\left(\frac{3}{4}+\frac{2}{4}\right)+\frac{3}{4}\cdot1\cdot\frac{1}{3}\right)$$

$$=\left[\frac{9}{32}+\frac{1}{4}+\frac{1}{4}+\frac{1}{3}+\frac{5}{32}+\frac{1}{4}\right]=\frac{1}{3}\left[\frac{7}{16}+\frac{13}{12}\right]$$

$$=\frac{1}{3}\left[\frac{21+52}{48}\right]=\frac{73}{3\times48}=\frac{m}{n}$$

So, $\frac{n}{m-1}=\frac{3\times48}{72}=2$

46. (10)

$\text{PQ} = \text{A X B B X}^\text{T} \text{A}\left[\text{Note X.X}^\text{T} = \text{I}\right]$

$$\text{B.B}=\begin{bmatrix}0 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}0 & 3\\2 & 0\end{bmatrix}=6\text{I}$$

$\therefore PQ = A.X.6I.X^T A = 6A^2 = 6\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

$(PQ)^{10} = \begin{bmatrix} 6^{10} & 0 \\ 0 & 24^{10} \end{bmatrix}$

$\therefore tr\left((PQ)^{10}\right) = 6^{10} + 24^{10} = 6^{10}\left(1 + 4^{10}\right)$

$a = 6; b = 4 \Rightarrow a + b = 10$

47. (4)

$\bar{b} = \frac{-\bar{a}}{2} + \frac{\sqrt{3}}{2}\hat{k}; \bar{b} = \frac{-1}{2\sqrt{2}}\hat{i} - \frac{1}{2\sqrt{2}}\hat{j} \pm \frac{\sqrt{3}}{2}\hat{k}$

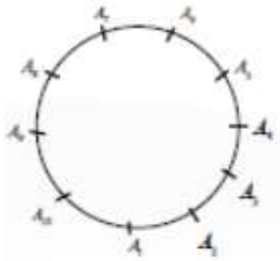
If $|\bar{b} \times \bar{c}| = |\bar{a} \times \bar{c}| \Rightarrow$ then $\bar{c}$ is perpendicular to both $\bar{a}$ and $\bar{b}$.

$\bar{c} = \lambda\left(\bar{a} \times \bar{b}\right) \& |\bar{c}| = 1$

$\therefore \bar{c} = \pm \frac{\left(\hat{i} - \hat{j}\right)}{\sqrt{2}}$

48. (C)

Question number of triangle that can be made using the vertices of polygon of 10 sides as their vertices and having exactly one side is common with the polygon is



When the side A_1A_2 associate with $A_4, A_5, A_6, A_7, A_8, A_9$ there are the triangle whose one side common with the side of polygon $\Rightarrow 6$ triangle so number of such triangles are (60)

(B) Number of triangle having 2 sides common with the polygon is (10) like
$\left(A_1, A_2, A_3\right), \left(A_2, A_3, A_4\right)........\left(A_{10}, A_1A_2\right)$

(C) Number of such Quadrilaterals are $10 \times 5_{C_1} + \frac{10 \times 5}{2} = 75$

(D) Number of such Quadrilaterals are = 10 (when four consecutive points are taken)

49. (D)

(A) $6x + 10 - x^2 > 3$
$\therefore x^2 - 6x - 7 < 0$
$(x + 1)(x - 7) < 0$
$\Rightarrow 0, 1, 2, 3, 4, 5, 6$

(B) $f(x) = (x - 1)(x^2 - 7x + 13)$ for f(x) to be prime at least one of the factors must be prime.
Hence $x - 1 = 1 \Rightarrow x = 2$ or $x^2 - 7x + 13 = 1 \Rightarrow x^2 - 7x + 12 = 0 \Rightarrow x = 3$ or 4
$\Rightarrow x = 2, 3, 4$

(C) $D = a^2 - 4(a + 1) = p^2$, where $p \in I$

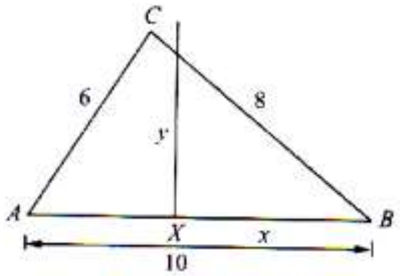
$\Rightarrow (a-2)^2 - p^2 = 8$

$\Rightarrow (a-2+p)$ and $(a-2-p)$ are even

$$\left.\begin{matrix} a-2+p=4 \\ a-2-p=2 \end{matrix}\right\} \text{ or } \begin{matrix} a-2+p=2 \\ a-2-p=2 \end{matrix} \quad \text{etc.}$$

$\Rightarrow a = 5, p = -1$, or $a = -1, p = 1$

$\Rightarrow$ number of integral value of 'a' = 2.



(D) $2\frac{x\cdot y}{2} = \frac{8\times 6}{2} = 24$

$\Rightarrow x\cdot x\tan B = 24 \quad \Rightarrow \quad x^2 \times \frac{3}{4} = 24$

$\Rightarrow x^2 = 32 \Rightarrow x = 4\sqrt{2}$

50. (B)

(A) From $\log_3(a+b) + \log_3(c+d) \ge 4$

We get $(a+b)(c+d) \ge 81$

Applying $AM \ge GM$ we get

$$\frac{(a+b)+(c+d)}{2} \ge ((a+b)(c+d))^{1/2} \ge 9$$

$(a+b+c+d) \ge 18$

(B) $\frac{(x+y)^{14}(1+xy)^{14}}{(xy)^{14}}$ both factors in numerator have15 independent terms. Hence total no of terms $= 15\times 15 = 225$

(C) $(23)^{86} = (529)^{43} = (530-1)^{43}$

$\Rightarrow \left[(530)^2 - {}^{43}C_1(530)^{42} .... {}^{43}C_{41}(530)^2\right] + (43\times 530) - 1.$

Hence last 2 digits are same as last two digits of $(43\times 530) +$ i.e.89

(D) $T_3 = 5C_2 \frac{1}{x^3}\left(x^{\log_{10} x}\right)^2 = 1000$

Taking $\log$ to the base 10 of both sides we get,

$-3\log_{10} x + 2(\log_{10} x)^2 - 2 = 0$

Put $\log_{10} x = t \Rightarrow 2t^2 - 3t - 2 = 0$

$$t = -\frac{1}{2}, 2$$

As $x > 1 \Rightarrow \log_{10} x = 2 \Rightarrow x = 100$

51. (A)

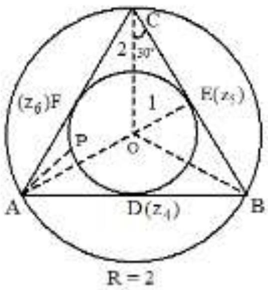
Centroid of eq. ΔABC is $z_G = \frac{z_1 + z_2 + z_3}{3} = 0 \Rightarrow z_1 + z_2 + z_3 = 0$ ...(i)

[$\because$ As circumcentre coincides with centroid for eq. Δ]

(P) $|z_1 + z_2 + z_3|^2 = \sum |z_1|^2 + 2\,\text{Re}\left(z_1 \bar{z}_2 + z_2 \bar{z}_3 + z_3 \bar{z}_1\right)$

$0 = 2^2 + 2^2 + 2^2 + 2\,\text{Re}\,(z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1)$

$\Rightarrow \text{Re}\left(z_1\,\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right) = -6$



(Q) $\frac{4z_1}{z_3} = a\left(-1 + i\sqrt{3}\right) \Rightarrow \left|\frac{4z_1}{z_3}\right| = |a|\,(2) \quad \Rightarrow 4\frac{|z_1|}{|z_3|} = |a|\,(2) \Rightarrow a = 2$

(R) $|z_1 + z_2|^2 + |z_2 + z_3|^2 + |z_3 + z_1|^2 = 2\left\{|z_1|^2 + |z_2|^2 + |z_3|^2\right\} + 2\,\text{Re}\left(z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right)$

$= 2\left(2^2 + 2^2 + 2^2\right) + 2(-6) = 12$

(S) $DP^2 + EP^2 + FP^2 = |z - z_4|^2 + |z - z_5|^2 + |z - z_6|^2$

$= 3|z|^2 + |z_4|^2 + |z_5|^2 + |z_6|^2 - 2\,\text{Re}\,\left\{\bar{z}\,(z_4 + z_5 + z_6)\right\}$

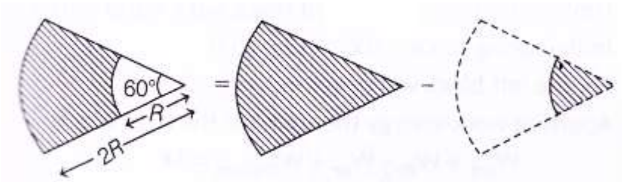
$= 3(1)^2 + 1^2 + 1^2 + 1^2 - 2\,\text{Re}\left\{\bar{z}\left(\frac{z_2 + z_3 + z_1 + z_3 + z_1 + z_2}{2}\right)\right\}$

$= 6 - 2\,\text{Re}\left\{\bar{z}\,(z_1 + z_2 + z_3)\right\} = 6 - 0$ ...[using (i)]

PART (A) : PHYSICS

1. (C)
Applying work-energy theorem
$$W_{mg} + W_N + W_{friction} + W_{spring} = \Delta K$$
$$\Rightarrow 0 + 0 - \int_0^{2L} (ax) mg\, dx + \frac{1}{2} k\left(0^2 - L^2\right)$$
$$= 0 - \frac{1}{2} m v_0^2$$
$$\Rightarrow v_0 = \sqrt{\frac{kL^2}{m} + 4aL^2 g}$$

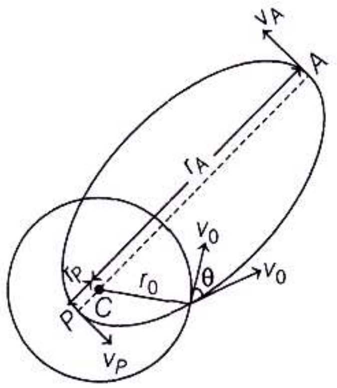
2. (A)
For a sector,
$$r_{CM} = \frac{4R}{3\theta}\sin\left(\frac{\theta}{2}\right) = \frac{4R}{3\left(\frac{\pi}{3}\right)}\sin\left(\frac{\pi/3}{2}\right) = \frac{2R}{\pi}$$



$$x_{CM} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$
$$= \frac{\left[\sigma(2R)^2\left(\frac{\pi}{3}\right)\right]\left[\frac{2(2R)}{\pi}\right] - \sigma R^2\left(\frac{\pi}{3}\right)\left(\frac{2R}{\pi}\right)}{\sigma(2R)^2\frac{\pi}{3} - \sigma R^2\left(\frac{\pi}{3}\right)}$$
$$= \frac{14R}{3\pi}$$

3. (B)
For the rod about the hinge, $\sum \tau = 0$
$$\Rightarrow N(1000) - F(2250) = 0$$
$$\Rightarrow N = 2.25F$$
$$f = \mu N = 0.3(2.25F)$$
For disc, $\sum \tau = I\alpha$
$$\Rightarrow -0.3(2.25F)(0.8) = 54(0.75)^2\alpha \Rightarrow \alpha = -\frac{4F}{225}$$
$$\omega = \omega_0 + \alpha t$$
$$\Rightarrow 0 = \left(\frac{600}{\pi}\times\frac{2\pi}{60}\right) + \left(-\frac{4F}{225}\times 25\right) \Rightarrow F = 45N$$

4. (B)



Applying conservation of angular momentum about C

$mv_p r_p = mv_A r_A = mv_0 r_0 \cos\theta$

$\Rightarrow v_A r_A = v_p r_p = \dfrac{3v_0 r_0}{5}$

Applying conservation of energy

$\dfrac{1}{2}mv_A^2 - \dfrac{GMm}{r_A} = \dfrac{1}{2}mv_0^2 - \dfrac{GMm}{r_0}$

$\Rightarrow \dfrac{9v_0^2 r_0^2}{50 r_A^2} - \dfrac{v_0^2 r_0}{r_A} + \dfrac{v_0^2}{2} = 0$

$\Rightarrow 9\left(\dfrac{r_0}{r_A}\right)^2 - 50\left(\dfrac{r_0}{r_A}\right) + 25 = 0$

$\Rightarrow \dfrac{r_0}{r_A} = \dfrac{5}{9} \Rightarrow r_A = \dfrac{9r_0}{5} \text{ and} r_p = \dfrac{r_0}{5}$

So, $\dfrac{v_p}{v_A} = \dfrac{r_A}{r_p} = 9$

5. (A, B, D)

(A) Loss in gravitational potential energy of M is Mgl as M falls down by l.

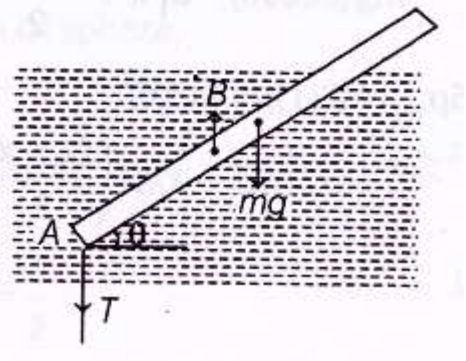
(B) Elastic potential energy stored $= \dfrac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

$= \dfrac{1}{2}\left(\dfrac{Mg}{A}\right)\left(\dfrac{l}{L}\right)(AL) = \dfrac{Mgl}{2}$

(D) Heat produced $= Mgl - \dfrac{Mgl}{2} = \dfrac{Mgl}{2}$

6. (A, B, C)

Let length of stick immersed in water be x.



Taking torque about A, $\sum \tau = 0$

$$\Rightarrow mg(1\cos\theta) - B\left(\frac{x}{2}\right)\cos\theta = 0$$

$$\Rightarrow \left(250\times10^{-3}\times2\right)g(1\cos\theta)$$

$$-\left(1000\times10^{-3}\times xg\right)\left(\frac{x}{2}\right)\cos\theta = 0$$

$$\Rightarrow x = 1m$$

So, $B = 1000\times10^{-3}\times1\times10 = 10N$

$T = B - mg = 10 - 250\times10^{-3}\times2\times10 = 5N$

When string is cut, $\sum F = ma$

$$\Rightarrow B - mg = ma$$

$$\Rightarrow 10 - 5 = 0.5a \Rightarrow a = 10\,m/s^2$$

7. (B, C)

Weight of ice cone $= 0.9\left(\frac{1}{3}\pi R^2(10)\right)g$

Buoyant force $= 1\left(\frac{1}{3}\pi\left(\frac{9R}{10}\right)^2 9\right)g$

$N = mg - B > 0$

Volume of water formed on melting of ice

$$= \frac{\text{Mass of ice melted}}{\text{Density of water}}$$

$$= \frac{0.9}{1}\left(\frac{1}{3}\pi R^2(10)\right)$$

Since, volume of water formed is greater than water displaced by the cone, level of water will increase.

Potential energy of ice-water system will decrease because work done by gravity on the system is positive and centre of mass of the system descends.

8. (4)

Speed just before hitting plate $A = \sqrt{2gh_0}$

Speed with which if rebounds from plate $A = e\sqrt{2gh_0}$

For plate $(B+C)$, using momentum conservation, $m\left(e\sqrt{2gh_0}\right)+0=(m+3m)v$

$$\Rightarrow v = \frac{e}{4}\sqrt{2gh_0}$$

$$h_2 = \frac{v^2}{2g} \Rightarrow 0.25 = \frac{e^2(9)}{16}$$

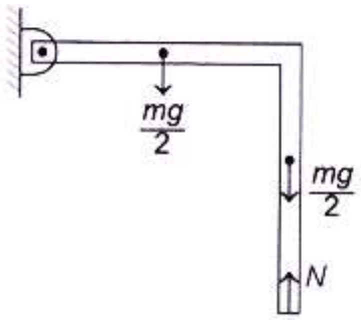
$$\Rightarrow e = \frac{2}{3}$$

$$h_1 = \frac{\left(e\sqrt{2gh_0}\right)^2}{2g}$$

$$\Rightarrow e^2h_0 = 4m$$

9. (4)

About the hinge, $\sum \tau = 0$



$$\Rightarrow \frac{mg}{2}\left(\frac{L}{2}\right)+\frac{mg}{2}L - N(L) = 0 \Rightarrow N = \frac{3mg}{4}$$

$$f = \mu N = 0.5\left(\frac{3mg}{4}\right) = \frac{3mg}{8}$$

For cylinder, $\tau = I\alpha \Rightarrow -\left(\frac{3mg}{8}\right)R = \left(\frac{mR^2}{2}\right)\alpha$

$$\Rightarrow \alpha = -\frac{3g}{4R} = -7.5 m/s^2$$

$$\omega = \omega_0 + \alpha t \Rightarrow 0 = 30 - 75t \Rightarrow t = 4s$$

10. (45)

Applying linear momentum conservation for system

$0+0=0.6(-u)+(0.3)v$ ……(i)

$\Rightarrow v = 2u$ ……..(i)

Applying angular momentum conservation about the cetnre of mass of rod,

$\left(\frac{(0.6)(0.9)^2}{12}\right)\omega = (0.3)vx \Rightarrow \frac{27}{200}\omega = vx$ …..(ii)

$e=\frac{v+u}{\omega x}=1\Rightarrow v+u=\omega x$ ....(iii)

Solving Eqs. (i), (ii) and (iii), we get

$x=45cm$

11. (2)

$K_1+U_1=K_2+U_2$

$\Rightarrow\frac{1}{2}m\left(1.5v_0\right)^2-\frac{GMm}{r}=\frac{1}{2}mv^2+0$

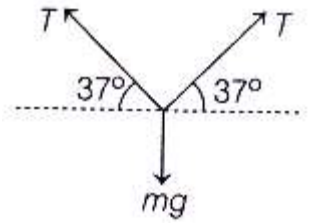
$\Rightarrow\frac{9}{8}mv_0^2-mv_0^2=\frac{1}{2}mv^2 \qquad \because\left[\frac{GM}{r}=v_0^2\right]$

$\Rightarrow v=\frac{v_0}{2}$

12. (12)

Compressive stress due to decrease in temperature,

$\sigma_1=Y\alpha\Delta T=5\times10^{11}\times2\times10^{-5}\times10=10^8\,N/m^2$



$F_y=0\Rightarrow 2T\sin37^\circ=mg\Rightarrow T=\frac{25m}{3}$

For junction to remain at initial position,

$\frac{T\ell}{Ay}=\ell\alpha\Delta\theta$

$F=Ay\alpha\Delta\theta$

$\frac{25M}{3}=10^{-6}\times5\times10^{11}\times2\times10^{-5}$

$\Rightarrow M=12\text{ kg}$

13. (2)

$F_1=F_2\Rightarrow p_0\left(\pi R_2^2\right)=\left(p_0+\rho gh\right)\pi R_1^2$

$\Rightarrow\frac{R_2}{R_1}=\sqrt{\frac{p_0+\rho gh}{p_0}}=2$

14. (0.75)

$mv_0+0=mv_1+mv_2$ ....(i)

$e=\frac{1}{2}=\frac{v_2-v_1}{v_0-0}$ ....(ii)

Solving Eqs. (i) and (ii), we get

$v_2=\frac{3v_0}{4}$

15. (0.6)

After collision B will start moving with speed $\frac{3v_0}{4}$ and C will also start moving when spring applies force on it. Subsequently speed of B decreases and that of C increases continuously. After some time, when the spring is again in unstretched position, speed of C becomes maximum. Applying mechanical energy conservation,

$$\frac{1}{2}m\left(\frac{3v_0}{4}\right)^2 = \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2 \quad \text{.........(i)}$$

Applying conservation of angular momentum about centre about centre of the track

$$m\left(\frac{3v_0}{4}\right)2R = mv_B(2R) + mv_C(R) \quad \text{..........(ii)}$$

Solving Eqs. (i) and (ii), we get

$$v_C = \frac{3v_0}{5}$$

16. (0)

Maximum acceleration of upper block is

$\mu g = 0.4(10) = 4\,m/s^2$

At $t = 5$, for lower block

$\sum f = ma$

$\Rightarrow 5t - 0.4(20) - (0.02t)(50) = 3a$

$\Rightarrow 25 - 8 - 5 = 3a$

$\Rightarrow a = 4m/s^2$

So, relative motion between the blocks will start at $t = 5$. So for $t < 5$ friction force between the blocks is static. Work done by static friction on upper block w.r.t. lower block will be zero.

17. (30)

For $t \le 5$,

$$a = \frac{5t - (0.02t)(50)}{5}$$

$$\Rightarrow a = \frac{4t}{5}$$

$$\Rightarrow \frac{dv}{dt} = \frac{4t}{5}$$

$$\Rightarrow v = \frac{2t^2}{5}$$

$$\Rightarrow K = \frac{1}{2}\times 2\times\left(\frac{2t^2}{5}\right)^2 = \frac{4t^4}{25}$$

$t > 5$

$a = \mu g = 4m/s^2$

$v = u + at$

$\Rightarrow v = 10 + 4(t - 5)$

$$\Rightarrow K = \frac{1}{2}\times 2\times(-10 + 4t)^2$$

$\Rightarrow K = \left(4t - 10\right)^2$

PART (B) : CHEMISTRY

18. (C)

19. (C)

20. (B)

21. (D)

22. (A, C, D)

23. (A, B, D)

24. (A, C)

25. (2)

26. (7)

27. (2)

28. (5)

29. (1)

30. (8)

31. (0)

32. (5)

33. (5.57)

34. (15)

PART (C) : MATHEMATICS

35. (C)

$$T_n = \frac{n}{(n^4+4n^2+4)-4n^2} = \frac{n}{(n^2+2)^2-(2n)^2} = \frac{n}{(n^2+2+2n)(n^2+2-2n)}$$

$$T_n = \frac{1}{4}\left[\frac{(n^2+2+2n)-(n^2-2n+2)}{(n^2+2+2n)(n^2-2n+2)}\right] = \frac{1}{4}\left[\frac{1}{(n-1)^2+1} - \frac{1}{(n+1)^2+1}\right]$$

$$\therefore\ S_n = \sum_{n=1}^{\infty} T_n = \frac{3}{8}$$

36. (B)

$$x^n - 1 = (x-1)(x-w)(x-w^2)....(x-w^{n-1})$$

$$1+x+x^2+......+x^{n-1} = (x-w)(x-w^2)(x-w^3)(x-w^{n-1})$$

Differentiate LHS and RHS both sides

$$1+2x+3x^2+......+(n-1)x^{n-2}$$

$$=\left[(x-w^3)(x-w^4)....(x-w^{n-1})\right](2x-(w+w^2))+(x-w)(x-w^2)\frac{d}{dx}\left[(x-w^3)....(x-w^{n-1})\right]$$

Put $x = w$

$$\frac{1+2w+3w^2+......+(n-1)w^{n-2}}{(w-w^2)}$$

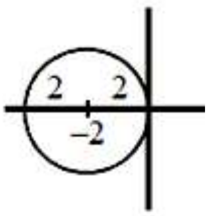
$$=(w-w^3)(w-w^4)........(w-w^{n-1})$$

37. (B)

$$\lambda f(0) < 0 \Rightarrow -2\lambda < 0 \Rightarrow \lambda > 0$$

38. (C)

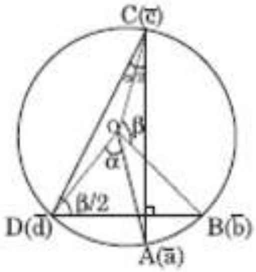
$$z = -2 + \frac{4}{\omega} \Rightarrow w = \frac{4}{z+2}$$



$$\therefore \left|\frac{4}{z+2}\right| = 2 \Rightarrow |z+2| = 2$$

$\Rightarrow$ centre $(-2,0)$ and $r = 2$

39. (A, C, D)



$\left|\bar{d}-\bar{a}\right|^2+\left|\bar{b}-\bar{c}\right|^2=\left|\bar{a}\right|^2+\left|\bar{b}\right|^2+\left|\bar{c}\right|^2+\left|\bar{d}\right|^2$

From figure, $\frac{\alpha}{2}+\frac{\beta}{2}=\frac{\pi}{2}$

$\Rightarrow \alpha+\beta=\pi$

Hence, $\bar{a}.\bar{d}+\bar{b}.\bar{c}$

$=R^2\cos\alpha+R^2\cos\beta$

$=R^2\left[\cos\alpha+\cos\beta\right]$

$=0$

Hence (i) becomes,

$\left|\bar{d}-\bar{a}\right|^2+\left|\bar{b}-\bar{c}\right|^2=4R^2$ i.e. $\lambda=4$

$\left[\bar{p}+\bar{q}-\bar{r}\ \ \bar{p}-\bar{q}+\bar{r}\ \ -\bar{p}+\bar{q}+\bar{r}\right]$

$-\begin{vmatrix}1 & 1 & -1\\ 1 & -1 & 1\\ -1 & 1 & 1\end{vmatrix}\left[\bar{p}\,\bar{q}\,\bar{r}\right]=-4\left[\bar{p}\,\bar{q}\,\bar{r}\right]$

Let $S=\left|2\hat{a}-3\hat{b}\right|^2+\left|2\hat{b}-3\hat{c}\right|^2+\left|2\hat{c}-3\hat{a}\right|^2$

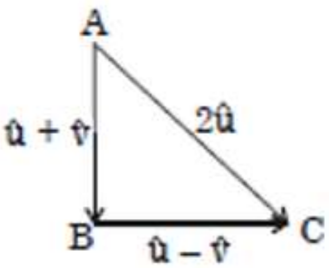
$39-12\left(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}\right)$

Now, $\left|\hat{a}+\hat{b}+\hat{c}\right|^2=3+2\left(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}\right)\geq 0$

$3+2\left(\frac{39-S}{12}\right)\geq 0\Rightarrow S\leq 57$

$S\big|_{max}=57$

(D)



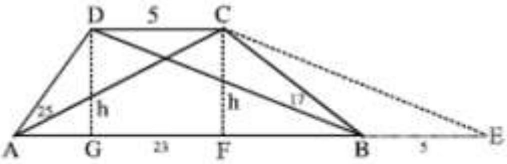
$\overline{AC}=2\hat{u}\ \&\ \overline{BC}=\hat{u}-\hat{v}\Rightarrow\overline{AB}=\hat{u}+\hat{v}$

$\overline{AB}.\overline{BC}=\left(\hat{u}+\hat{v}\right).\left(\hat{u}-\hat{v}\right)=0\Rightarrow\angle B=\frac{\pi}{2}$

We have,

$$\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$$
$$= -1 = \lambda - 5$$
$$= (8.10)$$

40. (A, D)



In ΔACE $\quad$ $a = AE = 23 + 5 = 28$

$b = AC = 25$

$c = CE = 17$

$$\therefore s = \frac{1}{2}(a+b+c) = \frac{1}{2}(28+25+17) = 35$$

$$\Delta = \frac{1}{2}AE.h = \sqrt{35(35-28)(35-25)(35-17)}$$

$$\Rightarrow \frac{1}{2}.28h = \sqrt{5.7.7.5.2.2.3.3}$$

$$\Rightarrow 14h = 5.7.2.3 \qquad \therefore h = 15$$

$$\therefore \text{In } \Delta CFE.EF = \sqrt{17^2 - 15^2} = 8$$

$$\therefore BF = 8 - 5 = 3$$

$$\therefore |BC| = \sqrt{h^2 + 3^2}$$

$$= \sqrt{225+9} = \sqrt{234}$$

$$= 3\sqrt{26}$$

From ΔACF $\quad$ $AF = \sqrt{25^2 - h^2}$

$$= \sqrt{25^2 - 15^2} = 20$$

$$\therefore AG = AF - GF = 20 - 5 = 15$$

$$\therefore AD = \sqrt{h^2 + AG^2} = \sqrt{15^2 + 15^2}$$

$$= 15\sqrt{2}$$

41. (A, B)

(A) $|z_1| + |z_2| = 4 \Rightarrow |z_1| + |-z_2| = |z_1 + (-z_2)|$

$\therefore z_1 \,\&\, (-z_2)$ are collinear with origin such that $z_1 \,\&\, z_2$ both lie towards the same side of origin.

$\therefore z_1, z_2$ and origin will also be collinear such that z_1 and z_2 will lie opposite to the side of origin

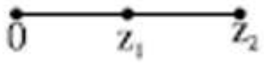
$$\therefore \arg\left(\frac{z_1}{z_2}\right) = \pi$$

(B) $|z_1| + |z_2| = |z_1 + z_2|$

$\therefore z_1 \& z_2$ are collinear with origin such that $z_1 \& z_2$ both lie towards the same side of origin

$\therefore \arg\left(\frac{z_1}{z_2}\right)=0$

(C) $|z_1|-|z_2|=-4 \Rightarrow |z_1|-|z_2|=-|z_1-z_2|$



$\therefore |z_1|+|z_2-z_1|=|z_2|$

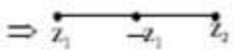
$\therefore z_1, z_2$ and origin will

Such that z_1 and z_2 will be towards the same side of origin

$\therefore \arg\left(\frac{z_1}{z_2}\right)=0$

(D) $|z_2|-|z_1|=|z_1+z_2|$

$\Rightarrow |z_1+z_2|+|z_1|=|z_2|$



$\therefore z_1, z_2$ and origin will be collinear such that z_1 and z_2 will lie opposite to the side of origin

$\therefore \arg\left(\frac{z_1}{z_2}\right)=\pi$

42. (7)

There are Maths-I, Maths-II and 6 other books.

If Maths-II is selected, Maths-I also selected. So number of ways $={}^6C_1$. If Maths-II not selected then number of ways $={}^7C_3$

$\lambda={}^6C_1+{}^7C_3=6+35=41$

Perfect squares less than 41 are 0,1,4,9,16,25,36.

43. (8)

$$\frac{(k-1)^2}{(k+1)(k+2)}=\frac{k^2+3k+2-5k-1}{(k+1)(k+2)}$$

$$=1-\frac{5k+1}{(k+1)(k+2)}=1-\frac{5(k+2)-9}{(k+1)(k+2)}$$

$$=1-\frac{5}{k+1}+\frac{9}{(k+1)(k+2)}$$

$$\therefore \sum_{k=0}^{10}\frac{(k-1)^2}{(k+1)(k+2)}\,{}^{10}C_k=\sum_{k=0}^{10}\left(1-\frac{5}{k+1}+\frac{9}{(k+1)(k+2)}\right){}^{10}C_k$$

$$=2^{10}-\frac{5}{11}.\left(2^{11}-1\right)+\frac{9}{11.12}.\left(2^{10}-1-12\right)$$

$$=2^{10}-\frac{10.2^{10}}{11}+\frac{5}{11}+\frac{36.2^{10}}{11.12}-\frac{9}{11.12}-\frac{9}{11}$$

$$=\frac{-9-4\times12}{11.12}+\left(\frac{11.12+36-10.12}{11.12}\right)2^{10}$$

$$=\frac{4}{11}.2^{20}-\frac{57}{11.12}=\frac{48.2^{10}-57}{132}$$

$$a=48, b=57, c=132, 11\left(\frac{a+b}{c}\right)=8.75$$

44. (1)

45. (42)
We have, $x+y+z=12$ ………. (i)
Assume $x<y<z$. Here, $x,y,z\geq1$
$\therefore$ Solution of Eq. (i) are
$(1,2,9),(1,3,8),(1,4,7),(1,5,6),(2,3,7),(2,4,6)$ and $(3,4,5)$.
Number of positive integral solutions of Eq. (i) = 7 but
x,y,z can be arranged in $3!=6$
Hence, required number of solutions $=7\times6=42$

46. (5)
We have,

$$\left(1+ax+bx^2\right)\left(1-2x\right)^{18}$$

$$=\left(1+ax+bx^2\right)\sum_{r=0}^{18}{}^{18}C_r\left(-1\right)^r2^rx^r$$

$$=\sum_{r=0}^{18}{}^{18}C_r\left(-1\right)^r2^rx^r+a\sum_{r=0}^{18}{}^{18}C_r\left(-1\right)^r2^rx^{r+1}+b\sum_{r=0}^{18}{}^{18}C_r\left(-1\right)^r2^rx^{r+2}$$

The coefficient of x^3 and x^4 in the expansion of $\left(1+ax+bx^2\right)\left(1-2x\right)^{18}$ are both zero.

$$\therefore{}^{18}C_3\left(-1\right)^3 2^3+a.{}^{18}C_2\left(-1\right)^2 2^2+b.{}^{18}C_1\left(-1\right)^1 2^1=0$$

And ${}^{18}C_4\left(-1\right)^4 2^4+a.{}^{18}C_3\left(-1\right)^3 2^3+b.{}^{18}C_2\left(-1\right)^2 2^2=0$

$\Rightarrow\quad 51a-3b=544$

And $32a-3b=240$

$\Rightarrow\quad a=16, b=\frac{272}{3}$

47. (1)

$$z^{12}-z+\left(z^{10}+z^9+.....+z^2+z+1\right)=0$$

$$z\left(z^{11}-1\right)+\frac{z^{11}-1}{z-1}=0$$

$\Rightarrow z^{11}-1=0$ has roots $1, z_1, z_2, z_3......z_{10}$

$$\therefore(1)^{2021}+\left(z_1\right)^{2021}+\left(z_2\right)^{2021}+.......+\left(z_{10}\right)^{2021}=0$$

$$\therefore\left(z_1\right)^{2021}+\left(z_2\right)^{2021}+.......+\left(z_{10}\right)^{2021}=-1$$

48. (3.00)

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}; B=\begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$A+\alpha B=\begin{bmatrix} a+\alpha p & b+\alpha q \\ c+\alpha r & d+\alpha s \end{bmatrix}$$

$$|A+\alpha B|=\begin{vmatrix} a+\alpha p & b+\alpha q \\ c+\alpha r & d+\alpha s \end{vmatrix}$$

$$\Rightarrow |A|+\alpha^2|B|+\alpha(as+pd-br-qc)$$

$$|\alpha A+B|=\begin{vmatrix} \alpha a+p & \alpha b+q \\ \alpha c+r & \alpha d+s \end{vmatrix}$$

$$=\alpha^2|A|+|B|+\alpha[as+pd-br-qc]$$

$$|A+\alpha B|-|\alpha A+B|$$

$$=(1-\alpha^2)|A|+(\alpha^2-1)|B|$$

$$=(1-\alpha^2)(|A|-|B|)$$

$$=\alpha^2-1$$

49. (4.00)

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}; B=\begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$A-\alpha B=\begin{bmatrix} a-\alpha p & b-\alpha q \\ c-\alpha r & d-\alpha s \end{bmatrix}$$

$$|A-\alpha B|=\begin{vmatrix} a-\alpha p & b-\alpha q \\ c-\alpha r & d-\alpha s \end{vmatrix}$$

$$\Rightarrow |A|+\alpha^2|B|-\alpha(as+pd-br-qc)$$

$$|-\alpha A+B|=\begin{vmatrix} -\alpha a+p & -\alpha b+q \\ -\alpha c+r & -\alpha d+s \end{vmatrix}$$

$$=\alpha^2|A|+|B|-\alpha[as+pd-br-qc]$$

$$|A-\alpha B|-|-\alpha A+B|$$

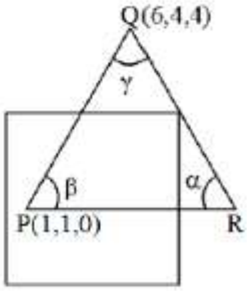
$$=(1-\alpha^2)|A|+(\alpha^2-1)|B|$$

$$=(1-\alpha^2)(|A|-|B|)$$

$$=\alpha^2-1$$

(C) option correct

50. (8)



If $\frac{QP+PR}{QR}$ is maximum then $\frac{\sin\alpha}{\sin\beta}+\frac{\sin(\pi-(\alpha+\beta))}{\sin\beta}$ is maximum.

Now $\frac{\sin\alpha}{\sin\beta}+\frac{\sin(\alpha+\beta)}{\sin\beta}=\frac{2\sin\left(\frac{2\alpha+\beta}{2}\right)\cos\frac{\beta}{2}}{\sin\beta}=\frac{\sin\left(\alpha+\frac{\beta}{2}\right)}{\sin\frac{\beta}{2}}$

$\Rightarrow$ If $\frac{PQ+PR}{QR}$ is maximum then

$\alpha+\frac{\beta}{2}=\frac{\pi}{2}$ and $\frac{\beta}{2}$ is minimum positive angle belongs to $\left(0,\frac{\pi}{2}\right)$

$\Rightarrow \alpha=\gamma$ and R lies on projection of PQ on plane

Now foot of Q on plane is $S(2,0,0)$. Direction cosine of $\overrightarrow{PS}$ is

$\left(\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}},0\right)$, Now $PQ=\sqrt{50}=PR$

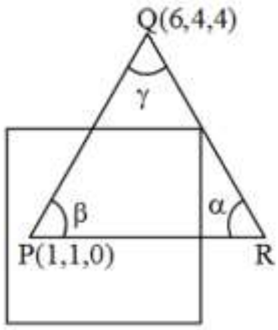
Cordinates of R is given by

$\frac{x-1}{\frac{1}{\sqrt{2}}}=\frac{y-1}{\frac{-1}{\sqrt{2}}}=\frac{z-0}{0}=\sqrt{50}$

$\Rightarrow x=6, y=-4, z=0 \Rightarrow R$ is $(6,-4,0)$

• $\ell=\frac{\sqrt{50}+\sqrt{50}}{\sqrt{80}}=\sqrt{\frac{5}{2}}\Rightarrow 2\ell^2=5$

51. (5)



If $\frac{QP+PR}{QR}$ is maximum then $\frac{\sin\alpha}{\sin\beta}+\frac{\sin(\pi-(\alpha+\beta))}{\sin\beta}$ is maximum.

Now $\dfrac{\sin\alpha}{\sin\beta}+\dfrac{\sin(\alpha+\beta)}{\sin\beta}=\dfrac{2\sin\left(\dfrac{2\alpha+\beta}{2}\right)\cos\dfrac{\beta}{2}}{\sin\beta}=\dfrac{\sin\left(\alpha+\dfrac{\beta}{2}\right)}{\sin\dfrac{\beta}{2}}$

$\Rightarrow$ If $\dfrac{PQ+PR}{QR}$ is maximum then

$\alpha+\dfrac{\beta}{2}=\dfrac{\pi}{2}$ and $\dfrac{\beta}{2}$ is minimum positive angle belongs to $\left(0,\dfrac{\pi}{2}\right)$

$\Rightarrow \alpha=\gamma$ and R lies on projection of PQ on plane

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$\Rightarrow x=6, y=-4, z=0 \Rightarrow R$ is $(6,-4,0)$

- $\ell=\dfrac{\sqrt{50}+\sqrt{50}}{\sqrt{80}}=\sqrt{\dfrac{5}{2}}\Rightarrow 2\ell^2=5$