

PACE-IIT & MEDICAL

MUMBAI/DELHI-NCR/PUNE/NASHIK/AKOLA/GOA/JALGOAN/BOKARO/AMRAVATI/DHULE

IIT – JEE: 2025

TW TEST (ADV)

DATE: 14/01/24

TOPIC: HEAT & THERMODYNAMICS

SOLUTIONS

1. (C)

Specific heat of ice is $2.1\text{J/g}^\circ\text{C}$. Total heat released by water is $10 \times 4.2 \times 50 = 2100\text{J}$. Total heat absorbed by ice from -20°C to $0^\circ\text{C} = 10 \times 2.1 \times 20 = 420\text{J}$

$\Delta\theta = 2100 - 420 = mL$ melted ice, $m = (2100 - 420) / 336 \approx 5\text{gm}$. Hence, at equilibrium, total water is 15 gm , total ice is 5gm .

2. (B)

At temperature t the heat energy required to raise temperature of unit mass by dt is

$$dq = at^3 \times 1 \times dt$$

So heat required to raise temperature from 1K to 2K is

$$\int_0^Q dq = \int_1^2 at^3 dt \Rightarrow Q = a \left. \frac{t^4}{4} \right|_1^2 = a(16 - 1)$$

$$\Rightarrow Q = 15a/4$$

3. (C)

Since specific heat of lead is given in Joules, hence use $W = Q$ instead of $W = JQ$.

$$\text{So, } \frac{1}{2} \times \left(\frac{1}{2} mv^2 \right) = m.c.\Delta\theta \Rightarrow \Delta\theta = \frac{v^2}{4c} = \frac{(300)^2}{4 \times 150} = 150^\circ\text{C}$$

4. (D)

$$\gamma_{ac} = \gamma_l - \gamma_c$$

$$\therefore C = \gamma_l - \gamma_c \quad \dots\dots(1)$$

$$\gamma_{as} = \gamma_l - \gamma_s$$

$$\therefore S = \gamma_l - \gamma_s \quad \dots\dots\dots(2)$$

From (1) and (2)

$$S + \gamma_s = C + \gamma_c$$

$$\gamma_s = C - S + \gamma_c$$

$$3\alpha_s = C - S + \gamma_c$$

$$\Rightarrow \alpha_s = \frac{C - S + \gamma_c}{3}$$

5. (A)

$$\therefore dl = \alpha l_0 dT$$

$$\begin{aligned} \therefore \Delta l &= \int dl = \int_{T_1}^{T_2} (aT - bT^2) l_0 dT \\ &= l_0 \left[\frac{a}{2} (T_2^2 - T_1^2) - \frac{b}{3} (T_2^3 - T_1^3) \right] \\ &= l_0 \left[\frac{3}{2} aT_1^2 - \frac{7b}{3} T_1^3 \right] \end{aligned}$$

6. (B)

We just need to insulate the system, and balance the heat. So this experiment is not dependent on time taken to reach equilibrium. If system is insulated then, heat lost by copper = heat gain by beaker and water.

7. (A,C,D)

$$\frac{\Delta A}{A} \times 100 = 2 \left(\frac{\Delta l}{l} \right) \times 100$$

$$\% \text{ increase in area} = 2 \times 0.2 = 0.4\%$$

$$\frac{\Delta V}{V} \times 100 = 3 \times 0.2 = 0.6\%$$

Since

$$\Delta l = l \alpha \Delta T$$

$$\frac{\Delta l}{l} \times 100 = \alpha \Delta T \times 100 = 0.2$$

$$\alpha = 0.25 \times 10^{-4} / ^\circ\text{C}$$

8. (B, C)

$$T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{l_0 + \alpha l_0 \Delta \theta_0}{g}}$$

$$= T_0 \left(1 + \frac{1}{2} \alpha \Delta \theta \right)$$

$$\text{At } 30^\circ\text{C, fraction loss of time} = \frac{T_{30^\circ} - T_{20^\circ}}{T_{20^\circ}}$$

$$= 5\alpha = 5 \times 19 \times 10^{-6}$$

$$\text{Time lost in 24 h} = 86400 \times 95 \times 10^{-6} = 8.2 \text{ s}$$

On a cold day at 10°C , fraction gain of time

$$= \frac{T_{10^\circ} - T_{20^\circ}}{T_{20^\circ}} = -5\alpha$$

$$\text{Time gains in 24 h} = 8.2 \text{ s}$$

9. (B, D)

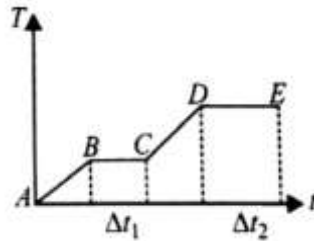
$$R = \frac{t}{(\alpha_B - \alpha_C) \Delta T}$$

10. (BC)

$$dQ = mCdT$$

$$\text{or } Hdt = mCdT$$

$$\therefore C = \left(\frac{H}{m}\right) \frac{1}{(dT/dt)}$$



As $\frac{dT}{dt}$ of CD is smaller, so $C_{\text{liquid}} > C_{\text{solid}}$

$$Q_1 = H(\Delta t_1) \text{ and } Q_2 = H(\Delta t_2)$$

As $\Delta t_2 > \Delta t_1$, $\therefore Q_2 > Q_1$

11. (B)

All parallel faces will expand by same amount. Therefore, there will not be any distortion in shape.

$$\beta_{\text{BCGH}} = \alpha_y + \alpha_z = 5 \times 10^{-5} / ^\circ\text{C} \text{ [Refer to example 9]}$$

Similarly, $\gamma = \alpha_x + \alpha_y + \alpha_z = 6 \times 10^{-5} / ^\circ\text{C}$

12. (A, B)

Let M and m be masses of water and ice, initially at temperature of 40°C and -40°C , respectively. To attain a temperature of 0°C , the heat lost by the water would be 4 units.

$$(Mg) (1 \text{ cal/g-}^\circ\text{C}) (40 - 0^\circ\text{C}) = 4 \text{ units}$$

$$\text{or } M \text{ cal} = \frac{1}{10} \text{ units} \quad \dots (i)$$

Similarly, to attain a temperature of 0°C , the heat gained by ice would be 1 unit.

$$(mg) \left(\frac{1}{2} \text{ cal/g}^\circ\text{C}\right) (0 + 40^\circ\text{C}) = 1 \text{ unit}$$

$$\text{or } m \text{ cal} = \frac{1}{20} \text{ unit} \quad \dots (ii)$$

$$\text{From (i) and (ii), } \frac{M}{m} = 2$$

Heat required for the complete ice to melt at 0°C will be $(mg) (80 \text{ cal/g}) = 80 m \text{ cal} = 4 \text{ unit}$.

By the time the temperature of the entire water had dropped to 0°C , the amount of heat ejected would be 4 unit, out of which 1 unit would be consumed by the ice to get heated up from -40°C to 0°C and the remaining heat of 3 units, would be consumed to melt.

Since of total mass of ice require a heat of 4 unit to melt completely, a heat of 3 units will be able to melt only (3/4)th of the ice.

13. (A, D)

$$\Delta V_L = \Delta V_V$$

$$\gamma_L V_L = \gamma_V V_V \text{ or } \frac{\gamma_L}{\gamma_V} = \frac{V_V}{V_L}$$

$$V_V > V_L \Rightarrow \gamma_L > \gamma_V$$

14. (A, B)

When the steam at 100°C transforms into water at 100°C , it releases heat given by

$$Q_1 = 100 \times 540 = 54000 \text{ cal}$$

200 g ice, for melting at 0°C needs an amount of heat given by

$$Q_2 = 200 \times 80 = 16000 \text{ cal.}$$

Water formed at 0°C , if heated to 100° , will need a heat given by

$$Q_3 = 200 \times 1 \times 100 = 20000 \text{ cal}$$

200g water at 55°C , if heated to 100°C , will need a heat given by

$$Q_4 = 200 \times 1 \times 45 = 9000 \text{ cal}$$

$$(Q_2 + Q_3 + Q_4) < Q_1$$

This implies that the entire steam will not condense, and the mixture will attain a temperature of 100°C .

Let mass of steam condensed by m

$$mL_v = Q_2 + Q_3 + Q_4$$

$$m \times 540 = 45000 \Rightarrow m = 83.3 \text{ g}$$

$$\begin{aligned} \therefore \text{Mass of water in the final mixture} &= 200 + 200 + 83.3 \\ &= 483.3 \text{ g} \end{aligned}$$

15. (C, D)

Thermal expansion is like photographic enlargement.

16. (8)

$$W_0 = mg = 46 \text{ g wt, } \theta_1 = 27^\circ \text{ C}$$

$$W_1 = 30 \text{ g} = W_0 - B_1$$

$$\Rightarrow B_1 = (46 - 30) \text{ g}$$

$$\Rightarrow B_1 = 16 \text{ g-wt} = V_1 \rho_1 g$$

$$\theta_2 = 42^\circ \text{ C}$$

$$W_2 = 30.5 \text{ g} = W_0 - B_2$$

$$\Rightarrow B_2 = 15.5 \text{ g} = V_2 \rho_2 g$$

$$\therefore \frac{B_2}{B_1} = \frac{V_2 \rho_2}{V_1 \rho_1}$$

$$\frac{15.5}{16} = (1 + 3\alpha_s \times 15) \times \frac{1.2}{1.24}$$

$$\alpha_s = \left[\left(\frac{15.5}{16} \times \frac{1.24}{1.2} \right) - 1 \right] \times \frac{1}{45}$$

$$\alpha_s = 2.31 \times 10^{-5} / ^\circ\text{C} = \frac{1}{43200} / ^\circ\text{C}$$

17. (12)

Heat released by steam = heat absorbed by water

$$m_1 L + m_1 \times S(100 - 90) = m_2 \times S(90 - 24)$$

$$540m_1 + 10m_1 = 66m_2$$

$$\Rightarrow m_1 = \frac{66 \times 100}{550} = 12 \text{ g}$$

18. (15)

$$V_C - V_{Hg} = V'_C = V'_{Hg} = \text{Volume of air}$$

$$\Rightarrow V'_C = V_C (1 + 3\alpha_s \Delta\theta)$$

$$V'_{Hg} = V_{Hg} (1 + \gamma_L \Delta\theta)$$

$$\text{So, } V_C \times 3\alpha_s = V_{Hg} \times \gamma_L$$

$$V_{Hg} = \frac{1 \times 3 \times 9 \times 10^{-6}}{1.8 \times 10^{-4}} = 0.15 \text{ L}$$

19. (0)
(Final mixture is 125g ice and 275 g water at 0°C.)
Say final mixture is 400 g water at 0°C.

$$Q_2 = (200)(1)(50) = 10,000 \text{ cal}$$

$$Q_1 = (200)(0.5)(40) + (200)(80) \\ = 20,000 \text{ cal}$$

$$Q = Q_2 - Q_1 = 10,000 - 20,000 \\ = -10,000 \text{ cal}$$

Since, $Q < 0$

$$\therefore |Q| = mL_F$$

$$10,000 = m \times 80$$

$$m = 125 \text{ g}$$

So final mixture is 125 g ice and 275 g water at 0°C.

20. (5)
Process A → B

$$W_{AB} = \int P \, dv = \int \frac{3}{2} T^{1/2} \, dv = \int \frac{3}{2} T^{1/2} \times \frac{1}{3} RT^{-1/2} \, dT$$

$$\text{On solving, } W_{AB} = \frac{115.2}{2} \times R = 480 \text{ J}$$

Process B → C

$$U = \frac{1}{2} V^{1/2} \quad \frac{3}{2} RT = \frac{1}{2} V^{1/2} \quad \Rightarrow 3PV^{1/2} = 1$$

$$\therefore P = \frac{1}{3\sqrt{V}} \quad \text{New } W_{BC} = \int P \, dv = \int_{100}^{1600} \frac{1}{3\sqrt{V}} = \frac{2}{3} \sqrt{V} = \frac{2}{3} [40 - 10] = \frac{2}{3} \times 30 = 20 \text{ J}$$

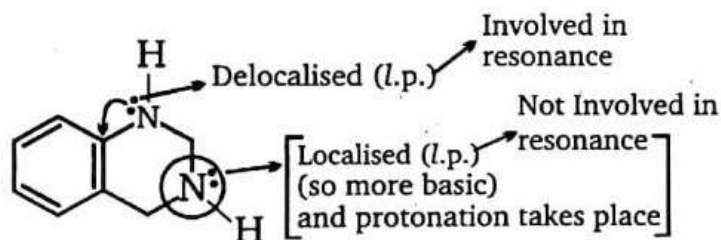
$$\text{Total work } W = 480 + 20 = 500 \text{ J}$$

Solutions

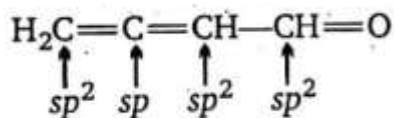
21. (B)

$$\text{Acidic strength} \propto -I; -H; -M \propto \frac{1}{+I; +H; +R}$$

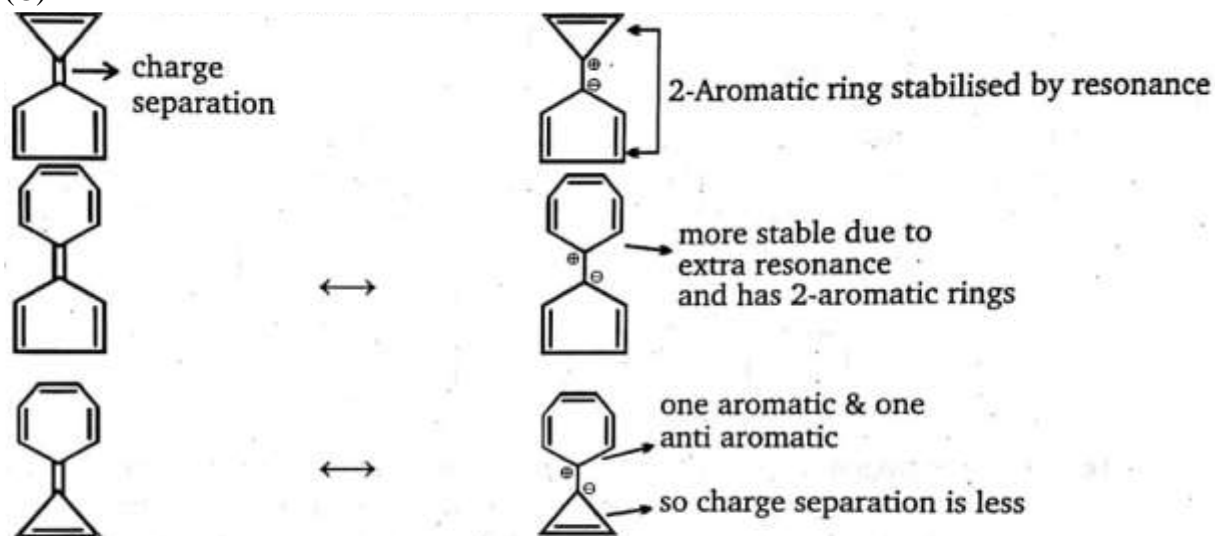
22. (B)



23. (C)



24. (C)



So bond rotation energy $C > A > B$

So order of rotation is $B > A > C$

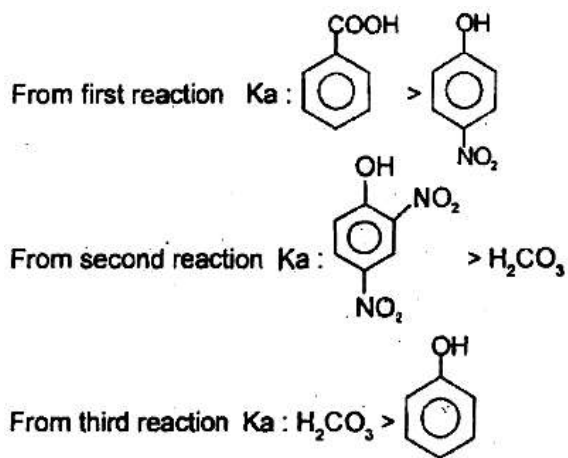
25. (B)

\propto 'H' atoms w.r.t. C=C bond take part in hyper conjugation.

26. (ABD)

Most Stable resonating structure contribute maximum & least stable resonating structure Contribute minimum in resonance hybrid.

27. (B,C,D)

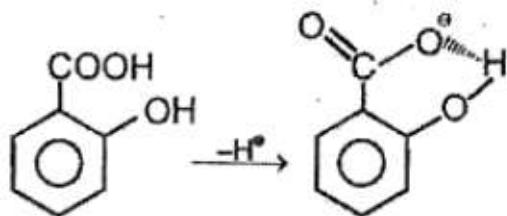


Since a strong acid displaces a weak acid from its salt and forms its own salt.

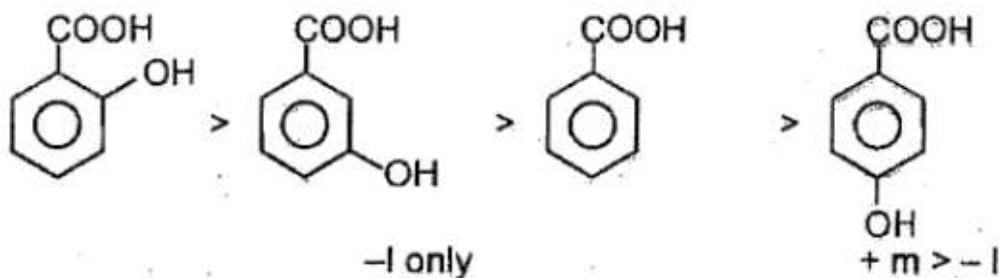
28. (ABD)

Due to steric hinderance of three groups $-NO_2$ group to out of the plane with benzene ring and so conjugation of $-NO_2$ group with benzene is slightly diminished. So bond length of $C_1 - N$ & $C_5 - N$ increases

29. (ABD)

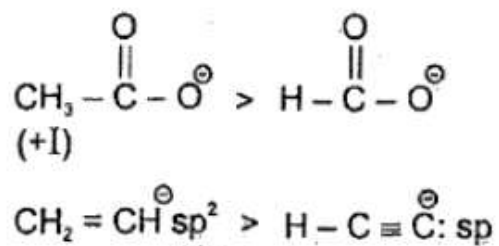


Anion is ore stable due to H-bonding \therefore shows ortho effect.



(D) Resonating structures are hypothetical

30. (ACD)



31. (ABD)

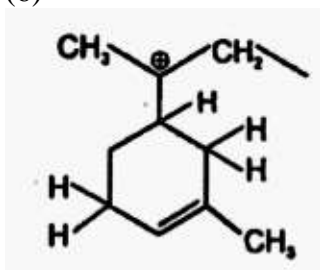
32. (ABC)

33. (BC)

34. (AB)

35. (BD)

36. (6)

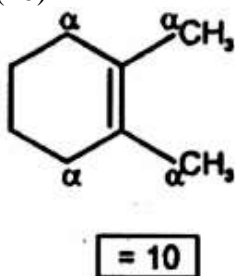


Total number of hydrogen involved in hyperconjugation with carbocation = 6

37. (3)

Compound has three acidic hydrogen.

38. (10)



39. (4)

40. (6)

1st, 3rd, 5th, 6th, 8th, 7th

SOLUTIONS

41. (D)

Out of $(2n + 1)$ tickets we can select 3 numbers in AP in n^2 number of ways

So number of favourable cases = 100

Total number of cases C_3^{31}

$$\text{Required probability} = \frac{100}{133}$$

42. (D)

A chess board is a square divided into 64 equal squares.

In 1st diagonal we have only 1 square

In 2nd diagonal we have only 2 squares

In 3rd diagonal we have 3 squares so selection can be done in 3C_3 ways

In 4th diagonal we have 4 squares so selection can be done in C_3^4 ways

And so on

Hence, the total number of ways in which 3 squares can be chosen

$$2({}^3C_3 + {}^4C_3 + {}^5C_3 + {}^6C_3 + {}^7C_3) + {}^8C_3$$

[Note that we do not have $2 \cdot {}^8C_3$]

Hence the total number of favourable ways $m = 4({}^3C_3 + {}^4C_3 + {}^5C_3 + {}^6C_3 + {}^7C_3) + 2 \cdot {}^8C_3 = 392$.

$$\text{And total number of ways} = {}^{64}C_3 = \frac{64 \cdot 63 \cdot 62}{1 \cdot 2 \cdot 3} = 32 \cdot 21 \cdot 62$$

Hence the required probability

$$= \frac{m}{n} = \frac{392}{32 \cdot 21 \cdot 62} = \frac{7}{744}$$

43. (C)

E_1 = denotes selection for 1st bag

E_2 = denotes selection for 2nd bag

$$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}$$

A = selected balls are 1 red & 1 black

$$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{1}{5}$$

$$P\left(\frac{A}{E_2}\right) = \frac{{}^3C_1 \times {}^2C_1}{{}^{(n+5)}C_2} = \frac{12}{(n+5)(n+4)}$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{10}}{\frac{1}{10} + \frac{6}{(n+5)(n+4)}} = \frac{6}{11}$$

$\Rightarrow n = 4$

44. (C)

(A) $P(E_1) \cdot P(E_2) = \frac{1}{6} \cdot \frac{1}{4} = \frac{1}{24} \neq P(E_1 \cap E_2)$

(B) $P(E'_1 \cap E'_2) = 1 - P(E_1 \cup E_2)$

$$= 1 - (P(E_1) + P(E_2) - P(E_1 \cap E_2))$$

$$= 1 - \left(\frac{1}{6} + \frac{1}{4} - \frac{1}{8}\right) = \frac{17}{24}$$

$$P(E'_1)P(E_2) = \frac{5}{6} \times \frac{1}{4} = \frac{5}{24}$$

(C) $P(E_1 \cap E'_2) = P(E_1) - P(E_1 \cap E_2) = \frac{1}{6} - \frac{1}{8} = \frac{1}{24}$

(D) $P(E'_1 \cap E_2) = P(E_2) - P(E_1 \cap E_2) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$

45. (B)

A = event that two defective machines are identified in first two tests out of four machines.

$$\therefore P(A) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}$$

46. (ABC)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.8 = 0.6 + 0.4 - P(A \cap B)$$

$$\therefore P(A \cap B) = 0.2$$

$$P(A \cup B \cup C) = (0.6 + 0.4 + 0.5) - (0.2 + P(B \cap C) + 0.3) + 0.2$$

$$= 1.5 - 0.3 - P(B \cap C)$$

$$\text{We know, } 0.85 \leq P(A \cup B \cup C) \leq 1$$

$$\text{or } 0.85 \leq 1.2 - P(B \cap C) \leq 1$$

$$\therefore 0.2 \leq P(B \cap C) \leq 0.35$$

47. (BC)

$$\text{At least one} \Rightarrow \text{Any one, any two, all the three} = 0.75 = \frac{3}{4}$$

$$\text{At least two} \Rightarrow \text{Any two, all the three} = 50\% = \frac{1}{2}$$

$$\text{Exactly two} \Rightarrow \text{Any two} = 40\% = \frac{2}{5}$$

$$\Sigma M(1-P)(1-C) + \Sigma MP(1-C) + MPC = \frac{3}{4} \quad \dots(1)$$

$$\Sigma MP(1-C) + MPC = \frac{1}{2} \quad \dots(2)$$

$$\Sigma MP(1-C) = \frac{2}{5} \quad \dots(3)$$

Solving (2) and (3),

$$MPC = \frac{1}{10} \Rightarrow (c) \quad \dots(4)$$

Solving (2) and (4),

$$M + P + C = \frac{27}{20}$$

48. (ACD)

Let $P(A)$ and $P(B)$ denote the percentage of city population who read newspapers A and B. Then

from given data, we have $P(A) = 25\% = \frac{1}{4}$, $P(B) = 20\% = \frac{1}{5}$

$$P(A \cap B) = 8\% = \frac{2}{25}$$

\therefore Percentage of those who read A but not B = $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$$= \frac{1}{4} - \frac{2}{25} = \frac{17}{100} = 17\%$$

Similarly $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$$= \frac{1}{5} - \frac{2}{25} = \frac{3}{25} = 12\%$$

If $P(C)$ denote the percentage of those who look into advertisement, then from the given data we obtain

$P(C) = 30\%$ of $P(A \cap B) + 40\%$ of $P(\bar{A} \cap B) + 50\%$ of $P(A \cap \bar{B})$

$$= \frac{3}{10} \times \frac{17}{100} + \frac{2}{5} \times \frac{3}{25} + \frac{1}{2} \times \frac{2}{25} = \frac{51 + 48 + 40}{1000} = \frac{139}{1000} = 13.9\%$$

Thus, the percentage of population who read an advertisement is 13.9%

49. (AC)

Let E_1 be the event of getting head, E_2 be the event of getting tail and let E be the event that noted number is 7 or 8 then

$$P(E_1) = \frac{1}{2}; P(E_2) = \frac{1}{2}$$

$P(E/E_1) = P$ (Getting either 7 or 8 when pair of unbiased dice is thrown)

$$= \frac{11}{36}$$

$P(E/E_2) = P$ (Getting either 7 or 8 when a card is picked from the pack of 11 cards)

$$= \frac{2}{11}$$

$\therefore E_1$ and E_2 are mutually exclusive and exhaustive events

$P(E) = P(E_1)P(E/E_1) + P(E_2)P(E/E_2)$

$$= \frac{1}{2} \cdot \frac{11}{36} + \frac{1}{2} \cdot \frac{2}{11}$$

$$= \frac{11}{72} + \frac{1}{11} = \frac{193}{792}$$

50. (ACD)

Probability that defective from A is $\frac{25}{100} \times \frac{5}{100} = \frac{125}{10000}$

Probability that defective from B is $\frac{35}{100} \times \frac{4}{100} = \frac{140}{10000}$

Probability that defective from C is $\frac{40}{100} \times \frac{2}{100} = \frac{80}{10000}$

A bulb is drawn and is found to be defective then Probability that it is manufactured by machine A is $\frac{125}{125+140+80} = \frac{125}{345} = \frac{25}{69}$

A bulb is drawn and is found to be defective then Probability that it is manufactured by machine B is $\frac{140}{125+140+80} = \frac{140}{345} = \frac{28}{69}$

A bulb is drawn and is found to be defective then Probability that it is manufactured by machine C is $\frac{80}{125+140+80} = \frac{80}{345} = \frac{16}{69}$

51. (AB)

The probability that both will be alive for 10 years, hence i.e., the prob. That the man and his wife Both be alive 10 years hence $0.83 \times 0.37 = 0.7221$

The prob. that at least one of them will be alive is $1 - P(\text{That none of them remains alive 10 years})$

$$= 1 - (1 - 0.83)(1 - 0.37)$$

$$= 1 - 0.17 \times 0.63$$

$$= 1 - 0.1071$$

$$= 0.8929$$

52. (ABCD)

$$P(A) = 0.7; P(B) = 0.4$$

$$P(A - B) = P(A) - P(AB)$$

$$\Rightarrow P(AB) = 0.2$$

$$\Rightarrow P(A \cup B) = 0.9 \Rightarrow P(B - A) = 0.2, \Rightarrow P(\bar{A} \cup \bar{B}) = 1 - P(AB) = 0.8$$

$$\Rightarrow P\left(\frac{B}{A \cup \bar{B}}\right) = \frac{P(A \cap B)}{P(A \cup \bar{B})} = \frac{1}{4}$$

53. (ABD)

Urn	Red Marbles	White marbles	Blue marbles
A	5	3	8
B	3	5	0

$$P(E_1) = P(R) = \left(\frac{2}{3}\right)\left(\frac{5}{16}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{8}\right) = \frac{10}{48} + \frac{6}{48} = \frac{1}{3}$$

$$P(E_2) = P(W) = \binom{2}{3} \binom{3}{16} + \binom{1}{3} \binom{5}{8} = \frac{6}{48} + \frac{10}{48} = \frac{1}{3}$$

$$P(E_3) = P(B) = \binom{2}{3} \binom{8}{16} = \frac{1}{3}$$

(C) Let A : event that urn A is chose

$$P\left(\frac{A}{R}\right) = \frac{P(A \cap R)}{P(R)} = \frac{\binom{2}{3} \binom{5}{16}}{\frac{1}{3}} = \binom{10}{48} (3) = \frac{5}{8} \Rightarrow \text{(C) is incorrect.}$$

$$(D) P\left(\frac{A}{W}\right) = \frac{P(A \cap W)}{P(W)} = \frac{\binom{2}{3} \binom{3}{16}}{\frac{1}{3}} = \binom{6}{48} (3) = \frac{3}{8}$$

$$P\left(\frac{\text{face five}}{W}\right) = \binom{3}{8} \binom{1}{4} = \frac{3}{32} \Rightarrow \text{(D) is correct.}$$

54. (AC)

$$P_K = \frac{{}^{15}C_K}{2^{15}}$$

$$\text{It is max where } K = \frac{15-1}{2} \text{ or } \frac{15+1}{2}$$

55. (BC)

Let S be the sample space. Then $|S| = 5 \cdot {}^5P_4$. If 0 is present then the number of 5 digit number divisible by 3 is $4 \underline{4} = 96$. If 0 is absent then the number of 5 digit number divisible by 3 is 120.

$$\text{Required Probability} = \frac{216}{600}$$

56. (4)

$$\text{The probability that he get marks} = \frac{1}{31}$$

$$\text{The probability that he get marks in second trial is } \frac{30}{31} \times \frac{1}{30} = \frac{1}{31}$$

$$\text{The probability that he get marks in third trial is } \frac{1}{31}$$

$$\text{Continuing this process the probability from } r \text{ trial is } \frac{r}{31} > \frac{1}{8}$$

$$\Rightarrow r > \frac{31}{8}$$

$$r = 4$$

57. (2)

$$n(X) = k + 1$$

$$\text{No. of ways to construct } A = 2^{k+1}$$

$$\text{No. of ways to construct } B = 2^{k+1}$$

$$\therefore \text{Total ways to construct } A \text{ and } B = 2^{k+1} \times 2^{k+1}$$

Favourable ways to construct $A = 2^{k+1}$

Favourable ways to construct B such that $B = A^C$ is = 1

\therefore Favourable ways = $2^{k+1} \times 1$

$$\text{Required Probability} = \frac{2^{k+1}}{(2^{k+1})^2} = \frac{1}{2^{k+1}}$$

$$\Rightarrow m-1 = k+1$$

$$\Rightarrow m-k = 2$$

58. (479)

$$A = \{1, 2, 3, 4\} : P(A) = \frac{3}{4} \rightarrow \text{Correct}$$

$$B = \{5, 6, 7, 8, 9, 10\} : P(B) = \frac{1}{4} \text{ Correct}$$

8 Correct Ans:

$$(4, 4) : {}^4C_4 \left(\frac{3}{4}\right)^4 \cdot {}^6C_4 \left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)^4$$

$$(3, 5) : {}^4C_3 \left(\frac{3}{4}\right)^3 \cdot \left(\frac{1}{4}\right)^1 \cdot {}^6C_5 \left(\frac{3}{4}\right)^4 \cdot \left(\frac{3}{4}\right)$$

$$(2, 6) : {}^4C_2 \left(\frac{3}{4}\right)^2 \cdot \left(\frac{1}{4}\right)^2 \cdot {}^6C_6 \left(\frac{1}{4}\right)^6$$

$$\text{Total} = \frac{1}{4^{10}} [3^4 \times 15 \times 3^2 + 4 \times 3^3 \times 6 \times 3 + 6 \times 3^2]$$

$$\Rightarrow k = 479$$

59. (5)

p	q
2	1
3	1, 2
4	1 to 4
5	1 to 6
6	1 to 9
7, 8, 9, 10	1 to 10

$$p^2 \geq 4q \Rightarrow$$

The total number of pairs (p, q) is $1 + 2 + 4 + 6 + 9 + 40 = 62$

$$\text{Probability} = \frac{62}{10 \cdot 10} = \frac{31}{50}$$

60. (33)

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

Divisible by 3.

Case – I : All 1 \rightarrow (1)

Case – II : All 8 \rightarrow (1)

Case – III : 3 ones & 3 eights

$$\frac{6!}{3! \times 3!} = 20$$

$$\text{Required probability} \quad \therefore p = \frac{22}{64}$$

$$96p = 96 \times \frac{22}{64} = 33$$