EXERCISE - 2 [A]

1. (C)

$$\lim_{x\to 3}\frac{\left(x^3+27\right)\ln\left(x-2\right)}{\left(x^2-9\right)}=\lim_{x\to 3}\frac{\left(x+3\right)\left(x^2-3x+9\right)\ln\left[1+\left(x-3\right)\right]}{\left(x-3\right)\left(x+3\right)}=\lim_{x\to 3}\left(x^2-3x+9\right).$$

$$\frac{\ln\left[1+\left(x-3\right)\right]}{\left(x-3\right)}$$

$$(9-9+9)(1)=9$$

2. (B)

$$\lim_{x\to 0}\frac{\left(4^x-1\right)^3}{\sin\left(\frac{x}{p}\right)\ln\left(1+\frac{x^2}{3}\right)}=\lim_{x\to 0}\frac{x^3\left(\frac{4^x-1}{x}\right)^3}{\left(\frac{x}{p}\right)\left[\frac{\sin\left(\frac{x}{p}\right)}{\frac{x}{p}}\right]\cdot\ln\left(1+\frac{x^2}{3}\right)}=3p\cdot\lim_{x\to 0}\frac{\left(\frac{4^x-1}{x}\right)^3}{\left[\frac{\sin\left(\frac{x}{p}\right)}{\frac{x}{p}}\right]}$$

$$\frac{\left(\frac{x^2}{3}\right)}{\ln\left(1+\frac{x^2}{3}\right)}=3p\left(\ln 4\right)^3$$

3. (D)

Let $x=2+h$ $\qquad x\to 2,\quad h\to 0$

$$=\lim_{h\to 0}\frac{\sin\left(e^h-1\right)}{\ln\left(1+h\right)}=\lim_{h\to 0}\frac{\sin\left(e^h-1\right)}{\left(e^h-1\right)}\times\frac{\left(e^h-1\right)}{h}\times\frac{1}{\frac{\ln\left(1+h\right)}{h}}=1\cdot 1\cdot 1=1$$

4. (D)

$$\lim_{x\to 0}\frac{\sin\left(\ln\left(1+x\right)\right)}{\ln\left(1+\sin x\right)}$$

$$=\lim_{x\to 0}\frac{\sin\left(\ln\left(1+x\right)\right)}{\ln\left(1+x\right)}\cdot\frac{\sin x}{\ln\left(1+\sin x\right)}\cdot\frac{\ln\left(1+x\right)}{x}\cdot\frac{x}{\sin x}=1\cdot 1\cdot 1\cdot 1=1$$

5. (D)

$$\lim_{x\to 1}\frac{\sqrt{1-\cos 2(x-1)}}{x-1};\ \lim_{x\to 1}\frac{\sqrt{2\sin^2(x-1)}}{x-1}$$

$$\lim_{x\to 1}\frac{\sqrt{2}\left|\sin(x-1)\right|}{x-1};\ \text{LHL}=\lim_{h\to 0}\frac{\sqrt{2}\sin h}{-h}=-\sqrt{2}$$

$$\text{RHL}=\lim_{h\to 0}\frac{\sqrt{2}\sin h}{h}=\sqrt{2}$$

$\because$ L.H.L. $\neq$ R.H.L.

So, limit does not exist.

6. (A)

$$\lim_{x\to 0}\frac{\left(1+x^2\right)^{1/3}-(1-1x)^{\frac{1}{4}}}{x+x^2}=\lim_{x\to 0}\frac{\left(1+\frac{x^2}{3}+....\right)-\left(1-\frac{x}{2}+\frac{\frac{1}{4}\left(\frac{1}{4}-1\right)}{2!}4x^2+....\right)}{x+x^2}=\frac{1}{2}$$

7. (D)

$$\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}=\lim_{x\to 0}\frac{2\sqrt{2}\sin^2\frac{x}{4}}{\sin^2 x}=\lim_{x\to 0}\frac{2\sqrt{2}\sin^2\frac{x}{4}}{16\cdot\frac{x^2}{16}\cdot\frac{\sin^2 x}{x^2}}=\frac{\sqrt{2}}{8}$$

8. (B)

$$\lim_{x\to 0^+}\frac{\cos^{-1}(1-x)}{\sqrt{x}}=\lim_{h\to 0}\frac{\cos^{-1}\left[1-(0+h)\right]}{\sqrt{0+h}}=\lim_{h\to 0}\frac{\cos^{-1}(1-h)}{\sqrt{h}}$$

Let $1-h=\cos\theta$

$$\sin\theta=\sqrt{1-(1-h)^2}$$

$$\therefore\ \theta=\sin^{-1}\sqrt{2h-h^2}=\lim_{h\to 0}\frac{\sin^{-1}\sqrt{2h-h^2}}{\sqrt{h}}=\lim_{h\to 0}\frac{\sin^{-1}\sqrt{2h-h^2}}{\sqrt{2h-h^2}}\cdot\frac{\sqrt{2h-h^2}}{\sqrt{h}}$$

$$=1\times\sqrt{2}=\sqrt{2}$$

9. (B)

$$\lim_{n\to\infty} n\cos\left(\frac{\pi}{4n}\right)\sin\left(\frac{\pi}{4n}\right)=\lim_{n\to\infty}\cos\left(\frac{\pi}{4n}\right)\lim_{n\to\infty}\frac{\sin\left(\frac{\pi}{4n}\right)}{\left(\frac{\pi}{4n}\right)}\times\frac{\pi}{4}=1\frac{\pi}{4}\cdot 1=\frac{\pi}{4}$$

10. (C)

$\sin h < h < \tan h,\ h \in \left(0, \frac{\pi}{2}\right)$

$\frac{h}{\sin h} > 1 \Rightarrow \frac{-h}{\sin h} < -1$

$$\text{LHL} = \lim_{h\to 0}\left[\frac{\frac{\pi}{2} - h - \frac{\pi}{2}}{\cos\left(\frac{\pi}{2} - h\right)}\right] = \lim_{h\to 0}\left[\frac{-h}{\sin h}\right] = -2$$

$$\text{RHL} = \lim_{h\to 0}\left[\frac{\frac{\pi}{2} + h - \frac{\pi}{2}}{\cos\left(\frac{\pi}{2} + h\right)}\right] = \lim_{h\to 0}\left[\frac{-h}{\sin h}\right] = -2$$

$\therefore\ \text{LHL} = \text{RHL} = -2$

11. (A)

$$\lim_{n\to\infty}\frac{-3n + (-1)^n}{4n - (-1)^n} = \lim_{n\to\infty}\frac{-3 + (-1)^n \frac{1}{n}}{4 - (-1)^n \cdot \frac{1}{n}} = \frac{-3+0}{4-0} = -\frac{3}{4}$$

12. (A)

$$\lim_{x\to 1}\left(\frac{2}{1-x^2} + \frac{1}{x-1}\right) = \lim_{x\to 1}\left(\frac{1}{1-x} + \frac{1}{1+x} + \frac{1}{x-1}\right) = \lim_{n\to 1}\left(\frac{1}{x+1}\right) = \frac{1}{2}$$

13. (D)

$$\lim_{n\to\infty}\frac{1}{n^4}([1^3x] + [2^3x] + \ldots + [n^3x])$$

$1^3x - 1 < [1^3x]\, 1^3x$

$2^3x - 1 < [2^3x]\, 2^3x$

. . .

. . .

. . .

$n^3x - 1 < [n^3x]\, n^3x$

Adding all these inequalities

$(1^3 + 2^3 + 3^3 \ldots.. + n^3)x - n < [1^3x] + [2^3x] + \ldots\ldots.[n^3x] \le (1^3 + 2^3 + \ldots\ldots n^3)x$

$$\frac{\frac{n^2(n+1)^2}{4}x - n}{n^4} < \frac{[1^3x] + [2^3x] + \ldots. + [n^3x]}{n^4} \le \frac{\left(\frac{n(n+1)}{2}\right)^2 x}{n^4}$$

$$\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^2 \frac{x}{4} - \frac{1}{n^3} < \lim_{n\to\infty}\frac{[1^3x] + [2^3x] + \ldots. + [n^3x]}{n^4} \le \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^2 \frac{x}{4}$$

$$\frac{x}{4} < \lim_{x\to\infty} \frac{[1^3 x]+[2^3 x]+.....+[n^3 x]}{n^4} = \frac{x}{4}$$

14. (B)

$$\lim_{x\to a^-}\left(\frac{|x|^3}{a} - \left[\frac{x}{a}\right]^3\right)(a>0)$$

$$x = a-h = \lim_{h\to 0}\left(\frac{|a-h|^3}{a} - \left(\frac{|a-h|}{a}\right)^3\right) = \lim_{h\to 0}\left(\frac{|a|^3}{a} - \left(1-\frac{h}{a}\right)^3\right) = a^2 - 0 = a^2$$

15. (C)

$$\lim_{n\to\infty} \cos\frac{x}{2}\cos\frac{x}{2^2}\cos\frac{x}{2^3}\cos\frac{x}{2^4}.....\cos\frac{x}{2^n}$$

$$= \lim_{n\to\infty} \frac{\sin x}{2^n \sin\frac{x}{2^n}} = \qquad \left(\because \lim_{n\to\infty}\frac{x}{2^n} = 0\right)$$

$$= \frac{\sin x}{x}$$

16. (C)

$$\sin\theta < \theta < \tan\theta,\ \theta \in \left(0, \frac{\pi}{2}\right)$$

$$\frac{\sin\theta}{\theta} < 1 < \frac{\tan\theta}{\theta}$$

$$\frac{n\sin\theta}{\theta} < n < \frac{n\tan\theta}{\theta}$$

$$\lim_{\theta\to 0}\left(\left[\frac{n\sin\theta}{\theta}\right] + \left[\frac{n\sin\theta}{\theta}\right]\right); \qquad n\in N$$

$$\text{L.H.L.} = \lim_{\theta\to 0^-}\left(\left[\frac{n\sin\theta}{\theta}\right] + \left[\frac{n\tan\theta}{\theta}\right]\right) = n-1+n = 2n-1$$

$$\text{R.H.L.} == \lim_{\theta\to 0^-}\left(\left[\frac{n\sin\theta}{\theta}\right] + \left[\frac{n\sin\theta}{\theta}\right]\right) = n-1+n = 2n-1$$

$\therefore$ L.H.L. = R.H.L. = $2n - 1$

17. (A)

$$\text{R.H.L.} = \lim_{x\to 0^-}\left[(1-e^x)\frac{\sin x}{x}\right]$$

When $x\in(0, h)$ and $h\to 0$ then $(1-e^x)\in(-1, 0)$ and $\frac{\sin x}{x} < 1$

So, $-1 < (1-e^x)\frac{\sin x}{x} < 0$; $\lim_{x\to 0^+}\left[(1-e^x)\frac{\sin x}{x}\right] = -1$

L.H.L. $= \lim_{x\to 0^-}\left[\left(1-e^x\right)\frac{\sin x}{-x}\right] = \lim_{x\to 0^-}\left[\left(e^x-1\right)\frac{\sin x}{x}\right]$

When $x\in(-h,0)$ and $h\to 0$, then $e^x-1\in(-1,0)$ and $\frac{\sin x}{x}<1$

18. (B)

$$\lim_{x\to\infty}\frac{e^x\left(\left(2^{x^n}\right)^{\frac{1}{e^x}}-\left(3^{x^n}\right)^{\frac{1}{e^x}}\right)}{x^n}, n\in N$$

$$\lim_{x\to\infty}\frac{(2)^{\frac{x^n}{e^x}}-(3)^{\frac{x^n}{e^x}}}{\frac{x^n}{e^x}}$$

When $x\to\infty$, $\frac{x^n}{e^x}\to 0$

Put $\frac{x^n}{e^x}=t$

$$\lim_{t\to 0}\left(\frac{2^t-3^t}{t}\right)=\ln 2-\ln 3=\ln\left(\frac{2}{3}\right)$$

19. (B)

$$\lim_{x\to 0}\frac{\ln\left(\tan\left(\frac{\pi}{4}+ax\right)\right)}{\sin bx}, b\neq 0$$

$$\lim_{x\to 0}\frac{\ln\left(1+\frac{2\tan ax}{1-\tan ax}\right)\cdot\left(\frac{2\tan ax}{1-\tan ax}\right)}{\left(\frac{2\tan ax}{1-\tan ax}\right)\sin bx(bx)} \Rightarrow \lim_{x\to 0}\frac{2\tan ax}{1-\tan ax}=\frac{2a}{b}$$

20. (A)

$$\lim_{x\to\infty}\frac{x}{a}\left(\frac{b}{x}-\left\{\frac{b}{x}\right\}\right)=\frac{b}{a}-\lim_{x\to 0}\frac{x}{a}\left\{\frac{b}{x}\right\}=\frac{b}{a}-\lim_{x\to 0}\frac{x}{a}\left\{\frac{b}{x}\right\}=\frac{b}{a}$$

21. (A)

$$\lim_{x\to 0^+}\frac{\left(\frac{\pi}{2}-\cot^{-1}\{x\}\right)x}{\operatorname{sgn}(x)-\cos x} \quad \text{as } x\to 0^+ \qquad \operatorname{sgn}(x)=1 \text{ and } \{x\}\to x$$

$$= \lim_{x\to 0^+} \frac{\left(\frac{\pi}{2} - \cot^{-1} x\right)}{1-\cos x} = \lim_{x\to 0^+} \frac{\tan^{-1} x.x}{1-\cos x} = \lim_{x\to 0^+} \frac{\frac{\tan^{-1} x}{x}}{\frac{1-\cos x}{x^2}} = 2$$

EXERCISE - 2 [B]

One or More than One Option(s) Correct

1. **AD**

2. (C, D)

$\lim_{n\to\infty}\left(\sqrt{n^2+1}-\sqrt{n^2-3n+1}\right)=\frac{3}{2}$, when $x \in \left(0, \frac{\pi}{2}\right]$ then $[\cos x]=0$

So $f(x)=\frac{3x}{2}$ when $x=0[\cos x]=1$

So $f(x)=\frac{3x}{2}$ when $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ then $[\cos x]=-1$

So $f(x)=0$

3.	**BC**	4.	**AC**	5.	**ABD**				
6.	**ABD**	7.	**ABCD**	8.	**AB**	9.	**BC**	10.	**AB**
11.	**ABC**	12.	**CD**	13.	**ABD**	14.	**AC**	15.	**BCD**

Comprehensions Type

1.	C	2.	B	3.	B	4.	A	5.	B
6.	A	7.	B	8.	C	9.	D		

Matrix – Match Type

1. (A) R; (B) S; (C) P; (D) Q
2. (A) Q; (B) R; (C) P; (D) P
3. (A) S; (B) P; (C) Q; (D) R
3. (A) S; (B) R; (C) P; (D) Q ; (E) P
5. (A) S; (B) R; (C) Q; (D) P

6. (A) S; (B) R; (C) P; (D) Q

(A) $\lim_{x\to 0}\left[\sin|x|-|x|\right]=$

LHL $=\lim_{x\to 0^-}[-\sin x+x]=-1 \quad (\therefore\ \sin x < x \text{ for } x<0)$

RHL $=\lim_{x\to 0^+}[\sin x - x]=-1$

(B) $\lim_{x\to 0}\left[\frac{x}{[x]}\right]=$

$$\text{LHL} = \lim_{x\to 0^-}\left[\frac{x}{[x]}\right] = \lim_{x\to 0^-}\left[\frac{x}{-1}\right] = 0 \qquad \therefore\ x \text{ is negative}$$

$$\text{RHL} = \lim_{x\to 0}\left[\frac{x}{[x]}\right] = \left[\frac{x}{0}\right] = \text{ does not exist}$$

(C) $\lim_{x\to\frac{1}{2}}\left[x\left[\frac{1}{x}\right]\right] =$

$$\text{LHL} = \lim_{x\to\frac{1}{2}^-}\left[x\left[\frac{1}{x}\right]\right]\lim_{h\to 0}\left[\left(\frac{1}{2}-h\right)\left[\frac{1}{\frac{1}{2}-h}\right]\right] = \lim_{h\to 0}\left[\left(\frac{1}{2}-h\right)(2)\right] = \lim_{h\to 0}$$

$$=[1-2h]=0$$

$$\text{RHL} = \lim_{x\to\frac{1}{2}^+}\left[x\left[\frac{1}{x}\right]\right] = \lim_{h\to 0}\left[\left(\frac{1}{2}+h\right)\left[\frac{1}{\frac{1}{2}+h}\right]\right] = \lim_{h\to 0}\left[\left(\frac{1}{2}+h\right)(1)\right] = 0$$

(D) $\lim_{x\to -1}\left[\frac{[x]}{x}\right]$

$$\text{LHL} = \lim_{h\to 0}\left[\frac{[-1-h]}{-1-h}\right] = \lim_{h\to 0}\left[\frac{-2}{-1-h}\right] = \lim_{h\to 0}\left[\frac{2}{1+h}\right] = 1$$

$$\text{RHL} = \lim_{h\to 0}\left[\frac{[-1+h]}{-1+h}\right] = \lim_{h\to 0}\left[\frac{-1}{-1+h}\right] = \lim_{h\to 0}\left[\frac{1}{1-h}\right] = 1$$

EXERCISE - 2 [C]

1.	**2**	2.	**10**	3.	**35**	4.	**4**	5.	**3**
6.	**8**	7.	**2**	8.	**1**	9.	**4**	10.	**9**
11.	**2**	12.	**8**	13.	**3**	14.	**4**	15.	**1**