PACE-IIT & MEDICAL

MUMBAI / DELHI-NCR / PUNE / NASHIK / AKOLA / GOA / JALGOAN / BOKARO / AMRAVATI / DHULE

IIT – JEE: 2024 **PART TEST - 2** **DATE: 02/12/23**

MAIN

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	C	**31.**	D	**61.**	C
2.	D	**32.**	D	**62.**	A
3.	C	**33.**	D	**63.**	D
4.	B	**34.**	B	**64.**	A
5.	C	**35.**	D	**65.**	B
6.	A	**36.**	A	**66.**	A
7.	A	**37.**	A	**67.**	D
8.	B	**38.**	D	**68.**	D
9.	D	**39.**	A	**69.**	D
10.	B	**40.**	A	**70.**	D
11.	B	**41.**	D	**71.**	D
12.	D	**42.**	C	**72.**	D
13.	A	**43.**	D	**73.**	D
14.	C	**44.**	A	**74.**	A
15.	B	**45.**	B	**75.**	A
16.	B	**46.**	C	**76.**	A
17.	D	**47.**	C	**77.**	C
18.	D	**48.**	C	**78.**	B
19.	B	**49.**	B	**79.**	C
20.	B	**50.**	A	**80.**	D
21.	4	**51.**	3	**81.**	3
22.	2	**52.**	200	**82.**	1
23.	90	**53.**	8	**83.**	4
24.	2	**54.**	1	**84.**	1
25.	2	**55.**	25	**85.**	5
26.	2	**56.**	3	**86.**	3
27.	30	**57.**	56	**87.**	4
28.	2	**58.**	2	**88.**	2
29.	6	**59.**	0	**89.**	5
30.	100	**60.**	3	**90.**	1

PART (A) : PHYSICS

ANSWER KEY

1.	(C)	2.	(D)	3.	(C)	4.	(B)	5.	(C)
6.	(A)	7.	(A)	8.	(B)	9.	(D)	10.	(B)
11.	(B)	12.	(D)	13.	(A)	14.	(C)	15.	(B)
16.	(B)	17.	(D)	18.	(D)	19.	(B)	20.	(B)
21.	(4)	22.	(2)	23.	(90)	24.	(2)	25.	(2)
26.	(2)	27.	(30)	28.	(2)	29.	(6)	30.	(100)

SOLUTIONS

1. (C)
Conceptual

2. (D)
$|E_{rms} = Ri - j\left(\omega_L - \frac{1}{\omega_C}\right)i$
$E_{rms} = Ri + j(V_L - V_C)$
$E_{rms} = RI$
$\therefore \quad V = 220\text{ V}$
$I = 220/100 = 2.2$ Amp

3. (C)
Magnetic induction due to AA' and BB' is given as $B_1 = \frac{\mu_0 i}{4\pi R}$
Magnetic induction due to CC' and DD' is zero at O.
Magnetic induction due to AB is given as $B_2 = \frac{\mu_0 i}{8R}$
Magnetic induction due to CD is given as $B_3 = \frac{-\mu_0 i}{8R}$
Thus net magnetic induction at O given as $B_0 = 2B_1 + B_2 + B_3$; $B_0 = \frac{\mu_0 i}{2\pi R}$

4. (B)
Conceptual

5. (C)
Apply KCL

6. (A)
$C_{eq} = \frac{K_1 K_3 C}{K_1 + K_3} + \frac{K_2 C}{2}$

7. (A)
Charge does not flow from one independent loop to another.

8. (B)
Conceptual

9. (D)
It is similar to the electric field at the axis of uniformly charged ring. In ring the electric field is maximum at R/$\sqrt{2}$. Here radius is equivalent to L/$\sqrt{2}$.

10. (B)

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\sigma r \frac{\pi}{2}.dr}{4\pi\varepsilon_0 r}$$

$$\therefore V = \frac{\sigma}{8\varepsilon_0}\int_{R/2}^{R} dr = \frac{\sigma R}{16\varepsilon_0}$$

11. (B)

12. (D)

Loss in electrostatic $PE = \frac{\sigma}{2\varepsilon_0}(y)\times q$

Loss in gravitational $PE = mg\ y \Rightarrow \frac{1}{2}mv^2 = \frac{\sigma q}{2\varepsilon_0}y + mg\ y \Rightarrow v^2 = 2y\left(\frac{q\sigma}{2m\varepsilon_0} + g\right)$

13. (A)

Induced emf $\int_a^b Bvd\ x = \int_a^b \frac{\mu_0 I}{2\pi x}vd\ x$

$\Rightarrow$ Induced emf $= \frac{\mu_0 Iv}{2\pi}\ell n\left(\frac{b}{a}\right) \Rightarrow$ Power dissipated $= \frac{E^2}{R}$

Also, power $= F.V \Rightarrow F = \frac{E^2}{VR}$

$$\Rightarrow \quad F = \frac{1}{VR}\left[\frac{\mu_0 IV}{2\pi}\ell n\left(\frac{b}{a}\right)\right]^2$$

14. (C)

$$L = \frac{e}{di/dt} = \frac{[W/q]}{(di/dt)} = \frac{ML^2T^{-2}/AT}{A/T^{-1}}$$

15. (B)
Force on semicircular wire = force on section *PQ*
$\therefore\ F = i(l\times B)$

Here, $l = PQ = (2\cos 45°)\hat{i} + (2\sin 45°)\hat{j}$

$= \left(\sqrt{2}\hat{i} + \sqrt{2}\hat{j}\right)m$

$\therefore\ F = (1)\left[\left(\sqrt{2}\hat{i} + \sqrt{2}\hat{j}\right)\times\left(3\hat{i} + 4\hat{j} + \hat{k}\right)\right] = \sqrt{2}\left(\hat{i} - \hat{j} + \hat{k}\right)$

16. (B)

$c = \frac{\omega}{k} = 45\ \text{m/s} = \sqrt{\frac{T}{\mu}}$

$T = \frac{0.12\times 10^{-3}}{10^{-2} m} kg \times 45\times 45 = 24.3\ \text{N}$

17. (D)

For 1st resonance, $\frac{\lambda}{4} = l + e = l + 0.3d$

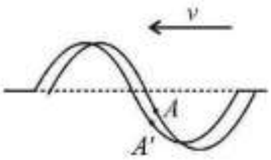
$= 15 + 0.3\times 10 = 18\text{cm}$

$\Rightarrow\ \lambda = 72\ \text{cm}$

$f = \frac{c}{\lambda} = \frac{330}{0.72} \simeq 460\ \text{Hz}$

18. (D)

Referring to the diagram right wave is initial position and left wave is final position. It is clearly visible that the particle is moving towards extreme position and in SHM acceleration is always towards mean position.



19. (B)

Phase difference at a point at a distance x from the central maxima, $\Delta\phi = 2\pi\left(\frac{x}{\beta}\right)$

Intensity at this point, $I = I_0\cos^2\left(\frac{\Delta\phi}{2}\right) = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$

20. (B)

$PN = d,\ PO = d\sec\theta$

$CO = PO\cos 2\theta = d\sec\theta\cos 2\theta$

Path difference $= CO + PO = d\sec\theta + d\sec\theta\cos 2\theta$

$= d\sec\theta(1+\cos 2\theta) = \frac{d}{\cos\theta}\times 2\cos^2\theta = 2d\cos\theta.$

The ray AO is incident on denser medium, so reflected ray suffers a phase difference of π or a path differences of $\frac{\lambda}{2}$

$\therefore$ Effective path difference $\Delta = 2d\cos\theta + \frac{\lambda}{2}$

For constructive interference path difference $\Delta = n\lambda$.

$\Rightarrow \quad 2d\cos\theta + \dfrac{\lambda}{2} = n\lambda$ or $2d\cos\theta = (2n-1)\dfrac{\lambda}{2}$

21. (4)

$x = 4$

$R = \dfrac{12}{4} = 3\Omega$

$Z = \dfrac{12}{2.4} = 5\Omega$

$R^2 + (\omega L)^2 = 25$

$(\omega L)^2 = 25 - 9 = 16$

$\omega L = 4 \Rightarrow L = \dfrac{4}{\omega}$

$\omega = 2\pi f = 2\pi \times \dfrac{50}{\pi} = 100$

$L = \dfrac{4}{100} = 4\times 10^{-2}$ H

22. (2)

The equivalent resistance of original grid about any two neighboring points will be $\dfrac{R}{2}$.

Let the resistance of the present grid is R_1.

We can consider the original grid as a circuit of two resistors R_1 and R connected in parallel, the latter being the removed resistor

$\therefore \quad \dfrac{R}{2} = \dfrac{RR_1}{R+R_1} \Rightarrow R_1 = R$.

$\Rightarrow R^2 + RR_1 = 2RR_1 \Rightarrow RR_1 = R_1^2 R_1 = R$

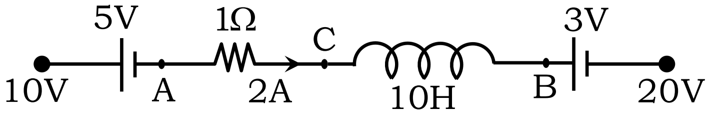
23. (90)

Since, f_1 & f_3 are equal.

$\dfrac{1}{f} = \dfrac{2}{f_1} + \dfrac{1}{f_2} = \left(\dfrac{4}{3}-1\right)\dfrac{2}{R} + \left(\dfrac{3}{2}-1\right)\dfrac{2}{-R}$

$= \dfrac{2}{R}\left(\dfrac{1}{3} - \dfrac{1}{2}\right) = \dfrac{-1}{3R}$

24. (2)



$V_A = 5V,\ V_C = 3V,\ V_B = 23V$

$\therefore \quad L\left|\dfrac{di}{dt}\right| = V_B - V_C = 20v$

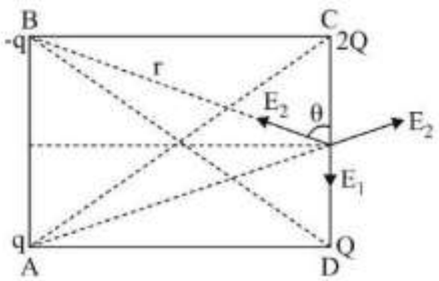
$\Rightarrow \left|\frac{di}{dt}\right| = 2\ A/s$

25. (2)
Flux passing from the small loop,
$\phi = BS = \left(\frac{\mu_0 i}{2R}\right)(a^2)$
$e = \frac{d\phi}{dt} = \left(\frac{\mu_0 a^2}{2R}\right)\frac{di}{dt}$ in volts
$= \left(\frac{\mu_0 a^2}{2R}\right)\left(\frac{i_0}{\tau}\right)\times 10^6$ (in microvolts)
$= (4\pi\times 10^{-7})(10^{-2})^2 \times \frac{2}{\pi}\times 100\times 10^6 \times \frac{1}{2\times 0.1\times 2\times 10^{-2}} = 2\ \mu V$

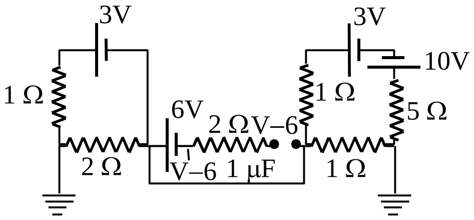
26. (2)
$\left(\frac{\mu_1}{\mu_0}-1\right)t_1 = \left(\frac{\mu_2}{\mu_0}-1\right)t_2$

27. (30)
Image forms on object, when rays from object travels same path after reflection by mirror so rays refracted by lens, will form image at cenre of curvature of mirror
By lens formula
$\frac{1}{v}-\frac{1}{u}=\frac{1}{f};\qquad \frac{1}{v}+\frac{1}{60}=\frac{1}{30}$
$v = 60$ cm
From lens distance of centre of curvature = 60 cm
Radius of curvature = 30 cm

28. (2)
$E_1 = 2E_2\cos\theta$
$\therefore\ \frac{KQ}{(a/2)^2} = 2\left[\frac{Kq}{r^2}\cdot\frac{a/2}{r}\right]$
$\therefore\ \frac{q}{Q} = 4\left(\frac{r}{a}\right)^3$
$= 4\left(\sqrt{\frac{5}{4}}\right)^3 = \frac{5\sqrt{5}}{2}$



29. (6)
In steady state the capacitor is fully charged and is treated as open circuit, so no current flows through branch containing capacitor in steady state. So, the circuit can be redrawn as



Potential difference across capacitor in steady state = V – 6 – V = –6V
(–ve sign signifies that left hand plate is of negative polarity)
Charge = CV = 1 × 6 = 6 μC

30. (100)

$$L = \frac{L_1 L_2}{L_1 + L_2} = 2H; \qquad i_0 = \frac{E}{R} = 1\,A$$

Now, $\dfrac{\frac{1}{2} L i_0^2}{i_0^2 R t} = \dfrac{L}{2Rt} = \dfrac{2}{2\times 10\times 10} = \dfrac{1}{100}$

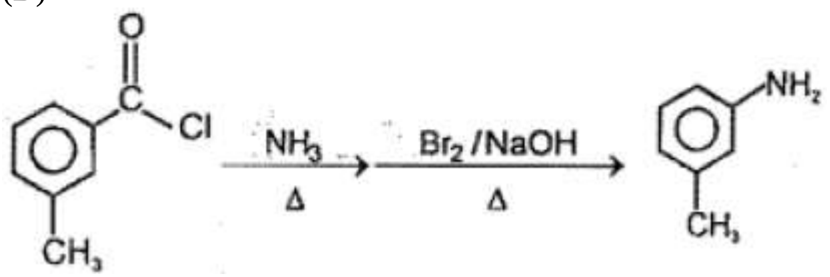
PART (B) : CHEMISTRY

ANSWER KEY

31.	(D)	32.	(D)	33.	(D)	34.	(B)	35.	(D)
36.	(A)	37.	(A)	38.	(D)	39.	(A)	40.	(A)
41.	(D)	42.	(C)	43.	(D)	44.	(A)	45.	(B)
46.	(C)	47.	(C)	48.	(C)	49.	(B)	50.	(A)
51.	(3)	52.	(200)	53.	(8)	54.	(1)	55.	(25)
56.	(3)	57.	(56)	58.	(2)	59.	(0)	60.	(3)

SOLUTIONS

31. (D)
Para position to N (least crowded)

32. (D)


33. (D)
Reaction of ester with excess GR gives tertiary alcohol.

34. (B)
Hunsdiecker reaction.

35. (D)
Hofman Bromamide reaction.

36. (A)
Primary alcohol gets converted into aldehyde by Cu/300.

37. (A)

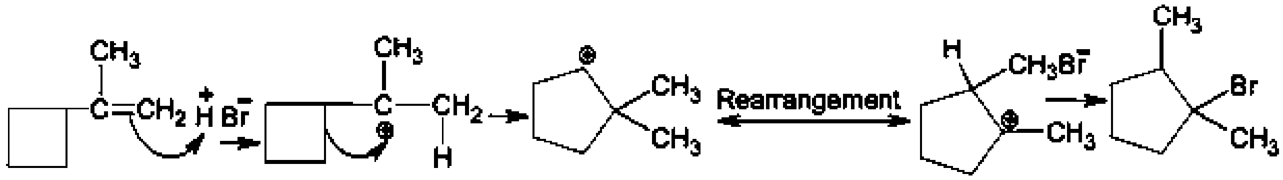
38. (D)
Activating group makes ring more reactive towards EAS.

39. (A)
Friedel Crafts Alkylation

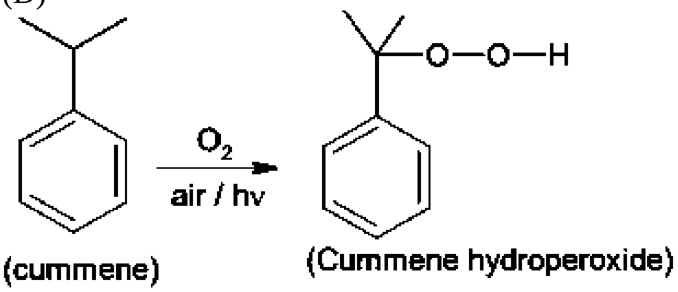
40. (A)
Birch reduction (benzene to diene)

41. (D)
Electrophilic Aromatic Substitution

42. (C)



43. (D)



Cummene hydroperoxide is an intermediate formed during phenol formation from cummene.

44. (A)

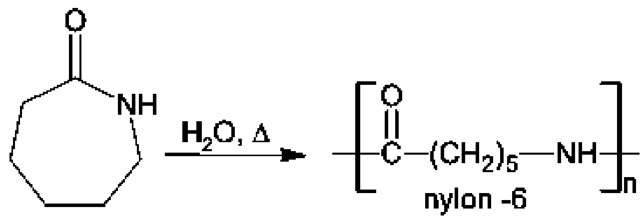
$Ph-Cl+CH_3-Cl \xrightarrow{2Na} Ph-CH_3$ (Wurtz Fittg)

$2Ph-Cl \xrightarrow{2Na} Ph-Ph$ (Fittig)

$Ph-N_2^+Cl^- \xrightarrow{Cu_2Cl_2} Ph-Cl+N_2$ (Sandmeyer)

$C_2H_5-Cl+NaI \xrightarrow{Acetone} C_2H_5I+NaCl$ (Finkelstein)

45. (B)

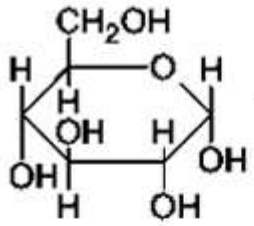


46. (C)

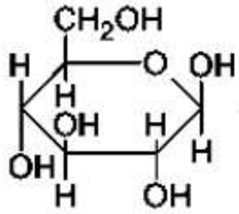
$Ph-CH_2-Br \xrightarrow{AgCN} \underset{\text{(Benzyl isocyanide)}}{Ph-CH_2-NC}$

$Ph-CH_2-NH_2 \xrightarrow[\text{aq. KOH}]{CHCl_3} \underset{\text{(Benzyl isocyanide)}}{Ph-CH_2-NC}$

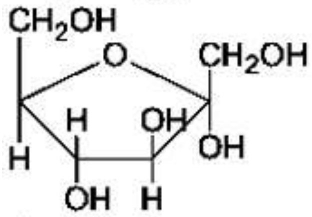
47. (C)



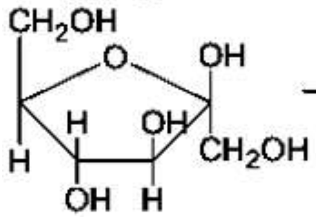
→ α-(D)(-)Glucopyranose



→ β-(D)(-)Glucopyranose

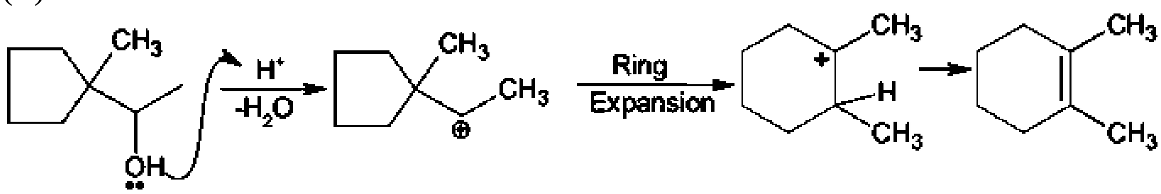


→ α-(D)(-) Fructofuranose



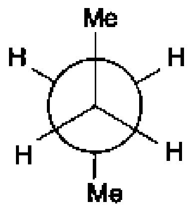
→ β-(D)(-) Fructofuranose

48. (C)



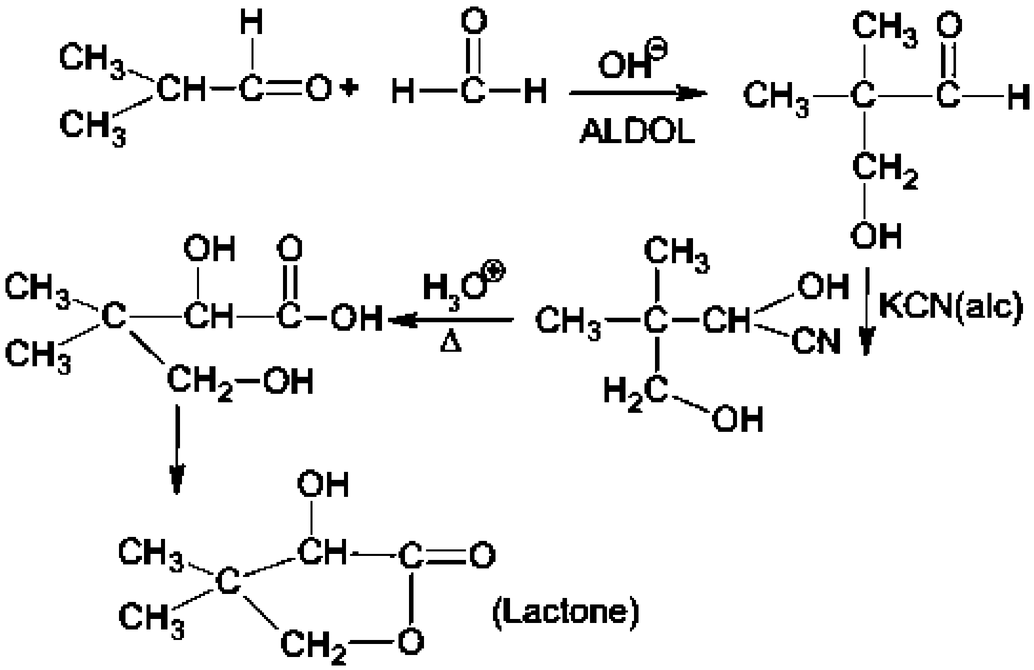
49. (B)

Anti form is most stable conformer



It has lowest Vanderwaal &torsional strain.

50. (A)



51. (3)
Increase is carbon increase boiling points. Boiling point $\propto$ molar mass $\propto$ Vander Wall force of attraction.

52. (200)
$\frac{0.01x \times 0.6}{12} = \frac{4.4}{44}$
$\Rightarrow\ x = 200\ \text{gm/mole}$

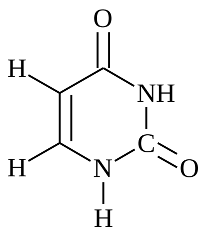
53. (8)
Total number of tripeptide by mixing valine and proline $= 2^3 = 8$

Val – Val – Val	Pro – Pro – Pro
Val – Pro – Pro	Pro – Val – Pro
Val – Val – Pro	Val – Pro – Val
Pro – Pro – Val	Pro – Val – Val

54. (1)
Degree of unsaturation $= 4 + 1 - \left(\frac{5-1}{2}\right) = 3$

$CH_3 - CH = CH - C \equiv N$
↑
sp^3 hybridised only one carbon

55. (25)
Molecular formula of Uracil



$= C_4N_2H_4O_2$

Molecular mass $= 12\times4+14\times2+1\times4+16\times2$
$= 48+28+4+32$
$= 112$

$\therefore$ % of N in uracil $= \frac{28}{112}\times100 = 25\%$

56. (3)
1 mole of hydrocarbon reacts with 3 moles of H_2 gas.

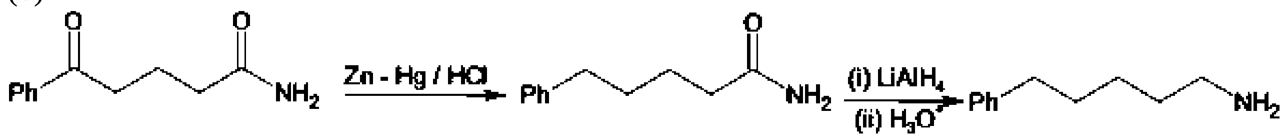
57. (56)
Organic compound $\rightarrow NH_3$
$2NH_3 + H_2SO_4 \rightarrow (NH_4)_2SO_4$
No. of milimoles of $NH_3 = 2\times$ no. milimoles of H_2SO_4
$= 2\times2\times2.5$
$= 10$ milimoles
= No. of milimoles of N

Mass of nitrogen $= \frac{10}{100}\times14 = 0.14$ gm

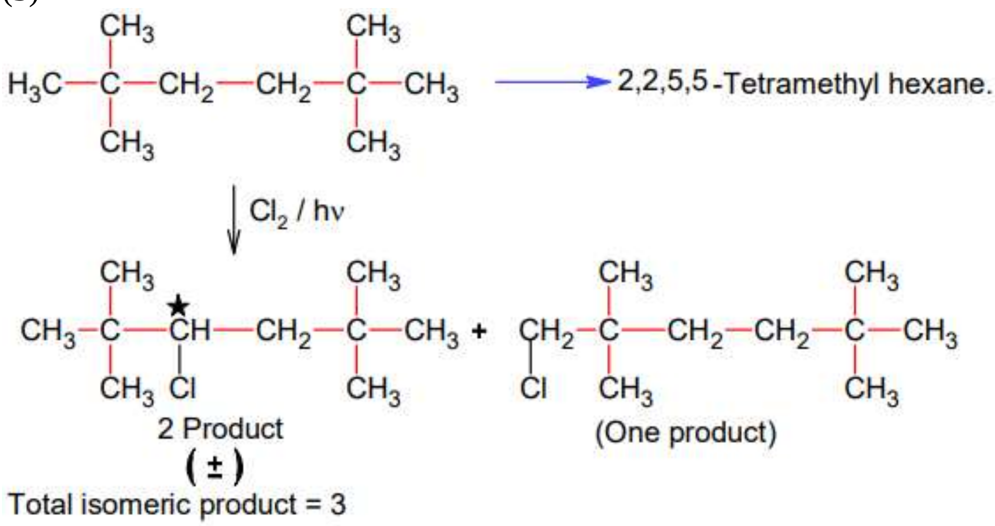
% of Nitrogen in compound $= \frac{0.14}{0.25}\times100$
$= 56\,\%$

58. (2)
2 (only 2 carbon will chiral in product form)

59. (0)



60. (3)


2,2,5,5-Tetramethyl hexane.
Cl2 / hv
2 Product
(±)
(One product)
Total isomeric product = 3

PART (C) : MATHEMATICS

ANSWER KEY

61.	(C)	62.	(A)	63.	(D)	64.	(A)	65.	(B)
66.	(A)	67.	(D)	68.	(D)	69.	(D)	70.	(D)
71.	(D)	72.	(D)	73.	(D)	74.	(A)	75.	(A)
76.	(A)	77.	(C)	78.	(B)	79.	(C)	80.	(D)
81.	(3)	82.	(1)	83.	(4)	84.	(1)	85.	(5)
86.	(3)	87.	(4)	88.	(2)	89.	(5)	90.	(1)

SOLUTIONS

61. (C)

Given, $f(x)=\dfrac{5^x}{5^x+5'}$ then

$$f(2-x)=\frac{5^{2-x}}{5^{2-x}+5}$$
$$=\frac{5}{5^x+5}$$

62. (A)

We have,

$$S=\lim_{x\to 2}\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}$$
$$=\sum_{n=1}^{9}\frac{2}{4(n^2+3n+2)}$$
$$=\frac{1}{2}\sum_{n=1}^{9}\frac{(n+2)-(n+1)}{(n+1)(n+2)}$$
$$=\frac{1}{2}\sum_{n=1}^{9}\left[\frac{1}{n+1}-\frac{1}{n+2}\right]$$
$$=\frac{1}{2}\left[\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+....+\left(\frac{1}{10}-\frac{1}{11}\right)\right]$$
$$=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{11}\right)=\frac{1}{2}\times\left(\frac{11-2}{2\times 11}\right)=\frac{9}{44}$$

63. (D)

$$\lim_{x\to 2}\frac{(a+2x)^{1/3}-(3x)^{1/3}}{(3a+x)^{1/3}-(4x)^{1/3'}},(a\neq 0)\left[\frac{0}{0}\text{form}\right]$$

Put x = a + h

So, $\displaystyle\lim_{h\to 0}\frac{(a+2a+2h)^{1/3}-(3a+3h)^{1/3}}{(3a+a+h)^{1/3}-(4a+4h)^{1/3}}$

$$=\lim_{h\to 0}\frac{(3a)^{1/3}\left[\left(1+\frac{2h}{3a}\right)^{1/3}-\left(1+\frac{3h}{3a}\right)^{1/3}\right]}{(4a)^{1/3}\left[\left(1+\frac{h}{4a}\right)^{1/3}-\left(1+\frac{4h}{4a}\right)^{1/3}\right]}$$

$$=\lim_{h\to 0}\left(\frac{3}{4}\right)^{1/3}\left[\frac{1+\frac{2h}{9a}-1-\frac{3h}{9a}+\text{higher degree terms}}{1+\frac{h}{12a}-1-\frac{4h}{12a}+\text{higher degree terms}}\right]$$

$$=\left(\frac{3}{4}\right)^{1/3}\left(\frac{\frac{2}{9}-\frac{3}{9}}{\frac{1}{12}-\frac{4}{12}}\right)=\left(\frac{3}{4}\right)^{1/3}\left(\frac{\frac{-1}{9}}{\frac{-3}{12}}\right)$$

64. (A)

f(x) is continuous at x = 0

LHL at x = 0 = f(0) = RHL at x = 0

$$\lim_{x\to 0}\frac{1}{2}\frac{\ln\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right)}{x}=\frac{\lim_{x\to 0^-}\left(\frac{1}{a}\right)\ln\left(1+\frac{x}{a}\right)}{\left(\frac{1}{a}\right)x}-\frac{\lim_{x\to 0^-}\left(-\frac{1}{b}\right)\ln\left(1-\frac{x}{b}\right)}{\left(-\frac{1}{b}\right)x}=\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$f(0)=k$$

$$\lim_{x\to 0^+}\frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1}$$

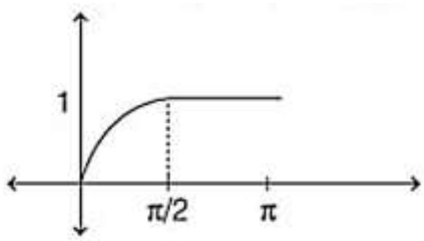
$$\lim_{x\to 0^+}\frac{-2\sin^2 x}{\sqrt{x^2+1}-1}=\lim_{x\to 0^+}-\left(\frac{2\sin^2 x}{x^2}\right)\quad \sqrt{x^2+1}+1=-4$$

$$\Rightarrow\ \frac{1}{a}+\frac{1}{b}=-4=K$$

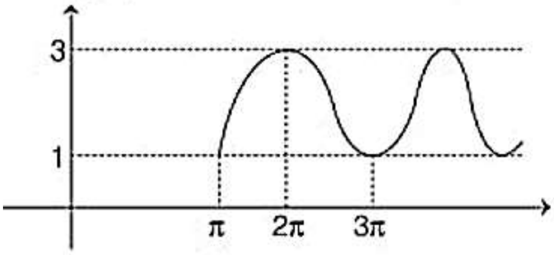
$$\left(\frac{1}{a}+\frac{1}{b}\right)+\left(\frac{4}{K}\right)=-4-1=-5$$

65. (B)

Graph of max $(\sin t : 0\le t\le x)$ in $x\in[0,\pi]$

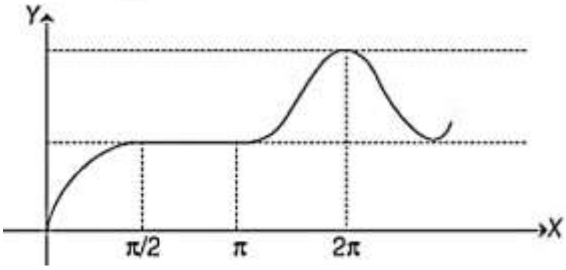


And graph of $2+\cos x$ for $x\in[\pi,\infty]$



So, graph of

$$f(x)=\begin{cases}\max[\sin t: 0\le t\le x], & 0\le x\le \pi\\ 2+\cos x, & x>\pi\end{cases}$$



So, f(x) is differentiable everywhere in $(0,\infty)$

66. (A)

We have, $f:S\to S, S=(0,\infty)$

$f(x+1)=x.f(x)$

$g:S\to R$

$g(x)=\log_e f(x)$

To find $|g''(5)-g''(1)|$

$\Rightarrow g(x+1)=\log_e f(x+1)$

$\Rightarrow g(x+1)=\log[x.f(x)]$

$\Rightarrow g(x+1)=\log x+\log f(x)$

$\Rightarrow g(x+1)=\log x+g(x)$

$\Rightarrow g(x+1)-g(x)=\log x$

$\Rightarrow g'(x+1)-g'(x)=1/x$

$\Rightarrow g''(x+1)-g''(x)=\dfrac{-1}{x^2}$

$x=1, g''(2)-g''(1)=-1$ ....(i)

$x=2, g''(3)-g''(2)=-1/4$ ....(ii)

$x=3, g''(4)-g''(3)=-1/9$ ....(iii)

$x=4, g''(5)-g''(4)=-1/16$ ....(iv)

Adding Eqs. (i), (ii), (iii) and (iv),

$g''(5)-g''(1)=-1-\dfrac{1}{4}-\dfrac{1}{9}-\dfrac{1}{16}$

$= -\left(\frac{144+36+16+9}{144}\right)$

$= \frac{-205}{144}$

So, $\left|g''(5) - g''(1)\right| = \frac{205}{144}$

67. (D)

Given, $\left|f(x) - f(y)\right| \le |x-y|^2$

$\Rightarrow \frac{\left|f(x)-f(y)\right|}{|x-y|} \le |x-y|$

Now, taking the limit,

$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right| \le \lim_{x\to y}|x-y|$

$\Rightarrow |f'(y)| \le 0$ [using the defination of $f'(y)$]

$\Rightarrow f'(y) = 0$ [since, modulus value can never be less than 0]

On integrating it, we get

$f(y) = c$ (constant)

Given, $f(0) = 1$ gives c = 1

$\therefore f(y) = 1 \forall y \in R$

From given options, $f(x) > 0 \forall x \in R$ is satisfied only.

Hence, answer will be option (D).

68. (D)

It is given that

$y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1+\tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}},\ \alpha \in \left(\frac{3\pi}{4}, \pi\right)$

$\Rightarrow y(\alpha) = \sqrt{2\cot\alpha\left(\frac{\tan^2\alpha+1}{1+\tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}$

$= \sqrt{2\cot\alpha + \operatorname{cosec}^2\alpha}$

$= \sqrt{2\cot\alpha + 1 + \cot^2\alpha}$

$= \sqrt{(1+\cot\alpha)^2} = |1+\cot\alpha|$

$\because \cot\alpha \in (-\infty, -1)$, for $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$

$\therefore y(\alpha) = -(1+\cot\alpha)$ $\left[\because |x| = -x, \text{ for } x < 0\right]$

$\therefore \frac{dy}{d\alpha} = -\left(0 - \operatorname{cosec}^2\alpha\right) = \operatorname{cosec}^2\alpha$

So, $\left.\frac{dy}{d\alpha}\right|_{\alpha=\frac{5\pi}{6}} = \operatorname{cosec}^2\left(\frac{5\pi}{6}\right)$

$= \operatorname{cosec}^2\left(\pi - \frac{\pi}{6}\right) = \operatorname{cosec}^2\left(\frac{\pi}{6}\right) = (2)^2 = 4$

69. (D)

$f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3$

$\Rightarrow f'(x) = \cos x\left(12\sin^3 x + 30\sin^2 x + 12\sin x\right)$

$\Rightarrow f'(x) = 6\sin x\cos x\left(2\sin^2 x + 5\sin x + 2\right)$

$\Rightarrow f'(x) = 3\sin 2x(2\sin x + 1)(\sin x + 2)$

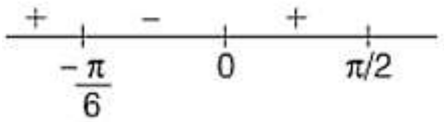
$f'(x) = 0$

$\sin 2x = 0$ or $2\sin x + 1 = 0$ $[\therefore \sin x \neq -2]$

$\Rightarrow x = 0$ or $x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

As, $x \in \left[\frac{-\pi}{6}, \frac{\pi}{2}\right]$

$x = 0, \frac{-\pi}{6}$



So, f(x) is increasing in the interval

$x \in \left(0, \frac{\pi}{2}\right)$

And f(x) is decreasing in the interval

$x \in \left(-\frac{\pi}{6}, 0\right)$

70. (D)

Here, $f(0) = 2 = g(1), g(0) = 0$ and $f(1) = 6$

$\because$ f and g are differentiable in (1, 0).

Let $h(x) = f(x) - 2g(x)$

$h(0) = f(0) - 2g(0)$

Now, $h(1) = f(1) - 2g(1) = 6 - 2(2)$

$h(1) = 2, h(0) = h(1) = 2$

Hence, using Rolle's theorem,

There exists $c \in]0,1[$, such that

$h'(c) = 0$

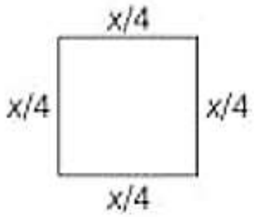
$\Rightarrow f'(c) - 2g'(c) = 0$, for some $c \in]0,1[$

$\Rightarrow f'(c) = 2g'(c)$

71. (D)

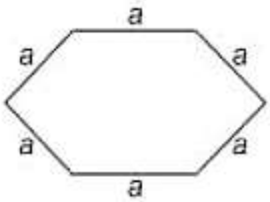
Let two pieces of wire one of length x and other of the length 20 – x.

Wire of length x is made into a square.



$\therefore$ Area of square $= \left(\frac{x}{4}\right)^2 = A_S$ (let)

Wire of length (20 – x) is made into a regular hexagon.



Area of hexagon $= 6 \times \frac{\sqrt{3}}{4} a^2$ [Let]

$A_H = \frac{3\sqrt{3}}{2}\left(\frac{20-x}{6}\right)^2 \quad \left[\because a = \frac{20-x}{6}\right]$

Sum of both area

$A = A_S + A_H = \frac{x^2}{16} + \frac{\sqrt{3}}{24}(20-x)^2$

$\frac{dA}{dx} = \frac{x}{8} - \frac{\sqrt{3}}{12}(20-x)$

$= \frac{3x - 40\sqrt{3} + 2\sqrt{3}x}{24}$

$\frac{dA}{dx} = 0 \Rightarrow x = \frac{40\sqrt{3}}{3+2\sqrt{3}} = \frac{40}{\sqrt{3}+2} = 40\left(2-\sqrt{3}\right)$

$\frac{d^2A}{dx^2} = \frac{3+2\sqrt{3}}{24} > 0$

$\Rightarrow$ Area will be minimum, when

$x = 40\left(2-\sqrt{3}\right)$

$\therefore$ Side of hexagon $= \frac{20 - 40\left(2-\sqrt{3}\right)}{6}$

$= \frac{20\sqrt{3} - 30}{3}$

$= \frac{10\left(2\sqrt{3}-3\right)}{3} = \frac{10}{2\sqrt{3}+3}$

72. (D)

As the point P(h, k) is the nearest point on the curve $y = x^2 + 7x + 2$, to the line $y = 3x - 3$, so the tangent to the parabola $y = x^2 + 7x + 7$ at point P(h, k) is parallel to the line y = 3x – 3

$\therefore \left.\frac{dy}{dx}\right|_0 = 2h + 7 = 3 \Rightarrow h = -2$ ......(i)

and the point P(h, k) on the curve, so

$k = h^2 + 7h + 2 = (-2)^2 + 7(-2) + 2$

$\Rightarrow k = 4 - 14 + 2 \Rightarrow k = -8$

$\therefore$ Point $P(-2, -8)$

Now, equation of normal to the parabola

$y = x^2 + 7x + 7$ At point $P(-2, -8)$ is

$$y + 8 = \frac{-1}{\left.\frac{dy}{dx}\right|_0}(x + 2)$$

$$= y + 8 = -\frac{1}{3}(x + 2) \Rightarrow x + 3y + 26 = 0$$

73. (D)

Given function,

$f(x) = (1 - \cos^2 x)(\lambda + \sin x)$

$= \sin^2 x(\lambda + \sin x)$

$\therefore f'(x) = \sin 2x(\lambda + \sin x) + \sin^2 x(\cos x)$

$= \sin 2x\left[\lambda + \sin x + \frac{1}{2}\sin x\right]$

$= \sin 2x\left[\lambda + \frac{3}{2}\sin x\right]$

For maxima and minima, as

$f'(x) = 0 \Rightarrow \sin 2x\left(\lambda + \frac{3}{2}\sin x\right) = 0$

So, either as $\sin 2x = 0 \Rightarrow x = 0$ as

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Or $\lambda + \frac{3}{2}\sin x = 0$ as there must exactly

One maxima and exactly one minima, so

$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$.

74. (A)

$$\text{Let } l = \int \frac{(2x - 1)\cos\sqrt{(2x - 1)^2 + 5}}{\sqrt{4x^2 - 4x + 6}}\,dx$$

$$= \int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{(2x-1)^2+5}}dx$$

Putting $(2x-1)^2+5 = z^2$

$\Rightarrow 2(2x-1)\times 2.dx = 2z\,dz$

$\Rightarrow (2x-1)dx = \frac{1}{2}z\,dz$

$\therefore I = \int \frac{\cos z}{z}.\frac{1}{2}z.dz$

$= \frac{1}{2}\int \cos z\,dz = \frac{1}{2}\sin z + c$

$= \frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

Note : You can also substitute

$\sqrt{(2x-1)^2+5} = z$ and then proceed.

75. (A)

Given integral

$I = \int \left(e^{2x}+2e^x-e^{-x}-1\right)e^{\left(e^x+e^{-x}\right)}dx$

Let $e^x = t \Rightarrow dx = \frac{dt}{t}$

So, $I = \int \left(t^2+2t-\frac{1}{t}-1\right)e^{\left(t+\frac{1}{t}\right)}\frac{dt}{t}$

$= \int \left(t+2-\frac{1}{t^2}-\frac{1}{t}\right)e^{\left(t+\frac{1}{t}\right)}dt$

$= \int \left[\left(t-\frac{1}{t}\right)+\left(1-\frac{1}{t^2}\right)+1\right]e^{\left(t+\frac{1}{t}\right)}dt$

$= \int \left[(1+t)+\left(1-\frac{1}{t^2}\right)+1\right]e^{\left(t+\frac{1}{t}\right)}dt$

$= \int (1+t)(1-1/t^2)e^{\left(t+\frac{1}{t}\right)}dt + \int e^{\left(t+\frac{1}{t}\right)}dt$

$= (1+t)e^{\left(t+\frac{1}{t}\right)} - \int 1.e^{\left(t+\frac{1}{t}\right)}dt + \int e^{\left(t+\frac{1}{t}\right)}dt$

$= (1+t)e^{\left(t+\frac{1}{t}\right)} + c = \left(1+e^x\right)e^{\left(e^x+e^{-x}\right)}+c$

$= g(x)e^{\left(e^x+e^{-x}\right)}+c$ [Given]

$\therefore\ g(x) = 1+e^x$

$\Rightarrow\ g(0) = 1+1 = 2$

76. (A)

Let $y=\lim_{n\to\infty}\left(U_n\right)^{\frac{-4}{n^2}}$

$y=\lim_{n\to\infty}$

$$\left[\left(1+\frac{1}{n^2}\right)^{\frac{-4}{n^2}}\left(1+\frac{2^2}{n^2}\right)^{\frac{-4}{n^2}.2}\left(1+\frac{3^2}{n^2}\right)^{\frac{-4}{n^2}.3}.....\right]$$

Taking log on both sides, we get

$$\ln y=\lim_{n\to\infty}\sum_{n=1}^{n}\left[\frac{-4}{n^2}.r\ln\left(1+\frac{r^2}{n^2}\right)\right]$$

Now, replace $\lim_{x\to\infty}\ \Sigma\to\int$

$\frac{1}{n}\to x,\frac{1}{n}\to dx$

Lower limit = 0

Upper limit = 1

$\therefore \ln y=\int_0^1 -4x\ln\left(1+x^2\right)dx$

Let $1+x^2=t$

$\Rightarrow xdx=\frac{dt}{2}$

When $x\to 0, t\to 1$

And $x\to 1, t\to 2$

$\therefore \ln y=\int_1^2 -2\ln t\,dt$

$=-2\left(t\ln t-t\right)_1^2$

$=-2\left(2\ln 2-2+1\right)$

$=-2\left(2\ln 2-1\right)$

$\Rightarrow \ln y=\ln\frac{1}{16}+2$

$\Rightarrow y=\frac{1}{16}e^2$

77. (C)

$$\int_{-1}^{1}\log_e\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$$

$$\int_{-a}^{a}f\left(x\right)dx=2\int_0^a f\left(x\right)dx,\ \text{if } f\left(-x\right)=f\left(x\right)$$

So, $2\int_0^1\log_e\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$

$$I=2\int_0^1\log_e\left(\sqrt{1-x}+\sqrt{1+x}\right).1dx$$

$$\Rightarrow \frac{1}{2}=\left[\log_e\left(\sqrt{1-x}+\sqrt{1+x}\right).x\right]_0^1-\int_0^1 \frac{\frac{1}{2\sqrt{1-x}}-\frac{1}{2\sqrt{1-x}}}{\sqrt{1-x}+\sqrt{1+x}}.x\,dx$$

$$\frac{1}{2}=\log_e\sqrt{2}-\frac{1}{2}\int_0^1\left(\frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}\right)\frac{x}{\sqrt{1-x^2}}dx$$

$$\Rightarrow \frac{1}{2}=\log_e\sqrt{2}-\frac{1}{2}\int_0^1\left(\frac{(1-x)+(1+x)-2\sqrt{1-x^2}}{(1-x)-(1+x)}\right)\frac{x}{\sqrt{1-x^2}}dx$$

$$\Rightarrow \frac{1}{2}=\log_e\sqrt{2}-\frac{1}{2}\int_0^1\frac{2\left(1-\sqrt{1-x^2}\right)}{-2x}.\frac{x}{\sqrt{1-x^2}}dx$$

$$\Rightarrow \frac{1}{2}=\log_e\sqrt{2}+\frac{1}{2}\int_0^1\left(\frac{1}{\sqrt{1-x^2}}-1\right)dx$$

$$\Rightarrow \frac{1}{2}=\log_e\sqrt{2}+\frac{1}{2}\left[\sin^{-1}x-x\right]_0^1$$

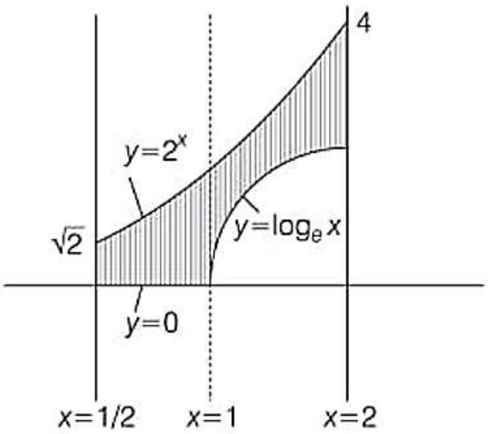
$$\Rightarrow \frac{1}{2}=\log_e\sqrt{2}+\frac{1}{2}\left(\frac{\pi}{2}-1\right)$$

$$\therefore\ l=\log_e 2+\frac{\pi}{2}-1$$

78. (B)

$R=\left\{(x,y)\right\}:\max\left\{0,\log_e x\right\}\le y\le 2^x,$

$\frac{1}{2}\le x\le 2$



$$\max\left\{0,\log_e x\right\}=\begin{cases}0 & , \frac{1}{2}\le x<1\\ \log_e x & , 1\le x\le 2\end{cases}$$

$$\text{Area }=\int_{\frac{1}{2}}^{1}2^x\,dx+\int_1^2\left(2^x-\log_e x\right)dx$$

$=\left[\frac{2^x}{\log_e 2}\right]_{\frac{1}{2}}^{1}+\left[\frac{2^x}{\log_e 2}\right]_{1}^{1}-\left[x\log_e x-x\right]_1^2$

$=\frac{2-\sqrt{2}}{\log_e 2}+\frac{4-2}{\log_e 2}-\left[\left(2\log_e 2-2\right)-\left(0-1\right)\right]$

$=\frac{4-\sqrt{2}}{\log_e 2}-2\log_e 2+1$

On comparing with expression,

$\alpha\left(\log_e 2\right)^{-1}+\beta\log_e 2+\gamma$

$\alpha=4-\sqrt{2},\beta=-2,\gamma=1$

$\therefore\left(\alpha+\beta-2\gamma\right)^2=\left(4-\sqrt{2}-2-2\right)^2=2$

79. (C)

$\cos ec^2x\,dy+2dx=\left(1+y\cos 2x\right)\cos ec^2x\,dx$

Divide L H S and R H S by $\cos ec^2x\,dx$,

$\frac{dy}{dx}+\left(\frac{2}{\cos ec^2x}\right)=\left(1+y\cos 2x\right)$

$\Rightarrow\ \frac{dy}{dx}+2\sin^2 x-y\cos 2x=1$

$\Rightarrow\ \frac{dy}{dx}-y\cos 2x=1-2\sin^2 x$

This is the form of

$\frac{dy}{dx}+py=q$

$lF=e^{\int P\,dx}$

Here, $P=-\cos 2x$

$e^{\int -\cos 2x}=e^{\left(\frac{-1}{2}\right)\sin 2x}$

So, $lF.y=\int q.lF\,dx$

$\Rightarrow\ ye^{\frac{-1}{2}\sin 2x}=\int e^{\frac{-1}{2}\sin 2x}\left(1-2\sin^2 x\right)dx$

$=\int e^{\frac{-1}{2}\sin 2x}\cos 2x\,dx$

Let $\frac{-1}{2}\sin 2x=t$

$\Rightarrow\ \frac{2}{-2}\cos 2x\,dx=dt$

$\Rightarrow\ \cos 2x\,dx=-dt$

$\Rightarrow\ ye^{\frac{-1}{2}\sin 2x}=e^{\frac{-\sin 2x}{2}}+C$

Now, $y\left(\frac{\pi}{4}\right)=0$

$\Rightarrow\ 0=-e^{\frac{-1}{2}}+C\quad C=\frac{1}{\sqrt{e}}$

$\therefore\ y(0)\Rightarrow y.1=-1+\frac{1}{\sqrt{e}}$

$\Rightarrow\ y+1=\frac{1}{\sqrt{e}}$

$\Rightarrow\ (y+1)^2=\frac{1}{e}$

80. (D)

Given differential equation is

$\frac{dy}{dx}-\frac{y+3x}{\log_e(y+3x)}+3=0$

Put, $y+3x=t\Rightarrow\frac{dy}{dx}+3=\frac{dt}{dx}$

$\therefore\ \frac{dt}{dx}-\frac{t}{\log_e(t)}=0\ \Rightarrow\ \int\frac{\log(t)}{t}dt=\int dx$

$\Rightarrow\ \frac{1}{2}(\log_e t)^2=x+c'$

$\Rightarrow\ x-\frac{1}{2}(\log_e(y+3x))^2=C,\ \text{where } c'=-C.$

81. (3)

Given differential equation for $x\geq 0$ is

$(x+1)dy=\left((x+1)^2+y-3\right)dx$

$\Rightarrow\frac{dy}{dx}-\frac{1}{x+1}y=(x+1)-\frac{3}{x+1}$

$\because$ The above differentiable equation is in the form of linear, so

$\text{I.F.}=e^{-\int\frac{dx}{1+x}}=\frac{1}{1+x}$

$\therefore$ Solution of the differential equation is

$\frac{y}{1+x}=\int\left[(x+1)-\frac{3}{x+1}\right]\frac{dx}{1+x}$

$\Rightarrow\ \frac{y}{1+x}=\int\left(1-\frac{3}{(x+1)^2}\right)dx$

$\Rightarrow\ \frac{y}{1+x}=x+\frac{3}{x+1}+C\qquad ....(i)$

82. (1)

Given, $(2xy^2-y)dx+xdy=0$

$\Rightarrow\ \frac{dy}{dx}-\frac{y}{x}=-2y^2$

$\Rightarrow \frac{-1}{y^2}\frac{dy}{dx}+\frac{1}{xy}=2$ (i) [Divide by y^2]

Let $\frac{1}{y}=v$, then $-\frac{1}{y^2}.\frac{dy}{dx}=\frac{dv}{dx}$, putting in Eq. (1)

$\frac{dv}{dx}+v\left(\frac{1}{x}\right)=2$ (this is a linear form)

Now, integrating factor (lF)

$=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

$\therefore (lF)v=\int 2.(lF)dx=\int 2xdx=2\frac{x^2}{2}+C$

$\therefore (lF)v=x^2+C$

Put $v=\frac{1}{y}$, this gives

$x^2+c=\frac{x}{y}$

Now, first find point of intersection of lines

$2x-3y=1$ and $3x=-2y+8$ by elimination method, we get x = 2, y = 1

$\therefore$ The curve $x^2+c=\frac{1}{y}$ passes through (2, 1)

Put x = 2, y = 1, we get c = - 2

$\frac{x}{y}=x^2-2$

Or $y=\frac{x}{x^2-2}$

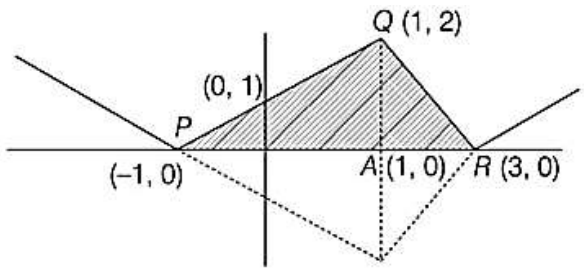
Put x = 1, we get $y(1)=\frac{1}{1-2}=-1$

$\therefore |y(1)|=1$

83. (4)

Given, $y=||x-1|-2|$

Required area us area of ΔPQR.



Area $=\frac{1}{2}\times(\text{Base})\times(\text{Height})$

$=\frac{1}{2}\times(PR)\times(AQ)$

84. (1)

Given, $I_n = \int_1^e x^{19} \left(\log|x|\right)^n dx$

$\Rightarrow I_n = \left[\frac{x^{20}}{20}\left(\ln|x|^n\right)\right]_1^e - \int_1^e n.\frac{\left(\ln|x|\right)^{n-1}}{x}.\frac{x^{20}}{20}dx$ (using integration by parts)

$\Rightarrow \ln = \frac{e^{20}}{20} - \frac{n}{20}\int_1^e \left(\ln|x|\right)^{n-1}.x^{19}dx$

$\Rightarrow \ln = \frac{e^{20}}{20} - \frac{n}{20}.I_{n-1}$

$\Rightarrow 20I_n + nI_{n-1} = e^{20}$

Put n = 10 and n = 9, we get

$20I_{10} + 10I_9 = e^{20}$ .....(i)

and $20I_9 + 9I_8 = e^{20}$ .....(ii)

85. (5)

$1 = \int_0^{\pi} \left(\sin^3 x\right) e^{-\sin^2 x} dx$

$= \int_0^{\pi} \left(1-\cos^2 x\right) \sin x \ e^{-\left(1-\cos^2 x\right)} dx$

If $f(x) = f(2a - x)$, then

$\int_0^{2a} f(x) dx = 2\int_0^a f(x) dx$

$1 = 2\int_0^{\pi/2} \left(1-\cos^2 x\right) \sin x \, e^{-\left(1-\cos^2 x\right)} dx$

Let $\cos^2 x = t \Rightarrow -2\cos x \sin x \, dx = dt$

$\Rightarrow \sin x \, dx = -\frac{dt}{2\sqrt{t}}$

$= -2\int_0^1 (1-t) e^{t-1} \frac{dt}{-2\sqrt{t}}$

$= \frac{1}{e}\int_0^1 \frac{(1-t)e^t}{\sqrt{t}} dt$

$= \frac{1}{e}\left\{\int_0^1 \frac{1}{\sqrt{t}} e^t \, dt - \int_0^1 \sqrt{t} \, e^t \, dt\right\}$

$= \frac{1}{e}\left\{\left(2\sqrt{t} \, e^t\right)_0^1 - 2\int_0^1 \sqrt{t} \, e^t \, dt - \int_0^1 \sqrt{t} \, e^t \, dt\right\}$

$= \frac{1}{e}\left\{2e - 3\int_0^1 \sqrt{t} \, e^t \, dt\right\}$

$= 2 - \frac{3}{e}\int_0^1 \sqrt{t} \, e^t \, dt$

On comparing, $\alpha - \frac{\beta}{e}\int_0^1 \sqrt{t} \ e^t \, dt$

$\Rightarrow \alpha = 2$ and $\beta = 3$

$\therefore \alpha + \beta = 2 + 3 = 5$

86. (3)
Let a cubic polynomial
$f(x) = ax^3 + bx^2 + cx + d$
$\because\ f(-1) = 10$
$\Rightarrow -a + b - c + d = 10$ ....(i)
$\because\ f(1) = -6$
$\Rightarrow a + b + c + d = -6$ .....(ii)
$\because\ f'(-1) = 0$
$\Rightarrow 3a - 2b + c = 0$ .....(iii)
$\because\ f''(1) = 0$
$\Rightarrow 6a + 2b = 0$

87. (4)
If the curves cut at right angle, then product of slopes will be – 1.
First curve $x = y^4$
Differentiate it, we get
$1 = 4y^3 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{4y^3}$
Slope of first curve $(m_1) = \frac{1}{4y_1^3}$ [at point (x, y)]
Second curve xy = k
Differentiate it, $0 = x\frac{dy}{dx} + y$
$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$
Slope of second curve $(m_2) = \frac{-y_1}{x_1}$ $[\text{at}(x_1, y_1)]$
$\Rightarrow m_1.m_2 = -1$
$\Rightarrow \frac{1}{4y_1^3}\left(\frac{-y_1}{x_1}\right) = -1$
$\Rightarrow \frac{-1}{4y_1^2 x_1} = -1$ $[\text{using } x_1 = y_1^4]$
$\Rightarrow \frac{-1}{4(y_1)^6} = -1$
$\Rightarrow y_1^6 = \frac{1}{4}$
Also, $x_1 y_1 = k$, using $x_1 = y_1^4$, we get $k = y_1^5$ or $k^6 = (y_1)^{30}$

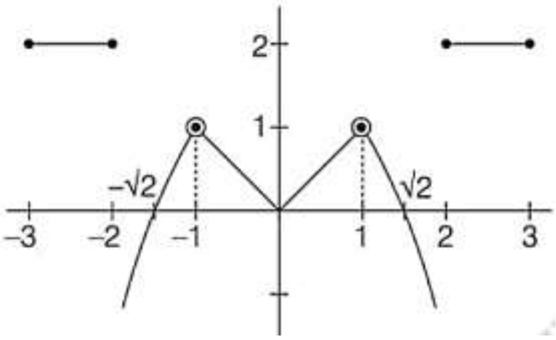
88. (2)
Given,
$f(x) = |2x+1| - 3|x+2| + |x^2 + x - 2|$

$= |2x+1| - 3|x+2| + |x+2| \times |x-1|$

Here, critical points are $x = \frac{-1}{2}, 1$

89. (5)

For this particular problem, try to draw graph in the region $(-3, 3)$, it will be as follows,



Thus, points of discontinuity will be at – 2, 2 because the curve breaks at these points and at – 1, 0, 1 because curve has sharp points.
Point of discontinuity are – 2, - 1, 0, 1, 2 i.e. 5 points.

90. (1)

Given, $\lim_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$

$= \tan\left(\lim_{n\to\infty} \sum_{r=1}^{n}\left[\tan^{-1}(r+1) - \tan^{-1} r\right]\right)$

$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1) - \frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1$

Hence, the required value is 1.