P& C Ex. 1(A)

18. (C)

$\Rightarrow$ _ _ _ _ _ unit's place can be filled in 3 different ways.

$\Rightarrow$ Required number of even numbers = ${}^{3}C_{1} \times (4!) = 72$

19. (C)

$\Rightarrow$ Required number of ways = $5! \times {}^{6}C_{5} \times 5! = 5! \times 6!$

20. (A)

$\Rightarrow$ _ ↓ _ ↓ _ ↓ _ ↓ _ ↓ _

B B B B B

$\Rightarrow$ Number of arrangement of boys = 5!

$\Rightarrow$ Number of possible permutations of girls = ${}^{4}P_{3}$.

$\Rightarrow$ Hence, Required number of arrangements = $5! \times {}^{4}P_{3} = 2880$

21. (C)

Arrangements of | L |AU|G|H | will be 4! i.e. 24

Arrangements of |A|U| will be 2! i.e. 2

Total arrangements = 48.

22. (D)

First digit from left may be selected in 3 ways

last digit may be selected in 2 ways,

other 4 digits may be selected in $5 \times 4 \times 3 \times 2$ ways.

Total number of numbers = $5 \times 4 \times 3 \times 2 \times 3 \times 2$ i.e. 720.

23. (D)

In {0, 1, 2, 3, 4, 5} sum of all the digits is 15 which is divisible by 3.

Now we have to choose 5 numbers out of these 6 given numbers such as their sum is also divisible by 3.

Clearly either 0 or three must be discarded.

In the case when 0 is discarded, using {1, 2, 3, 4, 5} we can form 5! numbers.

In the case when 3 is discarded, using {0, 1, 2, 4, 5} we can form $4 \times 4!$ numbers.

Total possible numbers are 216.

24. (A)

1, 3 & 5 we can put at 2[nd], 4[rd]& 6[th] place in 3! ways.
Now 1[st] place can be filled by 2 or 4 and 3[rd]& 5[th] places can be filled in 2! Ways.
Hence number of all such numbers is $3!\times 2\times 2!$ i.e. 24.

25. (B)

Keeping those three men as one entity we have 4! arrangements.
These three men can be arranged in 3! ways within themselves.
Hence all possible arrangements = $4!\times 3!$ i.e. 144.

26. (B)

We have to keep all the consonants together.

All the consonants as one entity and the 4 vowels may be arranged in $\frac{5!}{2!2!}$

Consonants with each other can be arranged in $\frac{5!}{2!}$ ways.

Hence all possible arrangements are $\frac{5!}{2!2!}\times\frac{5!}{2!}$ i.e. 1800.

27. (A)

For the number to be divisible by four the last two digits must be 12, 16, 32, 36, 52 or 56.
First two digits can be selected from remaining three numbers in 3 ways.
Hence number of such numbers = $6\times 3\times 2$ i.e. 36.

28. (C)

Each of the 9 digits can be selected in 9 ways.
Hence total number of such numbers is 9^5.

29. (A)

Let the sum of any five digits be N.
Now N can be any number from 1 to 45.
In each case the remaining digit can be chosen in two particular ways only so as sum of digits is divisible by 5.
{e.g. If N = 42, then remaining digit can be 3 or 8; if N = 23, then remaining digit is 2 or 7;…etc.}
Hence total possible number of such numbers is $9\times 10^4\times 2$ i.e. 180000.

30. (A)

Digits must be selected from {1, 3, 5, 7, 9}.

As the number must contain each of these so we need to permute 6 objects containing two identical objects and rest all distinct which can be done in $\frac{6!}{2}$ ways.

Also two identical digits can be selected in 5 ways.

Hence total number of such numbers is $5\times\frac{6!}{2}$.

31. (B)

For sum of digits to be even : we can choose first four digits in general in 9×10^3 ways
Now if sum of first four digits is even, then last digit must be even
and if sum of first four digits is odd, then last digit must be odd
Hence in any possibility last digit can be chosen in 5 ways.
Number of all such numbers = $9\times10^3\times5$ or 45000.

32. (A)

D, P, M, L can be arranged in 4! Ways.
Two pairs of vowels can be chosen in 2 ways i.e. {EE, AA}, {EA, AE}
Now these pairs can be put in 5 gaps between D, P, M, L in $2\times{}^5C_2\times2!$ ways.
Total number of arrangements = ${}^5C_2\times2\times2!\times4!$ i.e. 960.

33. (D)

$\Rightarrow$Required number = $(10-1)!\times3!=9!\times3!$

34. (B)

$\Rightarrow$Number of arrangement of men = (7 – 1)! = 6!

$\Rightarrow$Number of permutations of woman on the gaps created among man = 7!

$\Rightarrow$Hence, required number = $6!\times7!$

35. (C)

Number of arrangements of 8 boys around a circular table = 7!.
Number of ways to put the two particular boys in 8 gaps = ${}^8C_2\times2!$.
Hence all possible arrangements = $7!\times{}^8C_2\times2!$ i.e. 7 (8!).

36. (A)

Seats for the two specific persons can be chosen in 5 ways on any of the 2 sides between the master & mistress i.e. 10 ways.
Rest of the 10 places can be filled in $10!\times2$ ways (differentiating between left and right of master and mistress).
Hence total number of ways = $20\times10!$.

37. (C)

A & B can be seated in 1 way on two identical tables.
Rest of the 6 persons can be seated in 6! Ways.
Hence total ways to seat 8 persons = 720.

38. (B)

A & B along with any other two persons can sit on the straight table in ${}^{6}C_{2} \times 4!$ ways.
Remaining four persons can sit around the circular table in 3! Ways.
Total number of arrangements = ${}^{6}C_{2} \times 4! \times 3!$ i.e. 2160.

39. (D)

$\Rightarrow$Number of ways = $({}^{2}C_{1})^{10} = 1024$.

40. (D)

$\Rightarrow$Required number of ways = ${}^{11}C_{3} = 165$.

41. (D)

$\Rightarrow$Number of ways of appointing clerks = ${}^{20}C_{16} = 4845$.

42. (B)

$\Rightarrow$Number of ways ${}^{11}C_{4} = 330$.

43. (D)

$\Rightarrow$Required number of words = $\left({}^{5}C_{3} \times {}^{4}C_{2}\right) \times 5!$

44. (C)

$\Rightarrow$Number of ways of selection = ${}^{15}C_{1} \times {}^{10}C_{1} = 150$

45. (C)

$\Rightarrow {}^{n}C_{2} = 66$

$\Rightarrow n = 13$.

46. (A)

$\Rightarrow$Number of ways = ${}^{8}P_{5} = 6720$.

47. (B)

$\Rightarrow$Number of Greeting card exchanged = ${}^{20}C_{2} \times 2$

48. (C)

$\Rightarrow$The number of times he will go to the garden = ${}^{8}C_{3} = 56$.

49. (B)

$\Rightarrow$The number of ways person can make selection of fruits

$= (4 + 1)(5 + 1)(6 + 1) - 1 = 209$.

50. (C)

$\Rightarrow$Number of ways of selection = (10 + 1) (9 + 1) (7 + 1) – 1 = 879.

51. (B)

$\Rightarrow$Required number of ways = $2^6 - 1 = 63$.

52. (D)

$\Rightarrow$Number of ways in which a student can fail to get all answers correct = $4^3 - 1 = 63$.

53. (B)

To form a rectangle(including squares) we may choose any two vertical lines and any two horizontal lines.

Number of all possible rectangles, r = ${}^9C_2 \times {}^9C_2 = 1296$.

Number of squares of side length 1 = 8×8

Number of squares of side length 2 = 7×7

Number of squares of side length 3 = 6×6

Going by this pattern total number of squares, s = 1 + 4 + 9 + … + 64 = 204.

Hence $\frac{s}{r} = \frac{17}{108}$.

54. (A)

Number of ways to select any 3, 4 or all 5 out of first 5 questions and then to select 7, 6 or 5, respectively, out of the remaining 8 questions = ${}^5C_3 \times {}^8C_7 + {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$ i.e. 276.

55. (D)

To form a triangle we need to chose three non – collinear point.

We can chose 3 points in ${}^{18}C_3$ ways.

As each side of the triangle contains 6 points so we can choose all three points on any one side in $3\times{}^6C_3$ ways.

Total numbers of triangles = ${}^{18}C_3 - 3\times{}^6C_3$ i.e. 711.

56. (A)

$\Rightarrow$Number of ways $= 4\times4\times4\times4\times4\times4 = 4^6$

57. (B)

$\Rightarrow$Hence, required number = 3^4.

58. (A)

$\Rightarrow$Number of ways = $4^3 - 1 = 63$

59. (C)

$\Rightarrow$ Number of ways = 5^4, as each parcel has 5 options.

60. (C)

$\Rightarrow$ Required number of ways is the number of permutations of given 4 grades, taken 3 at a time. i.e., ${}^4P_3$.

61. (B)

$\Rightarrow$ The number of ways in which husbands can be selected = ${}^7C_2$.

$\Rightarrow$ The number of ways in which wives can be selected after selecting husbands = ${}^5C_2$.

$\Rightarrow$ Also, a set of two men and two women can play 2 games.

$\Rightarrow$ Hence, Required number of ways = ${}^7C_2 \times {}^5C_2 \times 2 = 420$.

62. (A)

$\Rightarrow$ Required number is number of circular permutations such that clockwise and anti-clockwise arrangements = $\frac{(5-1)!}{2} = \frac{4!}{2}$

63. (D)

To distribute books in given manner we need to select 5 persons and given one book each to these 5 persons.

Number of ways to do so will be ${}^{10}C_5 \times 5!$.

64. (C)

Sum of internal angles of an n sided polygon = $(n-2) \times 180^\circ$.

Hence $150^\circ \times n = (n-2) \times 180^\circ$ gives n = 12.

Now a diagonal will be formed by joining two of these points but not adjacent two vertices.

Number of diagonals will be ${}^{12}C_2 - 12$ i.e. 54.

65. (A)

Number of ways to choose 4 prizes out of 9 = ${}^9C_4$.

Rest of the 5 prizes can be distributed to remaining 4 students in 4^5 ways.

Total number of ways to distribute 9 prizes among 5 students = ${}^9C_4 \times 4^5$.

66. (C)

Number of ways in which m distinct objects can be permuted at n distinct places = ${}^mP_n$.

67. (D)

If one or more pair is there in the selection, then we can select shoes in the following manner
(i) Selection having at least one pair : can be formed by choosing one pair in 5 ways and then choosing 2 more shoes from remaining 8 in ${}^{8}C_2$ ways i.e. $5\times{}^{8}C_2$ or 140 ways.

(ii) Selection having two pairs : can be formed by choosing two pairs in ${}^{5}C_2$ or 10 ways.

Hence a selection having one or more pairs can be formed in 140 – 10 = 130 ways.

Number of all possible selections = ${}^{10}C_4$ or 210 ways.

Thus number of ways to make a selection having no pairs = 210 – 130 = 80.

68. (B)

Only possible way to distribute in given manner is in groups of 1, 4 & 2.

Hence number of ways to distribute $= \frac{7!}{1!\,2!\,4!}\times 3!$ or 630.

69. (A)

Number of ways to distribute m + n + p objects in three groups containing m, n & p objects is given by $\frac{(m+n+p)!}{m!n!p!}$.

Required number of ways $= \frac{10!}{2!3!5!}$.

70. (A)

Number of ways to distribute 'n' identical objects in 'r' distinct groups such that no group is empty $= {}^{n-1}C_{r-1}$.

Hence required number of ways $= {}^{10-1}C_{6-1}$ i.e. 126.

71. (C)

For each question we can have 3 choices namely (i) not selecting, (ii) selecting alternative 1 & (ii) selecting alternative 2.

Hence choices for 10 questions = 3^{10}.

But all the question cant be rejected hence required number of ways = $3^{10} - 1$.

72. (C)

Number of ways to divide 'nr' distinct objects into r groups containing equal number of objects

$= \frac{(nr)!}{r!\times(n!)^r}$.

Hence 52 cards can be distributed into 4 groups of 13 cards each $= \frac{(52)!}{4!\times(13!)^4}$.

73. (B)

Let the subset A contain r elements, then subset B must contain remaining n – r elements so as $A \cap B = \Phi$ & $A \cup B = S$.

Hence number of ordered pairs (A, B) = $\sum_{r=0}^{10} {}^{10}C_r$ i.e. 2^{10}

Total number of unordered pairs of subsets of S = $\frac{2^{10}}{2}$ i.e. 2^9.

74. (B)

Number of positive integers of r–digits not having any digit as 1 = $8 \times 9^{r-1}$.

Number of all the positive integers of r–digits = $9 \times 10^{r-1}$.

Number of numbers having at least one digit as 1 = $9 \times 10^{r-1} - 8 \times 9^{r-1}$.

Hence number of all the numbers up to 9-digit numbers = $\sum_{r=1}^{9} \left(9 \times 10^{r-1} - 8 \times 9^{r-1}\right)$ i.e. $10^9 - 9^9$.

Alternately

If we form a 9 digit number allowing consecutive 0s at places form beginning also, then r consecutive zeros from beginning will account for a number of (9 – r) digits.

Hence all numbers from 1 digit till 9 digit = 10^9.

Hence all numbers from 1 digit till 9 digit not including 1 = 9^9.

Number of numbers having at least one digit as 1 = $10^9 - 9^9$.

75. (C)

Number of outcomes on each die = 6.

Hence all the four dice will show same number I 6 ways.

76. (C)

Each suit can be arranged in 13! Ways and the suits can be arranged with each other in 4! Ways.

Hence total number of arrangements = $(13!)^4 \times 4!$.

77. (C)

12 dice can show any outcome in 12! Ways if all the outcomes could be distinct. Now as there are 2 each 1, 2, 3, 4, 5, 6 hence required number of ways = $\frac{12!}{(2!)^6}$.

78. (C)

If there are n number of B used, then required number of numbers will be $\frac{(a+n)!}{a!n!}$.

Hence total possible number of numbers where $0 \le n \le b$ will be $\sum_{n=0}^{b} \frac{(a+n)!}{a!n!} = \sum_{n=0}^{b} {}^{a+n}C_n$

i.e. ${}^{a+b+1}C_b$.

79. (A)

AAAAA, BBB, D, EE & F can be arranged in $\frac{12!}{5!\,3!\,2!}$ ways.

Now there will be 13 gaps where we can put 3 C in ${}^{13}C_3$ ways.

Hence total number of arrangements = ${}^{13}C_3 \times \frac{12!}{5!\,3!\,2!}$.

Ex. 1(B)

1. (A)

$$\Rightarrow n \cdot \frac{(n-1)!}{(n-r)!} = \frac{n!}{(n-r)!} = {}^{n}P_{r}$$

2. (D)

$$\Rightarrow \frac{(k+5)!}{4!} = \frac{11}{2}(k-1)\frac{(k+3)!}{3!}$$

$$\Rightarrow \frac{(k+5)(k+4)}{4} = \frac{11}{2}(k-1)$$

$\Rightarrow k = 6$ or 7.

3. (D)

$\Rightarrow n^2 - n = 2 + 10$

$\Rightarrow n = 4$ or -3

4. (A)

$$\Rightarrow {}^{n}C_3 + {}^{n}C_4 > {}^{n+1}C_3 \Rightarrow {}^{n+1}C_4 > {}^{n+1}C_3 \Rightarrow \left|\frac{n+1}{2} - 4\right| < \left|\frac{n+1}{2} - 3\right|$$

$$\Rightarrow |n-7| < |n-5|$$

$$\Rightarrow n > 6$$

5. (B)

$$\Rightarrow \text{Number of permutations} = \frac{(4+3+2)!}{2!4!3!} = \frac{9!}{2!4!3!}$$

6. (C)

$$\Rightarrow \begin{array}{ccccccccc} - & \underset{\substack{\downarrow \\ E}}{-} & - & \underset{\substack{\downarrow \\ E}}{-} & - & \underset{\substack{\downarrow \\ E}}{-} & - & \underset{\substack{\downarrow \\ E}}{-} & - \end{array}$$

$\Rightarrow$ Odd digits can occupy 2^{nd}. 4^{th}, 6^{th} & 8^{th} places.

$$\Rightarrow \text{The number of different nine-digit numbers} = \frac{5!}{2!3!} \times \frac{4!}{2!2!} = 60$$

7. (A)

$\Rightarrow \underline{C}\ \_\ \_\ \_\ \_\ \_\ \_\ \underline{Y}$

$\Rightarrow$ Number of words = 6!.

8. (B)

$\Rightarrow \_\ \_\ \underline{L}\ \_\ \_$

$\Rightarrow$ Required number of ways = 4!

9. (B)

$\Rightarrow$ Number of arrangement of boys = (7!)

$\Rightarrow$ Number of gaps created = 8.

$\Rightarrow$ Hence, Required number of different ways = $7! \times {}^8P_3$.

10. (B)

To form natural numbers from 1000 to 9999 having all 4 distinct digits

9 choices	9 choices	8 choices	7 choices

Number of such numbers = $9 \times 9 \times 8 \times 7 = 4536$
Number of all the numbers from 1000 to 9999 = 9000.
Hence number of natural numbers from 1000 to 9999 not having all 4 distinct digits
= 9000 – 4536
= 4464.

11. (A)

Two places to put 2s can be chosen in ${}^7C_2$ ways.
In each of the remaining 5 places we can put 1 or 3 hence these places can be filled in 2^5 ways.
Total number of numbers which can formed = ${}^7C_2 \times 2^5$ i.e. 672.

12. (C)

Four consonants may be chosen in ${}^7C_4$ ways.

Two vowels may be chosen in ${}^4C_2$ ways.

The selected 6 letters may be arranged in 6! ways.

Total number of words = ${}^7C_4 \times {}^4C_2 \times 6!$ i.e. 151200.

13. (D)

Case I – The required numbers will be of form 57N, where N must be an even number.
Total number of such numbers = 5.
Case II – If 5 is not considered, then
for first digit from left we have 8 choices (not taking 0 & 5),
for second digit we have 9 choices (not taking 5),
for last digit we have 5 choices (only even numbers).
Total number of such numbers = $8\times9\times5$ i.e. 360.
Hence the number of such numbers = 365.

14. (A)

Flag can be designed by having following choices for six strips.

4 choices	3 choices	3 choices	3 choices	3 choices	3 choices

Hence number of designs = 12×81.

15. (D)

All possible arrangements = 4!
Arrangements in which 'ab' kept together = $3!\times2!$
Arrangements in which 'cd' kept together = $3!\times2!$
Arrangements in which 'ab' as well 'cd' kept together = $2!\times2!\times2!$
Arrangements in which neither 'ab' nor 'cd' are together
$= 4! - 3!\times2! - 3!\times2! + 2!\times2!\times2!$
$= 8$.

16. (B)

Two consecutive digits we can have in two ways i.e. first two or last two.
Now if first two digits are identical, then we can chose these in 9 ways (excluding 0)
and then third digit may be selected in 9 ways (excluding the number already placed in first 2 places).
If last two digits are identical, then we can choose first digit in 9 ways (excluding 0)
and last two digits in 9 ways (excluding the number already placed in first place).
Hence number of all such numbers is $9\times9+9\times9$ i.e. 162.

17. (B)

As the number has 9 distinct digits hence the middle digit must be 5, digits in last four places must be 6,7,8,9 and first four digits must be 1,2,3,4.
Now for first four place we have $4!$ ways.
For the last 4 places we have $4!$ ways.
Hence total number of such numbers is $(4!)^2$.

18. (B)

6 digits can be selected in ${}^{9}C_{6}$.

Now as the digits are to be kept in decreasing order so no arrangement required.

Hence total number of such numbers is 84.

19. (D)

If one or more couple is there in the committee, then we can form the committee in the following manner

(i) committee having at least one couple : can be formed by choosing one couple in 4 ways and then choosing 2 more people from remaining 6 in ${}^{6}C_{2}$ ways i.e. $4\times{}^{6}C_{2}$ or 60 ways.

(ii) committee having two couples : can be formed by choosing two couples in ${}^{4}C_{2}$ or 6 ways.

Hence a committee having one or more couples can be formed in 60 – 6 = 54 ways.

Number of all possible committee = ${}^{8}C_{4}$ or 70 ways.

Thus number of ways to form a committee having no couples = 70 – 54 = 16.

20. (C)

Numbers formed using just 1 or 2 are only 2.

All possible numbers using 1 & 2 only = 2^n.

Hence numbers using at least one 1 & one 2 = $2^n - 2$.

Now $2^n - 2 = 510$ gives n = 9.

21. (A)

Number of ways to chose m objects out of (m + n) objects = ${}^{m+n}C_{m}$

Number of ways to arrange these m and other n on the two tables = $\frac{(m-1)!}{2}\times\frac{(n-1)!}{2}$.

Number of ways of arrangements = ${}^{m+n}C_{m}\times\frac{(m-1)!}{2}\times\frac{(n-1)!}{2}$ or $\frac{(m+n)!}{4mn}$.

22. (A)

Number of ways to arrange balls keeping balls of same color together $3!\times4!$ i.e. $6\times4!$.

Number of all possible arrangements = $\frac{9!}{2!3!}$ i.e. $6\times7!$.

Number of arrangements in which at least one ball of same color is separated from others of same color = $6(7!-4!)$.

23 (B)

Trials needed for first three digits = 2.

If Last digit is 9, then rest three digits can be tried in 9^3 ways.

If last digit is one of 1, 3, 5 or 7, then one of the remaining three is 9(chosen in 3 ways) and other two digits can be chosen in 9^2 ways, hence all possible ways are $4\times3\times9^2$ ways.

Total number of ways to try the number = $2\times\left(9^3+4\times3\times9^2\right)$ or 3402.

24. (A)

The required number is maximum possible number of points in which the diagonals of an nonagon intersect inside the shape.
Now Each selection of four of the points gives three pair of lines out which exactly one pair intersects in the region enclosed within the four points.
Hence number of required points is ${}^{9}C_{4}$ i.e. 126.

25. (D)

Put one of the distinct objects at 1 place in the circle. Now for remaining objects we can use linear permutations.

Hence 'r' identical & (n – r – 1) distinct objects can be arranged in $\frac{(n-1)!}{r!}$ ways.

26. (B)

Put ladies in (n – 1)! Ways.
Now put gentlemen in (n – 1) gaps between ladies in n! ways.
Total number of ways = $(n-1)! \times n!$.

27. (D)

Place for A can be selected in 2×3 ways (leaving corner seats on both the sides) and then place for B can be selected in 6 ways(leaving places adjacent to A and opposite to A).
Place for A can be selected in 4 ways(seats at corners) and then for place for B in 7 ways (leaving place adjacent to A and opposite to A).
All the other people can sit in 8! Ways.
Hence number of seating arrangements = $(2\times3\times6 + 4\times7)\times8!$ i.e. $64\times8!$.

28.

29. (C)

$\Rightarrow$ Total number of different combinations from the letters of word "MISSISSIPPI"
$(M\to1, S\to4, I\to4, P\to2)$ is (1 + 1) (4 + 1) (4 + 1) (2 + 1) – 1 = 149.

30. (A)

$\Rightarrow$ Total number of ways of selecting six coins = coefficient of x^6 in $(x^0 + x^1 + \ldots\ldots\ldots\ldots x^{20})$ $(x^0 + x^1 + \ldots\ldots + x^{10})$ $(x^0 + x^1 + \ldots\ldots x^7)$

$$\Rightarrow \text{coefficient of } x^6 \text{ in } \frac{(x^{21}-1)(x^{11}-1)(x^8-1)}{(x-1)(x-1)(x-1)}$$

$\Rightarrow$ coefficient of x^6 in $(x^{21} – 1)(x^{11} – 1)(x^8 – 1)(1 – x)^{-3}$

$$\Rightarrow {}^{6+3-1}C_{3-1} = {}^{8}C_{2} = \frac{8\times7}{2} = 28$$

31. (B)

Each match can result in 3 ways.

forecast for 5 matches can be made in 3^5 ways in which one forecast is completely correct.

Hence there must be at least 243 people.

32. (D)

Let the numbers to fill be a, b, c, d as shown

a	b
c	d

Now a + b + c + d = 10 & a + d = b + c = k, then k = 5.

Now a + d = 5 = b + c gives choices for (a, d) as (1, 4) & (2, 3).

As a & d are interchangeable hence 4 ways to choose (a, d).

Similarly there are 4 ways to choose (b, c).

But as numbers are all distinct therefore for a given selection of (a, d) we can choose (b, c) in 2 ways only.

altogether a, b, c, d can be chosen in $4\times2=8$ ways.

33. (A)

A student can answer 5 questions in two ways,

(i) one question each from two sections & 3 questions from one section. In this case

number of ways to chose two sections = 3,

number of ways to chose one question out of four from any section = 4,

number of ways to chose three questions out of four from any section = 4.

Hence number of ways to answer five questions = $3\times4^2\times4=192$.

(ii) two questions each from two sections and one question from one section. In this case

number of ways to chose two sections = 3,

number of ways to chose two question out of four from any section = 6,

number of ways to chose one questions out of four from any section = 4.

Hence number of ways to answer five questions = $3\times6^2\times4=432$.

Total number of ways = 192 + 432 = 624.

34. (B)

Any 5 digits from {0, 1, 2,…,9} can be chosen in ${}^{10}C_5$ ways and put in descending order in just one way, hence m = ${}^{10}C_5$.

Any 5 digits from {1, 2, 3,…,9} can be chosen in ${}^{9}C_5$ ways and put in ascending order in just one way, hence n = ${}^{9}C_5$.

Thus m – n = ${}^{10}C_5 - {}^{9}C_5$ or ${}^{9}C_4$.

35. (A)

(i) When A is excluded, to form a triangle we have to choose two points on one line & one point on one line number of triangles = ${}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1$

(ii) When A is included, other than the triangles formed in case (i) we can form more triangles by taking A as one vertex and choosing other two vertices one each on AB & AC.

Hence number of triangles = ${}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1 + {}^{n}C_1 \times {}^{m}C_1$

Ratio of number of triangles in (i) & (ii) = $\dfrac{{}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1}{{}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1 + {}^{n}C_1 \times {}^{m}C_1} = \dfrac{m+n-2}{m+n}$.

36. (C)

9 lines will intersect in ${}^{9}C_2$ points.

9 circles will intersect in $2 \times {}^{9}C_2$ points.

9 lines will intersect with 9 circles in $2 \times {}^{9}C_1 \times {}^{9}C_1$ points.

Total number of points = 270.

37. (D)

a_4	a_3	a_2	a_1

b_4	b_3	b_2	b_1

$a_1 + b_1 \le 9$, $a_2 + b_2 \le 9$, $a_3 + b_3 \le 9$ & $a_4 + b_1 \le 9$, where $a_1, a_2, a_3, b_1, b_2, b_3 \in W$ & $a_4, b_4 \in N$

Now number of solutions of each of the first three equations will be ${}^{9+3-1}C_{3-1}$ or 55 and those of the fourth equation will be ${}^{7+3-1}C_{3-1}$ or 36.

Hence total number of ways = 36×55^3.

38. (C)

Hand shakes by men with each other = ${}^{10}C_2$

Hand shakes by women with each other = ${}^{10}C_2$

Hand shakes by men with women = 9×5.

Total number of hand shakes = 135.

39.

40. (D)

Given $x+1$ divides $P(x)$ hence $P(-1)=0$ i.e. $a+c=2b$.

Now if we chose any two odd numbers or any two even numbers from the set of given numbers as 'a' & 'b', then c gets selected automatically.

Required number of selections = $\left({}^{7}C_2 + {}^{7}C_2\right) \times 2$.

41. (B)

A white square can be selected in 32 ways.

Leaving the 8 black squares in the row and column containing the previously selected white square, we are left with 24 black squares.

Number of ways to selected a white and a black square = 32×24.

42. (D)

Possible way to distribute apples are such that

(i) one boy gets one apple, one boy gets three and remaining one gets four.

Hence number of ways to distribute = ${}^{8}C_{1} \times {}^{7}C_{3} \times 3!$.

(ii) Two boy gets two apples each, remaining one gets four.

Hence number of ways to distribute = $\frac{{}^{8}C_{2} \times {}^{6}C_{2} \times 3!}{2!}$.

(iii) two boys get three apples each and remaining one gets two.

Hence number of ways to distribute = $\frac{{}^{8}C_{3} \times {}^{5}C_{3} \times 3!}{2!}$.

Total number of ways to distribute apples = ${}^{8}C_{1} \times {}^{7}C_{3} \times 3! + \frac{{}^{8}C_{2} \times {}^{6}C_{2} \times 3!}{2!} + \frac{{}^{8}C_{3} \times {}^{5}C_{3} \times 3!}{2!}$

i.e. 4620.

Now ${}^{7}P_{3} = 210$ gives k = 22.

43. (C)

There is only one way to distribute the toys i.e. 3 to youngest one and 2 each to others.

Number of ways to distribute = $\frac{9!}{(2!)^{3}\,3!}$.

44. (A)

Marbles are to be distributed in 1 : 2 ratio therefore one child will get 4 & other will bet 8

Required number of ways = $\frac{12!}{4!8!} \times 2!$ i.e. 990.

45. (B)

Number of ways to arrange n identical red & r identical green balls = $\frac{(n+r)!}{n!\,r!}$ or ${}^{n+r}C_{n}$.

Now range of r is 0 to m.

Hence number of arrangements = ${}^{n}C_{n} + {}^{n+1}C_{n} + {}^{n+2}C_{n} + \ldots + {}^{n+m}C_{n} = {}^{n+m+1}C_{n+1}$.

$x = n + m + 1$ & $y = n + 1$ or m.

46. (C)

$L = \frac{(p+q)!}{p!q!}, M = \frac{(p+q)!}{p!q!} \times 2!$ & $N = \frac{(p+q)!}{p!q!}$.

47. (C)

For 'm' choose 4 persons out of 10 and permute 4 books with 4 persons in ${}^{10}C_4 \times 4!$ ways.

For 'n' choose 4 persons out of 10 (no need to permute) in ${}^{10}C_4$ ways.

48. (C)
Standard Results

49. (A)
Number of non negative integral solutions of 3x + y + z = 24 is equal to number of no negative integral solutions of y + z = 24 – 3r, where r = 0, 1,…,8.

i.e. $\sum_{r=0}^{8} {}^{(24-3r)+2-1}C_{2-1}$ or $\sum_{r=0}^{8}(25-3r)$.

Hence required number of solutions are 225 – 3(1 + 2 +…+ 8) i.e. 117.

50. (D)
As $3^3+4^3+5^3=6^3$, hence for every $n \in N$, $(3n)^3+(4n)^3+(5n)^3=(6n)^3$.

There are infinitely many such numbers.

51. (A)

To form a number of r digits , where $1 \le r \le 9$, we have to chose r digits from {1, 2,…,9} in ${}^9C_r$ number of ways and arrange them in ascending order which can be done in just 1 way.

Number of numbers of r digits = ${}^9C_r$.

Hence number of all such numbers = $\sum_{r=1}^{9} {}^9C_r = 2^9 - 1$.

52. (B)
Red cards can be arranged with each other in 26! Ways.
Black cards can be arranged with each other in 26! Ways.
Ordering of red & black cards can be done in 2 ways.
Number of arrangements = $(26!)^2 \times 2$.

53. (A)
There are m number of ways to distribute each object and k objects are there so required number of ways = m^k.

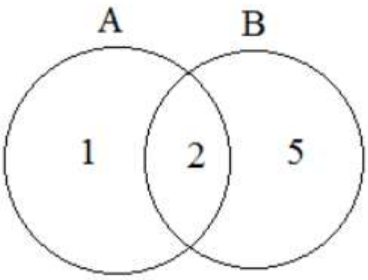
54. (D)
The word EQUATIONS contains {A, E, I, O, U} which are to be kept in the same order and {Q, T, N, S} which can be kept in any order.
Total number of arrangements are 9! But as vowels can't be arranged with each other hence required number of arrangements = $\frac{9!}{5!}$.

55. (A)

As targets in same column can be shot only in a certain order hence number of ways to shoot will be $\frac{8!}{3!2!3!}$ i.e. 560.

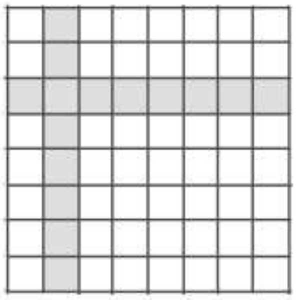
56. (A)

There are two common elements in A & B.
If A has n elements and B has m elements, then $m \times n = 21$.
Hence $\{m, n\} = \{3, 7\}$ or $\{7, 3\}$.
A possible arrangement is as shown in adjoining figure.
Hence $(A \cap B') \cup (A' \cap B) = 6$.



57. (B)

To place two rooks in attacking position they must be placed in the same row or column as shown in the adjoining figure.
As there are eight rows/columns and eight rooks hence each row/column must have exactly one rook.
So eight rooks can be placed in 8! Ways.



Ex. (1-C)

1. (D)

Number of ways to choose superintendent = 3

Number of ways to choose teachers for English school = ${}^{6}C_2 \times 4!$

Number of ways to choose teachers for Vernacular school = $4!$

Total number of ways = $3 \times {}^{6}C_2 \times 4! \times 4!$ or 25920.

2. (B)

Trials needed for first four digits = $\frac{4!}{2!}$.

Trials needed for fifth digit = 2.

Trials needed for seventh & eighth digit = 10.

Total number of trials needed = $\frac{4!}{2!} \times 2 \times 10 = 240$.

3. (B)

We can put 10 red balls at a gap of one each in 10! Ways, then green balls can be put in 11 gaps so formed in ${}^{11}C_9$ ways.

Hence number of arrangements are ${}^{11}C_9 \times 10!$.

4. (C)

Keeping volumes of the same together we have 7 objects to arrange.

Also keeping volumes of same book in order we can have two arrangements for each seat – ascending & descending.

Number of ways to arrange = $7! \times 2 \times 2 \times 2$.

5. (C)

Number of words starting with E = $\frac{4!}{2!}$.

Number of words starting with QE = $\frac{3!}{2!}$.

Number of words starting with QUEE = 1.

Rank of QUEUE = 17.

6. (B)

Numbers with 1 at first place = ${}^{8}C_4$ or 70.

Numbers with 23 at first two places = ${}^{6}C_3$ or 20.

Numbers with 245 at first three places = ${}^{4}C_2$ or 6.

As 70 + 20 + 6 = 96, hence 97^{th} number is 24678.

7. (A)

Dashes	5	4	3	2	1	0
Dots	2	3	4	5	6	7
Arrangements	${}^7C_2$	${}^7C_3$	${}^7C_4$	${}^7C_5$	${}^7C_6$	${}^7C_7$

Total Number of arrangements = $2^7 - {}^7C_0 - {}^7C_1$ or 120.

8. (A)

For India to win 5 matches before Pakistan does, there must be a minimum 5 matches and a maximum 9 matches.
In 5 matches – all must be won by India, hence 1 way.
In 6 matches – last match and in first 5 matches 4 must be won by India, hence ${}^5C_1$ ways.
In 7 matches – last match and in first 6 matches 4 must be won by India, hence ${}^6C_2$ ways.
In 8 matches – last match and in first 7 matches 4 must be won by India, hence ${}^7C_3$ ways.
In 9 matches – last match and in first 8 matches 4 must be won by India, hence ${}^8C_4$ ways.
Total number of ways = $1 + {}^5C_1 + {}^6C_2 + {}^7C_3 + {}^8C_4$ i.e. 126.

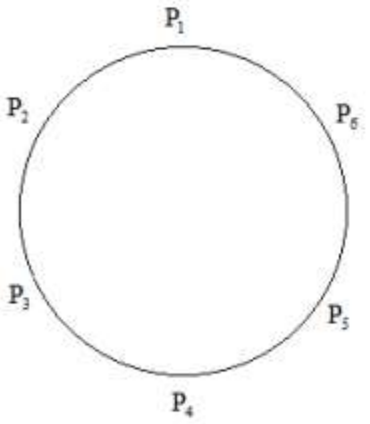
9. (D)

Number of ways in which delegates of A & B are together = $8! \times 2!$
Number of ways in which delegates of A & B as well of C & D are together = $7! \times 2! \times 2!$
Hence number of ways in which delegates of A & B are together but those of C & D are not together = $8! \times 2! - 7! \times 2! \times 2!$ i.e. $12 \times 7!$.

10. (D)

Number of ways to select 3 people from n sitting in a row, $P_n = {}^{n-3+1}C_3$
Number of ways to select 3 people from n sitting in a circle, $Q_n = {}^{n-3+1}C_3 - {}^{n-3-1}C_1$.
Hence ${}^{n-3-1}C_1 = 6$ or n = 10.
Alternately
Number of ways to select 3 people out of n sitting in a circle = ${}^nC_3$
Number of ways to select two adjacent and one separated = $n \times {}^{n-4}C_1$
Number of ways to select all three adjacent = n.
Hence number of ways to select 3 people out of n sitting in a circle such that no two are adjacent $Q_n = {}^nC_3 - n \times {}^{n-4}C_1 - n$.
$P_n - Q_n = {}^{n-2}C_3 - {}^nC_3 + n \times {}^{n-4}C_1 + n$
$\Rightarrow n - 4 = 6$ i.e. $n = 10$.

11. (D)



Let us put A at P_1.
If B is on right of A i.e. at P_2, then place on right of B i.e. at P_3 can be filled by C or D in 2 ways and remaining 3 places can be filled in 3! Ways. Hence this arrangements can be made in 12 ways.
If C is on right of A i.e. P_2, then we can make B & D sit $\{P_3, P_4\}$, {P4, P_5} or $\{P_5, P_6\}$ and the rest of the two places can be filled in 2 ways.
Hence this arrangements can be made in 6 ways.
Total number of arrangements = 18.

12. (C)

Let Miss C & Mr. B be included, then Mr. A cant be selected hence 1 more woman to be selected from 4 remaining and 2 more men to be selected from remaining 4.
Number of combinations = ${}^4C_1 \times {}^4C_2$ i.e. 24.
If Mr. A is included then Mr. B cant be selected hence 2 women to be selected from 5 choices and 2 more men to be selected from remaining 4.
Number of combinations = ${}^5C_2 \times {}^4C_2$ i.e. 60.
In none of A & B are included, then 2 women to be selected from 5 choices and 3 men to be selected from remaining 4.
Number of combinations = ${}^5C_2 \times {}^4C_3$ i.e. 40.
Total possible combinations = 124.

13. (B)

-	AB	-	E	-	F	-

We can place AB, E & F in $3! \times 2!$ i.e. 12 ways.
Now C & D can be placed in gaps in ${}^4C_2 \times 2!$ i.e. 12 ways.
Total number of arrangements = 144.

14. (C)

Let 'r' boxes be filled by red balls and rest '5 – r' with blue balls.
Blue balls must be put in r + 1 gaps between red balls.
Hence number of ways to put green balls = ${}^{r+1}C_{5-r}$.
Also $r+1 \geq 5-r \Rightarrow 2 \leq r \leq 5$.
Number of all possible arrangements = $\sum_{r=2}^{5} {}^{r+1}C_{5-r} = {}^{3}C_{3} + {}^{4}C_{2} + {}^{5}C_{1} + {}^{6}C_{0}$ i.e. 13.

15. (B)

Using 6 points which are not cyclic we can draw ${}^{6}C_{3}$ circles.
Using 2 of the 5 cyclic points and 1 of the 6 noncyclic points we can draw ${}^{5}C_{2} \times {}^{6}C_{1}$ circles.
Using 1 of the 5 cyclic points and 2 of the 6 noncyclic points we can draw ${}^{5}C_{1} \times {}^{6}C_{2}$ circles.
Using the 5 cyclic points only 1 circle can be drawn.
Total number of circles = ${}^{6}C_{3} + {}^{5}C_{2} \times {}^{6}C_{1} + {}^{5}C_{1} \times {}^{6}C_{2} + 1$ i.e. 156.
Alternately
Using 11 points we can draw ${}^{11}C_{3}$ circles, but 5 out of these are cyclic so ${}^{5}C_{3}$ circles will coincide into 1.
Total number of circles = ${}^{11}C_{3} - {}^{5}C_{3} + 1$.

16. (D)

Number of zeros at the end of 50! = exponent of 5 in 50!
i.e. $\left[\frac{50}{5}\right] + \left[\frac{50}{25}\right]$ or 12 , where [x] denotes greatest integer less than or equal to x.

17. (C)

Odd digits – 3,3,5,5.
Even places 2^{nd}, 4^{th}, 6^{th} & 8^{th}.
Number of arrangements of odd digits at even places = $\frac{4!}{2! \times 2!}$.
Number of arrangements of remaining digits at rest of the places = $\frac{5!}{2! \times 3!}$.
Number of all such numbers = $\frac{4!}{2! \times 2!} \times \frac{5!}{2! \times 3!}$ i.e. 60.

18. (A)

mn = 25! Gives mn = $2^{20} \times 3^{10} \times 5^{6} \times 7^{3} \times 11^{2} \times 13 \times 17 \times 19 \times 23$.
Hence mn as 9 prime factors.
For gcd(m, n) = 1, we need to chose m and n from these only.

Number of ways to chose m & n = $\sum_{r=0}^{9} {}^{9}C_r$ or 2^9.

Now number of selection such that $\frac{m}{n} > 1$ = Now number of selection such that $\frac{m}{n} < 1$

Hence number of ways to select m, n such that $\frac{m}{n} < 1 = \frac{2^9}{2}$ i.e. 2^8.

19. (A)

${}^{13}C_3 + {}^{13}C_4 = {}^{14}C_4$ & ${}^{13}C_4 + {}^{13}C_5 = {}^{14}C_5$

$\Rightarrow {}^{13}C_3 + 2\,{}^{13}C_4 + {}^{13}C_5 = {}^{14}C_4 + {}^{14}C_5$

$\Rightarrow {}^{n}C_r = {}^{15}C_5$

Hence r = 5 or 10.

20. (D)

To form a mixed doubles team we need to select 2 males and two females which can be done in ${}^{8}C_2 \times {}^{8}C_2$ ways and then the selected 4 players can be arranged in 2 ways.

Hence total number of ways = $2 \times {}^{8}C_2 \times {}^{8}C_2$ i.e. 1568.

21. (D)

Number of pairings = number of ways to distribute 8 distinct objects in 4 groups each containing 2 objects i.e. $\frac{8!}{2!2!2!2!4!} = \frac{8.7.6.5}{16} = 105$.

22. (C)

Let n parabola divide a plane in T_n regions.
1 parabola divides the plane in 2 regions.
If a second parabola is introduced, then 3 parts will increase.
If a third parabola is introduced, then 5 parts will increase.
If a fourth parabola is introduced, then 7 parts will increase.
Now if (n – 1) parabolas are already there and n^{th} parabola is introduced (2n – 1) new regions will be created, hence

$T_n = T_{n-1} + 2n - 1$

Also $T_1 = 2$.

Therefore $T_{10} = T_1 + 3 + 5 + ... + 19 = 101$.

23. (D)

We have to find umber of positive integral solutions of $3x + 5y = 283$.

For $y = \frac{283-3x}{5}$, $(283 - 3x)$ must be a multiple of 5, hence last digit of 3x must be 3 or 8.

Now multiples of 3 having last digit 3 and less than 283 are {3, 33, 63, 93,…,273} i.e. 10 and multiples of 3 having last digit 8 and less than 283 are {18, 48, 78, 93,…,258} i.e. 9

Hence total 19 such points are there.

24. (C)

n circles will intersect in $2 \times {}^nC_2$ ways.

n lines will intersect in ${}^nC_2$ ways.

n lines will intersect n circles in $2 \times {}^nC_1 \times {}^nC_1$ ways.

Hence $2 \times {}^nC_2 + {}^nC_2 + 2 \times {}^nC_1 \times {}^nC_1 = 80$ which gives $n = 5$.

25. (B)

Let there be x number of 2s, y number of 5s and z number of 7s are used in forming an n digit number, then the total number of numbers formed will be

$$\sum_{x=0}^{n}\sum_{y=0}^{n}\sum_{z=0}^{n}\frac{n!}{x!y!z!} \ge 900, \text{ where } x+y+z=n.$$

$\Rightarrow$ sum of coefficients in expansion of $(x+y+z)^n \ge 900$

$\Rightarrow 3^n \ge 900$.

Now $3^6 = 729$, hence least value of $n = 7$.

26. (D)

Given A = {1, 11, 21, …, 551}

There are 28 pairs whose sum is 552 such as (1, 551), (2, 550),…, (271, 281)

So if one number of a pair is taken in B then the other number can't be in B.

Hence B can contain maximum 28 elements.

27. (C)

EARTHQUAKE – {E,E}, {A,A}, {R, T, H, Q, U, K}

A 4 letter permutation can be of

(i) 2 alike & 2 alike = $\frac{4!}{2!\,2!}$ i.e. 6 ways

(ii) 2 alike & two distinct = $2 \times {}^7C_2 \times \frac{4!}{2!}$ i.e. 504 ways

(iii) all four distinct = ${}^8C_4 \times 4!$ i.e. 1680 ways

Hence total number of permutations = 6 + 504 + 1680 = 2190.

1.	**C**	2.	**D**	3.	**B**	4.	**D**	5.	**D**
6.	**A**	7.	**A**	8.	**A**	9.	**B**	10.	**D**
11.	**A**	12.	**A**	13.	**D**	14.	**B**	15.	**A**

16. **B**

$\because$ Each person gets at least one ball.

$\therefore$ 3 Persons can have 5 balls as follow.

Person	No. of balls	No. of balls
I	1	1
II	1	2
III	3	2

The number of ways of distribute balls, 1, 1, 3 in first to three persons

$= {}^5C_1 \times {}^4C_1 \times {}^3C_3$

Also 3, persons having 1, 1 and 3 balls can be arranged in $\frac{3!}{2!}$ ways.

$\therefore$ Total no. of ways of distribute 1, 1, 3 balls to the three persons.

$= {}^5C_1 \times {}^4C_1 \times {}^3C_3 \times \frac{3!}{2!} = 60$

Similarly, total no. of ways to distribute 1, 2, 2, balls to three person

$= {}^5C_1 \times {}^4C_2 \times {}^2C_2 \times \frac{3!}{2!} = 90$

$\therefore$ The required number of ways = 60 + 90 = 150

17. **B**

We know,

$T_n = {}^nC_3,\ T_{n+1} = {}^{n+1}C_3$

ATQ, $T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$

$\Rightarrow\ {}^nC_2 = 10 \Rightarrow n = 5$.

18. **A**

Number of diagonal $= 54 \Rightarrow \frac{n(n-3)}{2} = 54$

$\Rightarrow\ n^2 - 3n - 108 = 0 \Rightarrow n^2 - 12n + 9n - 108 = 0$

$\Rightarrow\ n(n-12) + 9(n-12) = 0$

$\Rightarrow\ n = 12, -9 \Rightarrow n = 12\ (\because n \neq 9)$

19. **A**

(a) Required number of ways

= Total arrangement – number of ways when B_1 and G_1 together = 7! – 6! 2! = 6! (7 – 2) = 5.6!

20. **A**

(a) The thousands place can only be filled with 2, 3 or 4, since the number is greater than 2000.
For the remaining 3 places, we have pick out digits such that the resultant number is divisible by 3.
It the sum of digits of the number is divisible by 3, then the number itself is divisible by 3.

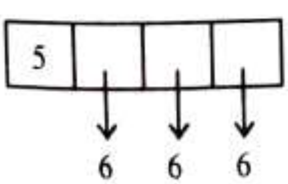
Case I: If we take 2 at thousands place.
The remaining digits can be filled as:
0, 1 and 3 as $2 + 1 + 0 + 3 = 6$ is divisible by 3.
0, 3 and 4 as $2 + 3 + 0 + 4 = 9$ is divisible by 3.
In both the above combinations the remaining three digits can be arranged in 3! ways.
$\therefore$ Total number of numbers in this case $= 2 \times 3! = 12$.

Case II: If we take 3 at thousands place. The remaining digits can be filled as:
0, 1 and 2 as $3 + 1 + 0 + 2 = 6$ is divisible by 3.
0, 2 and 4 as $3 + 2 + 0 + 4 = 9$ is divisible by 3.
In both the above combinations, the remaining three digits can be arranged in 3! ways. Total number of numbers in this case $= 2 \times 3! = 12$.

Case III: If we take 4 at thousands place.
The remaining digits can be filled as:
0, 2 and 3 as $4 + 2 + 0 - 3 = 9$ is divisible by 3.
In the above combination, the remaining three digits can be arranged in 3! ways.
$\therefore$ Total number of numbers in this case $= 3! = 6$.
$\therefore$ Total number of numbers between 2000 and 5000 divisible by 3 are $12 + 12 + 6 = 30$.

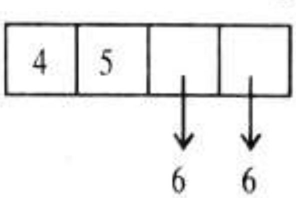
21. **D**

(d) 0, 1, 2, 3, 4, 5
Number of four-digit number starting with 5 is,



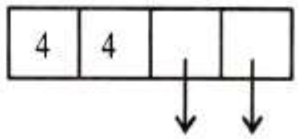
$= 6 \times 6 \times 6 = 216$

Number of four-digit numbers starting with 45 is,



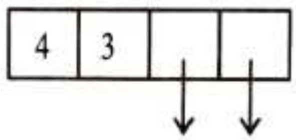
$= 6 \times 6 = 36$

Number of four-digit numbers starting with 44 is,



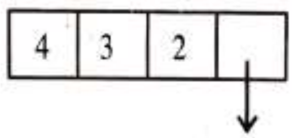
6 6 $= 6 \times 6 = 36$

Number of four-digit numbers starting with 43 and greater than 4321 is,



(5/4/3) 3 6 $= 3 \times 6 = 18$

Number of four-digit numbers starting with 432 and greater than 4321 is,



4 $= 4$

Hence, required numbers $= 216 + 36 + 36 + 18 + 4 = 310$.

22. **C**

(c) Number of ways of selecting 10 objects

$= (10I, 0D)$ or $(9I, 1D)$ or $(8I, 2D)$ or ... $(0I, 10D)$

Here, D signifies distinct object and I indicates identical object

$$= 1 + {}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10} = \frac{2^{21}}{2} = 2^{20}$$

23. **B**

Number of arrangement $= (3! \times 3! \times 4!) \times 3! = (3!)^3 4!$

24. **C**

(c) We know, $(r+1) \cdot {}^{r}P_{r-1} = (r+1) \cdot \frac{r!}{1!} = (r+1)!$

So, $(2 \cdot {}^{1}P_0 - 3 \cdot {}^{2}P_1 + \ldots..51 \text{ terms}) +$
$(1! - 2! + 3! - \ldots \text{ upto } 51 \text{ terms})$

$= [2! - 3! + 4! - \ldots + 52!] + [1! - 2! + 3! - \ldots + 51!]$

$= 52! + 1! = 52! + 1$

25. **D**

(d) $\frac{36}{r+1} \times {}^{35}C_r(k^2-3) = {}^{35}C_r.6$

$\Rightarrow k^2-3 = \frac{r+1}{6} \Rightarrow k^2 = 3+\frac{r+1}{6}$

r can be 5, 35 for $k \in I$

$r = 5, k = \pm 2;\ r = 35, k = \pm 3$

Hence, number of ordered pairs = 4.

26. **D**

(d) Since, a boy plays against a boy.

∴ Total matches of boys can be arranged in

$7 \times 4 = 28$ ways

Since, a girl plays against a girl.

∴ Total matches of girls can be arranged in

$n \times 6 = 6n$ ways

∵ $28 + 6n = 52 \Rightarrow n = 4$

27. **A**

(a)

Indians	Foreigners	Number of ways
2	4	${}^6C_2 \times {}^8C_4 = 1050$
3	6	${}^6C_3 \times {}^8C_6 = 560$
4	8	${}^6C_4 \times {}^8C_8 = 15$

Total number of ways = 1625

28. **D**

(d) Required number should be even and should be divisible by 6.

Then, number should also divisible by 2 & 3.

The possible choices for number divisible by 3 are (1, 2, 5, 6, 7) or (1, 2, 3, 5, 7)

Using 1, 2, 5, 6, 7, number of even numbers is

$= 4 \times 3 \times 2 \times 1 \times 2 = 48$

Using 1, 2, 3, 5, 7, number of even numbers is

$= 4 \times 3 \times 2 \times 1 \times 1 = 24$

Required answer is 72.

29. **309**

(309) M O T H E R

3 4 6 2 1 5

$\Rightarrow 2\times 5!+2\times 4!+3\times 3!+2!+1$

$=240+48+18+2+1=309$

30. **17**

(17) No. of ways of selecting 3 boys and 2 girls

$= {}^{b}C_3 \times {}^{g}C_2 = 168$

$b(b-1)(b-2)(g)(g-1) = 8\times 7\times 6\times 3\times 2$

$b+3g=17$

31. **240**

(240) $S \rightarrow 2,\ L \rightarrow 2,\ A, B, Y, U.$

$\therefore$ Required number of ways $= {}^{2}C_1 \times {}^{5}C_2 \times \frac{4!}{2!} = 240.$

32. **135**

(135)

Select any 4 correct questions in ${}^{6}C_4$ ways.

Number of ways of answering wrong question = 3

$\therefore$ Required number of ways $= {}^{6}C_4 (1)^4 \times 3^2 = 135.$

33. **54**

(54)

Let xyz be the three digit number

$x+y+z=10,\ x\ge 1,\ y\ge 0,\ z\ge 0$

$x-1=t \Rightarrow x=1+t$ $\quad$ $x-1\ge 0,\ t\ge 0$

$t+y+z=10-1=9$ $\quad$ $0\le t,\ y,\ z\le 9$

$\therefore$ Total number of non-negative integral solution

$= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = \frac{11\cdot 10}{2} = 55$

But for $t=9, x=10$, so required number of integers

$=55-1=54.$

34. **136**

(136) ${}^{1}P_1 + 2.{}^{2}P_2 + 3.{}^{3}P_3 + ... + 15.{}^{15}P_{15}$

$= 1! + 2.2! + 3.3! + ... 15 \times 15!$

$= 1! + (3!.2!) + ... + (16!\ 15!)$

$$\sum_{r=1}^{15}(r+1)! - (r)! = 16! - 1 = {}^{16}P_{16} - 1 \Rightarrow q = r = 16, s = 1$$

$${}^{q+s}C_{r-s} = {}^{17}C_{15} = \frac{17 \times 16}{2} = 136$$

35. **7744**

(7744) Numbers divisible by 11 will lie from 200 to 500

Now numbers are 209, 220, 231.....

$$\text{Sum} = \frac{27}{2}(209 + 495) = 9504$$

Number containing 1 at unit place $\underline{2}\ \underline{3}\ \underline{1}$, $\underline{3}\ \underline{4}\ \underline{1}$, $\underline{4}\ \underline{5}\ \underline{1}$

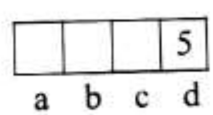
Number containing 1 at 10^{th} place $\underline{3}\ \underline{1}\ \underline{9}$, $\underline{4}\ \underline{1}\ \underline{8}$

Required $= 9504 - (231 + 341 + 451 + 319 + 418) = 7744$

36. **6**

(6) Here 4 digit numbers.

For divisibility by 55, no. should be div. by 5 and 11 both



Also, for divisibility by 11

$a + c = b + 5$

for $b = 1$ $\quad a = 2, c = 4$

$\quad a = 4, c = 2$

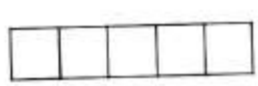
for $b = 2$ $\quad a = 3, c = 4$

$\quad a = 4, c = 3$

for $b = 3$ $\quad a = 6, c = 2$

$\quad a = 2, c = 6$

$\therefore$ 6 possible four digit no.s are div. by 55 (II) 5 digit number is not possible



(Not possible)

37. **180**

(180) Given a 5 digit number product is 36.

Multiple of $36 = 2^2 \times 3^2$

$= 2 \times 2 \times 3 \times 3$

We need to make five digit number by using the digits 2, 2, 3 & 3 and the fifth digit could be 7.

Case-I: 2, 2, 3, 3, 1

Case-I: 2, 2, 3, 3, 1

Total number of 5 digits numbers $= \dfrac{5!}{2!2!} = 30$

Case-II: 4, 3, 3, 1, 1

Total number of 5 digits numbers $= \dfrac{5!}{2!2!} = 30$

Case-III: 6, 2, 3, 1, 1

Total number of 5 digits numbers $= \dfrac{5!}{2!} = 60$

Case-IV: 9, 2, 2, 1, 1

Total number of 5 digits numbers $= \dfrac{5!}{2!2!} = 30$

Case-V: 4, 9, 1, 1, 1

Total number of 5 digits numbers $= \dfrac{5!}{3!} = 20$

Case-VI: 6, 6, 1, 1, 1

Total number of 5 digits numbers $= \dfrac{5!}{3!2!} = 10$

Total number of 5 digits numbers
$= 30 + 30 + 60 + 30 + 20 + 10 = 180.$

38. **1492**

(1492)

M	A	N	K	I	N	D

$$\left(\frac{4 \times 6!}{2!}\right) + (5! \times 0) + \left(\frac{4! \times 3}{2!}\right) + (3! \times 2) + (2! \times 1) + (1! \times 1) + (0! \times 0) + 1 = 1492$$

$\Rightarrow \quad 1440 + 36 + 12 + 4 = 1492$

39. **1086**

(1086) Let the number is *abcd*, where *a*, *b*, *c* are divisible by *d*.

	No. of such numbers
$d = 1,$	$9 \times 10 \times 10 = 900$
$d = 2$	$4 \times 5 \times 5 = 100$
$d = 3$	$3 \times 4 \times 4 = 48$
$d = 4$	$2 \times 3 \times 3 = 18$
$d = 5$	$1 \times 2 \times 2 = 4$
$d = 6, 7, 8, 9$	$4 \times 4 = 16$
	$= 1086$

40. **150**

(150) $36 = 2 \times 2 \times 3 \times 3$
We are required such three digit numbers whose G.C.D. with 36 is only 2.
⇒ Number should be odd multiple of 2 and does not having factor 3 and 9
Odd multiple of 2 are e.g. : {(36, 102), (36, 106) .....}
102, 106, 110, 114.....998 (225 no.)
No. of multiples of 3 are
102, 114, 126 .....990 (75 no.)
Which are also included multiple of 9
Hence, required $= 225 - 75 = 150$

41. **243**

(243)
Total choices of digits are (0, 1, 2, …, 9).
If 0 taken twice then ways = 9
If 0 taken once then ${}^{9}C_1 \times 2 = 18$
If 0 not taken then ${}^{9}C_1\,{}^{8}C_1 \times 3 = 216$
Total = 243

42. **40**

(40) Let ‘p’ and ‘q’ be the number of correct and incorrect answer is 5.
So, $x + y = 5$ and $3x - 2y = 5$
After solving the equations we have (3, 2).
Therefore, numbers of correct answers are 3 and number of incorrect answers are 2.
So, the number of possibilities are 3, 3, 3, –2, –2.

Number of ways $= \dfrac{5!}{3!2!} \times 2 \times 2 = 40$