PACE-IIT & MEDICAL

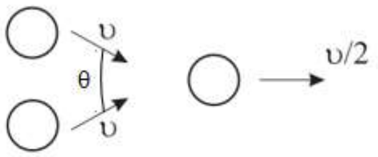
MUMBAI / AKOLA / DELHI / KOLKATA / GHAZIABAD / NASHIK / GOA / BOKARO / PUNE

IIT – JEE: 2025 | **TW TEST (ADV)** | **DATE: 24/12/23**

TOPIC: CENTRE OF MASS & COLLISIONS

SOLUTION

1. **(C)**



$$2\times m\upsilon\cos\frac{\theta}{2}=2m\frac{\upsilon}{2}$$

$$\Rightarrow \cos\frac{\theta}{2}=\frac{1}{2}$$

$$\Rightarrow \theta=120^{o}$$

2. **(C)**

Energy lost $=\frac{\left(1-e^{2}\right)}{2}\frac{m_1m_2}{m_1+m_2}\left(\vec{u}_1-\vec{u}_2\right)^2$

3. **(C)**

No external force in horizontal

4. **(A)**

$\vec{\upsilon}_{CM}=\frac{\vec{\upsilon}_1+\vec{\upsilon}_2}{2}$, *A* and *B* are ends.

5. **(D)**

Use rocket propulsion

$$\int_0^{\upsilon} d\upsilon=-\upsilon_r\int_{40}^{38}\frac{dn}{m}\Rightarrow \upsilon=10\ln\frac{20}{19}$$

6. **(ABD)**

Theory

7. **(AD)**

Use impule momentum theorem

8. **(BC)**

$$\vec{V}_{CM}=\frac{m_1\vec{\upsilon}_1+m_2\vec{\upsilon}_2}{m_1+m_2}$$

9. **(BC)**

$kx = \mu m_2 g$.

Then work-enerngy theorem on m_1 .

10. **(CD)**

Theory

11. **(BD)**

Theory

12. **(BCD)**

Theory

13. **(ABC)**

From momentum conservation

$$\text{mu} = \text{mv}\cos 30 + \text{mv}\cos 30;\ \text{v} = \frac{\text{u}}{\sqrt{3}}$$

So, choice (C) is correct.

For an oblique collision, we have to take components along normal i.e., along AB for sphere A and B.

$$\text{v}_\text{B} - \text{v}_\text{A} = \text{e}\left(\text{u}_\text{A} - \text{u}_\text{B}\right);\ \text{v} - 0 = \text{e}\left[\text{u}\cos 30 - 0\right]$$

$$\text{v} = \text{eu} \times \frac{\sqrt{3}}{2};\ \text{v} = \text{e.v}\sqrt{3}.\frac{\sqrt{3}}{2};\ \text{e} = \frac{2}{3}$$

So, choice (A) is correct. Also, loss of kinetic energy

$$\Delta\text{K} = \frac{1}{2}\text{mu}^2 - 2\left(\frac{1}{2}\text{mv}^2\right)$$

$$= \frac{1}{2}\text{mu}^2 - 2\frac{1}{2}\text{m}\left[\left(\frac{\text{u}}{\sqrt{3}}\right)^2\right] = \frac{1}{6}\text{mu}^2$$

14. **(CD)**

Theory

Elastic collision.

15. **(ABC)**

Theory.

Collision.

16. **(6)**

$$x_{\text{CM}} = \frac{M \times O - \frac{M}{4} \times \frac{a}{2}}{M - \frac{M}{4}} = -\frac{a}{6}$$

17. **(5)**

$$\frac{2d}{\frac{d}{\upsilon} + \frac{d}{e\upsilon}} = \frac{2\upsilon}{3} \quad \Rightarrow \frac{2e\upsilon}{e+1} = \frac{2\upsilon}{3} \Rightarrow e = 0.5$$

18. **(1)**

Conservation of linear momentum

$$m_0 \upsilon_0 = (m_0 + \rho A x)\upsilon \quad \Rightarrow \upsilon = \frac{m_0 \upsilon_0}{(m_0 + \rho A x)} = \frac{dx}{dt}$$

Then integrate.

19. **(5)**

Work energy theorem

$$\frac{1}{2} \times m \times \upsilon^2 = \mu m g x \qquad \Rightarrow x = 50\,\text{m}$$

20. **(5)**

Impulse momentum theorem along with thread

$$10\cos 30^\circ = 2\upsilon \qquad ...(1)$$

$$J = \upsilon \qquad ...(2)$$

$$\Rightarrow \qquad J = \frac{5\sqrt{3}}{2}$$

SOLUTION

21. **(A)**

As K_C is given by ratio of the product of the molar concentration of the products to that of the product of the molar concentration of reactants with each concentration term is raised to a power equal to a stoichiometric coefficient in the balanced chemical equation therefore for both the reaction, equilibrium constants are written as

$$K_c = \frac{[NH_3]^2}{[N_2][H_2]^3} \quad \text{and}\ K_C^{'} = \frac{[NH_3]}{[N_2]^{\frac{1}{2}}[H_2]^{\frac{3}{2}}}$$

Using rationale (3) relate K_c and $K_C^{'}$ is

$$K_C^{'} = \sqrt{K_C}$$

22. **(D)**

Gibb's free energy is given by

$$\Delta G^{\circ} = -2.303\ RT \log K_p$$

By putting the values $\Delta G^{\circ} = 1.7$ KJ (given)

$$1.7 = -2.303 \times 8.314 \times 10^{-3} \times 298\ \log K_p$$

Therefore $K_p = 0.5$

23. **(D)**

For equilibrium system,

$N_2O_4(g) \rightleftharpoons 2\ NO_2(g)$, the total pressure is 1.0 atm.

$\Rightarrow$ The total pressure = $P_{N_2O_4} + P_{NO_2}$ = 1.0 atm.

$P_{N_2O_4}$ = 0.5 atm, P_{NO_2} = 0.5 atm

(1) $$K_P = \frac{(P_{N_{O2}})^2}{P_{N_{2O_4}}} = \frac{0.5^2}{0.5} = 0.5 \text{ atm.}$$

According to Le – Chatlier's principle, when volume is decreased, the system moves in that direction where there is decrease in number of moles. Hence, the system will move in reverse direction, as there is a decrease in mole.

(Δ_{ng} = 2 – 1) i.e. , the NO_2 will be converted to $P_{N_2O_4}$

Pressure	N_2O_4	NO_2
Initially	1	1
At Equilibrium	1 + (x/2)	1 - x

$$K_P = \frac{(1-x)^2}{1+(x/2)} = 0.5$$

$4x^2 - 9x + 2 = 0$

X = 2 or 0.25

(x $\neq 2$, as initial pressure = 1.0)

X = 0.25

$P_{N_2O_4}$ = 1 + (x/2) = 1.125 atm.

P_{NO_2} = 1 – x = 0.75

24. **(C)**

$$PC\ell_5(g) \rightleftharpoons PC\ell_3(g) + C\ell_2(g)$$

Moles	PCl_5	PCl_3	Cl_2
Initially	a	0	0
At equlibrium	a - aα	aα	aα

Where α = degree of dissociation

α = Initial moles of reactants

Total moles = a + aα

Using the result

total moles = $\frac{a}{a + a\alpha} = \frac{d_{mix}}{d_0}$

where d_0 = density of PCl_5 (molar mass of PCl_5 = 208.5)

At T = 250°C

$$\frac{d_0}{d_{mix}} = \frac{Vd_{PCl5}}{V.d_{mix}} = \frac{M_0/2}{58} = \frac{a + a\alpha}{a} = \frac{208.5/2}{58} = 1 + \alpha$$

α = 0.80

25. **(B)**

The value of equilibrium constant remain same 4×10^{-4} because in this case value of temperature does not change, it remain same i.e. 2000K therefore, equilibrium constant remains therefore, equilibrium constant remains same.

26. **(D)**

Equilibrium constant K_a for H_2S

$H_2S = \frac{[H^+][HS^-]}{[H_2S]}$ …….(1)

According to Le-chatlier Principle,

An increase in $[H^+]$ ion concentration will increase the numerator value in the above equation no. 1 as a result $[HS^-]$ will decrease to maintain the constant value of K_a

27. **(D)**

Chemical equilibrium is the state when concentration of reactants and products do not change with time

It is attained when:Rate of forward reaction = rate of backward reaction.

In the formation of ammonia reaction is reversible so the amount of ammonia should be equal to the amount of ammonia decomposed.

28. **(A)**

$H_2(g) \rightarrow 2H(g)$, ΔH = +ve (endothermic reaction)
As the reaction is endothermic, it require high temperature. According to Le- chatlier's Principle reaction will shift to forward direction at high temperature and low pressure.

29. **(ABC)**
In the given reaction Cl_2 and F_2 are reactant and according to Le-Chatlier's Principle increase in any one of the reactants will make the reaction shifts in the forward direction i.e. more and more products are formed so adding F_2, which is one of the reactants will increase the concentration of ClF_3.

30. **(A)**
Solute + Solvent $\rightleftharpoons$ Solution; $\Delta H = +ve$
An increase in temperature always favours endothermic process i.e. heat is absorbed by the system and thus, solute having endothermic dissolution show an increase in their solubility with temperature hence solubility of NaCl increases with temperature and more and more solid will dissolve

31. **(ABCD)**

	$2NH_3$ (g) $\rightleftharpoons$	$N_2(g)$ +	$3H_2(g)$
At 400 K	20 atm	0	0
At 600K	30 atm	0	0
	−2x	x	3x
t = teq.	30 – 2x	x	3x

$P_T = 30 + 2x = 50$
$2x = 20$
$x = 10$

$$\alpha = \frac{2x}{30} = \frac{20}{30} = \frac{2}{3}$$

$$K_P = \frac{10 \times 30^3}{10^2} = \frac{27 \times 10^4}{10^2} = 2.7 \times 10^3$$

32. **(AC)**
At initial stage,
R contains twice as much of gas particle as S. If this ratio is maintained at different temperatures, the pressure in R and S will change but they will still be equal to one another, i.e., the Hg level will not change. Increase in temperature shifts the reaction backward and in (A) and forward in (C) which results in increase in pressure thus rise in right hand manometer.

33. **(ACD)**

34. **(ABC)**

35. **(C)**

36. **(1)**

37. **(9)**

$$\frac{K_{P_1}}{K_{P_2}} = \frac{4P_1}{P_2} = \frac{4 \times 1}{36} = \frac{1}{9} \Rightarrow K_{P_2} = 9K_{P1}$$

38. **(2)**

39. **(3)**

40. **(3)**

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TOPIC: FUNCTIONS

SOLUTION

41. **(C)**

Domain $[x] = -1, 0, 1$

If $[x] = -1$ then

$\text{LHS} = \frac{-\pi}{2}$ & $\cos^{-1}[x]$ cannot be less than $\frac{-\pi}{2}$

If $[x] = 0$ then $0 > \cos^{-1}[x]$, not possible then $\sin^{-1} > \cos^{-1}[x]$

$\frac{\pi}{2} > \cos^{-1}[x]$ (always true) $[x] \neq 0$

$\because$ $0 \leq [x] < 1$ $x \neq I$

$\frac{\pi}{2} \geq \cos^{-1}[x] > 0$

So, $x \in (1, 2)$

42. **(C)**

$f(x) = \underbrace{x^3 - x^2 + 4x}_{h(x)} + \underbrace{2\sin^{-1} x}_{g(x)}$

$h'(x) = 3x^2 - 2x + 4, \quad D = 4 - 4(4)(3) < 0$

$h'(x) > 0 \; \forall \; x$ so $h(x)$ is an increasing function.

$\sin^{-1} x$ is also increasing.

$f(0) = 0$

$f(1) = 1 - 1 + 4 + 2\sin^{-1}(1)$

$f(1) = 4 + 2 \times \frac{\pi}{2} = 4 + \pi$

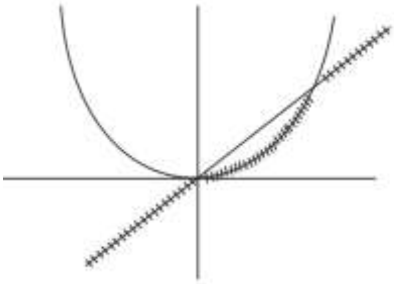
Range $\in [0, 4 + \pi]$

43. **(B)**

$\sqrt{x^2 - \frac{\pi^2}{9}} \in [0, \infty)$

$\cos\left(\sqrt{x^2 - \frac{\pi^2}{9}}\right) \in [-1, 1]$

44. **(A)**



$$f(x) = \min\{x, x^2\}$$

$$= \begin{cases} x^2 : x \in [0,1] \\ x : x \in (-\infty, 0] \cup [1, \infty) \end{cases}$$

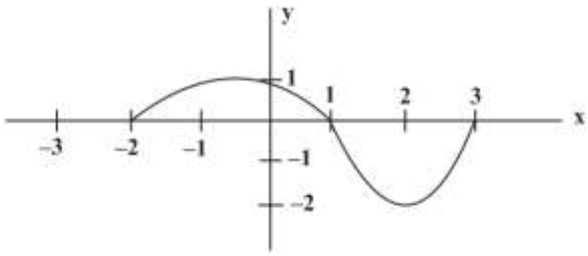
45. **(B)**

$$g(f(x)) = 2\sin x \cos x$$

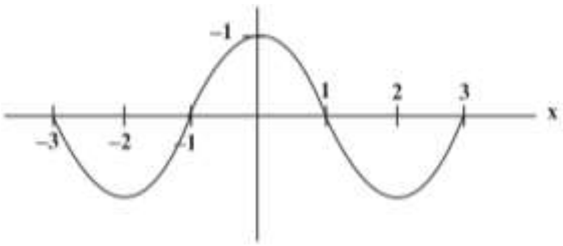
$$= \sin 2x \text{ is invertible in } \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

46. **(D)**

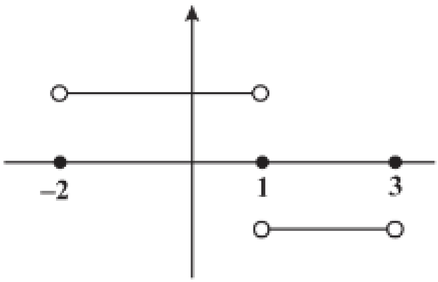
$f(x)$:



$f(|x|)$:

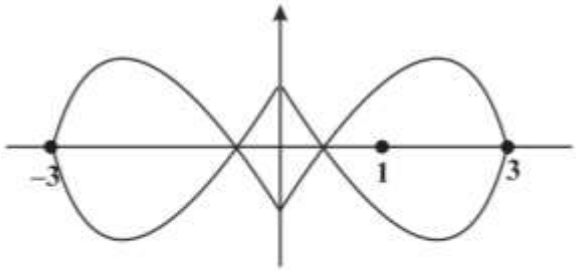


$\text{sgn}(f(x))$:

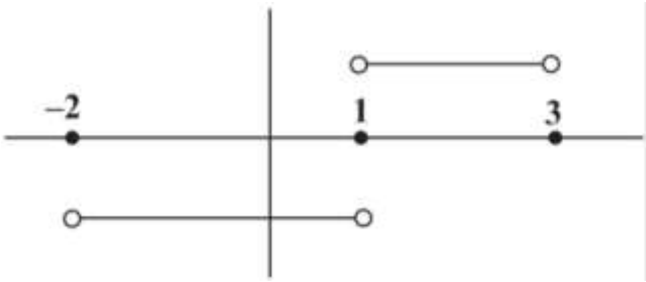


$$|y| = |f(|x|)|$$

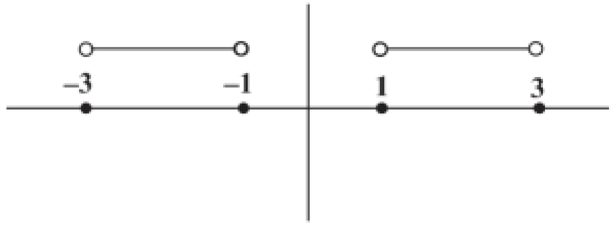
$$y = \pm f(|x|)$$



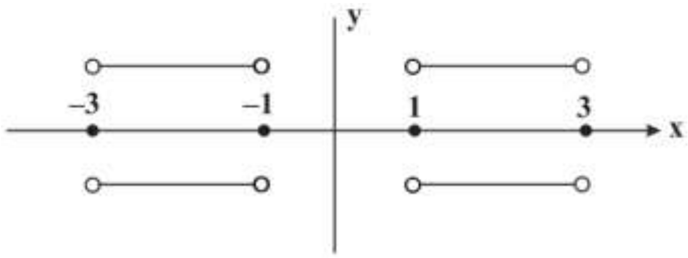
$y = \text{sgn}\left(-f(x)\right)$



$y = \text{sgn}\left(-f(|x|)\right)$



$|y| = \text{sgn}\left(-f(|x|)\right)$



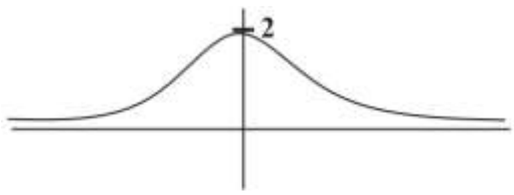
$y = x\,\text{sgn}(x)$

47. **(A)**

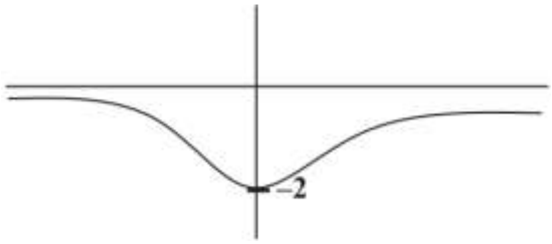
$$f(x) = \frac{x^2-1}{x^2+1} = \frac{(x^2+1)-2}{x^2+1}$$

$$f(x) = 1 - \left(\frac{2}{x^2+1}\right)$$

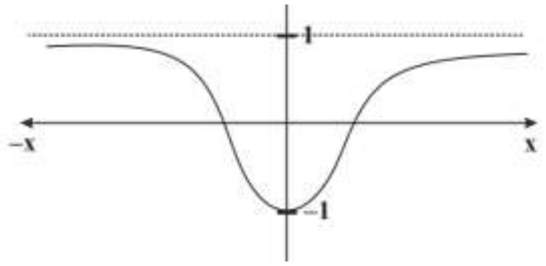
$$y = \frac{2}{x^2+1}$$



$$y = \frac{-2}{x^2+1}$$



$y = 1 - \frac{2}{x^2+1}$



Range $\in [-1, 1)$

Function is many one-onto

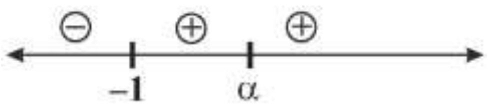
48. **(BC)**

$f(x) = \log\left(ax^3 + ax^2 + bx^2 + bx + cx + c\right)$

$f(x) = \log\left(\left(ax^2 + bx + c\right)(x+1)\right)$

$= \log\left(a(x-\alpha)(x-\alpha)(x+1)\right)$

$= \log\left(a(x-\alpha)^2(x+1)\right)$



$x \in (-1, \infty) - [\alpha]$

$x \in R - \left\{\frac{-b}{2a}\right\} \cap (-1, \infty)$

$x \in \left| R - \left(\left\{-\frac{b}{2a}\right\} \cup \{x \mid x \le -1\}\right)\right|$

49. **(AB)**

$f(0) = \max\{1, 1, 0\} = 1$

$f(1) = \max[1 + \sin 1, 1, 1 - \cos 1)$

$= 1 + \sin 1$

$g(0) = \max\{1, 1\} = 1$

$g(1) = \max\{1, 0\} = 1$

$g(f(0)) = g(1) = 1$

$g(f(1)) = g(1 + \sin 1)$

$= \max\{1, |\sin 1|\} = 1$

50. **(ACD)**

$$f(x) = \ln\left(1-\left(1-2x+x^2\right)\right)+\sin\left(\frac{\pi x}{2}\right)$$

$$= \ln\left(1-(x-1)^2\right)+\sin\left(\frac{\pi x}{2}\right)$$

$x=1$, function is maximum $f(1)=\ln(1)+1=1$

$$f(2-x)=\ln\left(1-(1-x)^2\right)+\sin\left(\pi-\frac{\pi x}{2}\right)$$

$$f(2-x)=f(x)$$

Minimum value $'f'$ doesn't exist.

51. **(D)**

$$y=f(x)-g(x)$$

$$f\left(\sqrt{2}\right)=0$$

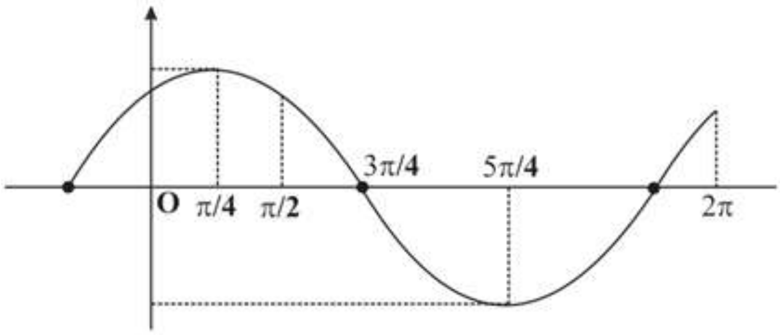
$$g\left(\sqrt{2}\right)=\sqrt{2}$$

$$f\left(\sqrt{2}\right)-g\left(\sqrt{2}\right)=0-\sqrt{2}$$

$$(f-g)(x)=\begin{cases} x & if \quad x\in Q \\ -x & if \quad x\notin Q \end{cases}$$

One-one function & onto.

52. **(AD)**



$$f(x)=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)+2\sqrt{2}$$

$$=\sqrt{2}\left(\sin\left(x+\frac{\pi}{4}\right)+2\right)$$

Range of $f(x)=\left[\sqrt{2},3\sqrt{2}\right]$

$\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$ is invertible in $\left[\frac{\pi}{4},\frac{5\pi}{4}\right]$

53. **(BC)**

$$f'(x)=-\sin x-2a$$

$$f'(x)\geq 0$$

$$\Rightarrow -\sin x-2a\geq 0$$

$$a \le \frac{-\sin x}{2}$$

$$\Rightarrow a \le -\frac{1}{2}$$

And $f'(x) \ge 0 \qquad \Rightarrow a \ge \frac{1}{2}$

54. **(AB)**

$$\{x\} + 2x = 4[x] - 2$$

$$\{x\} + 2[x] + 2\{x\} = 4[x] - 2$$

$$3\{x\} = 2[x] - 2$$

$$\{x\} = \frac{2}{3}([x] - 1)$$

$$0 \le \frac{2}{3}[x] - 1 < 1$$

$$0 \le [x] - 1 < \frac{3}{2}$$

$$1 \le [x] < \frac{5}{2}$$

$$[x] = 1, 2$$

55. **(AD)**

Conceptual

(A) Many-one

(B) Invertible by restricting the domain

56. **(3)**

$$f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)} \qquad ...(1)$$

$$x \to \frac{1}{1-x}$$

$$f\left(\frac{1}{1-x}\right) + f\left(1 - \frac{1}{x}\right) = -2x + \frac{2}{x} \qquad ...(2)$$

$$x \to 1 - \frac{1}{x}$$

$$f\left(1 - \frac{1}{x}\right) + f(x) = \frac{2x}{x-1} - 2x \qquad ...(3)$$

From (1), (2), (3)

$$f(x) = \frac{x+1}{x-1}$$

57. **(1)**

$$y = 2[x] + 3$$

$$(-) \qquad y = 3[x] - 1$$

$0=-[x]+4$

$[x]=4$

$\therefore\ y=12-1=11$

$y=11$

$x\in[4,5)$

$[x+3y]=4+33=37$

$\therefore\ \lambda=1$

58. **(6)**

$f(x)+f(x+3)=0$

$f(x+3)+f(x+6)=0$

$\therefore\ f(x)=f(x+6)$

$T=6$

59. **(5)**

$\{x\}=\{x^2\}$

$x-1=x^2-2$

$x^2-x-1=0$

$x=\frac{1\pm\sqrt{5}}{2}$

$\Rightarrow x=\frac{1+\sqrt{5}}{2}$

60. **(2)**

$f^2(x)\,f\left(\frac{1-x}{1+x}\right)=x^3$

$x\to\frac{1-x}{1+x}$

$$f^2\left(\frac{1-x}{1+x}\right)f\left(\frac{1-\left(\frac{1-x}{1+x}\right)}{1+\left(\frac{1-x}{1+x}\right)}\right)=\left(\frac{1-x}{1+x}\right)^3$$

$$f^2\left(\frac{1-x}{1+x}\right)f\left(\frac{2x}{2}\right)=\left(\frac{1-x}{1+x}\right)^3$$

$$\Rightarrow\ f^3(x)\,f^3\left(\frac{1-x}{1+x}\right)=x^3\left(\frac{1-x}{1+x}\right)^3$$

$$f(x)\,f\left(\frac{1-x}{1+x}\right)=x\left(\frac{1-x}{1+x}\right)$$

$$\therefore\ f(x)\left(x\left(\frac{1-x}{1+x}\right)\right)=x^3$$

$$f(x)=x^2\frac{(1+x)}{(1-x)}$$

$$f(-2)=\frac{4(-1)}{(+3)}=-\frac{4}{3}$$

$$\left|\left[f(-2)\right]\right|=2$$