1. (C)
Since events A and B are mutually exclusive
$\therefore\ P(A)+P(B)=1$
$\Rightarrow\ \frac{3x+1}{3}+\frac{1-x}{4}=1$
$\Rightarrow\ 12x+4+3-3x=12 \Rightarrow 9x=5 \Rightarrow x=\frac{5}{9}$
$\therefore\ x\in\left[-\frac{1}{3},\frac{5}{9}\right]$

2. (A)
Given $\frac{(x-10)(x-50)}{(x-30)}\geq 0$
Let $x\geq 10,\ x\geq 50$ equation will be true. $\forall\, x\geq 50$
As $\left(\frac{x-50}{x-30}\right)\geq 0,\ \forall\, x\in[10,30)$
$\frac{(x-10)(x-50)}{x-30}\geq 0\quad \forall x\in[10,30)$
Total value of x between 10 to 30 is 20
Total values of x between 50 to 100 including 50 and 100 is 51
Total values of x = 51 + 20 =71
$P(A)=\frac{71}{100}=0.71$

3. (A)
Let A and E be any two events with positive probabilities.
Consider statement – I :
$P(E/A)\geq P(A/E)P(E)$
$LHS: P(E/A)=\frac{P(E\cap A)}{P(A)}$ .....(i)
$RHS: P(A/E).P(E)=\frac{P(E\cap A)}{P(E)}.P(E)$
$=P(A\cap E)$ .....(ii)
Clearly, from (1) and (2), we have
$P(E/A)\geq P(A\cap E)$
Thus, statement-1 is true
Similarly, statement-2 is also true.

4. (C)
Note: The question should state ‘3 different’ boxes instead of '3 identical boxes' and one particular box has 3 balls. Then the solution would be:

Required probability = $\frac{{}^{12}C_3 \times 2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

5. (C)

P (exactly one of A or B occurs)

$= P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$ .....(i)

P (exactly one of B or C occurs)

$= P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$ .....(ii)

P (exactly one of C or A occurs)

$= P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$ .....(iii)

Adding (i), (ii) and (iii), we get

$2\sum P(A) - 2\sum P(A \cap B) = \frac{3}{4}$

$\therefore \quad \sum P(A) - \sum P(A \cap B) = \frac{3}{8}$

Now, $P(A \cap B \cap C) = \frac{1}{16}$

$\therefore \; P(A \cup B \cup C)$

$= \sum P(A) - \sum P(A \cap B) + \sum P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

6. (C)

Probability of 4 member committee which contain at least one women.

$\Rightarrow \; P(3M, 1W) + P(2M, W) + P(1M, 3W) + P(0M, 4W)$

$\frac{{}^{10}C_3\,{}^5C_1}{{}^{15}C_4} + \frac{{}^{10}C_2\,{}^5C_2}{{}^{15}C_4} + \frac{{}^{10}C_1\,{}^5C_3}{{}^{15}C_4} + \frac{{}^{10}C_0\,{}^5C_4}{{}^{15}C_4}$

$\frac{600}{1365} + \frac{450}{1365} + \frac{100}{1365} = \frac{1155}{1365}$

$\therefore$ Probability of committees to have more women than men.

$= \frac{P(1M, 3W) + P(0M, 4W)}{P(3M, 1W) + P(2M, 2W) + P(1M, 3W) + P(0M, 4W)}$

$= \frac{\frac{105}{1365}}{\frac{1155}{1365}} = \frac{1}{11}$

7. (D)

Let the number of children in each family be x

Thus the total number of children in both the families are 2x

Now, it is given that 3 tickets are distributed amongst the children of these two families

Thus, the probability that all the three tickets go to the children in family B

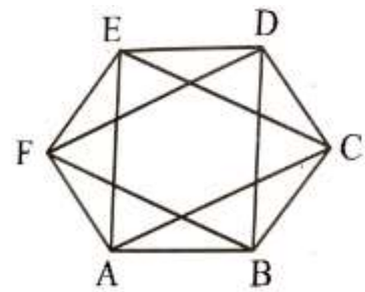
$$\frac{{}^{x}C_3}{{}^{2x}C_3} = \frac{1}{12} \Rightarrow \frac{x(x-1)(x-2)}{2x(2x-1)(2x-2)} = \frac{1}{12}$$

$$\Rightarrow \frac{(x-2)}{(2x-1)} = \frac{1}{3} \Rightarrow x = 5$$

Thus, the number of children in each family is 5.

8. (A)

Favorable no. of triangle i.e., equilateral triangles $(\Delta AEC \text{ and } \Delta BDF) = 2$



Hence, required probability = $\frac{2}{{}^{6}C_3} = \frac{1}{10}$

9. (A)

P = Set of students who opted for NCC
Q = Set of students who opted for NSS
n(P) = 40, n(Q) = 30, $n(P \cap Q) = 20$

$$n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$$

$$= 40 + 30 - 20 = 50$$

$\therefore$ Hence, required probability = $1 - \frac{50}{60} = \frac{1}{6}$

10. (B)

$\because A \subset B$; so $A \cap B = A$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \Rightarrow P\left(\frac{A}{B}\right) = \frac{P(A)}{P(B)}$

$\because P(B) \le 1 \Rightarrow P\left(\frac{A}{B}\right) \ge P(A)$

11. (C)

For an A.P. 2b = a + c (even), so both a and c even numbers or odd numbers from given numbers and b numbers will be fixed automatically.

Required probability = $\frac{{}^{6}C_2 + {}^{5}C_2}{{}^{11}C_3} = \frac{25}{165} = \frac{5}{33}$

12. (B)

$\because P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 1 - 0.8 = 0.2$

Now,

$\because P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$
$\Rightarrow \alpha = 0.6+0.4+0.5-0.2-\beta-0.3+0.2$
$\Rightarrow \beta = 1.2-\alpha$
$\because \alpha \in [0.85, 0.95]$ then $\beta \in [0.25, 0.35]$

13. (C)

$$P(\text{exactly one}) = \frac{2}{5}$$

$$\Rightarrow P(A)+P(B)-2P(A\cap B)=\frac{2}{5}$$

$$P(A \text{ or } B) = P(A\cup B) = \frac{1}{2}$$

$$\Rightarrow P(A)+P(B)-P(A\cap B)=\frac{1}{2}$$

$$\therefore P(A\cap B)=\frac{1}{2}-\frac{2}{5}=\frac{5-4}{10}=\frac{1}{10}=0.10$$

14. (B)

k= No. of times head occur consecutively

$$P(k=3)=\frac{5}{32}\{\text{HHHTH, HHHTT.THHT, TTHHH HT HHH}\}$$

$$P(k=5)=\frac{1}{32}\{\text{HHHHH}\}$$

$$P(\bar{3}\cap\bar{4}\cap\bar{5})=1-\left(\frac{5}{32}+\frac{2}{32}+\frac{1}{32}\right)=\frac{24}{32}$$

$\therefore$ Expected value of X

$$=\sum XP(x)=(-1)\frac{24}{32}+3\times\frac{5}{32}+4\times\frac{2}{32}+5\times\frac{1}{32}=\frac{1}{8}$$

15. (D)

Numbers of seven digit number = $\dfrac{7!}{2!3!2!}=7.6.5$

Numbers of seven digit even number = $\dfrac{6!}{2!2!2!}=6.5.3$

$$\therefore P=\frac{6.5.3}{7.6.5}=\frac{3}{7}$$

16. (A)

$$P(E)<\frac{1}{2}$$

$$\Rightarrow \sum_{r=n}^{8} {}^8C_r\left(\frac{1}{2}\right)^{8-r}\left(\frac{1}{2}\right)^{r}<\frac{1}{2}\Rightarrow \sum_{r=n}^{8} {}^8C_r\left(\frac{1}{2}\right)^{8}<\frac{1}{2}$$

$$\Rightarrow \left(\frac{1}{2}\right)^{8}\left({}^8C_n+{}^8C_{n+1}+......+{}^8C_n\right)<\frac{1}{2}$$

$$\Rightarrow {}^8C_n+{}^8C_{n+1}+......+{}^8C_8<128$$

$\Rightarrow 256-\left({}^{8}C_{0}+{}^{8}C_{1}+......+{}^{8}C_{n-1}\right)<128$

$\Rightarrow {}^{8}C_{0}+{}^{8}C_{1}+......+{}^{8}C_{n-1}>128$

$\Rightarrow n-1\geq 4\Rightarrow n\geq 5$

17. (D)

Total number of elements = 2022

$2022=2\times 3\times 337$

HGF (n, 2022) = 1

Is feasible when the value of 'n' and 2022 has no common factor.

A = Number which are divisible by 2 from {1, 2, 3,………., 2022}

$\Rightarrow \ n\left(A\right)\div 2=1011$

B = Number which are divisible by 3

From {1, 2, 3, …….., 2022}

n(B) = 674

$A\bigcap B$ = Number which are divisible by 6

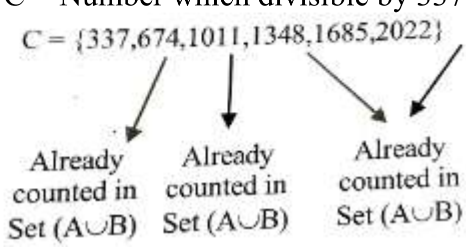
From {1, 2, 3,……., 2022}

6, 12, 18,………,2022

Here $n\left(A\bigcap B\right)=337$

$=1011+674-337=1348$

C = Number which divisible by 337 from {1,………………….,1022}



Total elements which are divisible by 2 or 3 or 337

= 1348 +2 = 1350

Favourable cases = Element which are neither divisible by 2, 3 or 337

=2022 – 1350 = 672

Required probability = $\dfrac{672}{2022}=\dfrac{112}{337}$

18. (C)

Let sample space n(S) = all 5 digit nos. = 9×10^{4}

Smallest 5 digit divisible by 7 is 10003

Largest 5 digit divisible by 7 is 99995

$\therefore\ 99995=10003+\left(n-1\right)7, n=12857$

Numbers divisible by 7 and 5 i.e. 35

99995 = 10010 + (P - 1) 35

$\Rightarrow\ 99995-10010+35=35P\Rightarrow P=2572$

$\therefore$ Numbers divisible by 7 but not by 35 are

12857 – 2572 = 10285

$\therefore\ P=\dfrac{10285}{90000} \qquad \therefore 9P=1.0285$

19. (B)

Given, $P(E_1) = \frac{2+3P}{6}, P(E_2) = \frac{2-P}{8}$ & $P(E_3) = \frac{1-P}{2}$

According to question,

$P(E_1) + P(E_2) + P(E_3) \le 1$

$\frac{2+3P}{6} + \frac{2-P}{8} + \frac{1-P}{2} \le 1$

$26 - 3P \le 24 \Rightarrow 2 \le 3P \Rightarrow P \ge \frac{2}{3}$

So, $\frac{2}{3} \le P \le 1$. Then, $P_1 = 1$ and $P_2 = \frac{2}{3}$

$P_1 + P_2 = \frac{5}{3}$

20. (A)

$x^2 + \alpha x + \beta > 0, \ \forall x \in R$

$D = \alpha^2 - 4\beta < 0$

$\alpha^2 < AB$

$\because \ \alpha, \beta \in \{1, 2, 3, 4, 5, 6\}$

Now, total cases = $6 \times 6 = 36$

Fav. Cases = $\beta = 1, \alpha = 1$ $\quad (\because \alpha^2 < 4)$

$\beta = 2, \alpha = 1, 2$ $\quad (\because \alpha^2 < 8)$

$\beta = 3, \alpha = 1, 2, 3$ $\quad$ so on

$\beta = 4, \alpha = 1, 2, 3, 4$

$\beta = 5, \alpha = 1, 2, 3, 4, 5$

$\beta = 6, \alpha = 1, 2, 3, 4, 5, 6$

Total favourable cases = 17

$P(x) = \frac{17}{36}$

21. (A)

Required case = exactly two digits are odd + exactly there 3 digits are odd

For exactly thres digits are odd then, the possible choices = $5 \times 5 \times 5 = 125$

For exactly two digits odd :

If 0 is used then : $2 \times 5 \times 5 = 50$

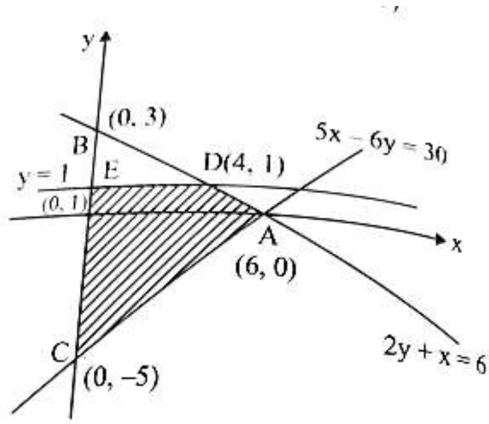
If 0 is not used then : ${}^3C_1 \times 4 \times 5 \times 5 = 300$

Favourable cases = 300 + 50 + 125 = 475

Required probability = $\frac{475}{900} = \frac{19}{36}$

22. (B)

Required probability = $\frac{ar(\Delta DEC)}{ar(ABC)}$



$$=1-\frac{ar(BDE)}{ar(ABC)}=1-\frac{\frac{1}{2}\times2\times4}{\frac{1}{2}\times8\times6}=1-\frac{1}{6}=\frac{5}{6}$$

23. (B)

Given conditions are

3R 4B 3W	2R 5B 2W

A : Drawn ball from boy II is black

B : Red ball transferred

Conditional Probability is $P\left(\frac{B}{A}\right)=\frac{P(A\cap B}{P(A)}$

$$=\frac{\frac{3}{9}\times\frac{5}{10}}{\frac{3}{9}\times\frac{5}{10}+\frac{4}{9}\times\frac{6}{10}+\frac{3}{9}\times\frac{5}{10}}$$

$$=\frac{15}{15+24+15}=\frac{15}{24}=\frac{5}{18}$$

24. (A)

Given that $P(A|B)=\frac{1}{7}\Rightarrow\frac{P(A\cap B)}{P(B)}=\frac{1}{7}$

$\Rightarrow\ P(B)=\frac{7}{9}$

$P(B|A)=\frac{2}{5}\Rightarrow\frac{P(A\cap B)}{P(A)}=\frac{2}{5}$

$\Rightarrow\ P(A)=\frac{5}{18}$

We know that $P(A'\cup B)=1-P(A\cup B)+P(B)$

$=1-P(A)+P(A\cap B)=\frac{5}{6}$

$P(A'\cap B)=1-P(A\cup B)$

$$= 1 - P(A) - P(B) + P(A \cap B) = \frac{1}{18}$$

$\Rightarrow$ Both (S1) and (S2) are true

25. (C)

Let matrix A is singular then |A|= 0

Let Prime Numbers are = (2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

Number of singular matrix = All entries are same + only two prime number are used in matrix.

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}\dots\begin{bmatrix} 29 & 29 \\ 29 & 29 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 2 & 5 \end{bmatrix}\dots\begin{bmatrix} 2 & 29 \\ 2 & 29 \end{bmatrix}$$

$$= 10 + 10 \times 9 \times 2 = 190$$

Required probability = $\frac{190}{10^4} = \frac{19}{10^3}$

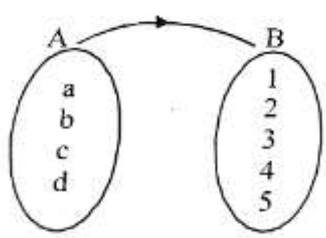
26. (A)

Total no. of relations = $2^{2\times 2} = 16$

Fav. Relation = $\phi, \{(x,x)\}, \{(y,y)\}, \{(x,x)(y,y)\}\{(x,x)(y,y)(x,y)(y,x)\}$

There relations are reflexive as well as transitive probability = $\frac{5}{16}$

27. (D)



Here no. of element in sample

$n(s) = 5 \times 4 \times 3 \times 2 = 120$

None for favourable cases

$f(a)$ +	$2f(b)$ =	$f(c)$ +	$f(d)$
5	2×1	3	4
4	2×2	3	5
1	2×3	2	5

$\therefore$ No. of favourable cases

$n(A) = 2! \times 3 = 6$

$$\therefore P(A) = \frac{\text{No. of favourable cases}}{\text{Total no. of cases}}$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{120} = \frac{1}{20}$$

28. (C)

No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1, 2, 3,…..,18.

$n(S) = {}^{18}C_5$

Fixed the value of x_2 and x_4 as 7 & 11.

$x_1(x_2)x_3(x_4)x_5$

Then, the number of choices of $x_1, x_3 \& x_5$ are shown below

$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$

$$P(E) = \frac{6\times3\times7}{{}^{18}C_5} \Rightarrow \frac{1}{17\times4} = \frac{1}{68}$$

29. (C)

Let $E_1 =$ denotes selection for 1^{st} bag

$E_2 =$ denotes selection for 2^{nd} bag

Then, $P(E_1) = P(E_2) = \frac{1}{2}$

F = selected balls are 1 red & 1 black

$$P\left(\frac{F}{E_1}\right) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{1}{5}$$

$$P\left(\frac{F}{E_1}\right) = \frac{{}^3C_1 \times {}^2C_1}{{}^{(n+5)}C_2} = \frac{12}{(n+5)(n+4)}$$

$$P\left(\frac{E_1}{F}\right) = \frac{P(E_1)\times P\left(\frac{F}{E_1}\right)}{P(E_1)\times P\left(\frac{F}{E_1}\right) + P(E_2)\times P\left(\frac{F}{E_2}\right)}$$

$$= \frac{\frac{1}{10}}{\frac{1}{10} + \frac{6}{(n+5)(n+4)}} = \frac{6}{11}$$

$11(n^2+9n+20) = 6(n^2+9n+80)$

$= 5n^2 + 45n - 260 = 0 \Rightarrow n = 4$

30. (D)

Let $\frac{P(\text{a prrime number})}{2} = \frac{P(\text{a composite})}{1} = \frac{P(1)}{3} = k$

So, P (a prime number) =2k,P(a composite number) $= k \& P(1) = 3k$

$\&3\times 2k + 2\times k + 3k = 1 \Rightarrow k = \frac{1}{11}$

P(success) =P(1 or 4) $= 3k + k = \frac{4}{11}$

Number of trials, $n = 2$

Therefore, mean $= np = 2\times\frac{4}{11} = \frac{8}{11}$

31. (C)

Given mean $=\alpha$, variance $=\dfrac{\alpha}{3}$ and

$P(x=1)=\dfrac{4}{243}$

$\overline{X}=nP=\alpha$ ....(i)

$\sigma^2=npq=\dfrac{\alpha}{3}$ [From (i)]

$\Rightarrow \alpha q=\dfrac{\alpha}{3}\Rightarrow q=\dfrac{1}{3}\Rightarrow P=1-q=\dfrac{2}{3}$

Here, $P(X=1)=\dfrac{4}{243}$

$${}^{n}C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^{n-1}=\frac{4}{243}$$

$$n\left(\frac{2}{3}\right)\left(\frac{1}{3^{n-1}}\right)=\frac{4}{243}=n=6$$

So, $P(X=4\,\text{or}\,X=5)={}^{6}C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2+{}^{6}C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1$

$$=\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)\times 9=\frac{16}{27}$$

32. (B)

Given mean and variance is $np=4$ and $npq=4/3$ respectively

Here $n=6, p=2/3, q=1/3$

$$54\left(P(X=2)+P(X=1)+P(X=0)\right)$$

$$54\left({}^{6}C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4+{}^{6}C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^5+{}^{6}C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6\right)$$

$$=\frac{146}{27}$$

33. (C)

Required binomial distribution is $B(7,p)$

So, $n=7; p=p$

Take $P(X=3)=5P(X=4)$

$${}^{7}C_3\times p^3(1-p)^4=5.{}^{7}C_4p^4(1-p)^3$$

$$\frac{{}^{7}C_3}{5\times{}^{7}C_4}=\frac{p}{1-p}\Rightarrow 1-p=5p\Rightarrow 6p=1$$

$\Rightarrow p=\dfrac{1}{6}\Rightarrow q=\dfrac{5}{6}$, Here $n=7$

Put the value of $n, p\,\&\,q$ in mean & variance

Mean $=np=7\times\dfrac{1}{6}=\dfrac{7}{6}$

Variance $= npq = 7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$

Sum $= \frac{7}{6} + \frac{35}{36} = \frac{42+35}{36} = \frac{77}{36}$

Therefore, the required sum is $\frac{77}{36}$.

34. (D)

We have probabilities as occurring of mark n is $\frac{1}{n}$.

Then, $P(n) = \frac{1}{n}; P(2) = \frac{1}{2}; P(B) = \frac{1}{8}$

$P(4) = \frac{1}{4}; P(16) = \frac{1}{16}; \Rightarrow P(32) = \frac{2}{32}$

Possible cases are 16,16,16 and 32,8,8

Therefore, Probability $= \frac{1}{16^3} + \frac{2}{12} \times \frac{1}{8} \times \frac{1}{8} \times 3 = \frac{13}{16^3} = \frac{13}{2^{12}}$

35. (D)

Given that,

A pair of dice is thrown 5 times.

Required Sample space is $\{(1,1),(1,2),(1,3)......(6,6)\}$

Total number of observations = 36

Now, a sum of is observed in $\{(1,4),(4,1),(2,3),(3,2)\}$

No. of favourable outcomes = 4

$P(\text{success})\, p = \frac{4}{36} = \frac{1}{9}$

$P(\text{failure})\, q = 1 - \frac{1}{9} = \frac{8}{9}$

We know that for a binomial distribution, $B(n, p)$

$P(X = k) = {}^nC_k\, p^k q^{n-k}$ where $k \geq 0$ and $p + q = 1$

$\therefore$ required probability

$\Rightarrow P(X \geq 4) = P(X = 4) + P(X = 5)$

$= {}^5C_4 \left(\frac{1}{9}\right)^4 \frac{8}{9} + {}^5C_5 \left(\frac{1}{9}\right)^5$

$= 5 \cdot \frac{8}{9^5} + \frac{1}{9^5} = \frac{41}{9^5}$

$= \frac{41 \times 3}{3^{10} \times 3} = \frac{123}{3^{11}}$

But given that $P(X \geq 4) = \frac{k}{3^{11}}$

$\Rightarrow \frac{k}{3^{11}} = \frac{123}{3^{11}}$

$\Rightarrow k = 123$

36. (A)

Let $X \equiv$ Event that product is defective, then

$$P\left(\frac{X}{A}\right)=\frac{3}{100}$$

$$P\left(\frac{X}{B}\right)=\frac{4}{100}$$

$$P\left(\frac{X}{C}\right)=\frac{2}{100}$$

Now, $P\left(\frac{C}{X}\right)=\frac{P(C)P\left(\frac{X}{C}\right)}{P(A)P\left(\frac{X}{A}\right)+P(B)P\left(\frac{X}{B}\right)+P(C)P\left(\frac{X}{C}\right)}$

$$\Rightarrow P\left(\frac{C}{X}\right)=\frac{\frac{50}{100}\times\frac{2}{100}}{\frac{20}{100}\times\frac{3}{100}+\frac{30}{100}\times\frac{4}{100}+\frac{50}{100}\times\frac{2}{100}}$$

$$\Rightarrow P\left(\frac{C}{X}\right)=\frac{100}{60+120+100}$$

$$\Rightarrow P\left(\frac{C}{X}\right)=\frac{5}{14}$$

37. (A)

Given, The probability that the random variable X takes values x is given by $P(X-x)=k(x+1)3^{-x}, x=0,1,2,3,.....,$ where k is a constant.

Now we know that,

$$P(X=0)+PP(x=1)+P(x=2)+.....=1$$

$$\Rightarrow \frac{k}{3^0}+\frac{2k}{3^1}+\frac{3k}{3^2}+.....=1$$

$$\Rightarrow k\left(1+\frac{2}{3}+\frac{3}{3^2}+.....\right)=1$$

Now finding,

$$S=1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+...... \quad ...(1)$$

$$\Rightarrow \frac{S}{3}=\frac{1}{3}+\frac{2}{3^2}+\frac{3}{2}+...... \quad ...(2)$$

Now on subtracting above two equation we get,

$$\Rightarrow \frac{2S}{3}=1+\frac{1}{3}+\frac{1}{3^2}+.....$$

$$\Rightarrow S=\frac{9}{4}$$

Hence, $k\left(1+\frac{2}{3}+\frac{3}{3^2}+....\right)=1$

$$\Rightarrow k=\frac{4}{9}$$

Now finding,

$P(X \geq 2) = P(2) + P(3) + ...$

$\Rightarrow\ P(X \geq 2) = 1 - P(0) - P(1)$

$\Rightarrow\ P(X \geq 2) = 1 - \left(\dfrac{k}{1} + \dfrac{2k}{3}\right) = 1 - \dfrac{20}{27} = \dfrac{7}{27}$

38. (D)

Given that,

$2^N < N!$

N is the sum of numbers of two dice.

$\Rightarrow\ 2 \leq N \leq 12$

Let us check the given condition $2^N < N!$

N = 1 (not possible) $\rightarrow$ 0

N = 2 (not possible) $\rightarrow$ 1

N = 3 (not possible) $\rightarrow$ 2

N = 4 (possible)

We need to find the probability for $N \geq 4$.

$\Rightarrow\ P(N \geq 4) = 1 - P(N = 2) - P(N = 3)$

$= 1 - \dfrac{1}{36} - \dfrac{2}{36}$

$= \dfrac{11}{12} = \dfrac{m}{n}$

Now let us find $4m - 3n$.

$= 4 \times 11 - 3 \times 12 = 8$.

39. (A)

Given that P (getting odd 7 times) = P (getting odd 9 times).

This is the case of binomial distribution with n successes and probability of success $= p = \dfrac{3}{6}$ and failure is $q = 1 - p = \dfrac{1}{2}$.

We know that $P(X = k) = {}^nC_k\,(p)^k\,(q)^{n-k}$

Now, $P(X = 7) = P(X = 9)$

$\Rightarrow\ {}^nC_7\left(\dfrac{1}{2}\right)^7\left(\dfrac{1}{2}\right)^{n-7} = {}^nC_9\left(\dfrac{1}{2}\right)^9\left(\dfrac{1}{2}\right)^{n-9}$

$\Rightarrow\ {}^nC_7 = {}^nC_9$

$\Rightarrow\ {}^nC_{n-7} = {}^nC_9$

$\Rightarrow\ n = 16$

$\Rightarrow\ P(2 \text{ times even}) = {}^{16}C_2\left(\dfrac{1}{2}\right)^{14}\left(\dfrac{1}{2}\right)^2$

$= \dfrac{15 \times 4}{2^{15}} = \dfrac{60}{2^{15}} = \dfrac{k}{2^{15}}$

$\Rightarrow\ k = 60$

40. (C)

Given that variance of $\alpha-\beta$ is $\frac{p}{q}$

$\alpha \in \{1, 2, 3, 4, 5, 6\}$

$\beta \in \{1, 2, 3, 4, 5, 6\}$

$(\alpha-\beta)=0$ (0 case)

$(\alpha-\beta)=-1$ (5 case)

$(\alpha-\beta)=-2$ (4 case)

$(\alpha-\beta)=-3$ (3 case)

$(\alpha-\beta)=-4$ (2 case)

$(\alpha-\beta)=-5$ (1 case)

$(\alpha-\beta)=1$ (5 case)

$(\alpha-\beta)=2$ (4 case)

$(\alpha-\beta)=3$ (3 case)

$(\alpha-\beta)=4$ (2 case)

$(\alpha-\beta)=5$ (1 case)

Mean = 0

We know that variance $=\dfrac{\sum_{i=1}^{n} f_i\left(x_i-\bar{x}\right)^2}{\sum_{i=1}^{n} f_i}$

Variance $=\sigma^2=\dfrac{0^2\times 6+2\times 1^2\times 5+2\times 2^2\times 4+2\times 3^2\times 3+2\times 4^2\times 2\times 5^2\times 1}{36}$

$$=\frac{2}{36}\times(5+16+27+32+25)=\frac{105}{18}=\frac{35}{6}$$

$\therefore\ p=35$

Sum of divisors of $p=1+5+7+35=48$

41. (C)

It is given that $P(H)=3P(T)$

$\therefore\ P(H)=\frac{3}{4},\ P(T)=\frac{1}{4}$

Since coin is tossed till either 1H or 3T occurs. So, process will end in the maximum of 3 throws.

X_i	1	2	3
$P(X_i)$	$\frac{3}{4}$	$\frac{1}{4}\cdot\frac{3}{4}$	$\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}$

Now we know that,

Mean is given by $\sum_{i=1}^{\infty} X_i P(X_i)$

Mean $= 1\times\frac{3}{4}+2\times\frac{3}{16}+3\times\frac{1}{16}=\frac{21}{16}$

42. (B)

Given that the difference between the mean and variance is 1.

$\Rightarrow\ np-npq=1$

$\Rightarrow\ np(1-q)=1$

$\Rightarrow\ np^2=1$

Also given that,

$\Rightarrow\ 2P(X=2)=3P(X=1)$

$\Rightarrow\ 2\cdot {}^nC_2 p^2 q^{n-2}=3\cdot {}^nC_1 p\cdot q^{n-1}$

$\Rightarrow\ 2\cdot\frac{n\cdot(n-1)}{2}\cdot p=3\cdot n\cdot q$

$\Rightarrow\ (n-1)p=3(1-p)$

$\Rightarrow\ \left(\frac{1}{p^2}-1\right)p=3(1-p)$

$\Rightarrow\ \frac{(1-p)(1+p)}{p}=3(1-p)$

$\Rightarrow\ 1+p=3p$

$\Rightarrow\ p=\frac{1}{2}$

$\therefore\ n=4$

Now,

$$n^2P(x>1)=n^2\left(1-P(x=1)-P(x=0)\right)=16\left(1-{}^4C_1\cdot\left(\frac{1}{2}\right)^4-\left(\frac{1}{2}\right)^4\right)=11$$

43. (C)

Given,

Bag have 6 white and 4 black balls.

Now also given die is rolled and the number denotes the number of ball drawn from the bag,

So, probability all drawn balls are white will be,

$$=\frac{1}{6}\left[\frac{{}^6C_1}{{}^{10}C_1}+\frac{{}^6C_2}{{}^{10}C_2}+\frac{{}^6C_3}{{}^{10}C_3}+\frac{{}^6C_4}{{}^{10}C_4}+\frac{{}^6C_5}{{}^{10}C_5}+\frac{{}^6C_6}{{}^{10}C_6}\right]$$

$$=\frac{1}{6}\left[\frac{6}{10}+\frac{15}{45}+\frac{20}{120}+\frac{15}{210}+\frac{6}{252}+\frac{1}{210}\right]$$

$$=\frac{504}{2520}$$

$$=\frac{1}{5}$$

44. (C)
Ω = sample space
$A=$ be an event
If $P(A)=0 \Rightarrow A=\phi$
If $P(A)=1 \Rightarrow A=\Omega$
Then both statement are true.

45. (B)
$M = 33\times 33$
$x(66-x)\geq \frac{5}{9}\times 33\times 33$
$11\leq x\leq 55$
$A:\{12,15,18,...54\}$
$p(A)=\frac{15}{45}=\frac{1}{3}$

46. (B)
$n(S)=36$
Given: $N-2, \sqrt{3N}$, $N+2$ are in G.P.
$3N=(N-2)(N+2)$
$3N=N^2-4$
$\Rightarrow N^2-3N-4=0$
$(N-4)(N+1)=0 \Rightarrow N=4$ or $N=-1$ rejected
(Sum = 4) $\equiv \{(1,3),(3,1),(2,2)\}$
$n(A)=3$
$P(A)=\frac{3}{36}=\frac{1}{12}=\frac{4}{48} \Rightarrow k=4$

47. (D)
Required probability $=1-\frac{D_{(15)}+{}^{15}C_1.D_{(14)}+{}^{15}C_2D_{(13)}}{15!}$
Taking $D_{(15)}$ as $\frac{15!}{e}$ $D_{(14)}$ as $\frac{14!}{e}$ $D_{(13)}$ as $\frac{13!}{e}$
We get, $1-\left(\frac{\frac{15!}{e}+15\cdot\frac{14!}{e}+\frac{15\times 14}{2}\times\frac{13!}{e}}{15!}\right)$
$=1-\left(\frac{1}{e}+\frac{1}{e}+\frac{1}{2e}\right)=1-\frac{5}{2e}\approx 0.08$

48. (B)
Let $P(w_1)=\lambda$, then $P(w_2)=\frac{\lambda}{2}$ .... $P(w_n)=\frac{\lambda}{m^{n-1}}$

As $\sum_{k=1}^{\infty} P(w_k) = 1 \Rightarrow \dfrac{\lambda}{1-\frac{1}{2}} = 1 \Rightarrow \lambda = \dfrac{1}{2}$

So, $P(w_n) = \dfrac{1}{2^n}$

$A = \{2k + 3\ell; k, \ell \in \mathbb{N}\} = \{5, 7, 8, , 9, 10, ...\}$

$B = \{w_n : n \in A\}$

$B = \{w_5, w_7, w_8, w_9, w_{10}, w_{11}, ...\}$

$A = \mathbb{N} - \{1, 2, 2, 4, 6\}$

$\therefore\ P(B) = 1 - \left[P(w_1) + P(w_2) + P(w_3) + P(w_4) + P(w_6)\right]$

$= 1 - \left[\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \dfrac{1}{16} + \dfrac{1}{64}\right]$

$= 1 - \dfrac{32 + 16 + 8 + 4 + 1}{64} = \dfrac{3}{64}$

49. (B)

Either all outcomes are positive or any two are negative.

Now, $p = P$ (positive) $= \dfrac{3}{6} = \dfrac{1}{2}$

$q = p$ (negative) $= \dfrac{2}{6} = \dfrac{1}{3}$

Required probability $= {}^5C_5\left(\dfrac{1}{2}\right)^5 + {}^5C_2\left(\dfrac{1}{3}\right)^2\left(\dfrac{1}{2}\right)^3 + {}^5C_4\left(\dfrac{1}{3}\right)^4\left(\dfrac{1}{2}\right)^1$

$= \dfrac{521}{2592}$

$\therefore$ Option (B) is correct.

50. (A)

$$\frac{{}^5C_2 + {}^6C_2}{{}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^8C_2} = \frac{10 + 15}{1 + 3 + 6 + 10 + 15} = \frac{25}{35} = \frac{5}{7}$$

51. (B)

$np + npq = 5, np \cdot npq = 6$

$np(1+q) = 5, n^2p^2q = 6$

$n^2p^2(1+q)^2 = 25, n^2p^2q = 6$

$\dfrac{6}{q}(1+q)^2 = 25$

$6q^2 + 12q + 6 = 25q$

$6q^2 - 13q + 6 = 0$

$6q^2 - 9q - 4q + 6 = 0$

$(3q-2)(2q-3) = 0$

$q=\frac{2}{3},\frac{3}{2}, q=\frac{2}{3}$ is accepted

$p=\frac{1}{3}\Rightarrow\ n\cdot\frac{1}{3}+n\cdot\frac{1}{3}\cdot\frac{2}{3}=5$

$\frac{3n+2n}{9}=5$

$n=9$

So, $6(n+p-q)=6\left(9+\frac{1}{3}-\frac{2}{3}\right)=52$

52. (A)

A : no. on 1st die < no. on 2nd die

A : no. on 1st die = even & no. of 2nd die = odd

C : no. on 1st die = odd & no. on 2nd die = even

$n(A)=5+4+3+2+15$

$n(B)=9$

$n(C)=9$

$n((A\cup B)\cap C)=(A\cap C)\cup(B\cap C)$

$=(3+2+1)+0=6$

53. (D)

Given that three dice are thrown.

The total number of outcomes when three dice are thrown together is $6^3=6\times6\times6$

The number of outcomes such that all the outcomes are different is $=6\times5\times4$.

For ex: If the outcome in dice 1 is "6" then the number of outcomes for dice 2 should be $5(1, 2, 3, 4, 5)$ which excludes the outcome "6".

Hence, the required probability is $=\frac{6\times5\times4}{6\times6\times6}$

$=\frac{20}{36}=\frac{5}{9}=\frac{p}{q}$

$\Rightarrow\ q-p=9-5=4$

Therefore, the required answer is 4.

54. (30)

$\because\sum \mathrm{P}(\mathrm{X})=1$

$\therefore \mathrm{k}+2\mathrm{k}+2\mathrm{k}+3\mathrm{k}+\mathrm{k}=1\Rightarrow \mathrm{k}=\frac{1}{9}$

Now, $\mathrm{P}=\mathrm{P}\left(\frac{1<\mathrm{X}<4}{\mathrm{X}<3}\right)=\frac{\mathrm{P}(\mathrm{X}=2)}{\mathrm{P}(\mathrm{X}<3)}=\frac{\frac{2\mathrm{k}}{9\mathrm{k}}}{\frac{\mathrm{k}}{9\mathrm{k}}+\frac{2\mathrm{k}}{9\mathrm{k}}}=\frac{2}{3}$

$\Rightarrow \mathrm{p}=\frac{2}{3}$

Now, $5\mathrm{p}=\lambda\mathrm{k}$

$\Rightarrow (5)\left(\frac{2}{3}\right) = \lambda(1/9)$

$\Rightarrow \lambda = 30$

55. (6)

Let $p(E_1) = x, p(E_2) = y$ and $p(E_3) = z$

$\alpha = p(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = p(E_1).p(\bar{E}_2).p(\bar{E}_3)$

$\Rightarrow \alpha = x(1-y)(1-z)$ ....(i)

Similarly,

$\beta = (1-x).y(1-z)$ .....(ii)

$\gamma = (1-x)(1-y)(1-z)$ .....(iii)

$p = (1-x)(1-y)(1-z)$ .....(iv)

From (i) and (iv),

$\frac{x}{1-x} = \frac{\alpha}{p} \Rightarrow x = \frac{\alpha}{\alpha + p}$

From (iii) and (iv),

$\frac{z}{1-z} = \frac{\gamma}{p} \Rightarrow z = \frac{\gamma}{\gamma + p}$

$\frac{p(E_1)}{p(E_2)} = \frac{x}{z} = \frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}} = \frac{\frac{\gamma+p}{\gamma}}{\frac{\alpha+p}{\alpha}} = \frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}}$ ....(v)

Given that

$(\alpha - 2\beta)p = \alpha\beta \Rightarrow \alpha p = (\alpha + 2p)\beta$ .....(vi)

$(\beta - 2\gamma)p = 2\beta\gamma \Rightarrow 3\gamma p = (p - 2\gamma)\beta$ .....(vii)

From (vi) and (vii),

$\frac{\alpha}{3\gamma} = \frac{\alpha + 2p}{p - 2\gamma} \Rightarrow p\alpha - 6p\gamma = 5\gamma\alpha$

$\Rightarrow \frac{p}{\gamma} - \frac{6p}{\alpha} = 5 \Rightarrow \frac{p}{\gamma} + 1 = 6\left(\frac{p}{\alpha} + 1\right)$ .....(viii)

From (v) and (viii),

$\frac{p(E_1)}{p(E_3)} = 6$

56. (33)

Total number of numbers according to the condition

n(S) = $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Divisible by 21 when divided by 3.

Case – I : All $1 \rightarrow (1)$

Case – II : All $8 \rightarrow (1)$

Case – III : 3 Ones & 3 eights

$\frac{6!}{3!\times 3!}=20$

Required probability

$p=\frac{22}{64}$

$96p=96\times\frac{22}{64}=33$

57. (75)

$0\le\frac{1-d}{4}\le 1 \quad\Rightarrow\quad -3\le d\le 1$ .....(i)

$0\le\frac{1-2d}{4}\le 1 \quad\Rightarrow\quad -\frac{3}{4}\le d\le\frac{3}{2}$ .....(ii)

$0\le\frac{1-4d}{4}\le 1 \quad\Rightarrow\quad -\frac{3}{4}\le d\le\frac{1}{4}$ .....(iii)

$0\le\frac{1-3d}{4}\le 1 \quad\Rightarrow\quad -\frac{1}{3}\le d\le 1$ .....(iv)

From (i), (ii) and (iv)

$-\frac{1}{3}\le d\le\frac{1}{4}$ Minimum value of d = $-\frac{1}{3}$

Mean = $0+\frac{1+2d}{4}+\frac{2(1-4d)}{4}+\frac{3(1+3d)}{4}$

$\bar{X}=\frac{6+3d}{4}=\frac{1}{4}\left(6-3\times\frac{1}{3}\right)=\frac{5}{4}\Rightarrow 60\bar{X}=60\times\frac{5}{4}=75$

58. (19)

Given that A is subset of S hence, A can have elements:

Type 1: {}

Type 2: $\{E_1\},\{E_2\},.....,\{E_8\}$

Type 3: $\{E_1,E_2\},\{E_1,E_3\},.....,\{E_1,E_8\}$

⋮

⋮

Type 6 : $\{E_1,E_2,....E_5\},......,\{E_4,E_5,E_6,E_7,E_8\}$

Type 7: $\{E_1,E_2,....E_6\},......,\{E_3,E_4,......,E_8\}$

Type 8: $\{E_1,E_2,....E_7\},......,\{E_2,E_3,......,E_8\}$

Type 9: $\{E_1,E_2,....E_7\}$

As $P(A)\ge\frac{4}{5}$;

Note: Type 1 to Type 4 elements can not be in set A as maximum probability of type 4 elements.

$\{E_5,E_6,E_7,E_8\}$ is $\frac{5}{36}+\frac{6}{36}+\frac{7}{36}+\frac{8}{36}=\frac{13}{18}<\frac{4}{5}$

Now for type 5 acceptable elements let's call probability as P_5

$P_5=\frac{n_1+n_2+n_3+n_4+n_5}{36}\le\frac{4}{5}$

$\Rightarrow\ n_1+n_2+n_3+n_4+n_5 \ge 29$

Hence, 2 possible ways $\{E_5, E_6, E_7, E_8, E_3 \text{ or } E_4\}$

$P_6 = n_1+n_2+n_3+n_4+n_5 \ge 29$

$\Rightarrow$ 9 possible ways

$P_7 = n_1+n_2+........+n_7 \ge 29$

$\Rightarrow$ 7 possible ways

$P_8 = n_1+n_2+........+n_8 \ge 29$

$\Rightarrow$ 1 possible ways

Total number ways = $2+9+7+1=19$

59. (27)

Given a quadratic equation $64x^2+5nx+1=0$ has no real roots, so

$D<0$

$\Rightarrow\ N^2-4\times 64<0$

$\Rightarrow\ N<\frac{16}{5}\ \Rightarrow\ N=1,2,3$

And, For a biased coin, the probability of getting head is $P=\frac{1}{4}$, and probability of getting tail will be $Q=\frac{3}{4}$

So, required probability is

$$\frac{1}{4}+\frac{3}{4}\cdot\frac{1}{4}+\left(\frac{3}{4}\right)^2\cdot\frac{1}{4}+\left(\frac{3}{4}\right)^3\cdot\frac{1}{4}=\frac{p}{q}$$

$$\Rightarrow \frac{\frac{1}{4}\left(1-\left(\frac{3}{4}\right)^3\right)}{1-\left(\frac{3}{4}\right)}=\frac{p}{q}$$

$$\Rightarrow 1-\left(\frac{3}{4}\right)^3=\frac{p}{q}$$

$$\Rightarrow 1-\frac{27}{64}=\frac{p}{q}$$

$$\Rightarrow \frac{p}{q}=\frac{37}{64}$$

$q-p=27$

60. (10)

Given,

A fair $n(n>1)$ faces die is rolled repeatedly until a number less than n appears,

x_i	1	2	3	4	.....
p_i	$\frac{n-1}{n}$	$\frac{1}{n}\cdot\left(\frac{n-1}{n}\right)$	$\frac{1}{n^2}\cdot\frac{n-1}{n}$	$\frac{1}{n^3}\cdot\left(\frac{n-1}{n}\right)$	.....

Now we know that, mean is given by,

$$\text{Mean} = \sum_{i=1}^{\infty} p_i x_i = 1 \cdot \frac{n-1}{n} + \frac{2}{n} \cdot \left(\frac{n-1}{n}\right) + \frac{3}{n^2}\left(\frac{n-1}{n}\right) + ...$$

$$\Rightarrow \frac{n}{9} = \left(1 - \frac{1}{n}\right) S \qquad ...(1)$$

Where, $S = 1 + \frac{2}{n} + \frac{3}{n^2} + \frac{4}{n^3} + ...$ ...(2)

$$\frac{1}{n} S = \frac{1}{n} + \frac{2}{n^2} + \frac{3}{n^3} + ... \qquad ...(3)$$

Now subtracting equation (2) – (3) we get,

$$\left(1 - \frac{1}{n}\right) S = 1 + \frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} + ...$$

$$\Rightarrow \left(1 - \frac{1}{n}\right) S = \frac{1}{1 - \frac{1}{n}}$$

Now putting the value of S in equation (1) we get,

$$\Rightarrow \frac{n}{9} = \left(1 - \frac{1}{n}\right) \times \frac{1}{\left(1 - \frac{1}{n}\right)^2} = \frac{n}{n-1}$$

$$\Rightarrow n = 10$$

61. (120)

$x + y = 5\lambda$

Cases :

x	**y**	**Number of ways**
5λ	5λ	20
$5\lambda + 1$	$5\lambda + 4$	25
$5\lambda + 2$	$5\lambda + 3$	25
$5\lambda + 3$	$5\lambda + 2$	25
$5\lambda + 4$	$5\lambda + 1$	25

62. (9)

$E_1 =$ Smokers

$$P(E_1) = \frac{1}{4}$$

$E_2 =$ non-smokers

$$P(E_2) = \frac{3}{4}$$

E = diagnosed with lung cancer

$$P\left(\frac{E}{E_1}\right) = \frac{27}{28}$$

$$P\left(\frac{E}{E_2}\right) = \frac{1}{28}$$

$$P\left(\frac{E_1}{E}\right)=\frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E)}$$

$$=\frac{\frac{1}{4}\times\frac{27}{28}}{\frac{1}{4}\times\frac{27}{28}+\frac{3}{4}\times\frac{1}{28}}=\frac{27}{30}=\frac{9}{10}$$

$K=9$

63. (14)

$$p=\frac{{}^6C_1}{6\times6}=\frac{1}{6}$$

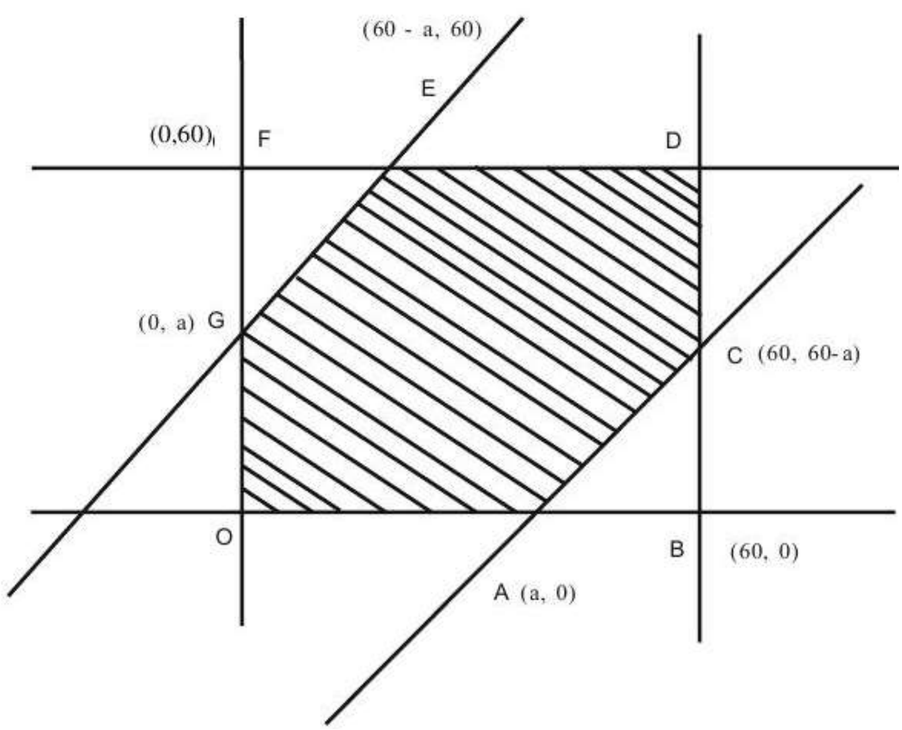
$$q=\frac{{}^6C_1\times{}^5C_1\times4}{6\times6\times6\times6}=\frac{5}{54}$$

$\therefore\ p:q=9:5\ \Rightarrow\ m+n=14$

64. (10)

$|x-y|<a\ \Rightarrow\ -a<x-y<a$

$\Rightarrow\ x-y<a$ and $x-y>-a$



$$P(A)=\frac{\text{ar}(OACDEG)}{(OBDF)}$$

$$=\frac{\text{ar}(OBDF)-\text{ar}(ABC)-\text{ar}(EFG)}{\text{ar}(OBDF)}$$

$$\Rightarrow\ \frac{11}{36}=\frac{(60)^2-\frac{1}{2}(60-a)^2-\frac{1}{2}(60-a)^2}{3600}$$

$\Rightarrow 1100 = 3600 - (60 - a)^2$

$\Rightarrow (60 - a)^2 = 2500 \Rightarrow 60 - a = 50$

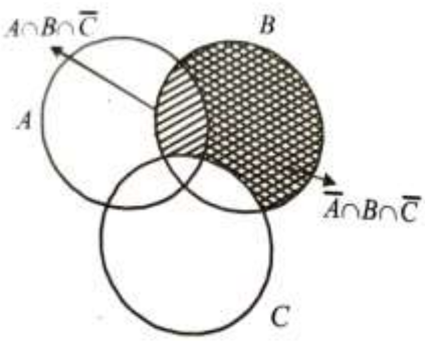
$\Rightarrow a = 10$

Only One Option Correct

1. (A)

Given: $P(B)=3/4, P(A\cap B\cap \bar{C})=1/3$

$P(\bar{A}\cap B\cap \bar{C})=1/3$



From above venn diagram, we see

$B\cap C=B-(A\cap B\cap \bar{C})-(\bar{A}\cap B\cap \bar{C})$

$\Rightarrow P(B\cap C)=P(B)-P(A\cap B\cap \bar{C})-P(\bar{A}\cap B\cap \bar{C})$

$\Rightarrow P(B\cap C)=\frac{3}{4}-\frac{1}{3}-\frac{1}{3}=\frac{9-4-4}{12}=\frac{1}{12}$

2. (C)

Given that E, F, G are pairwise independent events.

$=P(E^C\cap F^C/G)=\frac{P((E^C\cap F^C)\cap G)}{P(G)}$

$=\frac{P((E^C\cap F^C)\cap G)}{P(G)}=\frac{P(G)-P((E\cup F)\cap G)}{P(G)}$

$=\frac{P(G)-P((E\cap F)\cup(F\cap G))}{P(G)}$

$=\frac{P(G)-P(E\cap F)-P(F\cap G)+P(E\cap F\cap G)}{P(G)}$

$=\frac{P(G)-P(E)-P(G)-P(F).P(G)+0}{P(G)}$

$=\frac{P(G)-P(E)-P(G)-P(F).P(G)+0}{P(G)}$ $\quad [\because P(E\cap F\cap G)=0]$

$=\frac{P(G)[1-P(E)-P(F)]}{P(G)}$

$=P(E^C)-P(F)$

3. (D)

We know that $P(H_i/E)=\dfrac{P(H_i\cap E)}{P(E)}$

$=\dfrac{P(E/H_i)P(H_i)}{P(E)}$

$\Rightarrow \quad P(E)=\dfrac{P(E/H_i)P(H_i)}{P(H_i/E)}$

Now, given that 0 < P (E) < 1

$\Rightarrow \; 0<\dfrac{P(E/H_i)P(H_i)}{P(H_i/E)}<1$

$\Rightarrow P(E/H_i)P(H_i)<P(H_i/E)$

But if $P(H_i\cap E)=0$ then

$P(H_i/E)=P(E/H_i)=0$

Then $P(E/H_i)=P(H_i)<P\,P(H_i/E)$ is not true.

$\therefore$ Statement – I is not always true.

Also as $H_1,H_2,.....,H_n$ are mutually exclusive and exhaustive events.

$\therefore \; P(H_1)+P(H_2)+.....+P(H_n)=\sum_{i=1}^{n}P(H_i)=1$

$\therefore$ Statement – II is true.

4. (D)

Given that A and B to be independent events

$\therefore \; P(A\cap B)=P(A)P(B)\Rightarrow \dfrac{n(A\cap B)}{n(S)}=\dfrac{n(A)}{n(S)}\times\dfrac{n(B)}{n(S)}$

$\Rightarrow \; \dfrac{n(A\cap B)}{10}=\dfrac{4}{10}\times\dfrac{a}{10}\Rightarrow n(A\cap B)=\dfrac{5}{2}y$

$\Rightarrow \; n(A\cap B)$ has to be integer, we have b = 5 or 10

$\therefore \; n(B)=5$ or 10

5. (B)

The given system of equations are

ax + by = 0, cx + dy = 0, where $a,\ b,\ c,\ d\in\{0,\ 1\}$

$\because$ the system of equations have unique solution,

$\therefore \; \dfrac{a}{c}\neq\dfrac{b}{d}$ i.e. $\begin{vmatrix} a & b \\ c & d \end{vmatrix}\neq 0$

This condition is satisfy by the following cases –

$\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix},\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix},\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix},\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix},\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix},\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$\therefore$ Number of favourable cases for the system of equations have unique solution = 6.

Total possible cases of $a,b,c,d\in\{0,1\}=2^4=16$

$\therefore$ Required probability = $\frac{6}{16} = \frac{3}{8}$

$\therefore$ Statement - 1 is true.

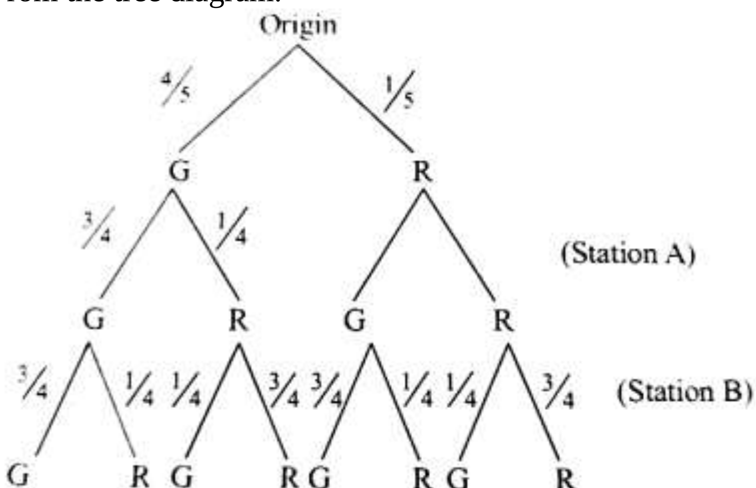
$\therefore$ Homogeneous system of equations always has a solution
(Trival solution x = 0, y = 0 or many solution)

$\therefore$ The probability that the system of equations has a solution is 1.

Hence the statement-2 is true but is not a correct explanation of statement- 1.

6. (C)

From the tree diagram.



P (original signal is green / signal received at *B* is green)

$$= \frac{P(GGG)+P(GRG)}{P(GGG)+P(GRG)+P(RGG)+P(RRG)}$$

$$= \frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}}$$

$$= \frac{\frac{4}{5}\times\frac{10}{16}}{\frac{4}{5}\times\frac{10}{16}+\frac{1}{5}\times\frac{6}{16}} = \frac{40}{40+6} = \frac{40}{46} = \frac{20}{23}$$

7. (C)

Given, $\omega^{r_1}+\omega^{r_2}+\omega^{r_3}=0$. If ω is a complex cube root of unity then,

Sum of consecutive power ω is zero

$\omega^{3m}+\omega^{3m+1}+\omega^{3m+2}=0$

Where m, is integer.

r_1, r_2, r_3 are the numbers obtained on die, these can take any value from 1 to 6.

$\therefore$ m can take values 1 or 2 for r_1, values o or for r_2 and values o or 1 for r_3

$\therefore$ Number of ways of selecting r_1, r_2, r_3

$= {}^2C_1 \times {}^2C_1 \times {}^2C_1 \times 3!$

Also the total number of ways of getting r_1, r_2, r_3 on die = $6\times6\times6$

$\therefore$ Required probability = $\dfrac{{}^2C_1 \times {}^2C_1 \times {}^2C_1 \times 3!}{6\times 6\times 6} = \dfrac{2}{9}$

8. (A)

D_4 can show a number appearing on one of D_1, D_2 and D_3 in the following cases.

Case I: D_4 shows a number which is shown by exactly one of D_1, D_2 and D_3.

D_4 shows a number in ${}^6C_1$ ways.

One out of D_1, D_2 and D_3 can be selected in ${}^3C_1$ ways.

[The selected die shows the same number as on D_4 in one way and rest two dice show the different number in 5 ways each.]

$\therefore$ Number of ways

$= {}^6C_1 \times {}^3C_1 \times 1 \times 5 \times 5 = 450$

Case II: D_4 shows a number which is shown by exactly two of D_1, D_2 and D_3.

Number of ways

$= {}^6C_1 \times {}^3C_2 \times 1 \times 1 \times 5 = 90$

Case III: D_4 shows a number which is shown by all three dice D_1, D_2 and D_3.

$= {}^6C_1 \times {}^3C_3 \times 1 \times 1 \times 1 = 6$

$\therefore$ Total number of favourable ways = 450 + 90 + 6 = 546

Total ways = $6\times 6\times 6\times 6$

$\therefore$ Required probability = $\dfrac{546}{6\times 6\times 6\times 6} = \dfrac{91}{216}$

9. (A)

P (at least one of them solves the problem)

$= 1 - P$(none of them solves it)

$= 1 - \left(1-\frac{1}{2}\right)\left(1-\frac{3}{4}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{8}\right)$

$= 1 - \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}\times\frac{7}{8} = 1 - \frac{21}{256} = \frac{235}{256}$

10. (A)

According to given condition, following cases may arise

GGBBB BGGBB

GBGBB, BGBGB,

GBBGB

Thus favourable cases are $= 5\times 2!\times 3! = 60$

Total ways in which 5 persons can be seated $= 5! = 120$

$\therefore$ Required probability $= \dfrac{60}{120} = \dfrac{1}{2}$

11. (C)

Given $P(T_1) = \dfrac{20}{100}, P(T_2) = \dfrac{80}{100}, P(D) = \dfrac{7}{100}$

Let $P\left(\dfrac{D}{T_2}\right) = P$, then $P\left(\dfrac{D}{T_1}\right) = 10P$

By total probability,

$$P(D)=P(T_1)P\left(\frac{D}{T_1}\right)+P(T_2)P\left(\frac{D}{T_2}\right)$$

$$\Rightarrow \frac{7}{100}=\frac{20}{100}\times 10P+\frac{80}{100}\times P$$

$$\Rightarrow \frac{7}{280}=P \Rightarrow P=\frac{1}{40}$$

$$\therefore P\left(\frac{D}{T_1}\right)=\frac{10}{40} \text{ and } P\left(\frac{D}{T_2}\right)=\frac{1}{40}$$

$$\Rightarrow P\left(\frac{\bar{D}}{T_1}\right)=1-\frac{10}{40}=\frac{30}{40} \text{ and } P\left(\frac{\bar{D}}{T_2}\right)=1-\frac{1}{40}=\frac{39}{40}$$

$$P\left(\frac{T_2}{\bar{D}}\right)=\frac{P\left(\frac{\bar{D}}{T_2}\right)P(T_2)}{P\left(\frac{\bar{D}}{T_1}\right)P(T_1)+P\left(\frac{\bar{D}}{T_2}\right)P(T_2)}$$

$$=\frac{\frac{80}{100}\times\frac{39}{40}}{\frac{20}{100}\times\frac{30}{40}+\frac{80}{100}\times\frac{39}{40}}=\frac{156}{186}=\frac{78}{93}$$

12. (B)

We know the total number of ways of dividing n identical things among r persons $= {}^{n+r-1}C_{r-1}$

Total number of non negative solutions of x + y + z = 10 are $= {}^{12}C_2 = 66$

If z is even then there can be following cases arise:

Z= 0 $\Rightarrow$ No. of ways of solving x + y = 10 $\Rightarrow$ ${}^{11}C_1$

Z = 2 $\Rightarrow$ No. of ways of solving x + y = 8 $\Rightarrow$ ${}^{9}C_1$

Z = 4 $\Rightarrow$ No. of ways of solving x + y = 6 $\Rightarrow$ ${}^{7}C_1$

Z = 6 $\Rightarrow$ No. of ways of solving x + y = 4 $\Rightarrow$ ${}^{5}C_1$

Z = 8 $\Rightarrow$ No. of ways of solving x + y = 2 $\Rightarrow$ ${}^{3}C_1$

Z = 10 $\Rightarrow$ No. of ways of solving x + y = 0 $\Rightarrow$ 1

$\therefore$ Total ways when z is even = 11 + 9 + 7 + 5 + 3 + 1 = 36

$\therefore$ Required probability = $\frac{36}{66}=\frac{6}{11}$

13. (B)

Probability of getting head on coin $C_1 = P(H)=\frac{2}{3}$

Probability of getting head on coin $C_2 = P(H)=\frac{1}{3}$

For the coin C_1

No. of Heads (α)	0	1	2

Probability	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$

For the coin C_2

No. of Heads (α)	0	1	2
Probability	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

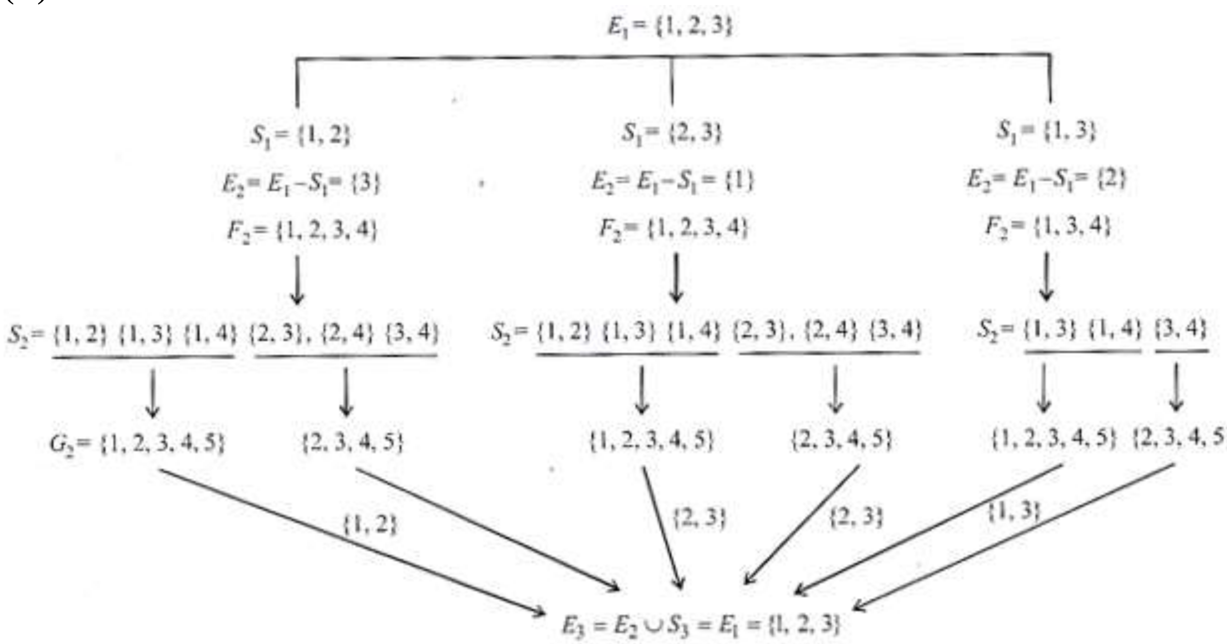
For real and equal roots of the given polynomial

$\alpha^2 - 4\beta = 0 \Rightarrow \alpha^2 = 4\beta$

$\therefore\ (\alpha,\beta) = (0,0),(2,1)$

So, probability = $\frac{1}{9}\times\frac{4}{9}+\frac{4}{9}\times\frac{4}{9}=\frac{20}{81}$

14. (A)



$$P=\frac{P\left(S_1\cap\left(E_1=E_3\right)\right)}{P\left(E_1=E_3\right)}=\frac{P\left(A_1,2\right)}{P(A)}$$

$$P(A)=P\left(A_{1,2}\right)+P\left(A_{1,3}\right)+P\left(A_{2,3}\right)$$

$$P(A)=P\left(A_{1,2}\right)+P\left(A_{1,3}\right)+P\left(A_{2,3}\right)$$

$$\begin{array}{ccc}\uparrow & \uparrow & \uparrow \\ \text{If } 1,2 & \text{If } 1,3 & \text{If } 2,3 \\ \text{Chosen} & \text{Chosen} & \text{Chosen} \\ \text{at start} & \text{at start} & \text{at start}\end{array}$$

$$P\left(A_{1,2}\right)=\frac{1}{3}\times\frac{1\times {}^3C_1}{\underbrace{{}^4C_2}_{\substack{\text{1 is definitely}\\\text{chosen from } F_2}}}\times\frac{1}{\underbrace{{}^5C_2}_{\substack{1,2\text{ chosen}\\\text{from } G_2}}}$$

$$=\frac{1}{3}\times\frac{1}{2}\times\frac{1}{10}=\frac{1}{60}$$

$$P\left(A_{1,3}\right)=\frac{1}{3}\times\underbrace{\frac{1\times{}^{2}C_{1}}{{}^{3}C_{2}}}_{\text{1 is definitely chosen from } F_2}\times\underbrace{\frac{1}{{}^{5}C_{2}}}_{\text{1,2 chosen from } G_2}$$

$$=\frac{1}{3}\times\frac{2}{3}\times\frac{1}{10}=\frac{1}{45}$$

$$P\left(A_{2,3}\right)=\frac{1}{3}\times\left[\underbrace{\frac{{}^{3}C_{1}\times1}{{}^{4}C_{2}}}_{\text{1 is not chosen from } F_2}\times\frac{1}{{}^{4}C_{2}}+\underbrace{\frac{1\times{}^{3}C_{1}}{{}^{4}C_{2}}}_{\text{1 is chosen from } F_2}\times\frac{1}{{}^{5}C_{2}}\right]$$

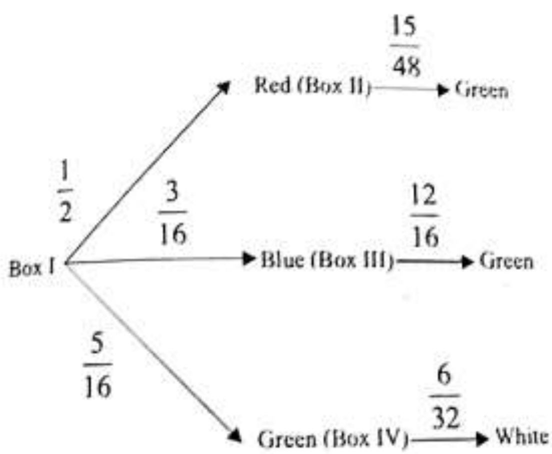
$$=\frac{1}{3}\left(\frac{1}{12}+\frac{1}{20}\right)=\frac{2}{45}$$

$$P(A)=\frac{1}{60}+\frac{1}{45}+\frac{2}{45}=\frac{1}{12}$$

$$\frac{P\left(A_{1,2}\right)}{P(A)}=\frac{1}{5}$$

15. (C)

According to question



Event E : One of the chosen ball is white

F : At least one of the chosen ball is green

$$\therefore\quad P\left(\frac{E}{F}\right)=\frac{\frac{5}{16}\times\frac{6}{32}}{\frac{1}{2}\times\frac{15}{48}+\frac{3}{16}\times\frac{12}{16}+\frac{5}{16}\times1}=\frac{5}{52}$$

16. (A)

	Box -1	Box-2	Box-3	Box-4
	3R 2B	3R 2B	3R 2B	3R 2B
Case I	4 Balls	2 Balls	2 Balls	2 Balls

Case II 3 Balls 3 Balls 2 Balls 2 Balls

2R 1B 2B 1R 2R 1B 2B 1R

$$^4C_1\left[{}^3C_3.{}^2C_1+{}^3C_2.{}^2C_2\right]\left({}^3C_1\,{}^2C_1\right)^3+{}^4C_2\left[{}^3C_2\,{}^2C_1+{}^3C_1\,{}^2C_2\right]^2\left[{}^3C_1\,{}^2C_1\right]^2$$

$$=4(5)(6)^3+6(3\times2+3)^2(6)^2$$

$$=4320+17496$$

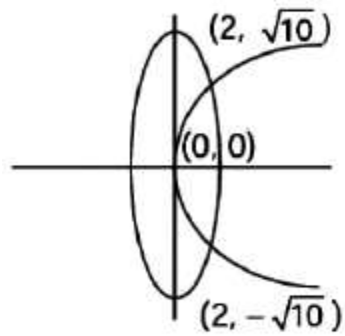
$$=21816 \text{ (Option a)}$$

17. (B)

$$\frac{x^2}{8}+\frac{y^2}{20}<1 \text{ and } y^2<5x$$

$$\frac{x^2}{8}+\frac{y^2}{20}=1 \quad \ldots(1)$$

$$y^2=5x \quad \ldots(2)$$



On solving (1) and (2)

$$\frac{x^2}{8}+\frac{x}{4}=1$$

$$x^2+2x=8$$

$$x^2+2x-8=0$$

$$x=2,-4$$

$$X=\{(1,1),(1,0),(1,-1),(1,2),(1,-2),(2,1),(2,-1),(2,3),(2,-3),(2,-2),(2,2),(2,0)\}$$

$$n(S)={}^{12}C_3$$

A is event of selecting 3 points for which area of Δ is positive integer.

$$n(A)=4\times7+9\times5=73$$

$$P(A)=\frac{73}{{}^{12}C_3}=\frac{73}{220}$$

18. (B)

$$P(H)=\frac{1}{3},\ P(T)=\frac{2}{3}$$

$$P=P(H\,H)+\{P(T\,H\,H)+P(T\,H\,T\,H\,H)+P(T\,H\,T\,H\,T\,H\,H)+\ldots.\infty\}$$
$$+\{P(H\,T\,H\,H)+P(H\,T\,H\,T\,H\,H)+P(H\,T\,H\,T\,H\,T\,H\,H)+\ldots.\infty\}$$

$$=\frac{1}{9}+\left(\frac{2}{3}\times\frac{1}{9}+\frac{2}{3}\times\frac{1}{3}\times\frac{2}{3}\times\frac{1}{9}+\ldots..\infty\right)+\left(\frac{1}{3}\times\frac{2}{3}\times\frac{1}{9}+\frac{1}{3}\times\frac{2}{3}\times\frac{1}{3}\times\frac{2}{3}\times\frac{1}{9}+\ldots..\infty\right)$$

$$=\frac{1}{9}+\frac{\frac{2}{27}}{1-\frac{2}{9}}+\frac{\frac{1}{3}\times\frac{2}{3}\times\frac{1}{9}}{1-\frac{2}{9}}$$

$$=\frac{1}{9}+\frac{2}{3\times7}+\frac{2}{7\times9}=\frac{5}{21}$$

One or More than One Option Correct

1. (B, C)

P (passing at least in one subject)

$= P(P \cup C \cup M) = 1 - P(\overline{P \cup C \cup M}) = 0.75$

$\Rightarrow \ P.(\bar{P}).P.(\bar{C}).P.(\bar{M}) = 1 - 0.75 = 0.25 = \frac{1}{4}$

$\Rightarrow \ (1-m)(1-P)(1-C) = \frac{1}{4}$ .....(i)

P (passing exactly in two subjects) = 0.4

$\Rightarrow \ P(P \cap C \cap \bar{M}) + P(P \cap \bar{C} \cap M) + P(\bar{P} \cap C \cap M) = \frac{2}{5}$

$\Rightarrow \ P.C(1-m) + pm(1-c) + cm(1-p) = \frac{2}{5}$ .....(ii)

P (passing at least in two subjects) = 0.5

$\Rightarrow \ pm(1-c) + pc(1-m) + cm(1-p) + pcm = \frac{1}{2}$ .....(iii)

$\Rightarrow \ pcm = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$ [From (ii)]

$\therefore$ (C) is true.

From (i), (ii) and (iii), we get

$p + c + m = \frac{27}{20}$

$\therefore$ (b) is true.

2. (A, D)

Given that E and F are independent events

$\therefore \ P(E \cap F) = P(E).P(F)$ .....(i)

$P(\text{exactly one}) = P(E \cap \bar{F}) + P(\bar{E} \cap F) = \frac{11}{25}$

$\Rightarrow \ P(E)P(\bar{F}) + P(\bar{E})P(F) = \frac{11}{25}$

$\Rightarrow \ P(E)(1 - P(F)) + (1 - P(E))P(F) = \frac{11}{25}$

$\Rightarrow \ P(E) - P(E)P(F) + P(F) - P(E)P(F) = \frac{11}{25}$

$\Rightarrow \ P(E) + P(F) - 2P(E).P(F) = \frac{11}{25}$ .....(ii)

$P(\text{none of them}) = P(\bar{E} \cap \bar{F}) = \frac{2}{25} \Rightarrow P(\bar{E})\ P(\bar{F}) = \frac{2}{25}$

$\Rightarrow [1 - P(E)][1 - P(F)] = \frac{2}{25}$

$\Rightarrow \ 1 - P(E) - P(F) + P(E)P(F) = \frac{2}{25}$ ....(iii)

Adding equation (ii) and (iii) we get

$1-P(E)P(F)=\frac{13}{25}$ or $P(E)P(F)=\frac{12}{25}$ ....(iv)

Using the result in equation (ii) we get

$P(E)+P(F)=\frac{35}{25}$ .....(v)

Solving (iv) and (v) we get

$P(E)=\frac{3}{5}$ and $P(F)=\frac{4}{5}$ or $P(E)=\frac{4}{5}$ and $P(F)=\frac{3}{5}$

$\therefore$ (a) and (d) are the correct options.

3. (A, B)

$\because\ P(X/Y)=\frac{P(X\cap Y)}{P(Y)}\Rightarrow\frac{1}{2}=\frac{1/6}{P(Y)}\Rightarrow P(Y)=\frac{1}{3}$

Similarly, $P(Y/X)=\frac{P(X\cap Y)}{P(X)}$

$\Rightarrow\ \frac{1}{3}=\frac{1/6}{P(X)}\Rightarrow P(X)=\frac{1}{2}$

(a) $P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{2}{3}$

$\therefore$ (a) is true

(b) $\because\ P(X\cap Y)=P(X)P(Y)$

$\Rightarrow$ X and Y are independent events.

$\therefore$ (b) is true

But (c) is not true.

(d) $P(X^C\cap Y)=P(X^C)\times P(Y)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$

$\therefore$ (d) is true

4. (B, D)

Given that $P(X_1)=\frac{1}{2},P(X_2)=\frac{1}{4},P(X_3)=\frac{1}{4}$

$P(X)=P$ (at least 3 engines are functioning)

$=P(X_1\cap X_2\cap X_3^C)+P(X_1\cap X_2^C\cap X_3)+P(X_1^C\cap X_2\cap X_3)+P(X_1\cap X_2\cap X_3)$

$\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}=\frac{1}{4}$

(a) $P(X_1^C/X)=\frac{P(X_1^C/X)}{P(X)}=\frac{P(X_1^C\cap X_2\cap X_3)}{P(X)}$

$=\frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}}{\frac{1}{4}}=\frac{1}{8}$

$\therefore$ (a) is not true

(b) P[Exactly two engines are functioning / X]

$$= \frac{P\left[\left(\text{Exactly two engines are functioning}\right)\cap X\right]}{P(X)}$$

$$= \frac{P\left(X_1^C \cap X_2 \cap X_3\right) + P\left(X_1 \cap X_2^C \cap X_3\right) + P\left(X_1 \cap X_2 \cap X_3^C\right)}{P(X)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{2}\times\frac{3}{4}\times\frac{1}{4} + \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}}{\frac{1}{4}} = \frac{7}{8}$$

$\therefore$ (b) is true

(c) $P\left(X / X_2\right) = \frac{P\left(X \cap X_2\right)}{P\left(X_2\right)}$

$$= \frac{P\left(X_1 \cap X_2 \cap X_3\right) + P\left(X_1^C \cap X_2 \cap X_3\right) + P\left(X_1 \cap X_2 \cap X_3^C\right)}{P\left(X_2\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{2}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}}{\frac{1}{4}} = \frac{5}{8}$$

$\therefore$ (c) is not true

(d) $P\left(X / X_1\right) = \frac{P\left(X \cap X_1\right)}{P\left(X_1\right)}$

$$= \frac{P\left(X_1 \cap X_2 \cap X_3\right) + P\left(X_1 \cap X_2^C \cap X_3\right) + P\left(X_1 \cap X_2 \cap X_3^C\right)}{P\left(X_1\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{2}\times\frac{3}{4}\times\frac{1}{4} + \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}}{\frac{1}{2}} = \frac{7}{16}$$

$\therefore$ (d) is true

5. (A, B)

Given that $P(X) = \frac{1}{3}, P(X/Y) = \frac{1}{2}, P(Y/X) = \frac{2}{5}$

We have $P(X \cap Y) = P(Y/X)P(X) = \frac{2}{5}\times\frac{1}{3} = \frac{2}{15}$

$\therefore$ (c) is not true

And $P(Y) = \frac{P(X \cap Y)}{P(X/Y)} = \frac{\frac{2}{15}}{\frac{1}{2}} = \frac{4}{15}$

$\therefore$ (a) is true

$$P(X'/Y) = \frac{P(X' \cap Y)}{P(Y)} = \frac{P(Y) - P(X \cap Y)}{P(Y)}$$

$$= 1 - P(X/Y) = \frac{1}{2}$$

$\therefore$ (b) is true

$$P(X \cup Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$$

$\therefore$ (d) is not true

6. (B, D)

$$\begin{vmatrix} R & -5 \\ G & -5 \end{vmatrix}_{B_1} \quad \begin{vmatrix} R & -3 \\ G & -5 \end{vmatrix}_{B_2} \quad \begin{vmatrix} R & -5 \\ G & -3 \end{vmatrix}_{B_3}$$

$$\because \ P(B_1) = \frac{3}{10}, P(B_2) = \frac{3}{10}, P(B_3) = \frac{4}{10}$$

$$P(G/B_1) = \frac{5}{10}, P(G/B_2) = \frac{5}{8}, P(G/B_3) = \frac{3}{8}$$

(a) $P(B_3 \cap G) = P(B_3)P(G/B_3) = \frac{4}{10} \times \frac{3}{8} = \frac{3}{20}$

$\therefore$ (a) is not true

(b) $P(G/B_3) = \frac{3}{8}$

$\therefore$ (b) is true

(c) $\because \ P(B_3/G)$

$$= \frac{P(G/B_3)P(B_3)}{P(G/B_1)P(B_1) + P(G/B_2)P(B_2) + P(G/B_3)P(B_3)}$$

$$= \frac{\frac{3}{8} \times \frac{4}{10}}{\frac{5}{10} \times \frac{3}{10} + \frac{5}{8} \times \frac{3}{10} + \frac{3}{8} \times \frac{4}{10}} = \frac{\frac{12}{80}}{\frac{15}{100} + \frac{15}{80} + \frac{12}{80}}$$

$$= \frac{12}{80} \times \frac{400}{60+75+60} = \frac{60}{195} = \frac{4}{13}$$

$\therefore$ (c) is not true

(d) $P(G) = P(G/B_1)P(B_1) + P(G/B_2)P(B_2) + P(G/B_3)P(B_3)$

$$= \frac{5}{10} \times \frac{3}{10} + \frac{5}{8} \times \frac{3}{10} + \frac{3}{8} \times \frac{4}{10}$$

$$= \frac{60+75+60}{400} = \frac{195}{400} = \frac{39}{80}$$

$\therefore$ (d) is true

7. (A, B, C)

Given that

$$P(E) = \frac{1}{8}; P(F) = \frac{1}{6}; P(G) = \frac{1}{4}; P(E \cap F \cap G) = \frac{1}{10}$$

(c) $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(G \cap E) + P(E \cap F \cap G)$

$$= \frac{1}{8} + \frac{1}{6} + \frac{1}{4} - \sum P(E \cap F) + \frac{1}{10}$$

$$= \frac{13}{24} + \frac{1}{10} - \sum P(E \cap F)$$

$\Rightarrow \ P(E\cup F\cup G)\le \dfrac{13}{24}$ [Option (C) is correct]

(d) Now, $P(E^C\cap F^C\cap G^C)$

$=1-P(E\cup F\cup G)\ge 1-\dfrac{3}{24}$

$\Rightarrow \ P(E^C\cap F^C\cap G^C)\ge \dfrac{11}{24}$ [Option (D) is correct]

(a) $\because P(E)\ge P(E\cap F\cap G^C)+P(E\cap F\cap G)$

$\Rightarrow \dfrac{1}{8}\ge P(E\cap F\cap G^C)+\dfrac{1}{10}$

$\Rightarrow \dfrac{1}{8}-\dfrac{1}{10}\ge P(E\cap F\cap G^C)$

$\Rightarrow \dfrac{1}{40}\ge P(E\cap F\cap G^C)$ [Option (A) is correct]

(b) $\because \ P(F)\ge P(E^C\cap F\cap G)+P(E\cap F\cap G)$

$\Rightarrow \ \dfrac{1}{6}\ge P(E^C\cap F\cap G)+\dfrac{1}{10}$

$\Rightarrow \ \dfrac{1}{6}-\dfrac{1}{10}\ge P(E^C\cap F\cap G)$

$\Rightarrow \ \dfrac{4}{60}\ge P(E^C\cap F\cap G)$

$\Rightarrow \ \dfrac{1}{15}\ge P(E^C\cap F\cap G)$ [Option (B) is correct]

Paragraph Type

Passage – I

1. (B)

Given: $P(u_1)\propto i$. Let $P(u_i)=ki$, we have $\sum P(u_i)=1$

$\Rightarrow \sum ki=1 \Rightarrow k\sum i=1 \Rightarrow k=\dfrac{2}{n(n+1)} \Rightarrow P(u_i)=\dfrac{2i}{n(n+1)}$

By total probability theorem

$P(w)=\sum_{i=1}^{n}P(u_i)P(w/u_i)=\sum_{i=1}^{n}\dfrac{2i}{n(n+1)}\times\dfrac{i}{n+1}$

$=\dfrac{2}{n(n+1)^2}.\dfrac{n(n+1)(2n+1)}{6}=\dfrac{2n+1}{3n+3}$ $\left[\because \sum n^2 \dfrac{n(n+1)(2n+1)}{6}\right]$

$\therefore \lim\limits_{\substack{\delta x\\ n\to\infty}} P(w)=\lim\limits_{n\to\infty}\dfrac{2n+1}{3n+3}=\lim\limits_{n\to\infty}\dfrac{2+1/n}{3+3/n}=\dfrac{2}{3}$

2. (A)

Given that $P(u_i)=c$

By Baye's theorem, $P\left(u_n / w\right)=\dfrac{P\left(w / u_n\right) P\left(u_n\right)}{\sum_{i=1}^{n} P\left(w / u_i\right) P\left(u_i\right)}$

$$=\frac{c \times \frac{n}{n+1}}{\sum_{i=1}^{n} \frac{i}{n+1}}=\frac{n}{n+1} \times \frac{n+1}{\frac{n(n+1)}{2}}=\frac{2}{n+1}$$

3. (B)

$$P(w / E)=\frac{P\left(w \cap u_2\right)+P\left(w \cap u_4\right)+\ldots\ldots.+\left(w \cap u_n\right)}{P\left(u_2\right)+P\left(u_4\right)+\ldots..+\left(u_n\right)}$$

$$=\frac{\frac{1}{n} \times \frac{2}{n+1}+\frac{1}{n} \times \frac{4}{n+1}+\frac{1}{n} \times \frac{6}{n+1}+\ldots\ldots+\frac{1}{n} \times \frac{n}{n+1}}{\frac{1}{n}+\frac{1}{n}+\ldots\ldots+\left(\frac{n}{2} \text{ times}\right)}$$

$$=\frac{\frac{2}{n(n+1)}\left[1+2+3 \ldots..+\frac{n}{2}\right]}{\frac{1}{n} \times \frac{n}{2}} \quad (\because \text{even})$$

$$=\frac{4}{n(n+1)}\left[\frac{\frac{n}{2}\left(\frac{n}{2}+1\right)}{2}\right]=\frac{n+2}{2(n+1)}$$

Passage – II

1. (A)

Let E: getting 6

$P(X=3)$

$$=P\left(\bar{E} \cap \bar{E} \cap E\right)=P\left(\bar{E}\right).P\left(\bar{E}\right).P(E)$$

$$=\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}=\frac{25}{216}$$

2. (B)

$P(X \geq 3)=1-[P(X=1)+P(X=2)]$

$$=1-\left[\frac{1}{6}+\frac{5}{6} \times \frac{1}{6}\right]=1-\frac{11}{36}=\frac{25}{36}$$

3. (D)

$$P\left(E_1\right)=P(X \geq 6)=\left(\frac{5}{6}\right)^5 \times \frac{1}{6}+\left(\frac{5}{6}\right)^6 \times \frac{1}{6}+\ldots\ldots.\infty$$

$$=\left(\frac{5}{6}\right)^5\times\frac{1}{6}\left[1+\frac{5}{6}+\left(\frac{5}{6}\right)^2+......\infty\right]=\left(\frac{5}{6}\right)^2\times\frac{1}{6}\times\frac{1}{1-\frac{5}{6}}=\left(\frac{5}{6}\right)^5$$

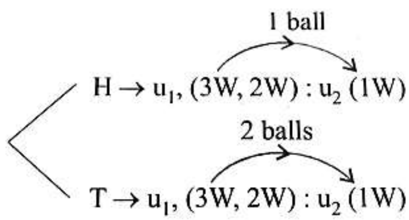
And $P(E_2)=P(X>3)=\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}+.......\infty=\left(\frac{5}{6}\right)^3$

$\because E_1\cap E_2=X\geq 6=E_1$

$$\therefore P(E_1/E_2)=\frac{P(E_1\cap E_2)}{P(E_2)}=\frac{P(E_1)}{P(E_2)}=\frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3}=\frac{25}{36}$$

Passage – III

1. (B)



$P(w)=P(H\cap W)+P(T\cap W)$

$=P(H)+P(W/H)+P(T)P(W/T)$

$$=\frac{1}{2}\left\{\frac{3}{5}\times1+\frac{2}{5}\times\frac{1}{2}\right\}+\frac{1}{2}\times\left\{\frac{{}^3C_2}{{}^5C_2}\times1+\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}+\frac{{}^3C_1.{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right\}$$

$$=\frac{1}{2}\times\frac{8}{10}+\frac{1}{2}\times\left(\frac{3}{10}+\frac{1}{30}+\frac{12}{30}\right)=\frac{4}{10}+\frac{11}{30}=\frac{23}{30}$$

2. (D)

$$P(H/W)=\frac{P(H\cap W)}{P(W)}=\frac{\frac{1}{2}\left[\frac{3}{5}\times1+\frac{2}{5}\times\frac{1}{2}\right]}{\frac{23}{30}}=\frac{\frac{4}{10}}{\frac{23}{30}}=\frac{12}{23}$$

Passage – IV

1. (A)

B_1 [1W, 3R, 2B] B_2 [2W, 3R, 4B] B_3 [3W, 4R, 5B]

Probability that all three balls are of same colour = P (RRR) + P (WWW) + P(BBB)

$$=\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}+\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}+\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}=\frac{82}{648}$$

2. (D)

B_1	1W, 3R, 2B
B_2	2W, 3R, 4B
B_3	3W, 4R, 5B

Let E_1, E_2, E_3 be the events to select bag B_1, B_2 and B_3 respectively.
Let F be the event of getting one white and one red ball.
Then by baye's theorem,

$$= P(E_2/F) = \frac{P(F/E_2)P(E_2)}{P(F/E_1)P(E_1)+P(F/E_2)P(E_2)+P(F/E_3)P(E_3)}$$

$$= \frac{\frac{1}{3}.\frac{2\times3}{{}^9C_2}}{\frac{1}{3}.\left(\frac{1\times3}{{}^6C_2}+\frac{2\times3}{{}^9C_2}+\frac{3\times4}{{}^{12}C_2}\right)} = \frac{55}{181}$$

Passage – V

1. (B)

$x_1 + x_2 + x_3$ will be odd. If two of them are even and one is odd or all three are odd.

E_i and O_i denotes the even and odd number resp. from i^{th} box.

$\therefore$ Required probability

$$= P(E_1E_2O_3) + P(E_1O_2E_3) + P(O_1E_2E_3) + P(O_1O_2O_3)$$

$$= \frac{1}{3}\times\frac{2}{5}\times\frac{4}{7}+\frac{1}{3}\times\frac{3}{5}\times\frac{3}{7}+\frac{2}{3}\times\frac{2}{5}\times\frac{3}{7}+\frac{2}{3}\times\frac{3}{5}\times\frac{4}{7}$$

$$= \frac{8+9+12+14}{105} = \frac{53}{105}$$

2. (C)

Let x_1, x_2, x_3 are in AP $\Rightarrow 2x_2 = x_1 + x_3$

$\because$ LHS is even, that means x_1 & x_3 can be both even or both odd.

Required probability = $P(E_1E_3) + P(O_1, O_3)$

$$= \frac{{}^1C_1 \times {}^3C_1 \times {}^2C_1 \times {}^4C_1}{{}^3C_1 \times {}^5C_1 \times {}^7C_1} = \frac{3+8}{3\times5\times7} = \frac{11}{105}$$

Passage - VI

1. (A, B)

Let E_1 = Box I is selected

E_2 = Box II is selected

F $\equiv$ Ball drawn is red

$$P(E_2/F) = \frac{P(F/E_2).P(E_2)}{P(F/E_1).P(E_1)+P(F/E_2).P(E_2)}$$

$$=\frac{\frac{n_3}{n_3+n_4}\times\frac{1}{2}}{\frac{n_1}{n_1+n_2}\times\frac{1}{2}+\frac{n_3}{n_3+n_4}\times\frac{1}{2}}=\frac{1}{3}$$

or $$\frac{\frac{n_3}{n_3+n_4}}{\frac{n_1}{n_1+n_2}+\frac{n_3}{n_3+n_4}}=\frac{1}{3}$$

On putting the values of the options we observed that (a) and (b) are the correct options.

2. (C, D)

$E_1 \equiv$ Red ball is selected from box I

$E_2 \equiv$ Black ball is selected from box I

$F \equiv$ Second red ball is drawn from box I

$\therefore\ P(F)=P(E_1)P(F/E_1)+P(E_2)P(F/E_2)$

$$=\frac{n_1}{n_1+n_2}\times\frac{n_1-1}{n_1+n_2-1}+\frac{n_2}{n_1+n_2}\times\frac{n_1}{n_1+n_2-1}$$

On putting the value of the options, we observed that (c) and (d) have the correct values.

Passage – VII

1. (A)

Total number of arrangements of seating of 5 students = 5! = 120

No. of de-arrangements of S_2, S_3, S_4 and S_5 not get previously seats.

$$=4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)$$

$=12-4+1=9$

$\therefore$ Required probability = $\frac{9}{120}=\frac{3}{40}$

2. (C)

Total number of arrangement of seating of 5 students = 5! = 120

Favourable cases:

$=\{(S_1S_3S_5S_2S_4),(S_1S_4S_2S_5S_3),(S_2S_4S_1S_3S_5),$

$(S_2S_5S_3S_1S_4),(S_2S_4S_1S_5S_3),(S_3S_1S_4S_2S_5),$

$(S_3S_5S_1S_4S_2),(S_3S_5S_2S_4S_1),(S_3S_1S_5S_2S_4),$

$(S_4S_2S_5S_1S_3),(S_4S_2S_5S_3S_1),(S_4S_1S_3S_5S_2),$

$(S_5S_2S_4S_1S_3),(S_5S_3S_1S_4S_2)\}$

$\therefore$ Favourable cases = 14

$\therefore$ Required probability = $\frac{14}{120}=\frac{7}{60}$

<u>Numerical Stem Questions</u>

1. (76.25)

Since, P_1 = probability that maximum of chosen numbers is at least 81

P_2 = 1-probability that maximum of chosen number is less than or equal to 80

$$P_1 = 1 - \frac{80\times80\times80}{100\times100\times100} = 1 - \frac{64}{125}$$

$$P_1 = \frac{61}{125}$$

$$\therefore \frac{625p_1}{4} = \frac{625}{4}\times\frac{61}{125} = \frac{305}{4} = 76.25$$

2. (24.50)

Since P_2 = probability that minimum of chosen numbers is at most 40

= l - probability that minimum of chosen number is greater than or equal to 41

$$= 1 - \frac{60\times60\times60}{100\times100\times100} = 1 - \frac{27}{125} = \frac{98}{125}$$

$$\therefore \frac{125}{4}p_2 = \frac{125}{4}\times\frac{98}{125} = 24.50$$

3. **(24)**

x	$P(x)$	$x.P(x)$
0	$P(0)=0$	0
1	$P(1)=0$	0
2	$P(2)=\frac{4}{49}$	$\frac{8}{49}$
3	$P(3)=\frac{20}{49}$	$\frac{60}{49}$
4	$P(4)=\frac{25}{49}$	$\frac{100}{49}$

$$E(x) = E\times P(x) = \frac{168}{49} = \frac{24}{7}$$

$$7E(x) = 24$$

4. (0.5)

2 consecutive points can be chosen in $2\times6\times7$ ways = 84 ways

So, $n(E)=84; n(S) = {}^{49}C_2$

So, $7p = 7\times\frac{84}{{}^{49}C_2} = 0.5$

Numerical Value Answer

1. (8)

Given that $P(X\geq2)\geq0.96$

$\Rightarrow 1-P(X=0)-P(X=1) \geq 0.96$

$\Rightarrow P(X=0)+P(X=1) \leq 0.04$

$\Rightarrow \left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^n \leq 0.04$

$\Rightarrow \frac{n+1}{2^n} \leq \frac{1}{25} \Rightarrow \frac{2^n}{n+1} \geq 25$

$\Rightarrow$ Minimum value of n is 8.

2. (0.50)

Total number of 3×3 matrices with 0 or $1 = 2^9 = 512$

E_2 contains those matrices in which sum of entries is 7.

$\therefore$ It will be contains 7 one's and 2 zeroe's

$\therefore$ $n(E_2) = {}^9C_2 = 36$

$E_1 \cap E_2$ contains those matrices in which 7 ones, 2 zeroes and its det is zero.

Det (A) = 0. This can be occurs when two rows/columns are identical.

e.g.

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \text{ or } \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \text{ or } \begin{vmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$\therefore$ $n(E_1 \cap E_2) = {}^3C_1 \times {}^3C_1 \times 2 = 18$

$\therefore$ $p(E_1/E_2) = \frac{p(E_1 \cap E_2)}{p(E_2)} = \frac{18/512}{36/512} = \frac{1}{2} = 0.50$

3. (422)

Let $n(A) = a, n(B) = b, n(A \cap B) = c$

$\therefore 1 \leq b < a$

Also given that A and B are independent events

$\therefore$ $p(A \cap B) = p(A)p(B)$

$\Rightarrow \frac{n(A \cap B)}{n(S)} = \frac{n(A)}{n(S)} \times \frac{n(B)}{n(S)}$

$\Rightarrow \frac{c}{6} = \frac{a}{6} \times \frac{b}{6} \Rightarrow ab = 6c$

If a = 6 then b = c = 5, 4, 3, 2, 1 $\quad (\because b < a)$

There is only one way to select all 6 elements of set A

Number of ways of selecting 5, 4, 3, 2 or 1 elements in B and $A \cap B$ are

${}^6C_5 + {}^6C_4 + {}^6C_3 + {}^6C_2 + {}^6C_1 = 2^6 - 2 = 62$

If a = 5 then b = $\frac{6c}{5}$, which is not possible because if c = 5 then b = 6, while b < a

If a = 4 then b = $\frac{6c}{5} = \frac{3c}{2}$, which is not possible because if c = 2 then b = 3

2 elements in $A \cap B$ can be selected in ${}^6C_2$ ways.

2 additional elements in A can be selected in ${}^4C_2$ ways.
1 additional elements in B can be selected in ${}^2C_1$ ways.
$\therefore$ No. of ways for a = 4, b = 3, c = 2 are
${}^6C_1 \times {}^4C_1 \times {}^2C_1 = 15 \times 6 \times 2 = 180$
If a = 3 then b = 2c $\Rightarrow$ c = 1, b = 2
Which can be done in ${}^6C_1 \times {}^5C_1 + {}^4C_2 = 6 \times 5 \times 6 = 180$ ways.
If a = 2 then b = 3c which is not possible
$\therefore$ Total number of required ways
$= 62 + 180 + 1800 = 422$

4. (6)
Given that P = Probability of hit the target = 0.75 = $\frac{3}{4}$
Z = Probability of miss the target = $1 - \frac{3}{4} = \frac{1}{4}$
$\therefore P(X = r) =$ Probability of r success = ${}^nC_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{n-r}$
$P(X \geq 3) = 1 - (P(0) + P(1) + P(2)) \geq 0.95$
$\Rightarrow 1 - {}^nC_0 \left(\frac{1}{4}\right)^n - {}^nC_1 \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{n-1} - {}^nC_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^{n-2} \geq 0.95$
$\Rightarrow 1 - \left(\frac{1 + 3n + \frac{9n(n-1)}{2}}{4^n} \right) \geq 0.95$
$\Rightarrow \frac{2 + 6n + 9n^2 - 9n}{2.4^n} \geq 1 - 0.95$
$\Rightarrow 9n^2 - 3n + 2 \leq 0.05 \times 4^n \times 2 \leq \frac{4^n}{10}$
For $n = 5, 212 \leq 102.4$ (Not true)
For $n = 6, 308 \leq 409.6$ True
Hence least value of n = 6

5. (8.00)
$\text{Prime}(2,3,5,7,11) = \{(1,1),(1,2),(2,1),(1,4),(2,3),(3,2),(4,1),$
$(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(5,6),(6,5)\}$
n(odd prime) $= 14$
$\therefore P(\text{odd prime}) = \frac{14}{36}$
Perfect square $= (4,9) = \{(1,3),(2,2),(3,1),(3,6),(4,5),(5,4),(6,3)\}$
N(perfect square) $= 7$
$\therefore$ P (Perfect square) $= \frac{7}{36}$

And P (odd perfect square) $=\frac{4}{36}$

Required probability

$$=\frac{\frac{4}{36}+\frac{14}{36}\times\frac{4}{36}+\left(\frac{14}{36}\right)^2\frac{4}{36}+....}{\frac{7}{36}+\frac{14}{36}\times\frac{7}{36}+\left(\frac{14}{36}\right)^2\frac{7}{36}+........}=\text{P}=\frac{4}{7}$$

$$\therefore 14\text{P}=14.\frac{4}{7}=8$$

6. (214)

E = Set of numbers divisible by 3

$\text{E}=\{3,6,9,12.........,1998\}$

$\therefore\ \ \text{n}(\text{E})=666$

F = Set of numbers divisible by 7

$\text{F}=\{7,14,21,.........,1995\}$

$\text{E}\cap\text{F}$ = Set of numbers divisible by 3 and 7

$=\{21,42,......,1995\}$

$\therefore\ \ \text{n}(\text{E}\cup\text{F})=606+285-95=856$

Required probability = $\frac{856}{2000}=\text{P}$

So, $500\,\text{P}=\frac{856}{2000}\times 500=214$

7. (0.8)

8. (31)

0, 1, 2, 2, 2, 4, 4

Number divisible by 5 (event *A*)

= Coefficient of x^4 in $4!\left(x^0+x^1\right)\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}\right)\left(1+x+\frac{x^2}{2!}\right)=38$

Number of divisible by 20 must end in 20 or 40 = 31 (event *B*)

$$P(A\cap B)=P(A)\cdot P\left(\frac{B}{A}\right)=P(B)P\left(\frac{A}{B}\right)$$

$$\Rightarrow\ P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{31}{38}$$

$$\therefore\ 38p=31$$

Matrix Match Type

1. (A)

$$\text{P}(\text{X}_\text{i}>\text{Y}_\text{i})+\text{P}(\text{X}_\text{i}<\text{Y}_\text{i})+\text{P}(\text{X}_\text{i}=\text{Y}_\text{i})=1$$

And $\text{P}(\text{X}_\text{i}>\text{Y}_\text{i})+\text{P}(\text{X}_\text{i}<\text{Y}_\text{i})=\text{P}$

For i = 2

$P(X_2 = Y_2) = P(5,5) + P(4,4)$

$= \frac{5}{12} \times \frac{5}{12} \times 2 + \frac{1}{6} \times \frac{1}{6}$

$= \frac{25}{72} + \frac{1}{36} = \frac{27}{72} = \frac{3}{8}$

$P(X_2 > Y_2) = P(10,0) = \frac{5}{12} \times \frac{5}{12} + \frac{5}{12} \times \frac{1}{6} \times 2 = \frac{5}{16}$

$P(X_2 > Y_2) \frac{5}{16} + \frac{3}{8} = \frac{11}{16}$

$I \to Q, II \to R$

For i = 3

$P(X_3 = Y_3) = P(6,6) + P(7,7)$

$= \frac{1}{6 \times 6 \times 6} + \frac{5}{12} \times \frac{1}{6} \times \frac{5}{12} \times 6 = \frac{77}{432}$

$P(X_3 > Y_3) = \frac{1}{2}\left(1 - \frac{77}{432}\right) = \frac{355}{864}$

$III \to T, IV \to S$

Subjective Problems

1. $\frac{{}^{12}C_2 \times {}^{6}C_4}{{}^{18}C_6} \cdot \frac{{}^{10}C_1 \times {}^{2}C_1}{{}^{12}C_2} + \frac{{}^{12}C_1 \times {}^{6}C_5}{{}^{18}C_6} \cdot \frac{{}^{11}C_1 \times {}^{1}C_1}{{}^{12}C_2}$

Let us consider the events

$E_1 \equiv$ 4 white balls are drawn in first six draws

$E_2 \equiv$ 5 white balls are drawn in first six draws

$E_3 \equiv$ 6 white balls are drawn in first six draws

$F \equiv$ Exactly one white ball is drawn in next two draws (i.e. one white and one red)

Then $P(F) = P(F/E_1)P(E_1) + P(F/E_2)P(E_2) + P(F/E_3)P(E_3)$

$P(F) = P(F/E_1)P(E_1) + P(F/E_2)P(E_2) \quad [\because P(F/E_3) = 0]$

$= \frac{{}^{12}C_2 \times {}^{6}C_4}{{}^{18}C_6} \cdot \frac{{}^{10}C_1 \times {}^{2}C_1}{{}^{12}C_2} + \frac{{}^{12}C_1 \times {}^{6}C_5}{{}^{18}C_6} \cdot \frac{{}^{11}C_1 \times {}^{1}C_1}{{}^{12}C_2}$

2. $P(A \cup B)P(\bar{A} \cap \bar{B})$

$= [P(A) + P(B) - P(A \cap B)]P(\bar{A}).P(\bar{B})$

$\le [P(A) + P[B]]P(\bar{A}).P(\bar{B})$

$= P(A)P(\bar{A})P(\bar{B}) + P(\bar{A})P(B)P(\bar{B})$

$= P(A)[1 - P(A)]P(\bar{B}) + P(\bar{A})P(B)[1 - P(B)]$

$= P(A)P(\bar{B}) - P(A)P(A)P(\bar{B}) + P(\bar{A})P(B)$

$-P(\bar{A})P(B)P(B)$

$\le P(A)P(\bar{B})+P(\bar{A})P(B)=P(C)$

Thus $P(C)\ge P(A\cup B)P(\bar{A}\cap\bar{B})$ is proved.

3. $\mathbf{\frac{1}{7}}$

Let us consider the events

$E_1 \equiv$ Person goes by car,

$E_2 \equiv$ Person goes by scooter,

$E_3 \equiv$ Person goes by bus,

$E_4 \equiv$ Person goes by train,

$F \equiv$ Person reaches late.

Then according to question

$$P(E_1)=\frac{1}{7};P(E_2)=\frac{3}{7};P(E_3)=\frac{2}{7};P(E_4)=\frac{1}{7}$$

$$P(F/E_1)=\frac{2}{9}\Rightarrow P(\bar{F}/E_1)=1-\frac{2}{9}=\frac{7}{9};$$

$$P(F/E_2)=\frac{1}{9}\Rightarrow P(\bar{F}/E_2)=1-\frac{1}{9}=\frac{8}{9};$$

$$P(F/E_3)=\frac{4}{9}\Rightarrow P(\bar{F}/E_3)=1-\frac{4}{9}=\frac{5}{9};$$

$$P(F/E_4)=\frac{1}{9}\Rightarrow P(\bar{F}/E_4)=1-\frac{1}{9}=\frac{8}{9};$$

$P(E_1/\bar{F})$

$$=\frac{P(\bar{F}/E_1).P(E_1)}{P(\bar{F}/E_1).P(E_1)+P(\bar{F}/E_2).P(E_2)+P(\bar{F}/E_3).P(E_3)+P(\bar{F}/E_4).P(E_4)}$$

$$P(E_1/\bar{F})=\frac{\frac{7}{9}\times\frac{1}{7}}{\frac{7}{9}\times\frac{1}{7}+\frac{8}{9}\times\frac{3}{7}+\frac{5}{9}\times\frac{2}{7}+\frac{8}{9}\times\frac{1}{7}}$$

$$=\frac{7}{7+24+10+8}=\frac{7}{49}=\frac{1}{7}$$